Track functions and TMD physics

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NWO

SIDIS with jets

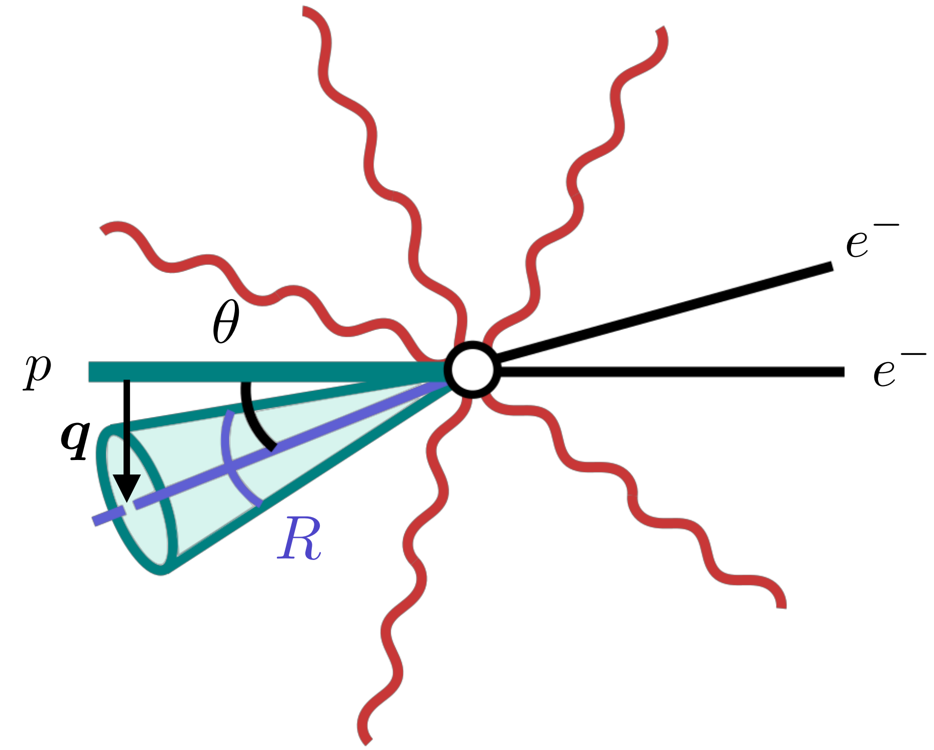
- In SIDIS, replacing the final hadron by a jet, involves replacing the TMD FF by a TMD jet function

$$\frac{\mathrm{d}\sigma(ep \to ehX)}{\mathrm{d}Q\,\mathrm{d}x\,\mathrm{d}z\,\mathrm{d}\vec{q}_T} = H(Q) F_q(\vec{q}_T, x) \otimes \sout{D_{q\to h}(\vec{q}_T, z)}$$

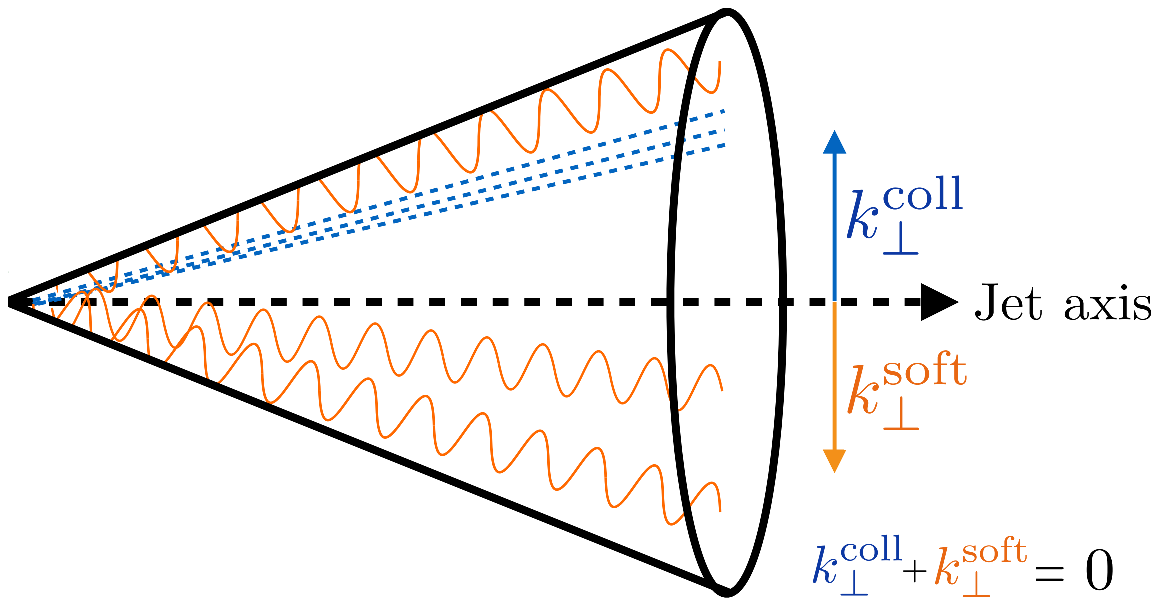
$$\delta(1-z) J_q(\vec{q}_T)$$

[Gutierrez-Reyes, Scimemi, WW, Zoppi]

- Conditions:
  - Use winner-take-all scheme
  - $\theta \sim q_T/Q \ll R$, otherwise jet boundary effects and $z < 1$



Winner-Take-All scheme



- Default: Jet axis along jet momentum, so recoiled by soft radiation **in jet** → non-global logarithms
- Winner-Take-All (WTA) recombination [Salam; Bertolini, Chan, Thaler]

$$E_r = E_1 + E_2 \qquad \hat{n}_r = \begin{cases} \hat{n}_1 & \text{if } E_1 > E_2 \\ \hat{n}_2 & \text{if } E_2 > E_1 \end{cases}$$

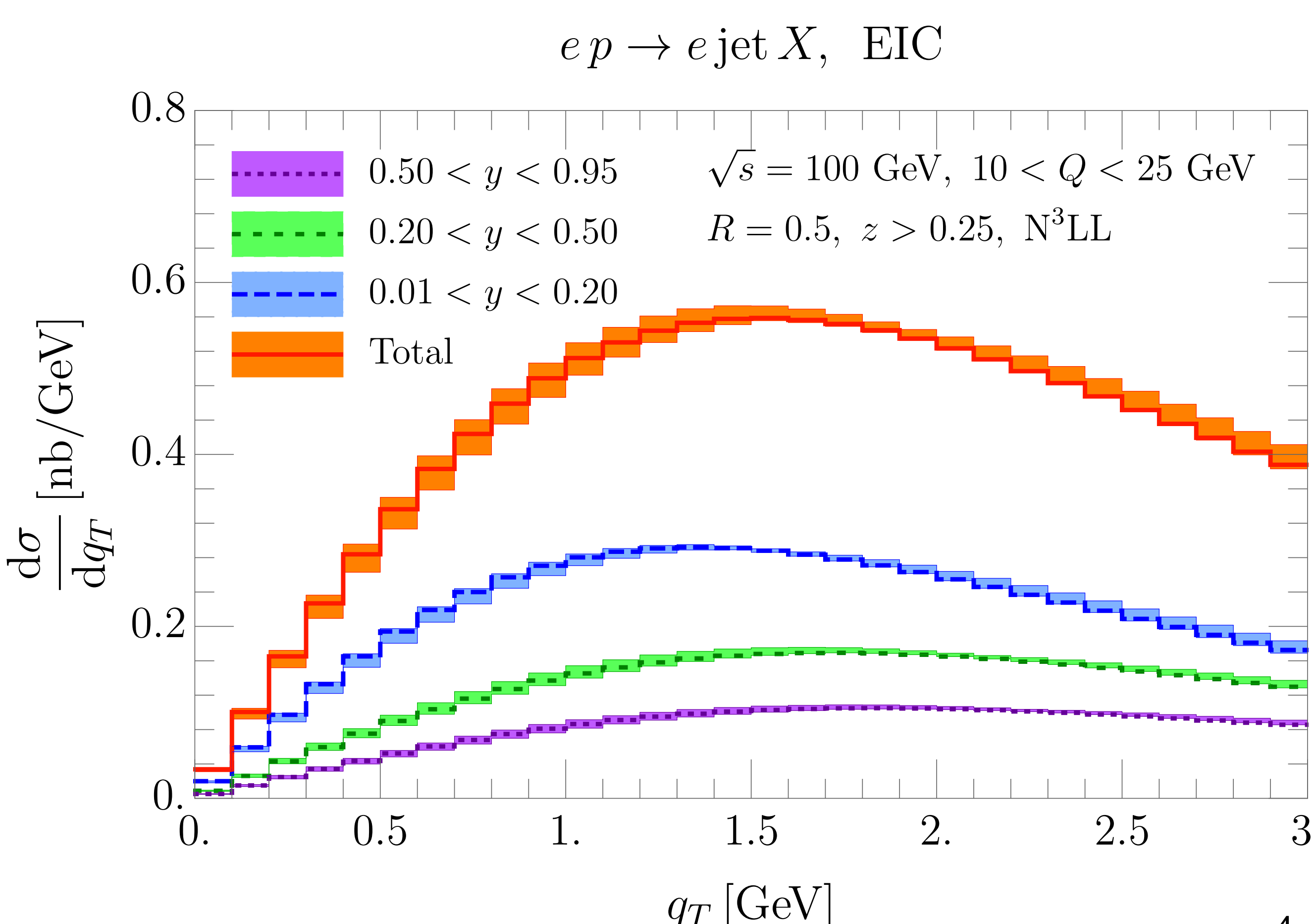
- Now jet recoiled by **all** soft radiation → sensible large *R* limit

(Dis)advantages of using jets

✓ Nonperturbative momentum fraction *z* does not enter

✓ Same N3LL accuracy can be achieved [Gutierrez-Reyes, Scimemi, WW, Zoppi] (two-loop jet function extracted from EVENT2)

✓ Leading nonp. effects from Collins-Soper kernel



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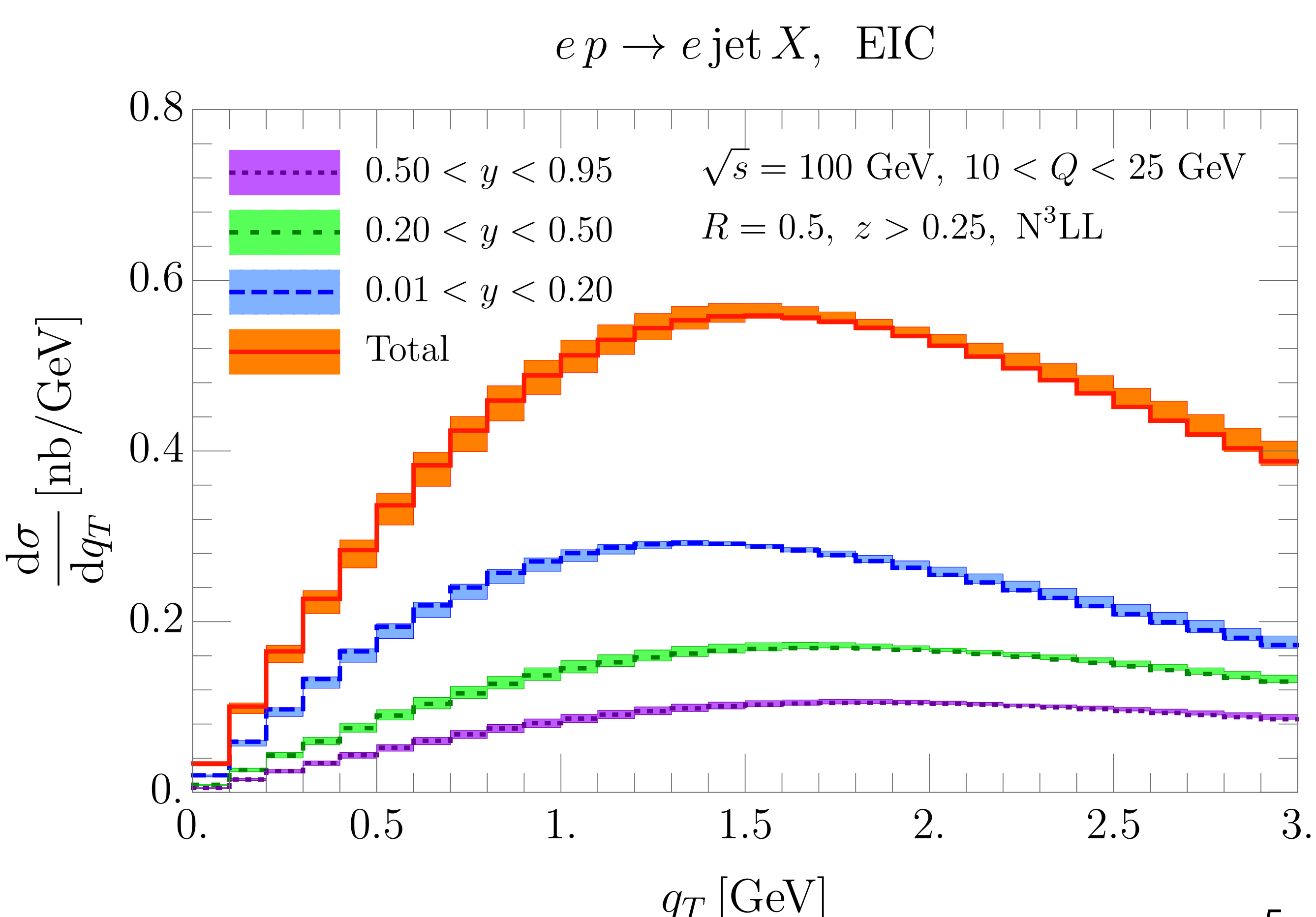
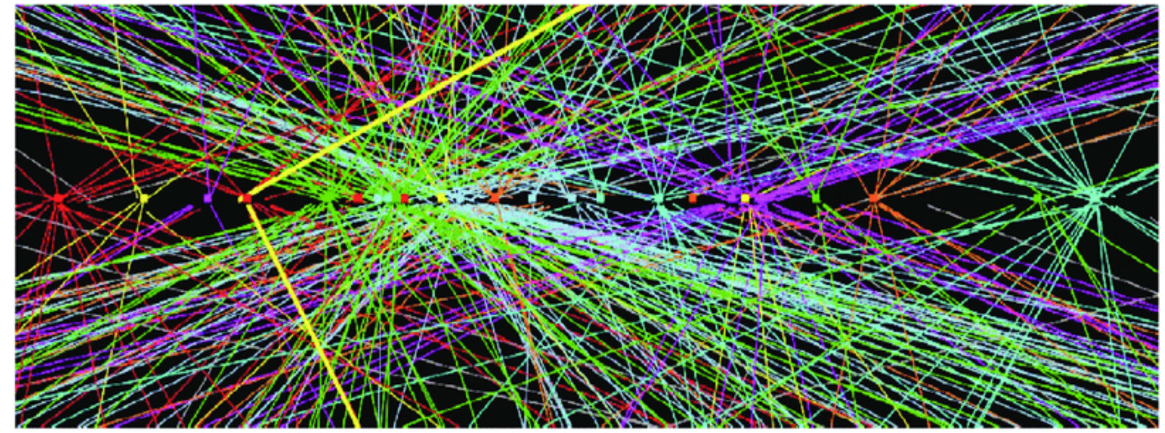
x Angular resolution for neutral particles (in jets) much worse than charged hadrons

Use jets based on charged particles?



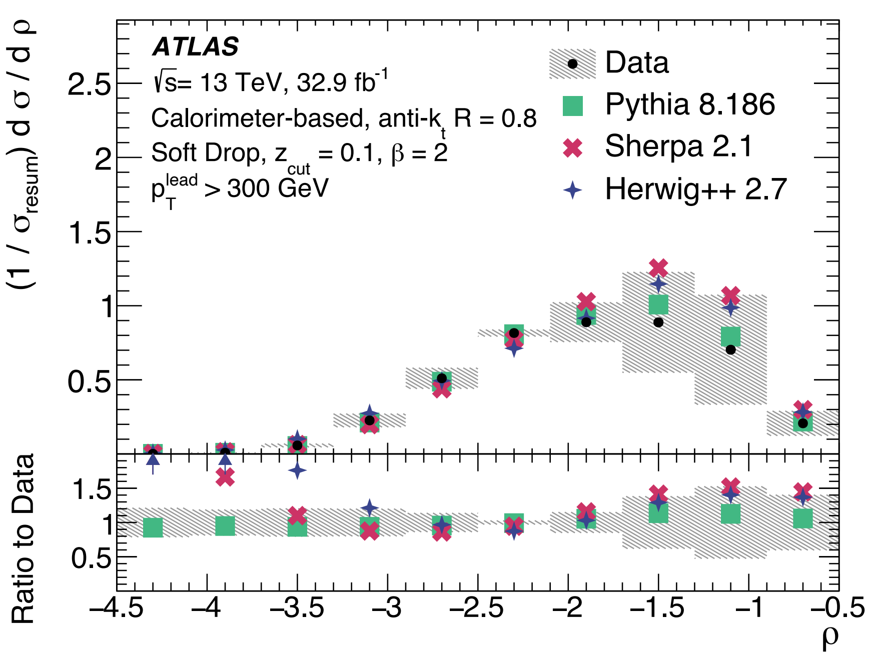
Benefits of track-based measurements at the LHC

- Superior angular resolution
- Pile-up removal

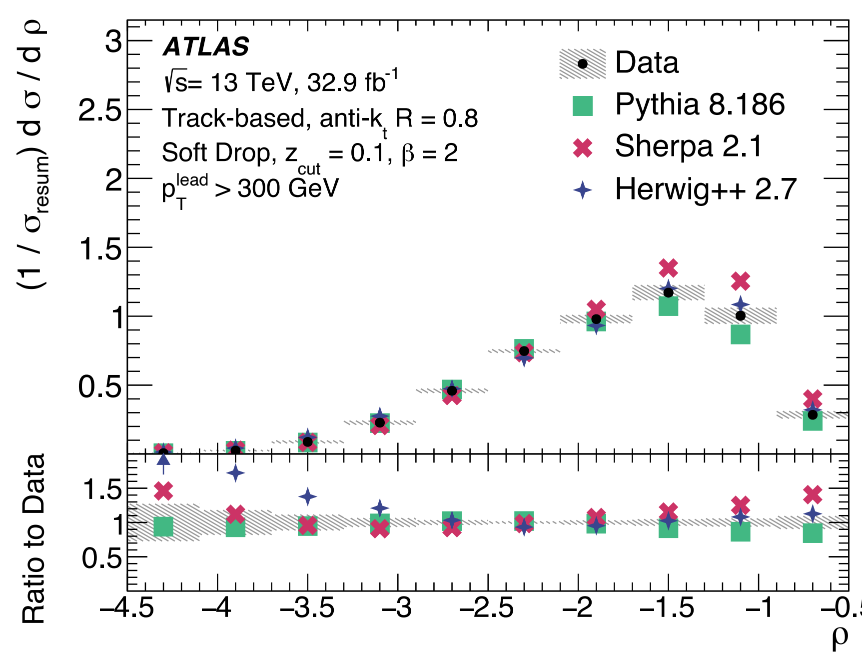


- Example of groomed jet mass $\rho = \ln(m^2/p_T^2)$

All particles



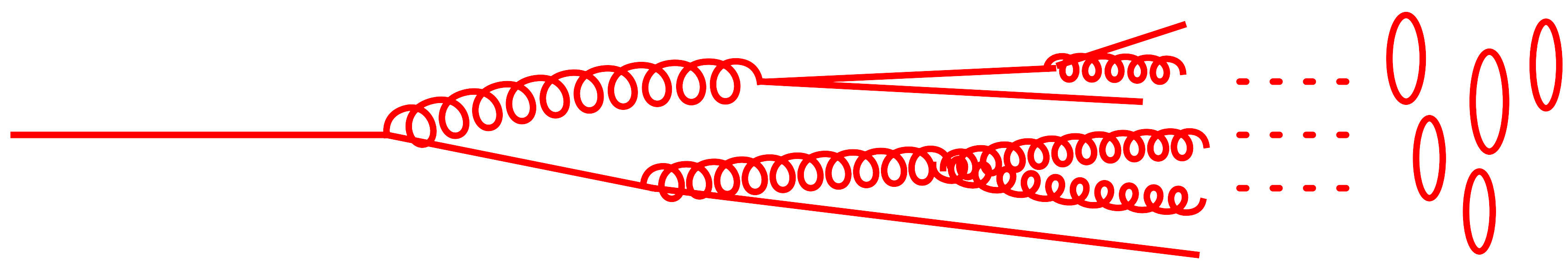
Tracks



Calculations for track-based observables

- Track function formalism for calculating track-based observables has been studied at $\mathcal{O}(\alpha_s)$ [Chang, Procura, Thaler, WW]

- Track function $T_i(x, \mu)$ describes momentum fraction x of initial parton i converted to tracks, i.e. $\bar{p}^\mu = x p^\mu + \mathcal{O}(\Lambda_{\mathrm{QCD}})$

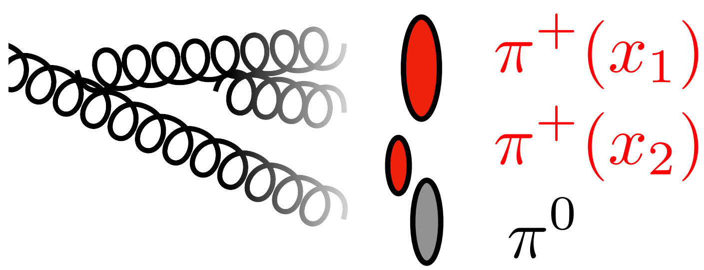


- Nonperturbative, process-independent function

- Sum rule: $\int_0^1 \mathrm{d}x \, T_i(x) = 1$

Similar but different from fragmentation functions

- Track function encodes correlations between hadrons



- Connection to fragmentation functions:

$$\int_0^1 \mathrm{d}x\, x\, T_i(x,\mu) = \sum_{\text{charged }h} \int_0^1 \mathrm{d}x\, x\, D_{i\to h}(x,\mu)$$

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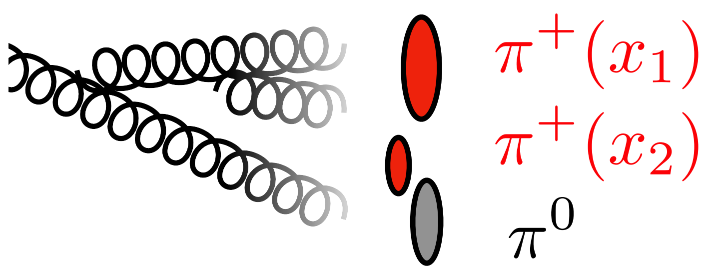

$$(x_1 + x_2)^2 = x_1^2 + x_2^2 + 2x_1 x_2$$

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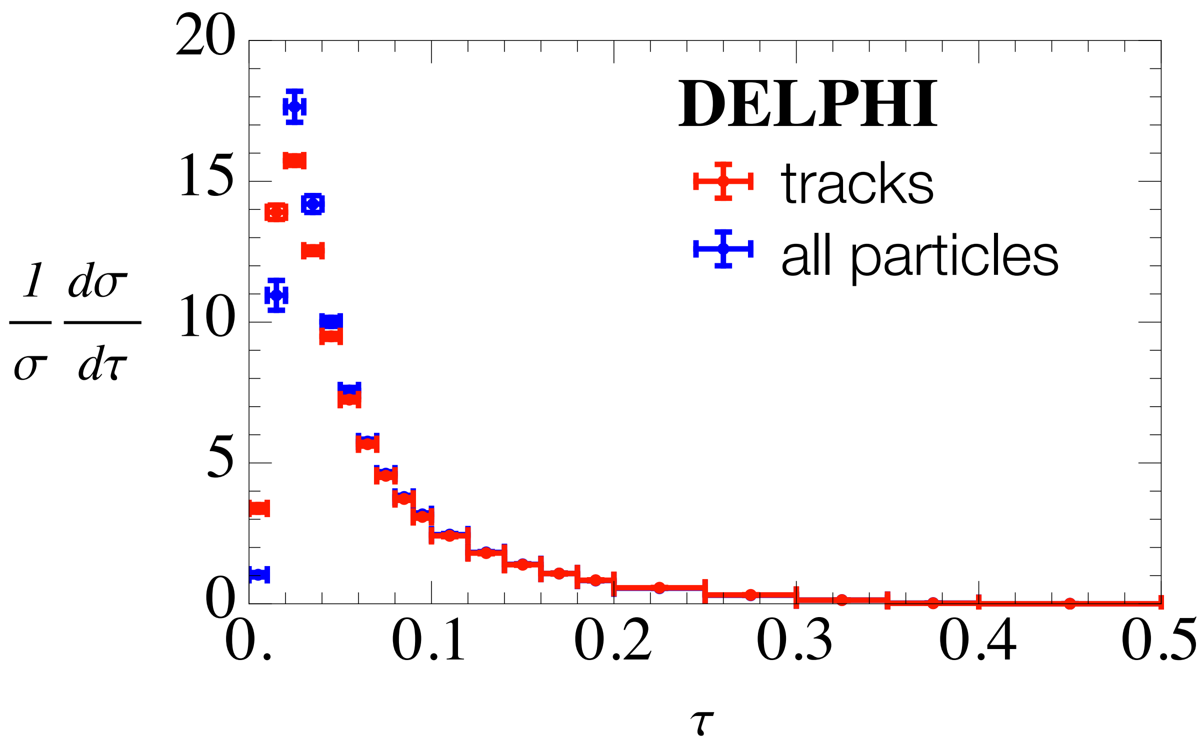
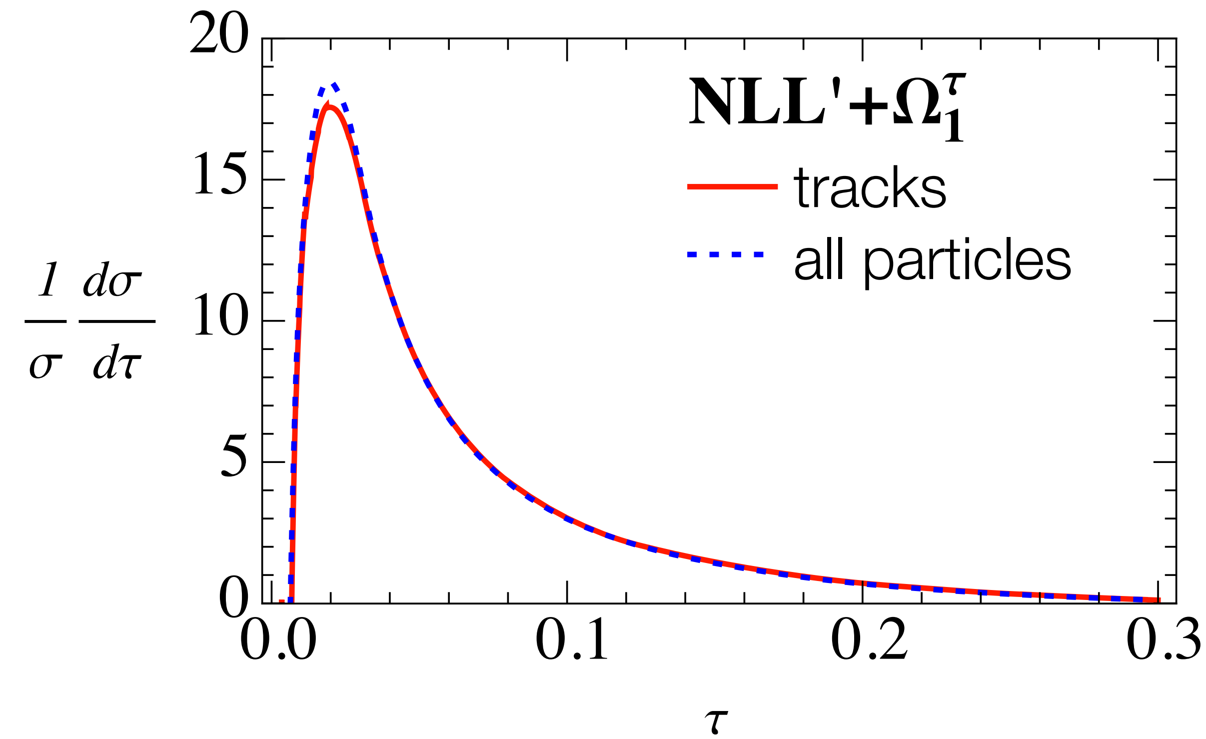
$$\int_0^1 \mathrm{d}x\, x\, T_i(x,\mu) = \sum_{\text{charged } h} \int_0^1 \mathrm{d}x\, x\, D_{i\to h}(x,\mu)$$

$$\int_0^1 \mathrm{d}x\, x^2\, T_i(x,\mu) = \sum_{\text{charged } h} \int_0^1 \mathrm{d}x\, x^2\, D_{i\to h}(x,\mu)$$

$$+ \sum_{\text{charged } h_1,h_2} \int_0^1 \mathrm{d}x_1\, \mathrm{d}x_2\, x_1\, x_2\, D_{i\to h_1 h_2}(x_1, x_2, \mu)$$

Example application: track thrust





[Chang, Procura, Thaler, WW]

- Small effect from switching to tracks except in nonpert. peak region (due to phenomenological track functions).
- Complicated dependence on track functions:

$$\begin{aligned}\bar{J}(\bar{s},x,\mu) &= \left(\delta(\bar{s}) + \frac{\alpha_s C_F}{2\pi}\left[\frac{2}{\mu^2}\mathcal{L}_1\Big(\frac{\bar{s}}{\mu^2}\Big) - \frac{2g_1^L}{\mu^2}\mathcal{L}_0\Big(\frac{\bar{s}}{\mu^2}\Big) + \delta(\bar{s})\Big(g_2^L - \frac{\pi^2}{6}\Big)\right]\right)T_q(x) + \frac{\alpha_s C_F}{2\pi}\int_0^1 \mathrm{d}x_2 \int_0^1 \frac{\mathrm{d}z}{z} \\ &\times \left\{\frac{1}{\mu^2}\mathcal{L}_0\Big(\frac{\bar{s}}{\mu^2}\Big)(1+z^2)\mathcal{L}_0(1-z) + \delta(\bar{s})\left[(1+z^2)\mathcal{L}_1(1-z) + \ln\Big(\frac{xz^2}{[x-(1-z)x_2]x_2}\Big)(1+z^2)\mathcal{L}_0(1-z) + 1-z\right]\right\} \\ &\times T_q\Big(\frac{x-(1-z)x_2}{z}\Big)T_g(x_2)\,.\end{aligned}$$

Track-based TMD jet function

- Due to WTA scheme, switching to tracks only modifies jet function → Anomalous dimension must be the same.
- Indeed, it only changes by a constant:

$$\bar{J}_q^{(1)}(\vec{q}_T) = J_q^{(1)}(\vec{q}_T) + \delta^2(\vec{q}_T) 4C_F \int_0^1 \mathrm{d}x \, \frac{1+x^2}{1-x} \ln \frac{x}{1-x} \int_0^1 \mathrm{d}z_1 \, T_q(z_1,\mu) \times \int_0^1 \mathrm{d}z_2 \, T_g(z_2,\mu) [\Theta(z_1 x - z_2(1-x)) - \Theta(x - \tfrac{1}{2})]$$

[Chien, Rahn, Schrijnder van Velzen, Shao, WW, Wu]

- Effect of switching to tracks will be small, and can treated in a simple manner

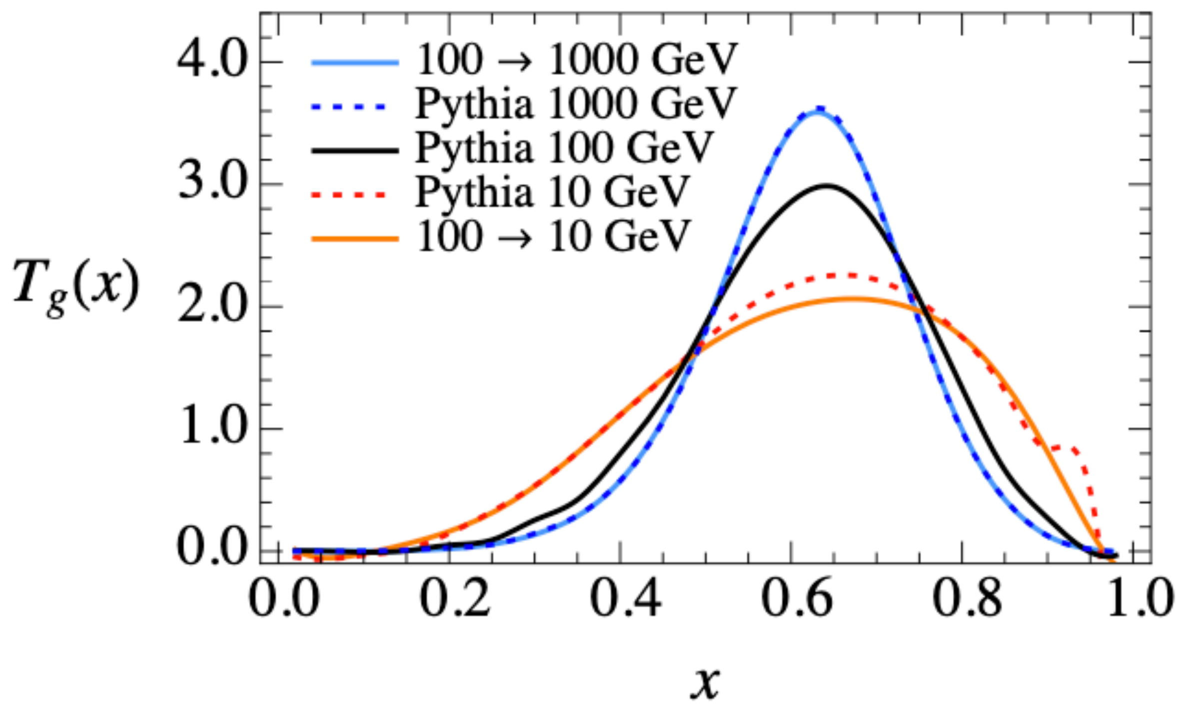
Track function formalism at $\mathcal{O}(\alpha_s^2)$

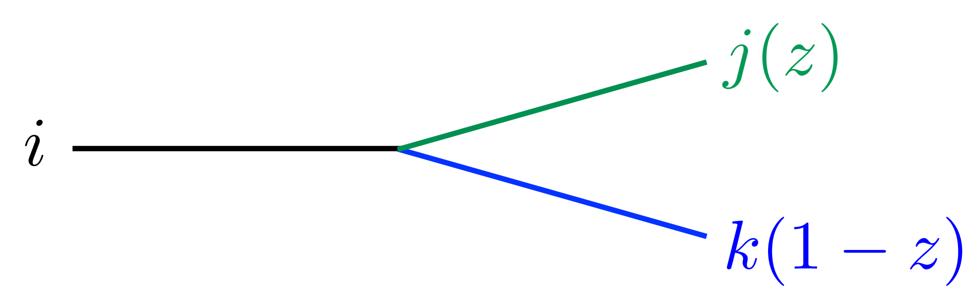
- Direct track function calculation in dimensional regularization results in scaleless integrals $1/\epsilon_{\rm UV} - 1/\epsilon_{\rm IR} = 0$
- Track jet function $J_i(s,x)$ differential in invariant mass s of **all** particles and momentum fraction x of **charged** particles
  - Same renormalization as invariant mass jet function $J_i(s)$ from consistency of factorization in SCET
  - Remaining $1/\epsilon$ poles are infrared and cancel when matching $\mathcal{G}$ onto track functions

$$J_i^{(2)} = T_i^{(2)} + \sum_j J_{i\to jk}^{(1)} \otimes [T_j^{(1)} T_k^{(0)}] + \sum_{j,k} J_{i\to jk\ell}^{(2)} \otimes [T_j^{(0)} T_k^{(0)} T_\ell^{(0)}]$$

  - This fixes the IR poles, and thus UV poles, of $T_i^{(2)}$

Track function evolution at $\mathcal{O}(\alpha_s)$





- Nonlinear evolution (leading order):

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu^2} T_i(x,\mu) = \sum_{j,k} \int \mathrm{d}z \, \frac{\alpha_s}{4\pi} P_{ji}(z) \int \mathrm{d}x_1 \, T_j(x_1,\mu) \int \mathrm{d}x_2 \, T_k(x_2,\mu) \times \delta[x - z x_1 - (1-z) x_2]$$

[Chang, Procura, Thaler, WW]

- Compare to extraction from Pythia (in absence of data)

Track function evolution in moments

- Taking **integer** moments of evolution equation:

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu^2} T_i(N,\mu) = \sum_{j,k} \int \mathrm{d}z\, \frac{\alpha_s}{4\pi} P_{ji}(z) \int \mathrm{d}x_1\, T_j(x_1,\mu) \int \mathrm{d}x_2\, T_k(x_2,\mu) [z x_1 - (1-z) x_2]^N$$

$$= \sum_{j,k} \int \mathrm{d}z\, \frac{\alpha_s}{4\pi} P_{ji}(z) \sum_n T_j(n,\mu)\, T_k(N-n,\mu) \binom{N}{n} z^n (1-z)^{N-n}$$

  - Fragmentation function evolution only has $n = 0$ term, and this term has same coefficient for track functions

- Beyond leading order (ignoring flavor)

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu^2} T(3,\mu) = c_3 T(3,\mu) + c_{21} T(2,\mu) T(1,\mu) + c_{111} T(1,\mu) T(1,\mu) T(1,\mu)$$

- Goal to determine unknown coefficients

Shift symmetry of evolution

- Energy conservation implies evolution has symmetry $x \to x + a$

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu^2} T_i(x+a) = \sum_{\{i_m\}} \int \left[\prod_m \mathrm{d}z_m\,\mathrm{d}x_m\, T_{i_m}(x_m+a)\right] \gamma_{i\to i_1\cdots i_m\cdots}(\{z_m\})$$

$$\times\, \delta\left(1 - \sum_m z_m\right) \delta(x - \sum x_m z_m)$$

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$$\times\, \delta\Big(1 - \sum_m z_m\Big)\, \delta(x - \sum x_m z_m)$$

- Make manifest by using shift-invariant central moments

$$\Delta = T_q(1) - T_g(1), \quad \sigma_i(2) = T_i(2) - T_i(1)^2, \quad \ldots$$

- E.g. evolution of the first two moments reads:

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu^2}\Delta = [-\gamma_{qq}(2) - \gamma_{gg}(2)]\Delta$$

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu^2}\begin{pmatrix}\sigma_g(2)\\ \sigma_q(2)\end{pmatrix} = \begin{pmatrix}-\gamma_{gg}(3) & -2n_f\gamma_{qg}(3)\\ -\gamma_{gq}(3) & -\gamma_{qq}(3)\end{pmatrix}\begin{pmatrix}\sigma_g(2)\\ \sigma_q(2)\end{pmatrix} + \begin{pmatrix}\gamma_g^{\Delta^2}\\ \gamma_q^{\Delta^2}\end{pmatrix}\Delta^2$$

Results for evolution $\mathcal{O}(\alpha_s^2)$

- Central moments: [Li, Moult, Schrijnder van Velzen, WW, Zhu]

$$\left.\frac{\mathrm{d}\sigma_g(2)}{\mathrm{d}\ln\mu^2}\right|_{\alpha_s^2} = -\gamma_{gg}^{(1)}(3)\sigma_g(2) + \sum_i \Big\{ -\gamma_{qg}^{(1)}(3)(\sigma_{q_i}(2) + \sigma_{\bar{q}_i}(2) + \Delta_{q_i}^2 + \Delta_{\bar{q}_i}^2) + T_F\Big[\Big(\frac{12413}{1350} - \frac{52}{45}\pi^2\Big)C_A + \frac{1528}{225}C_F - \frac{16}{25}n_f T_F\Big]\Delta_{q_i}\Delta_{\bar{q}_i}\Big\}$$

$$\left.\frac{\mathrm{d}\sigma_g(3)}{\mathrm{d}\ln\mu^2}\right|_{\alpha_s^2} = -\gamma_{gg}^{(1)}(4)\sigma_g(3) + \sum_i \Big\{ -\gamma_{qg}^{(1)}(4)(\sigma_{q_i}(3) + \sigma_{\bar{q}_i}(3) + 3\sigma_{q_i}(2)\Delta_{q_i} + 3\sigma_{\bar{q}_i}(2)\Delta_{\bar{q}_i} + \Delta_{q_i}^3 + \Delta_{\bar{q}_i}^3)$$

$$+ T_F\Big[\Big(-\frac{638}{45} + \frac{8}{3}\pi^2\Big)C_A - \frac{3803}{250}C_F\Big]\sigma_g(2)(\Delta_{q_i} + \Delta_{\bar{q}_i})$$

$$+ T_F\Big[\Big(\frac{5321}{3000} - \frac{2}{5}\pi^2\Big)C_A + \frac{1523}{240}C_F - \frac{12}{25}n_f T_F\Big](\sigma_{q_i}(2)\Delta_{\bar{q}_i} + \sigma_{\bar{q}_i}(2)\Delta_{q_i} + \Delta_{q_i}^2\Delta_{\bar{q}_i} + \Delta_{\bar{q}_i}^2\Delta_{q_i})$$

- Regular moments:

$$\left.\frac{\mathrm{d}T_g(2)}{\mathrm{d}\ln\mu^2}\right|_{\alpha_s^2} = -\gamma_{gg}^{(1)}(3)T_g(2) - \sum_i \gamma_{qg}^{(1)}(3)(T_{q_i}(2) + T_{\bar{q}_i}(2))$$

$$+ \Big[C_A^2\Big(-8\zeta_3 + \frac{26}{45}\pi^2 + \frac{2158}{675}\Big) - \frac{4}{9}C_A n_f T_F\Big] T_g(1)T_g(1)$$

$$+ \sum_i \Big[T_F\Big(-\frac{299}{225}C_A - \frac{4387}{900}C_F\Big)\Big] T_g(1)(T_{q_i}(1) + T_{\bar{q}_i}(1))$$

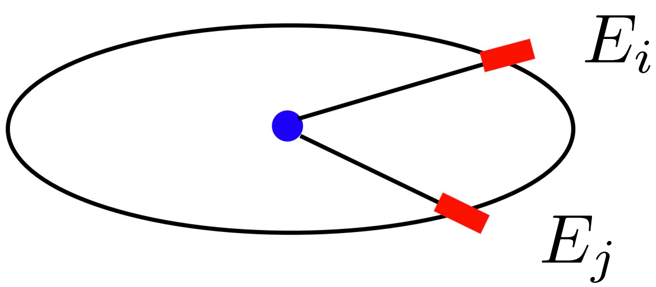
$$+ \sum_i T_F\Big[\Big(\frac{12413}{1350} - \frac{52}{45}\pi^2\Big)C_A + \frac{1528}{225}C_F - \frac{16}{25}n_f T_F\Big] T_{q_i}(1)T_{\bar{q}_i}(1)$$

Example application: Energy-energy correlator

- Weighted cross section in e^+e^- collisions

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\chi} = \sum_{i,j} \int \mathrm{d}\sigma \, \frac{E_i E_j}{Q^2} \, \delta(\chi - \theta_{ij})$$

[Basham, Brown, Ellis, Love]



- Tracks are essential to measure EEC at small angles.

- Conversion to tracks is simple:

$$E_i \to \int \mathrm{d}x_i \, T_i(x_i) \, x_i E_i = T_i(1) E_i$$

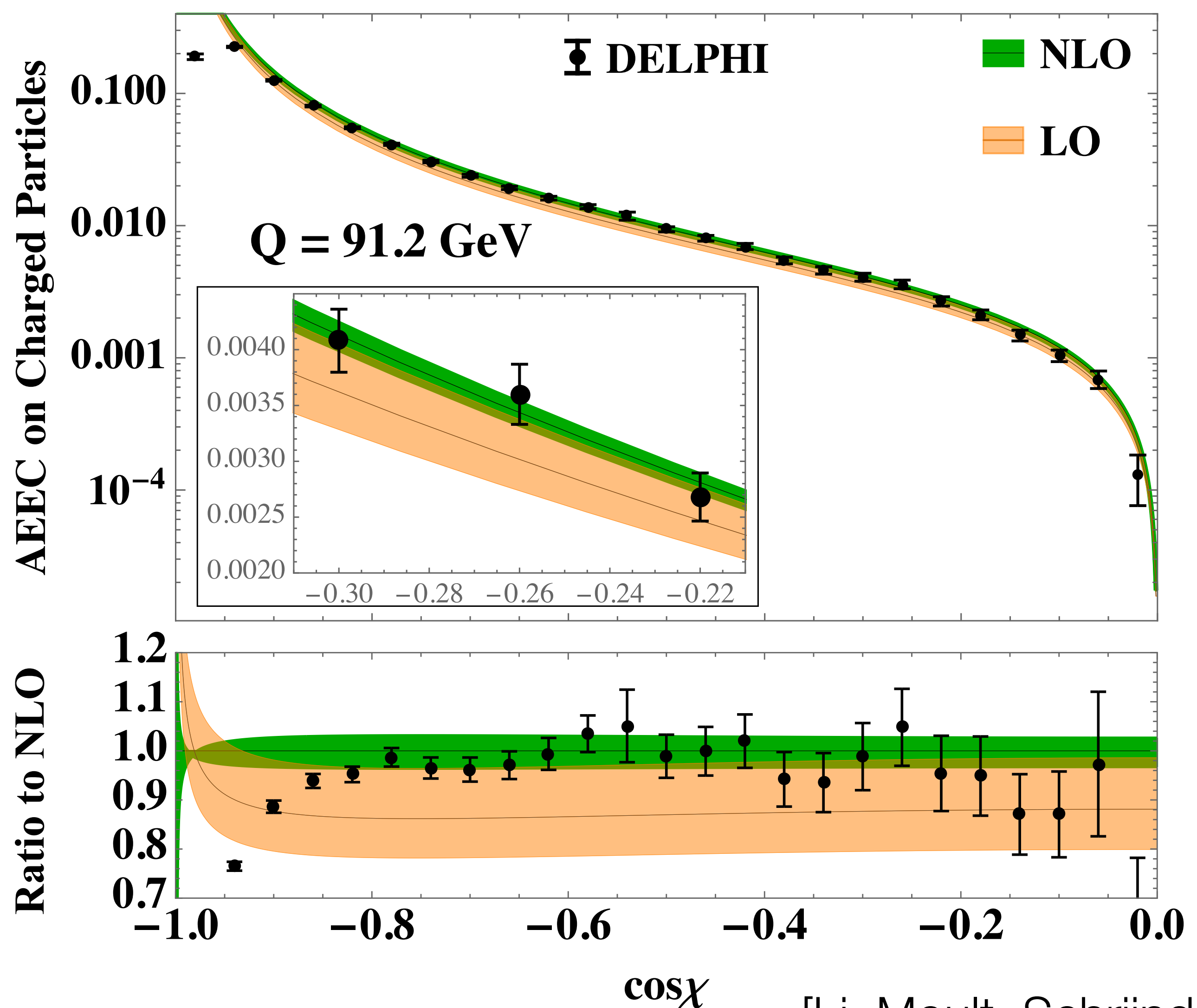
[Chen, Moult, Zhang, Zhu]

- Collinear limit $\chi = 0$ involves $T_i(2)$

- Generalizing, *N*-point energy correlators involve at most the *N*th moment of track functions

Results for track-based EEC

- First $\mathcal{O}(\alpha_s^2)$ result for track-based measurement:



[Li, Moult, Schrijnder van Velzen, WW, Zhu]

$$\mathrm{AEEC}(\cos\chi) = \mathrm{EEC}(\cos\chi) - \mathrm{EEC}(-\cos\chi)$$

- Cancellation of IR poles provides check on NLO evolution

Conclusions and outlook

- Using jets, instead of hadrons, in SIDIS avoids nonperturbative momentum fraction *z*, with same N^{3}LL precision
- Track-based measurements overcome limited resolution for neutral particles, and only modify TMD jet function constant
- Track function formalism is being extended to $\mathcal{O}(\alpha_s^2)$, and has been applied to the EEC

Thank you!