RESUMMATION, EVOLUTION, FACTORIZATION WORKSHOP

Dijets at the EIC beyond TMDs

Farid Salazar

November 16th, 2021

R. Boussarie, H. Mäntysaari, FS, and B. Schenke. 2106.11301 (JHEP09(2021)178)

P. Caucal, FS, and R. Venugopalan. 2108.06347 (To appear on JHEP)

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Outline

- Dijet production in the TMD formalism
  Observables at the EIC

- Dijet production at EIC beyond TMDs
  Resummation of kinematic and genuine saturation corrections

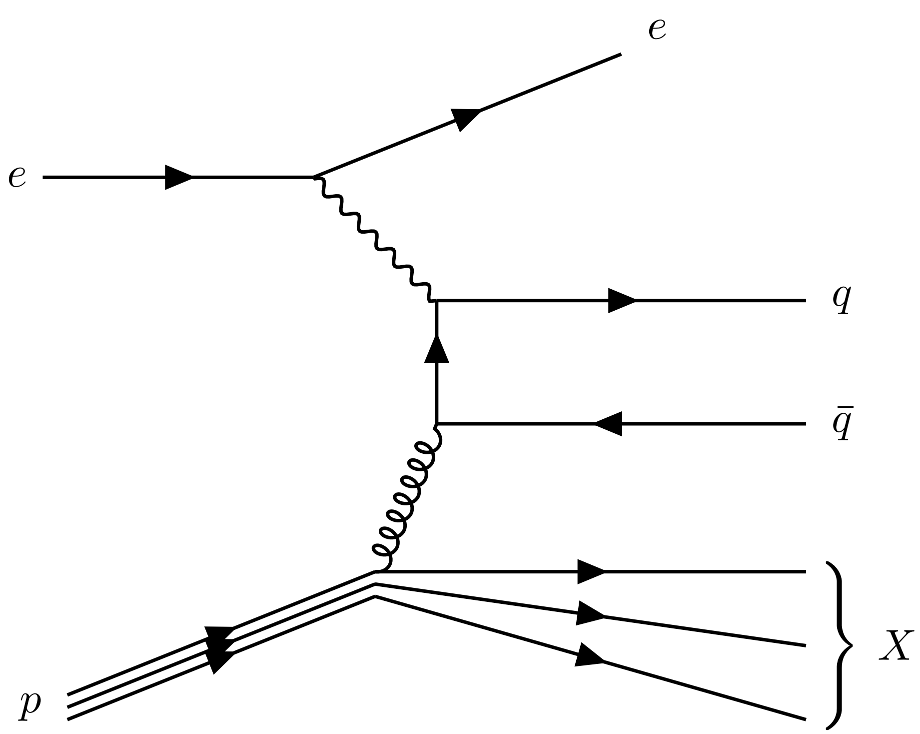
- Dijet production at EIC in the CGC at NLO
  JIMWLK rapidity **factorization**, and finite impact factor

- Outlook

Dijet and dihadron production: TMD formalism

The Weizsäcker-Williams gluon TMD

Bomhof, Mulders, Pijlman (2006)
Dominguez, Marquet, Xiao, Yuan (2011)
Dominguez, Qiu, Xiao, Yuan (2011)



LO diagram for $q\bar{q}$ production

$$d\sigma^{\gamma^* A \to q\bar{q}X} \sim \mathcal{H}^{ij}_{\rm TMD}(\boldsymbol{P}_\perp)\, \alpha_s x G^{ij}_{\rm WW}(x, \boldsymbol{k}_\perp)$$

Perturbatively calculable on-shell matrix element

WW gluon TMD

$$xG^{ij}_{\rm WW}(x,\boldsymbol{k}_\perp) = \frac{1}{2}\delta^{ij} xG^0_{\rm WW}(x,k_\perp) + \Pi^{ij}(\boldsymbol{k}_\perp) x h^0_{\rm WW}(x,k_\perp)$$

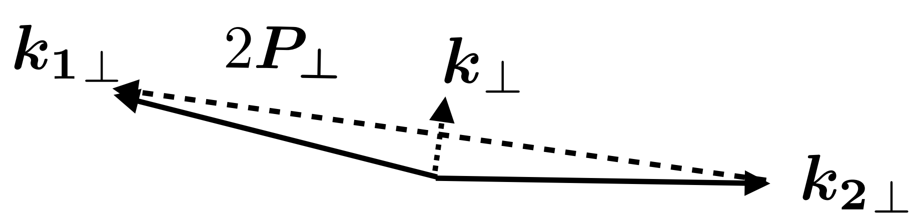
Unpolarized

Linearly polarized

$$\Pi^{ij}(\boldsymbol{k}_\perp) = \left(2\frac{\boldsymbol{k}^i_\perp \boldsymbol{k}^j_\perp}{\boldsymbol{k}^2_\perp} - \delta^{ij}\right)$$

Validity of TMD approach:

$k_\perp \ll P_\perp$ (i.e. back-to-back configuration)



$$\boldsymbol{k}_\perp = \boldsymbol{k}_{1\perp} + \boldsymbol{k}_{2\perp}$$

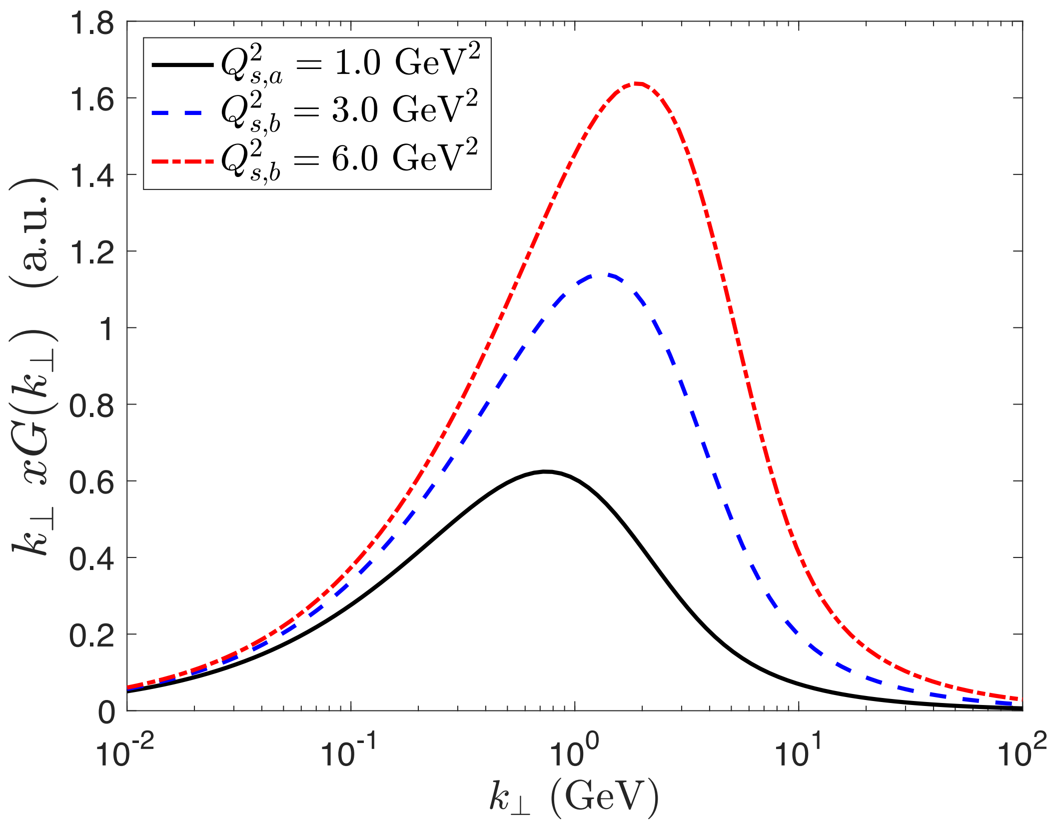
$$\boldsymbol{P}_\perp = z_2 \boldsymbol{k}_{1\perp} - z_1 \boldsymbol{k}_{2\perp}$$

See Rafael's talk for factorization within SCET!

Dijet and dihadron production: TMD formalism

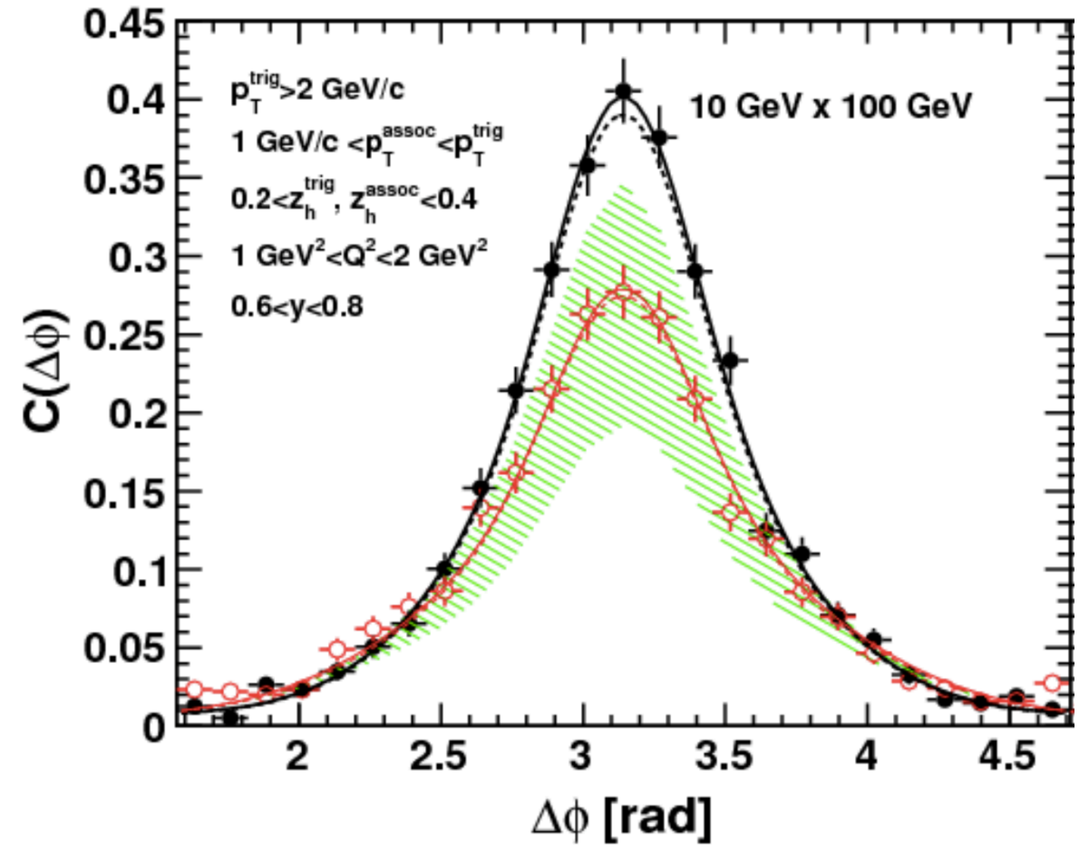
Forward dihadron azimuthal correlations and gluon saturation

Weizsäcker-Williams TMD at small-x → Dihadron suppression back-to-back peak

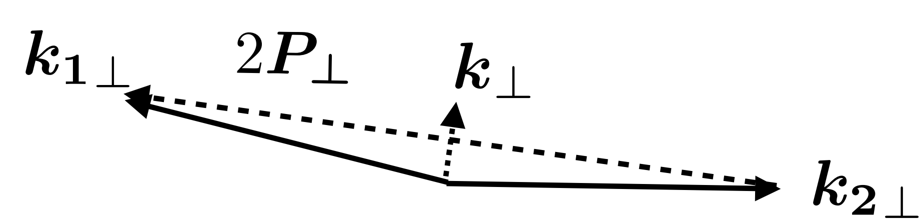


Typical momentum transfer from hadron/nucleus to

Momentum imbalance → $k_\perp \sim Q_s$ ← Saturation scale



Zheng, Aschenauer, Lee, Xiao (2014)

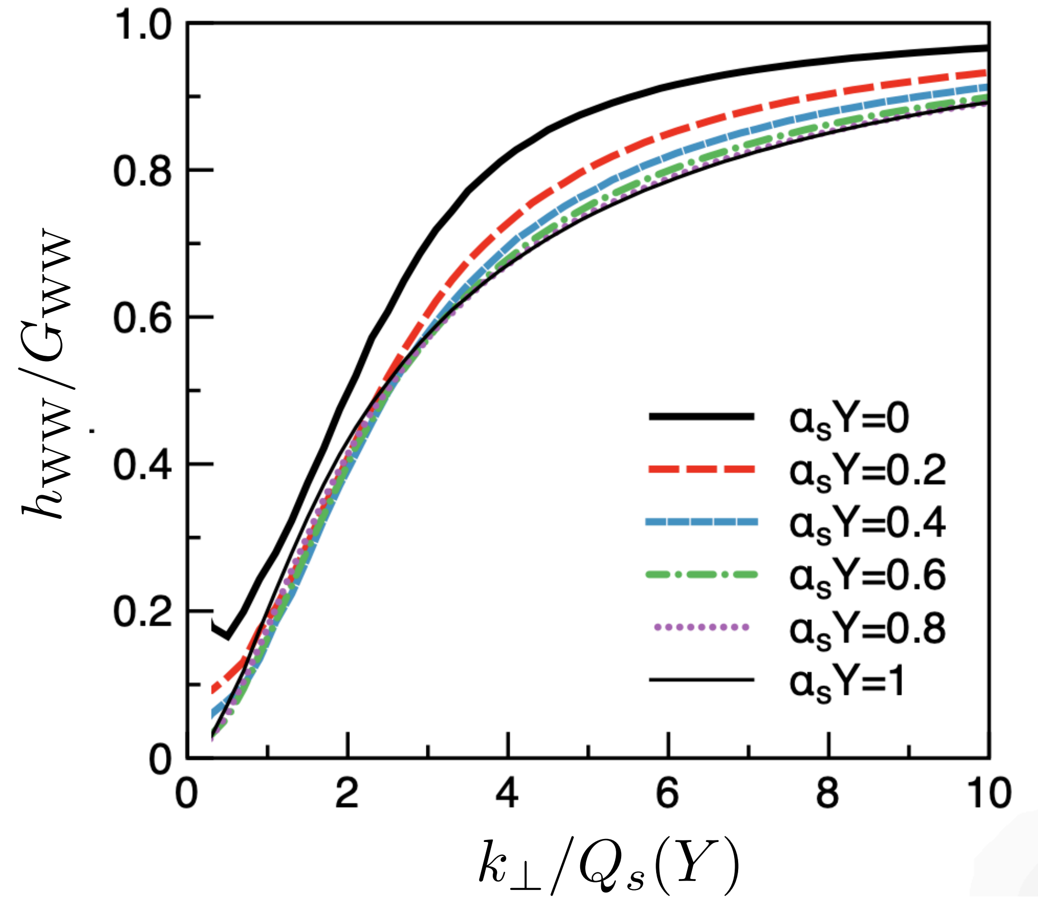


$\Delta\phi$ angle between $k_{1\perp}$ and $k_{2\perp}$

Dijet and dihadron production: TMD formalism

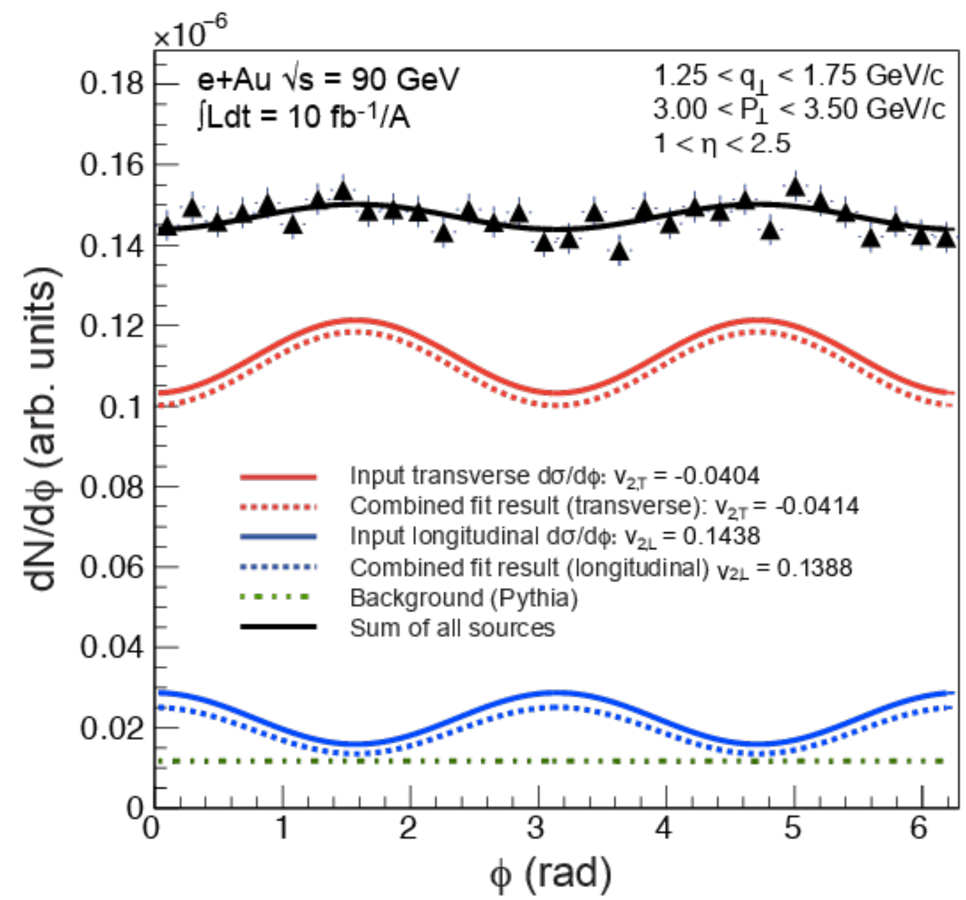
Forward dijet azimuthal asymmetries and linearly pol WW gluon TMD

Ratio of linearly polarized to unpolarized gluon WW

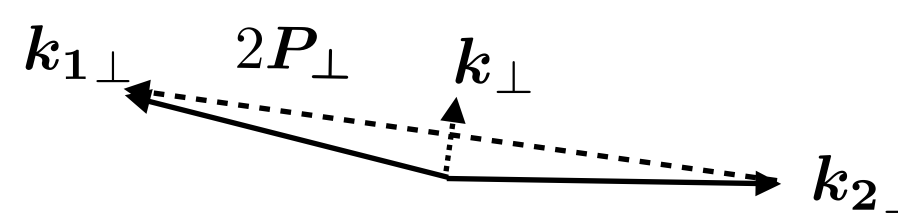


Dumitru, Lappi, Skokov (2015)

→ Dijet azimuthal asymmetries in momentum imbalance



Dumitru, Skokov, Ullrich (2018)



ϕ angle between $\boldsymbol{P}_\perp$ and $\boldsymbol{k}_\perp$

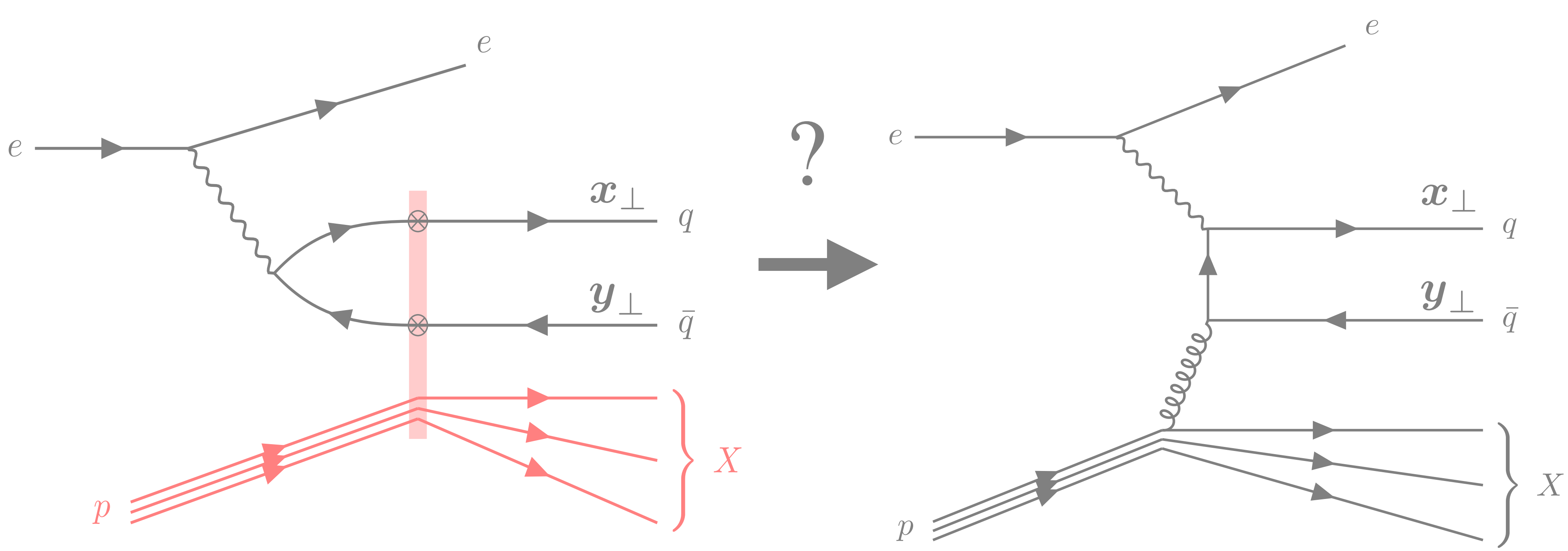
$$|v_2| \sim \frac{x h_{WW}}{x G_{WW}}$$

Accessed in azimuthal asymmetry of momentum imbalance in dijet pairs

Dominguez, Qiu, Xiao, Yuan (2011)

See Feng's talk tomorrow effects of soft gluon radiation!

Dijet production beyond TMDs



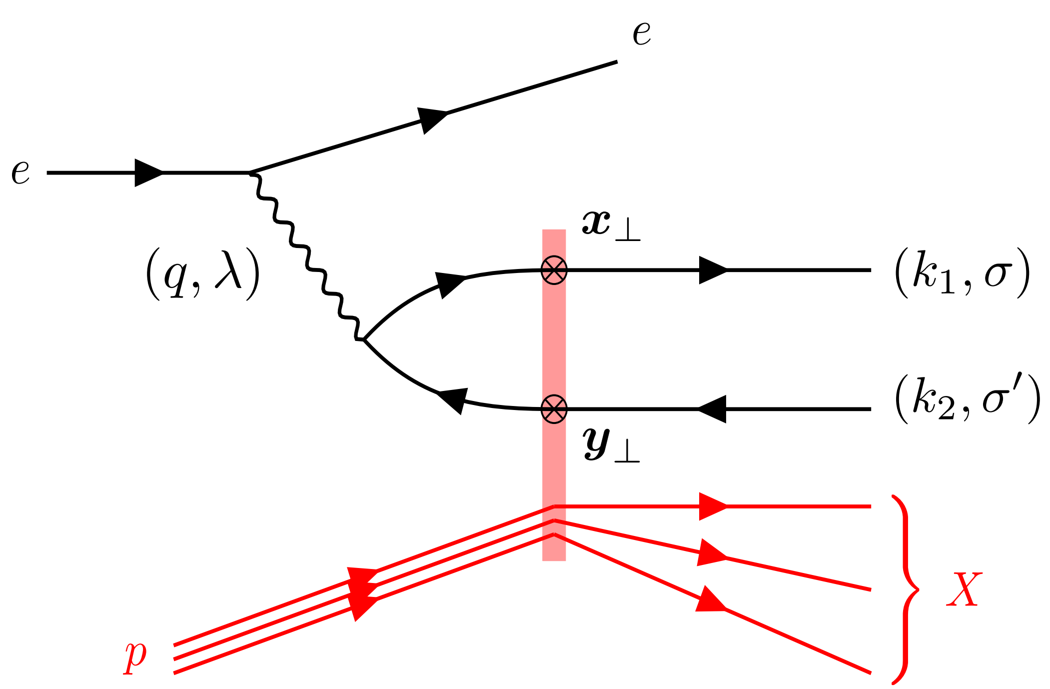
A comprehensive numerical study of the TMD/CGC correspondence

R. Boussarie, H. Mäntysaari, FS, and B. Schenke. 2106.11301 (JHEP09(2021)178)

Dijet production beyond TMDs

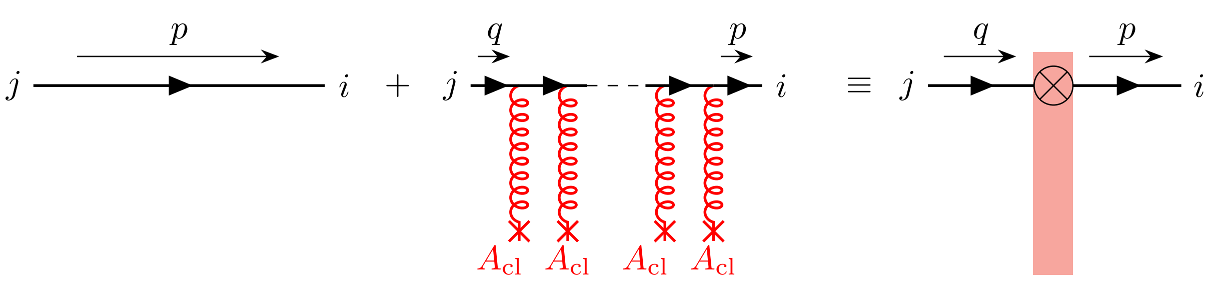
Computation in the CGC: **resummation** of multiple scatterings

Dominguez, Marquet, Xiao, Yuan (2011)



LO diagram for $q\bar{q}$ production in the CGC EFT

Dense gluon field $A_{\rm cl} \sim 1/g$ **needs resummation of multiple gluon interactions**



$$V_{ij}(\boldsymbol{x}) = P\exp\left\{ig\int dx^- A^{+,a}_{cl}(\boldsymbol{x},x^-)t^a\right\}$$

Amplitude (modulo leptonic part):

$$\mathcal{M}^{\lambda\sigma\sigma'}_{\rm LO} = \Psi^{\gamma^*_\lambda\to q\bar{q}}(Q,\mathbf{r}_{xy},z_q) \otimes_{\rm LO} \left[1 - V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{y}_\perp)\right]$$

perturbatively computable

non-perturbative

$$\otimes_{\rm LO} \equiv \frac{ee_f q^-}{\pi}\int \mathrm{d}^2\boldsymbol{x}_\perp \mathrm{d}^2\boldsymbol{y}_\perp e^{-i\boldsymbol{k_1}_\perp\cdot\boldsymbol{x}_\perp} e^{-i\boldsymbol{k_2}_\perp\cdot\boldsymbol{y}_\perp}$$

Dijet cross-section in the CGC will contain dipoles and quadrupole:

$$\frac{1}{N_c}\left\langle \mathrm{Tr}\left[V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{y}_\perp)\right]\right\rangle_Y$$

$$\frac{1}{N_c}\left\langle \mathrm{Tr}\left[V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{y}_\perp)V(\boldsymbol{y}'_\perp)V^\dagger(\boldsymbol{x}'_\perp)\right]\right\rangle_Y$$

Building blocks of CGC observables!

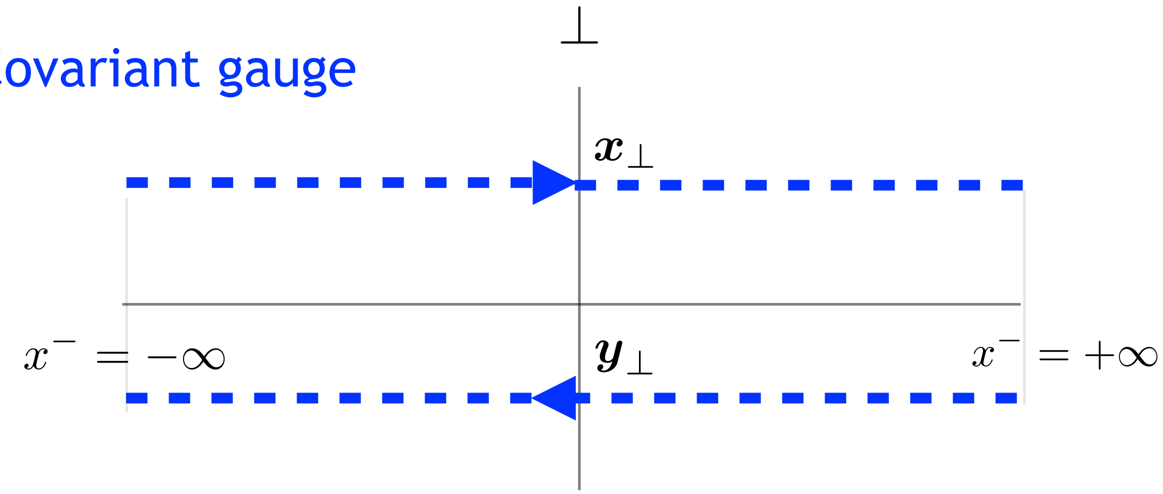
Dijet production beyond TMDs

Choosing the gauge: light-like Wilson lines vs transverse gauge link

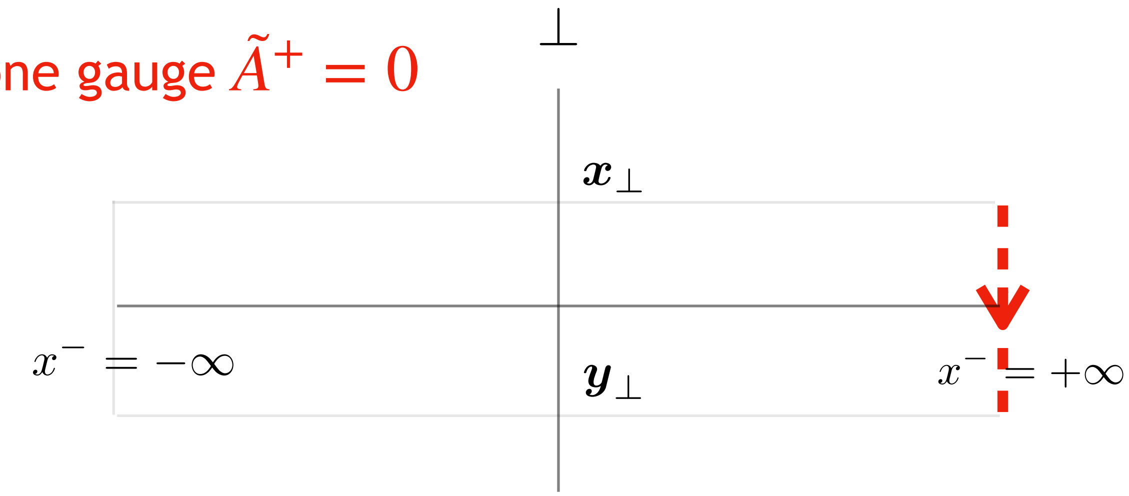
Boussarie, Mehtar-Tani (2020)

Pair of Wilson lines as transverse gauge link

Covariant gauge



Light-cone gauge $\tilde{A}^+ = 0$



$$V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{y}_\perp) = \mathcal{P}\,\exp\left[-ig\int_{\boldsymbol{y}_\perp}^{\boldsymbol{x}_\perp} \mathrm{d}\boldsymbol{z}_\perp \cdot \tilde{\boldsymbol{A}}_\perp(\boldsymbol{z}_\perp)\right]$$

gA expansion:

$$= 1 - ig\int_{\boldsymbol{y}_\perp}^{\boldsymbol{x}_\perp} \mathrm{d}\boldsymbol{z}_\perp \cdot \tilde{\boldsymbol{A}}_\perp(\boldsymbol{z}_\perp) + \ldots$$

Small dipole expansion:

Altinoluk, Boussarie, Kotko (2019)

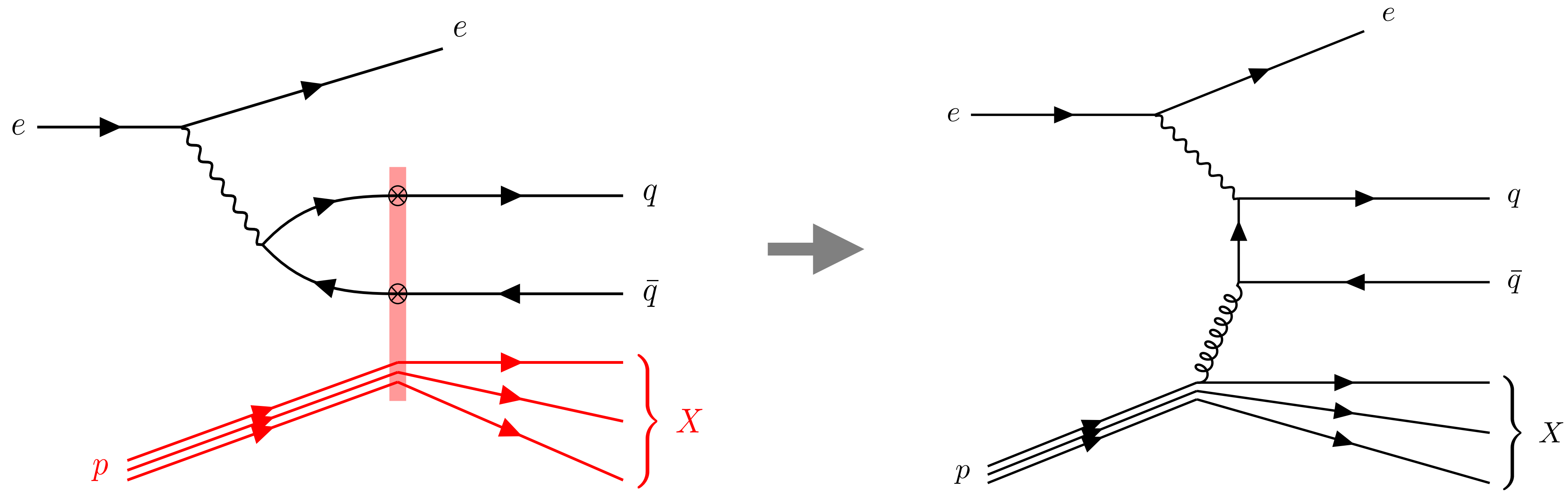
$$= 1 + ig\,\boldsymbol{r}_\perp \cdot \tilde{\boldsymbol{A}}_\perp(\boldsymbol{z}_\perp) + \ldots$$

$\boldsymbol{r}_\perp = \boldsymbol{x}_\perp - \boldsymbol{y}_\perp$

Dominguez, Marquet, Xiao, Yuan (2011)

Dijet production beyond TMDs

From CGC to Improved TMD



CGC

$$V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{y}_\perp) = \mathcal{P}\,\exp\left[-ig\int_{\boldsymbol{y}_\perp}^{\boldsymbol{x}_\perp} \mathrm{d}\boldsymbol{z}_\perp \cdot \tilde{\boldsymbol{A}}_\perp(\boldsymbol{z}_\perp)\right]$$

Boussarie, Mehtar-Tani (2020)

Improved TMD

$$= 1 - ig\int_{\boldsymbol{y}_\perp}^{\boldsymbol{x}_\perp} \mathrm{d}\boldsymbol{z}_\perp \cdot \tilde{\boldsymbol{A}}_\perp(\boldsymbol{z}_\perp) + \ldots$$

TMD

Altinoluk, Boussarie, Kotko (2019)

$$= 1 + ig\,\boldsymbol{r}_\perp \cdot \tilde{\boldsymbol{A}}_\perp(\boldsymbol{z}_\perp) + \ldots$$

$\boldsymbol{r}_\perp = \boldsymbol{x}_\perp - \boldsymbol{y}_\perp$

Dominguez, Marquet, Xiao, Yuan (2011)

Dijet production beyond TMDs

Resummation of power corrections and genuine saturation corrections

$$d\sigma_{\rm CGC} = \underbrace{d\sigma_{\rm TMD} + \overbrace{\mathcal{O}\left(\frac{k_\perp}{Q_\perp}\right)}^{\text{kinematic}}}_{d\sigma_{\rm ITMD}} + \overbrace{\mathcal{O}\left(\frac{Q_s}{Q_\perp}\right)}^{\text{genuine}}$$

Dominguez, Marquet, Xiao, Yuan (2011)

TMD valid $k_\perp, Q_s \ll Q_\perp$

back-to-back hadrons/jets and transverse momenta larger than sat scale

Hard factor

Weizsäcker-Williams gluon TMD

$$d\sigma^{\gamma^* A \to q\bar{q}X} \sim \mathcal{H}^{ij}_{\rm TMD}(\boldsymbol{P}_\perp)\, \alpha_s x G^{ij}_{\rm WW}(x, \boldsymbol{k}_\perp)$$
$$+\mathcal{O}(k_\perp/P_\perp) + \mathcal{O}(Q_s/P_\perp)$$

Altinoluk, Boussarie, Kotko (2019)
For massive quarks see Altinoluk, Marquet, Taels. (2021)

Improved TMD valid $Q_s \ll Q_\perp$

transverse momenta larger than sat scale

Hard factor resums kinematic powers $k_\perp/P_\perp$

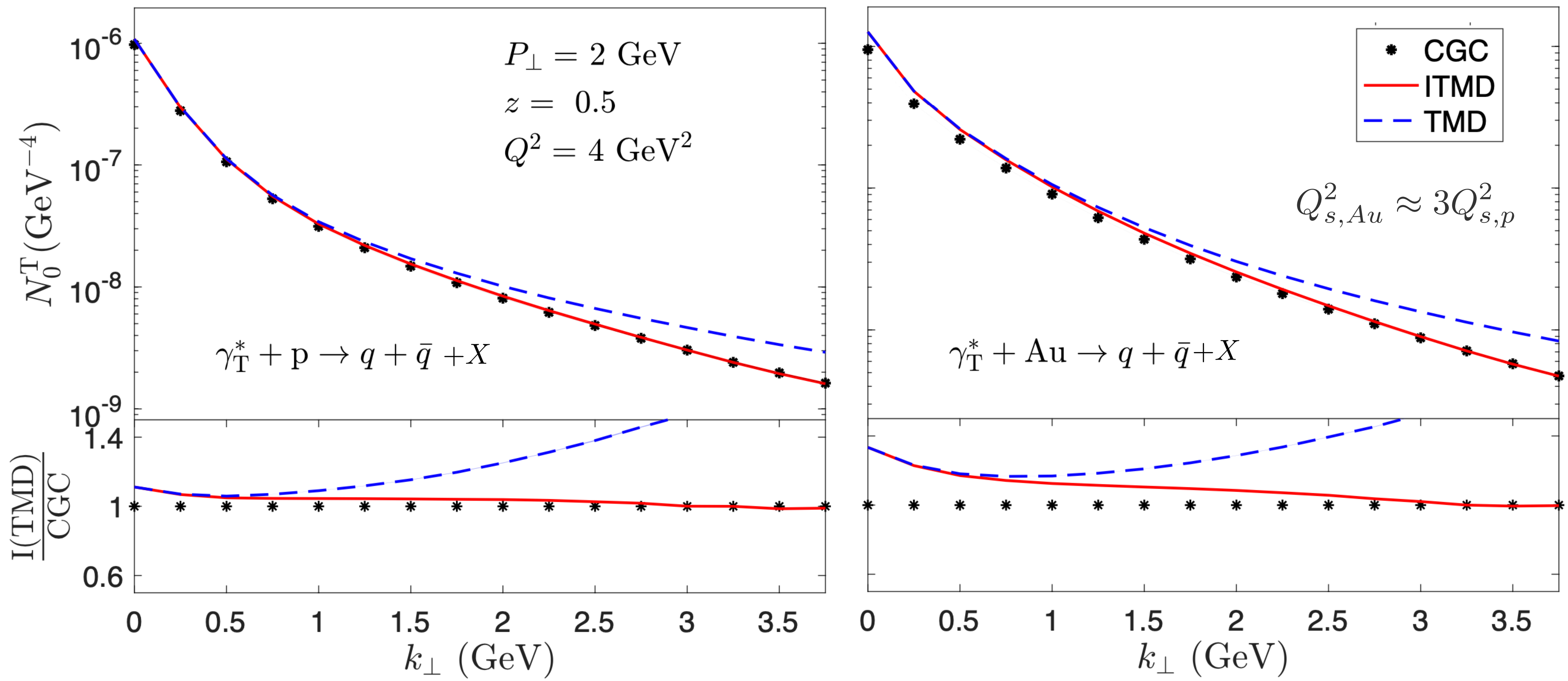
$$d\sigma^{\gamma^*_\lambda A \to q\bar{q}X} \sim \mathcal{H}^{\lambda,ij}_{\rm ITMD}(\boldsymbol{P}_\perp, \boldsymbol{k}_\perp)\, \alpha_s x G^{ij}_{\rm WW}(x, \boldsymbol{k}_\perp)$$
$$+\mathcal{O}(Q_s/P_\perp)$$

See Krzysztof's talk for more on ITMD at the EIC!

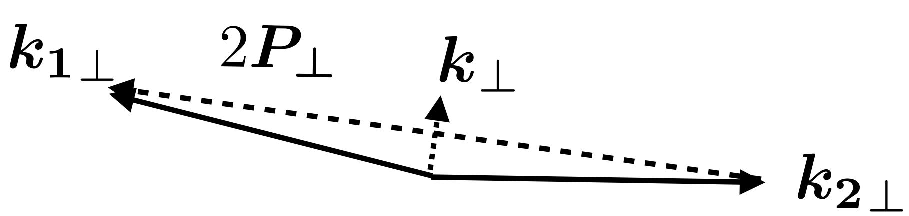
For ITMD in proton-nucleus collisions see
Kotko, Kutak, Marquet, Petreska, Sapeta, van Hameren (2015)

Dijet production beyond TMDs

Differential yield: TMD, ITMD and CGC



proton ~ smaller Q_s^2 **Gold nucleus ~ larger** Q_s^2



$$\frac{\mathrm{d}N^{\gamma_\lambda^* + A \to q\bar{q} + X}}{\mathrm{d}^2 \boldsymbol{P}_\perp \mathrm{d}^2 \boldsymbol{k}_\perp \mathrm{d}\eta_1 \mathrm{d}\eta_2} = N_0^\lambda(P_\perp, k_\perp)\left[1 + 2\sum_{k=1}^{\infty} v_{k,\lambda}(P_\perp, k_\perp)\cos(k\phi)\right]$$

$$\phi \equiv \phi_{\boldsymbol{k}_\perp} - \phi_{\boldsymbol{P}_\perp}$$

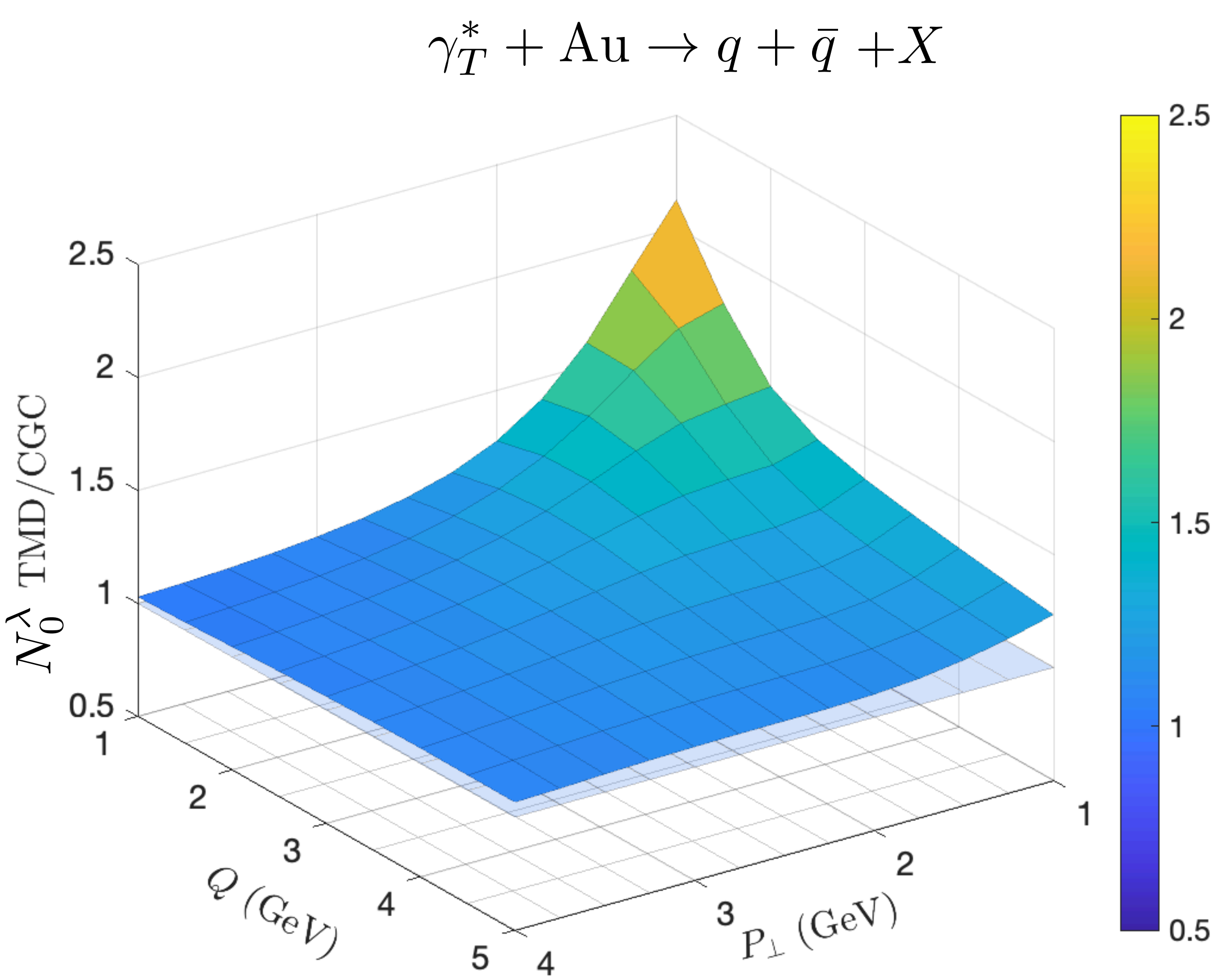
R. Boussarie, H. Mäntysaari, FS, B. Schenke (2021)

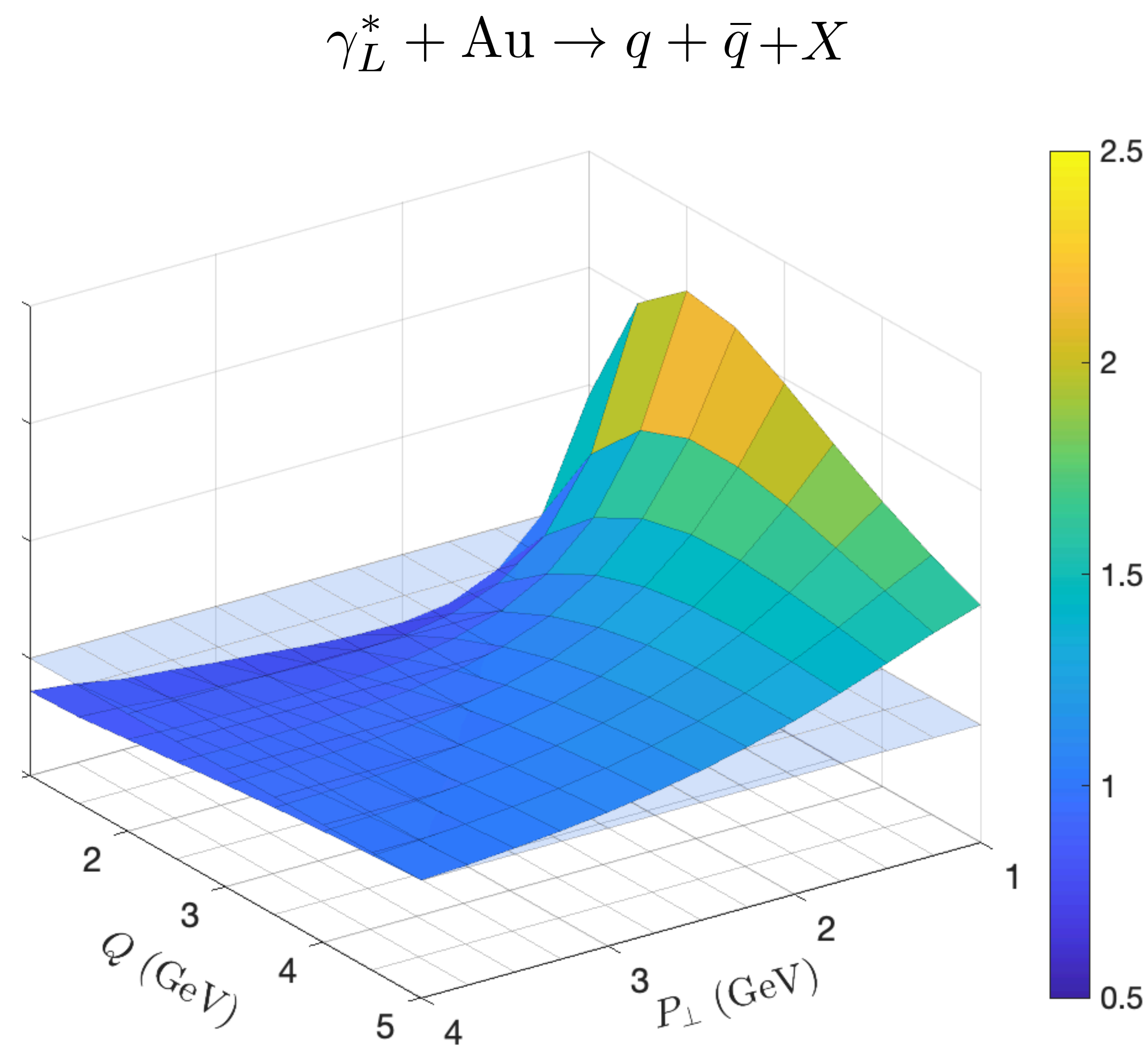
For a comparison of TMD, ITMD and CGC in pA
see Fuji, Watanabe, Marquet (2020)

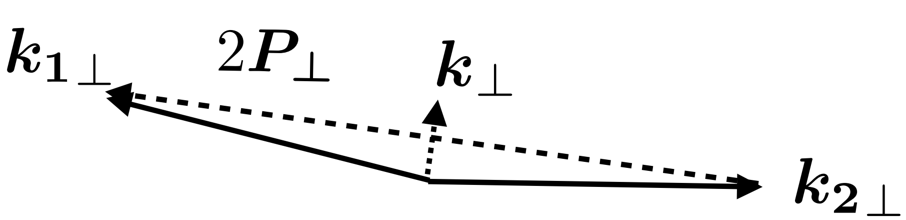
Dijet production beyond TMDs

Q^2 and $P_\perp$ dependence of genuine saturation

At exactly back-to-back $k_\perp \approx 0$ the ratio of CGC/TMD is sensitive to genuine twists







$$\frac{\mathrm{d}N^{\gamma_\lambda^* + A \to q\bar{q} + X}}{\mathrm{d}^2\boldsymbol{P}_\perp \mathrm{d}^2\boldsymbol{k}_\perp \mathrm{d}\eta_1 \mathrm{d}\eta_2} = N_0^\lambda(P_\perp, k_\perp)\left[1 + 2\sum_{k=1}^{\infty} v_{k,\lambda}(P_\perp, k_\perp)\cos(k\phi)\right]$$

$$\phi \equiv \phi_{\boldsymbol{k}_\perp} - \phi_{\boldsymbol{P}_\perp}$$

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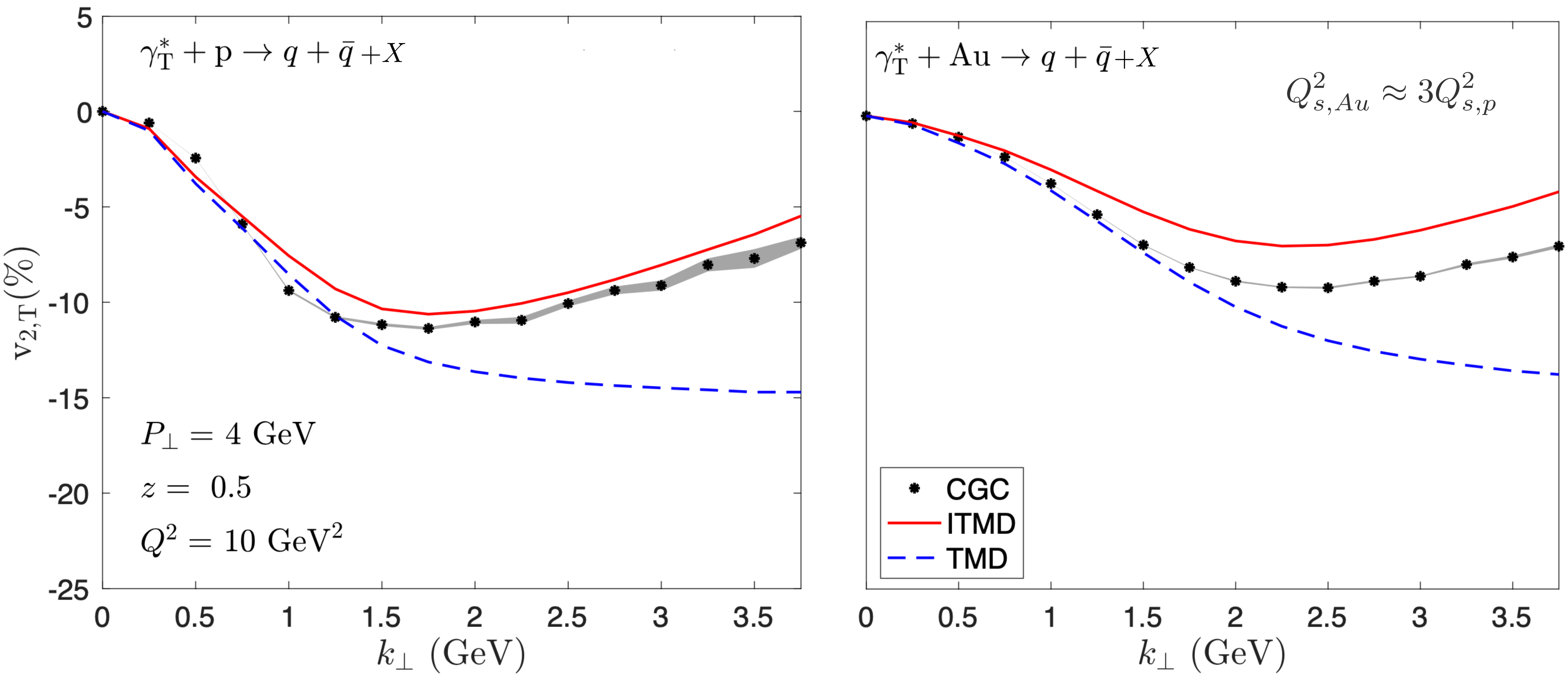
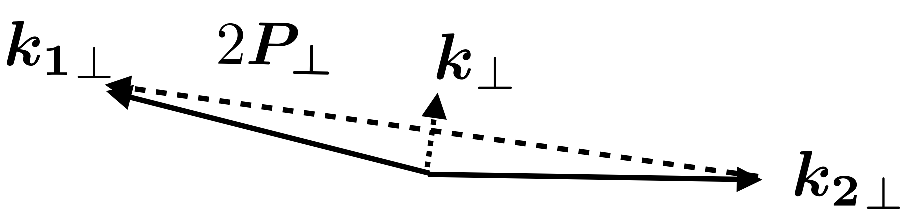
FS, B. Schenke (2021)

Dijet production beyond TMDs

Momentum imbalance **elliptic** anisotropies:
TMD vs ITMD vs CGC



proton ~ smaller Q_s^2 **Gold nucleus ~ larger** Q_s^2



$$\frac{\mathrm{d}N^{\gamma^*_\lambda + A \to q\bar{q}+X}}{\mathrm{d}^2\boldsymbol{P}_\perp \mathrm{d}^2\boldsymbol{k}_\perp \mathrm{d}\eta_1 \mathrm{d}\eta_2} = N_0^\lambda(P_\perp, k_\perp)\left[1 + 2\sum_{k=1}^{\infty} v_{k,\lambda}(P_\perp, k_\perp)\cos(k\phi)\right]$$

$$\phi \equiv \phi_{\boldsymbol{k}_\perp} - \phi_{\boldsymbol{P}_\perp}$$

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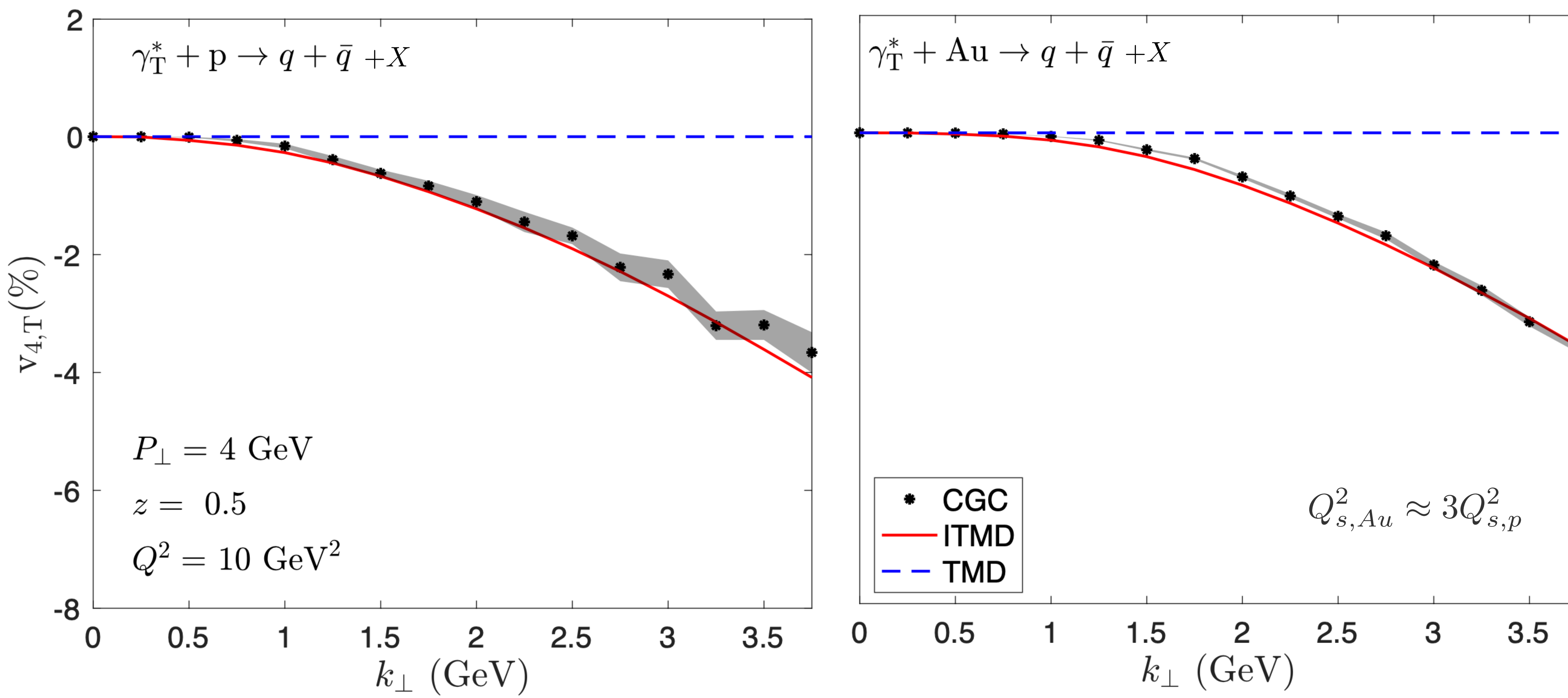
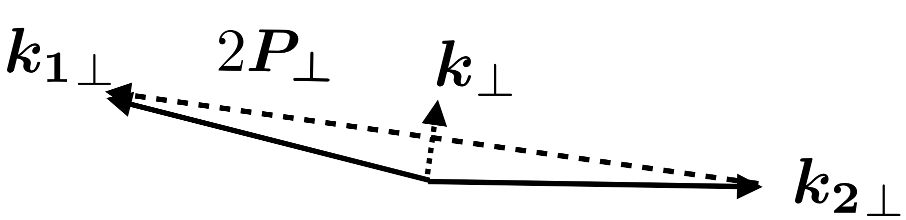
FS, B. Schenke (2021)

Dijet production beyond TMDs

Momentum imbalance **quadrangular** anisotropies:
TMD vs ITMD vs CGC



proton ~ smaller Q_s^2 **Gold nucleus ~ larger Q_s^2**

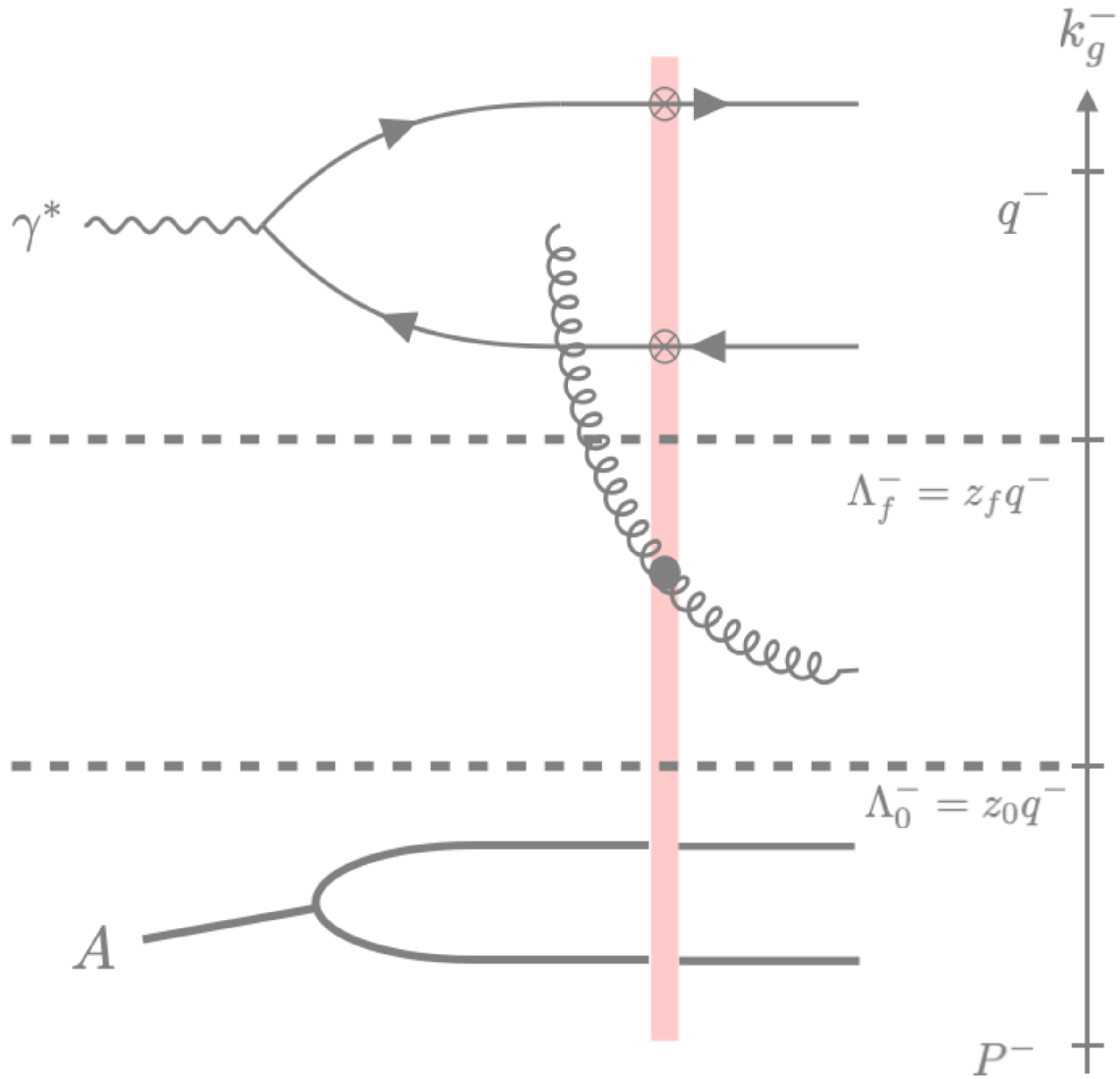


$$\frac{\mathrm{d}N^{\gamma_{\lambda}^{*}+A\rightarrow q\bar{q}+X}}{\mathrm{d}^2\boldsymbol{P}_{\perp}\mathrm{d}^2\boldsymbol{k}_{\perp}\mathrm{d}\eta_1\mathrm{d}\eta_2} = N_0^{\lambda}(P_{\perp},k_{\perp})\left[1+2\sum_{k=1}^{\infty}v_{k,\lambda}(P_{\perp},k_{\perp})\cos(k\phi)\right]$$

$$\phi \equiv \phi_{\boldsymbol{k}_{\perp}} - \phi_{\boldsymbol{P}_{\perp}}$$

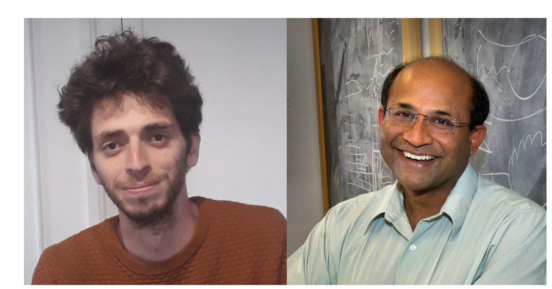
R. Boussarie, H. Mäntysaari,
FS, B. Schenke (2021)

Dijet production in the CGC at NLO



Rapidity factorization and NLO impact factor

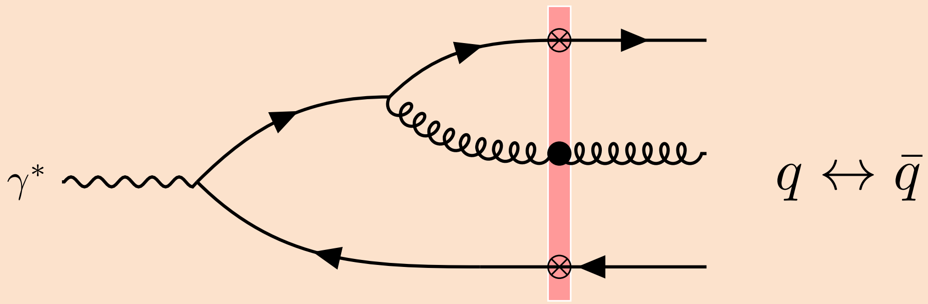
P. Caucal, FS, and R. Venugopalan. 2108.06347
(To appear on JHEP)



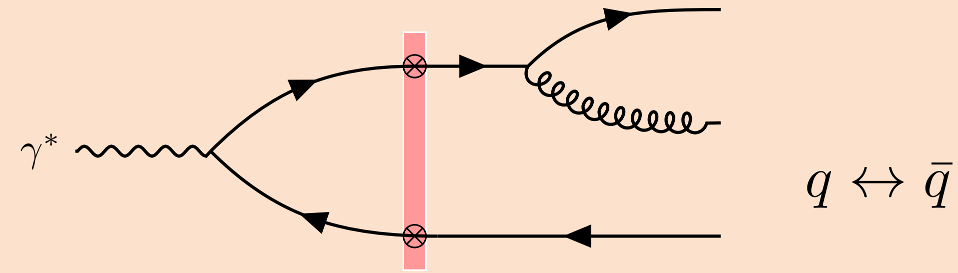
Dijet production in the CGC at NLO

Real and virtual emissions

Real emission diagrams (loop opens in DA and closes in the CCA)



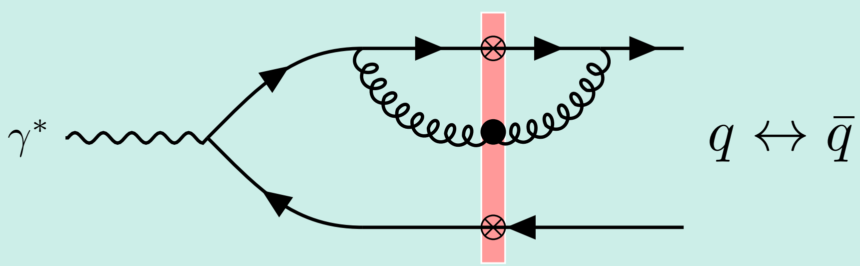
R1



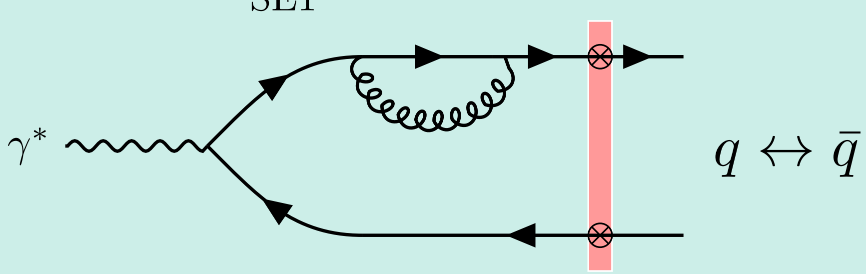
R2

Virtual emission (loop open and closes in DA or CCA)

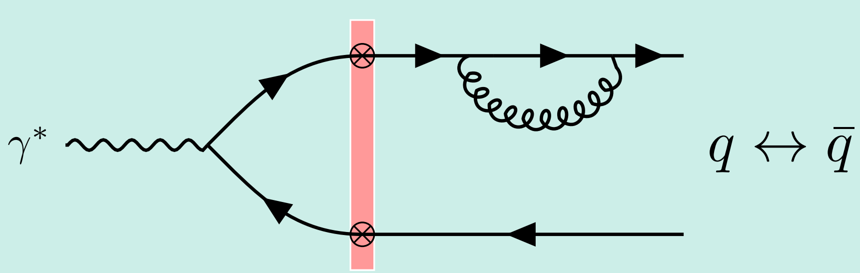
Self-energy contributions



SE1

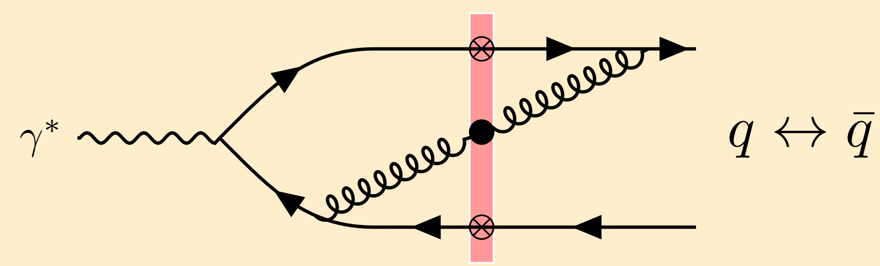


SE2

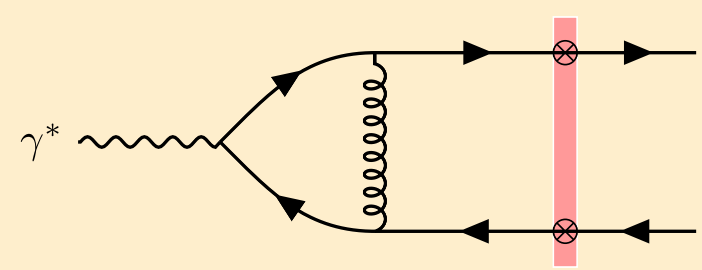


SE3

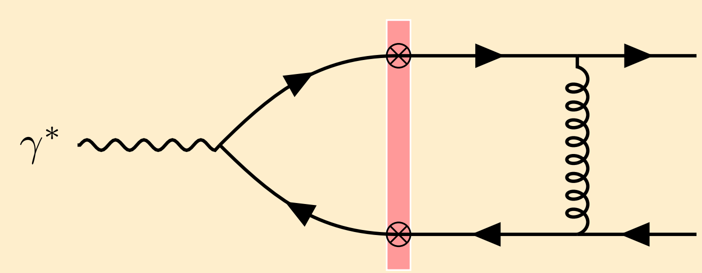
Vertex contributions



V1



V2



V3

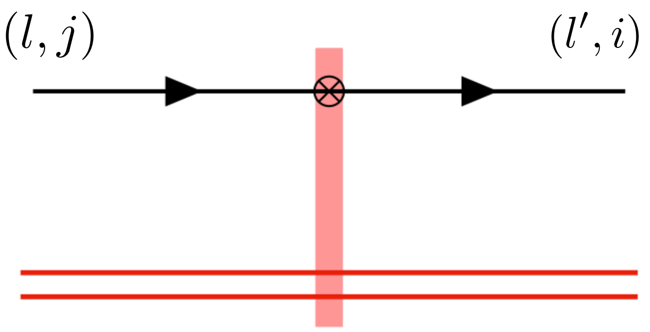
Dijet production in the CGC at NLO

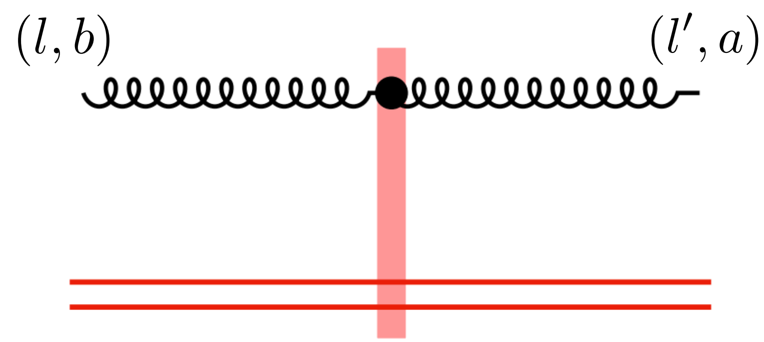
Setup for the calculation

- Covariant perturbation theory in momentum space
  (another popular approach is LCPT)

  Standard QED, QCD rules: propagators, vertices, polarization vectors, etc

- Vertices for (eikonally) coupling to the CGC background field (in $A^-_{\rm cl} = 0$ gauge)





$$\mathcal{T}^q_{ij}(l, l') = (2\pi)\delta(l^- - l'^-)\gamma^- \mathrm{sgn}(l^-) \times \int \mathrm{d}^2\boldsymbol{z}_\perp e^{-i(\boldsymbol{l}'_\perp - \boldsymbol{l}_\perp)\cdot \boldsymbol{z}_\perp} V^{\mathrm{sgn}(l^-)}_{ij}(\boldsymbol{z}_\perp)$$

$$\mathcal{T}^g_{ab}(l, l') = -(2\pi)\delta(l^- - l'^-)(2l^-) g_{\mu\nu} \mathrm{sgn}(l^-) \times \int \mathrm{d}^2\boldsymbol{z}_\perp e^{-i(\boldsymbol{l}'_\perp - \boldsymbol{l}_\perp)\cdot \boldsymbol{z}_\perp} U^{\mathrm{sgn}(l^-)}_{ab}(\boldsymbol{z}_\perp)$$

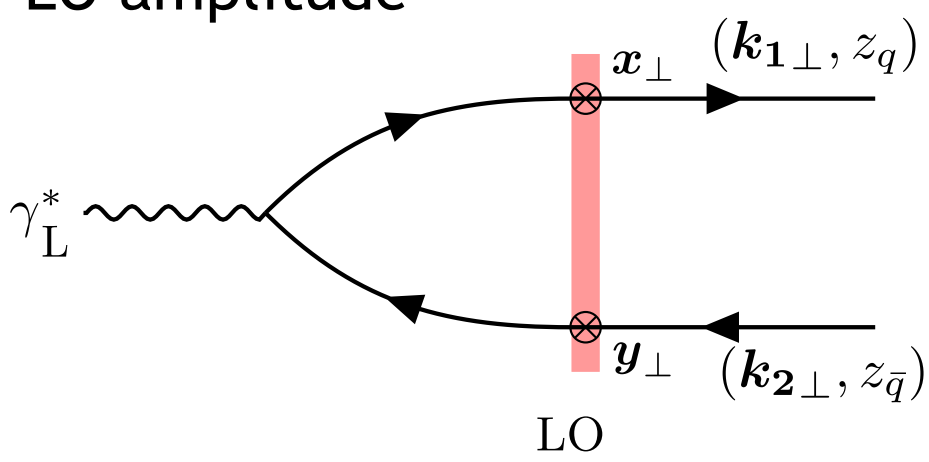
- Regularization schemes

  dimensional regularization + rapidity cut-off

Dijet production in the CGC at NLO

An example of structure of LO vs NLO amplitudes

LO amplitude



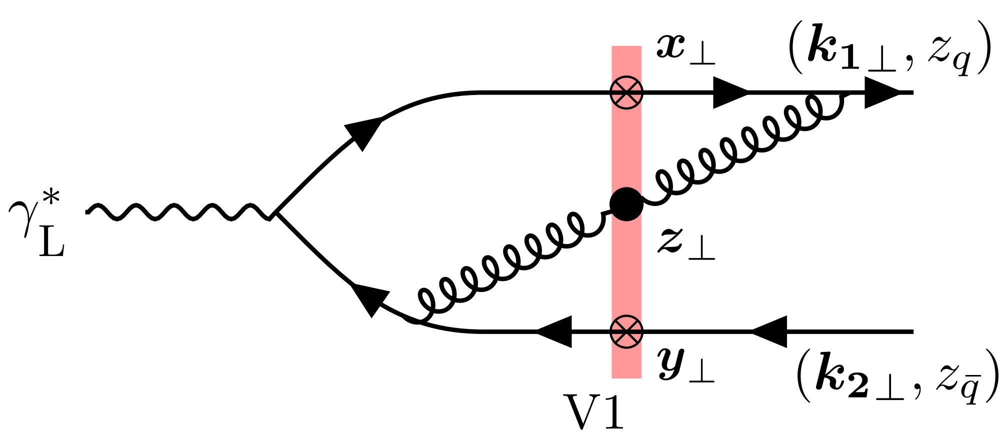
$$\otimes_{\rm LO} \equiv \frac{ee_f q^-}{\pi}\int \mathrm{d}^2\boldsymbol{x}_\perp \mathrm{d}^2\boldsymbol{y}_\perp e^{-i\boldsymbol{k_1}_\perp\cdot\boldsymbol{x}_\perp}e^{-i\boldsymbol{k_2}_\perp\cdot\boldsymbol{y}_\perp}$$

$$X^2_{q\bar{q}} = z_q z_{\bar{q}} r^2_{xy}$$ effective dipole size

$$\mathcal{M}^{{\rm L},\sigma\sigma'}_{\rm LO} = \left[1 - V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{y}_\perp)\right] \otimes_{\rm LO} 2(z_q z_{\bar{q}})^{3/2} Q K_0(QX_{q\bar{q}})\delta^{\sigma,-\sigma'}$$

non-perturbative — perturbatively computable

Dressed vertex amplitude



$$\otimes_{\rm V} \equiv \frac{ee_f q^-}{\pi}\int \mathrm{d}^2\boldsymbol{x}_\perp \mathrm{d}^2\boldsymbol{y}_\perp \mathrm{d}^2\boldsymbol{z}_\perp e^{-i\boldsymbol{k_1}_\perp\cdot\boldsymbol{x}_\perp}e^{-i\boldsymbol{k_2}_\perp\cdot\boldsymbol{y}_\perp}$$

$$X^2_{q\bar{q}g} = (z_q - z_g) z_{\bar{q}} r^2_{xy} + (z_q - z_g) z_g r^2_{xz} + z_g z_{\bar{q}} r^2_{zy}$$ effective dipole size

$$\mathcal{M}^{{\rm L},\sigma\sigma'}_{\rm V1} = \left[C_{\rm F}\mathbb{1} - t^a V(\boldsymbol{x}_\perp) t^b V^\dagger(\boldsymbol{y}_\perp) U_{ab}(\boldsymbol{z}_\perp)\right] \otimes_{\rm V} \frac{\alpha_s}{\pi^2}\int_{z_0}^{z_q}\frac{\mathrm{d}z_g}{z_g}\frac{\mathbf{r}_{zx}\cdot\mathbf{r}_{zy}}{\mathbf{r}^2_{zx}\mathbf{r}^2_{zy}}\left[\left(1-\frac{z_g}{z_q}\right)\left(1+\frac{z_g}{z_{\bar{q}}}\right)\left(1-\frac{z_g}{2z_q}-\frac{z_g}{2(z_{\bar{q}}+z_g)}\right)+\dots\right] e^{-i\frac{z_g}{z_q}\boldsymbol{k}_\perp\cdot\mathbf{r}_{zx}}$$

$$\times\, 2(z_q z_{\bar{q}})^{3/2} Q K_0(QX_{q\bar{q}g})\delta^{\sigma,-\sigma'}$$

non-perturbative — perturbatively computable

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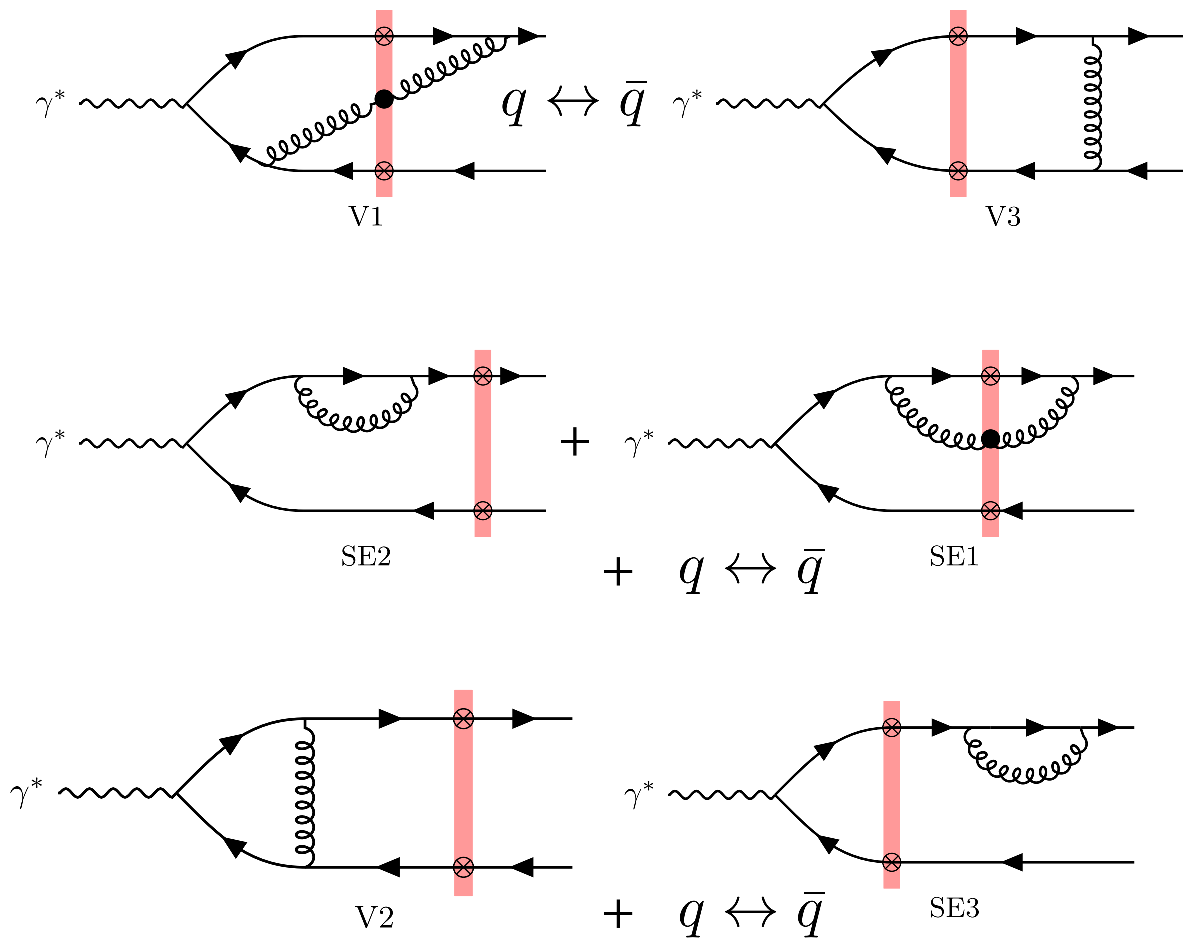
Dijet production in the CGC at NLO

Cancellation of divergences of UV divergences



- UV finite diagrams
- UV divergences cancel among self energies contributions (before SW and crossing SW)
- UV divergence cancel in vertex contribution before SW and self energy contribution after SW

- <u>UV finite</u>, no need for counter-terms at this order in PT.
- Overall <u>IR divergence</u> is left in sum of virtual diagrams, and <u>soft divergence</u> left in V3. Both <u>cancel with real emissions</u>.

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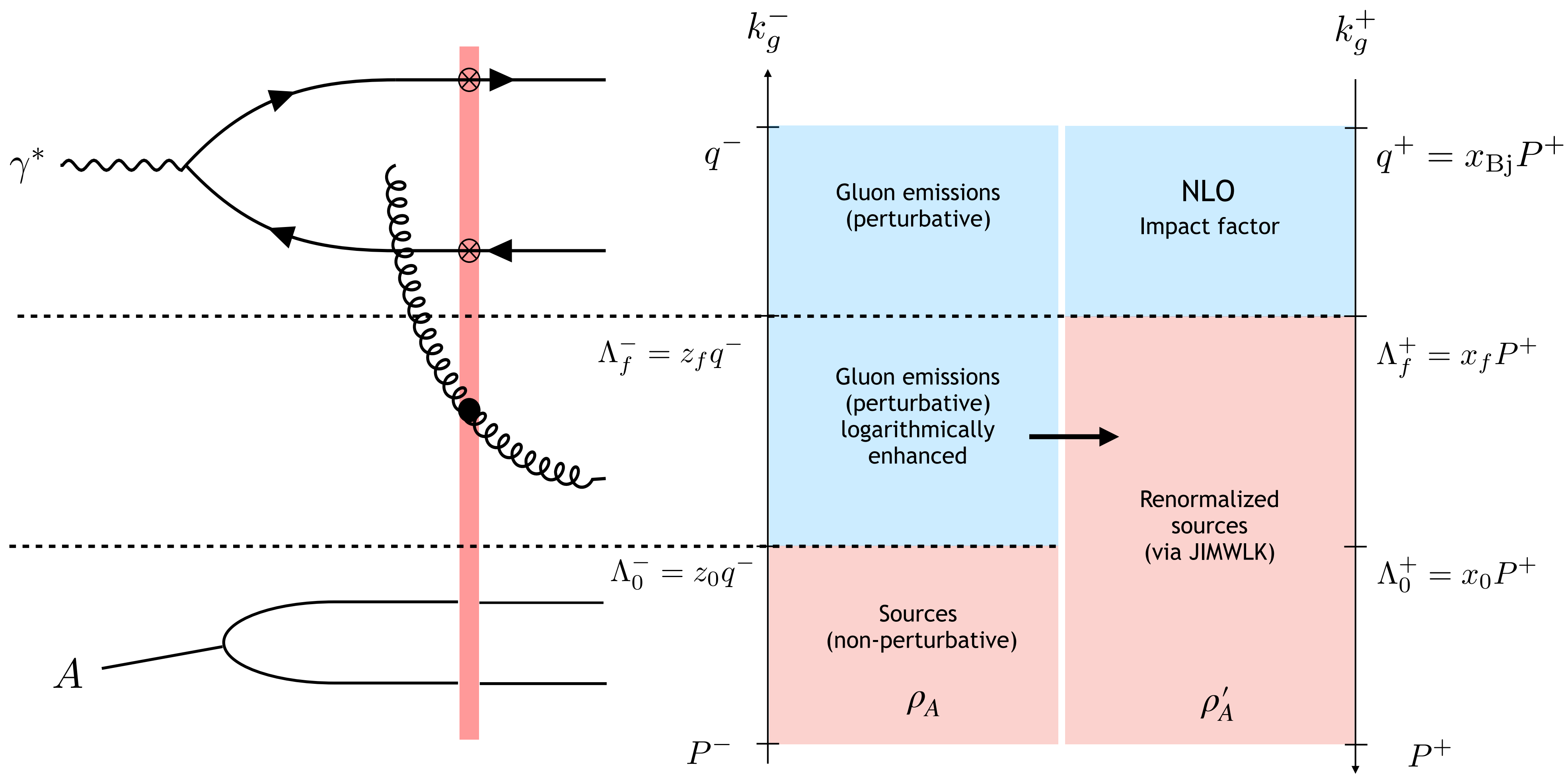
Inclusive dijet production at NLO

Rapidity (slow gluon) divergences and JIMWLK **factorization**



JIMWLK LL Hamiltonian

$$\mathrm{d}\sigma_{\rm NLO} = \alpha_s \ln\left(\frac{z_f}{z_0}\right) \mathcal{H}_{\rm LL} \mathrm{d}\sigma_{\rm LO} + \alpha_s \mathrm{d}\sigma_{\rm NLO,i.f.}$$

Large logs need to be **resummed**!

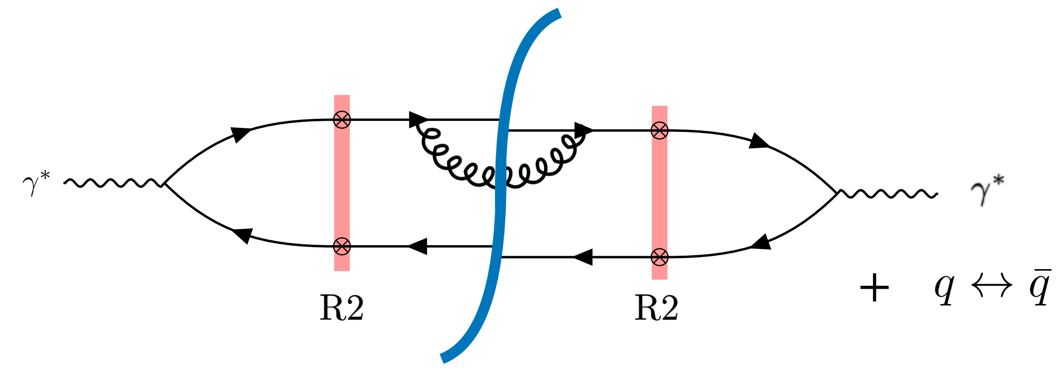
$$\alpha_s \ln\left(\frac{z_f}{z_0}\right) \sim \alpha_s \ln(s)$$

Evolution of sources (weight functional)

$$W_{\Lambda_0^-}[\rho_A] \to W_{\Lambda_f^-}[\rho'_A]$$

Dijet production in the CGC at NLO

Infrared and collinear safety



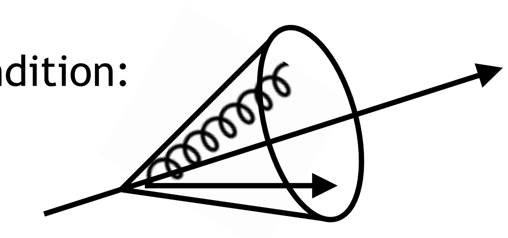
Collinear non-slow divergences

- Implement a jet algorithm* (small cone) excluding slow gluon divergence

Phase space for collinear non-slow gluon

$$\int_{z_f}^{z_j} \frac{\mathrm{d}z_g}{z_g}\mu^{\varepsilon} \int \frac{\mathrm{d}^{2-\varepsilon}\mathcal{C}_{qg,\perp}}{(2\pi)^{2-\varepsilon}} \frac{1}{\mathcal{C}^2_{qg,\perp}}$$

Small-cone condition:

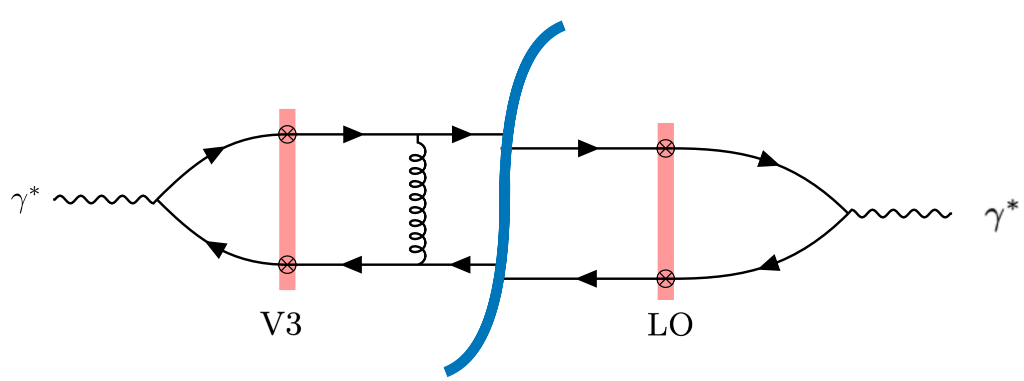


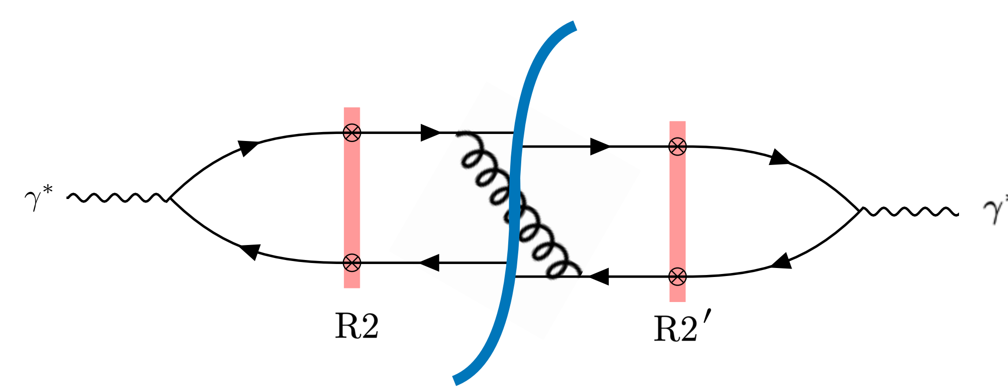
$$\mathcal{C}^2_{qg,\perp} \leq \mathcal{C}^2_{qg,\perp}\big|_{\mathrm{max}} = R^2 \boldsymbol{p}_j^2 \mathrm{min}\left(\frac{z_g^2}{z_j^2}, \frac{(z_j - z_g)^2}{z_j^2}\right)$$

- Collinear divergence cancels against IR divergence left in virtual contributions

Soft divergence

- Remaining soft divergence cancel between vertex correction after SW, and cross term real gluon emission after SW





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Summary

- Dijet production in the TMD formalism
  Observables at the EIC

- Dijet production at EIC beyond TMDs
  Resummation of kinematic and genuine saturation corrections

- Dijet production at EIC in the CGC at NLO
  JIMWLK rapidity **factorization**, and finite impact factor

Outlook

- Couple our partonic cross-sections to event generators

  How much of the kinematic power and genuine saturation corrections survives in the actual observable?

- Investigate dijet production at NLO in the back-to-back limit

  Match to TMD factorization at NLO

  Xiao, Yuan, Zhou (2017)
  Hentschinski (2021)

  See Martin's talk for high energy factorization at NLO!

  Is the Improved TMD framework valid at NLO?

- Numerical implementation of dijet production at NLO

  Promoting saturation physics to a precision science

- Employ modern techniques such SCET to the CGC at NLO

  Extend existing SCET studies for dijet TMD factorization to the small-x regime

  del Castillo, Echevarria, Makris, Scimemi (2020)
  Kang, Reiten, Shao, Terry (2020)