Curing high-energy instability of the NLO heavy-quarkonium hadroproduction cross section with High-Energy Factorization

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REF-2021
November $16^{\text{th.}}$, 2021

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Outline

1. Why η_c ?
2. High-energy instability of the NLO cross section
3. High-Energy Factorization $\Rightarrow$ resummation of $\ln 1/z$
4. Reproducing NLO and NNLO results at $z \to 0$ from HEF
5. Matching HEF and NLO CPM calculations

Do we understand quarkonium production?

Understanding of hadronisation of the $Q\bar{Q}$-pair ($Q = b, c$) into heavy quarkonium turned-out to be challenging theoretical/phenomenological problem (recent reviews: hep-ph/2012.14161, hep-ex/2011.15005, hep-ph/1903.09185). Do we understand which states of the $Q\bar{Q}$-pair are important?

For some processes – yes! The importance of the **color-singlet** $Q\bar{Q}$-state has been established:

- Photoproduction of prompt J/ψ at $0 < z < 0.9$.
- Bulk of the double prompt hadroproduction of J/ψ

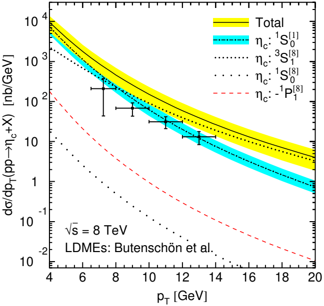
and (somewhat unexpectedly...)

- Prompt η_c hadroproduction

Probability (=LDME) of hadronisation of the color-singlet $Q\bar{Q}$-pair to the physical state is $\propto |R^{(')}(r = 0)|^2$, which is relatively well constrained, in comparison to CO LDMEs. **No free parameters!?**

Is ${}^1S_0^{(1)}$-dominance in η_c-production a bug or a feature?

The η_c hadroproduction was found to be dominated by the $c\bar{c}\left[{}^1S_0^{(1)}\right]$ state:

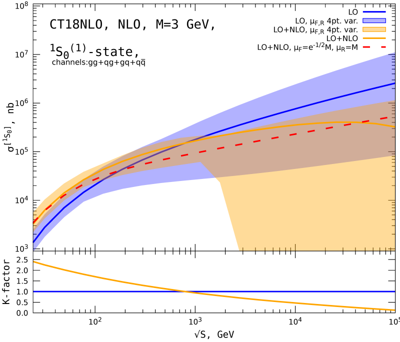


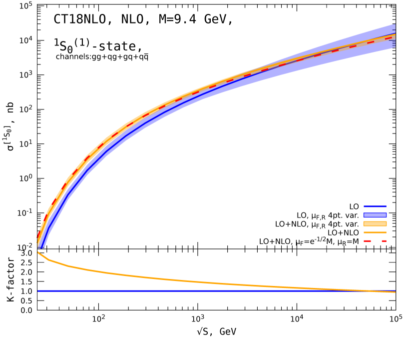
[Butenschoen, He, Kniehl, 2015] This is a problem for NRQCD factorization, because roughly the same contribution of color-octet states as for J/ψ was expected.

- Color-octet LDMEs for η_c are related (up to v^2 corrections) to LDMEs of J/ψ by *heavy-quark spin symmetry* (long wavelength gluons do not "see" heavy quark's spin). Strong HQSS violation?
- HQSS is quite "good" symmetry, manifests itself e.g. in hadron spectrum as closeness of D and D^* (B and B^*) masses.
- **My speculation:** HQSS maybe inapplicable to production, because m_Q is not the largest scale in the problem. There is a lot of "soft" gluons with $m_{c,b} \ll E_g \ll p_T$.

Is NLO QCD prediction for η_c production perturbatively stable?

If we want to calculate the p_T**-integrated** total or y-dfferential cross section, then **NO:**





Why?

Collinear factorization for total CS for the state $m = {}^{2S+1}L_J^{(0)}$:

$$\sigma^{[m]}(\sqrt{S}) = \int\limits_{z_{\min}}^{1} \frac{dz}{z} \mathcal{L}_{ij}(z,\mu_F) \hat{\sigma}_{ij}^{[m]}(z,\mu_F,\mu_R),$$

where $i,j = q,\bar{q},g$, $\boxed{z = M^2/\hat{s}}$ and *partonic luminosity*:

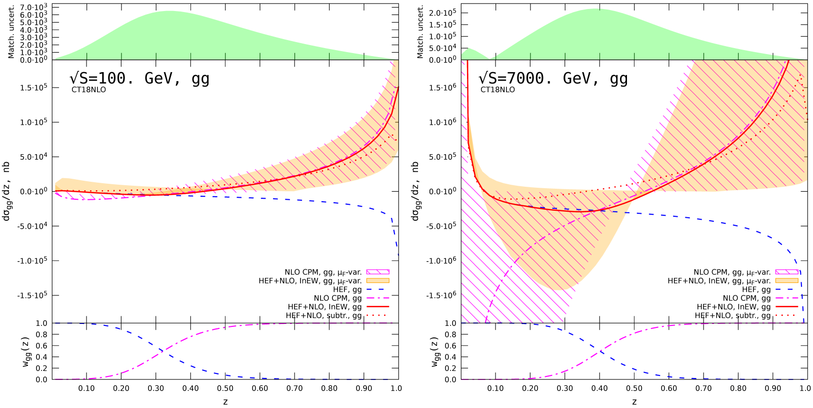
$$\mathcal{L}_{ij}(z,\mu_F) = \int\limits_{-y_{\max}}^{+y_{\max}} dy\ \tilde{f}_i\left(\frac{M}{\sqrt{Sz}}e^{y},\mu_F\right) \tilde{f}_j\left(\frac{M}{\sqrt{Sz}}e^{-y},\mu_F\right),$$

with $\tilde{f}_j(x,\mu_F)$ – momentum density PDFs.
NLO coefficient function [Kuhn, Mirkes, 93'; Petrelli *et.al.*, 98'] in the $z \to 0$ limit

$$\hat{\sigma}_{ij}^{[m]} = \sigma_{\rm LO}^{[m]} \left[A_0^{[m]} \delta(1-z) + C_{ij} \frac{\alpha_s(\mu_R)}{\pi} \left(A_0^{[m]} \ln \frac{M^2}{\mu_F^2} + A_1^{[m]} \right) + O(z\alpha_s, \alpha_s^2) \right],$$

where $C_{gg} = 2C_A = 2N_c$, $C_{qg} = C_{gq} = C_F = (N_c^2-1)/(2N_c)$, $C_{q\bar{q}} = 0$ and $A_1^{[m]} < 0$.

Why?



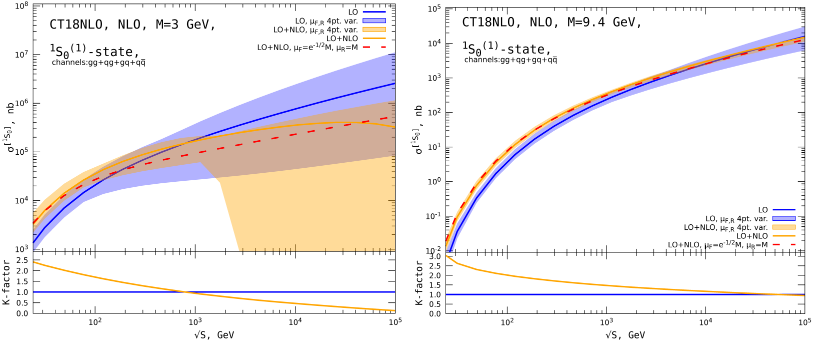
√S=100. GeV, gg
CT18NLO
√S=7000. GeV, gg
CT18NLO
Match. uncert.
dσgg/dz, nb
wgg(z)
z
NLO CPM, gg, μF-var.
HEF+NLO, InEW, gg, μF-var.
HEF, gg
NLO CPM, gg
HEF+NLO, InEW, gg
HEF+NLO, subtr., gg

Optimal μ_F choice?

It is natural to choose μ_F such a way, that the negative $A_1^{[m]}$ is cancelled [Lansberg, Ozcelik, 2020]:

$$\hat{\mu}_F = M \exp\left[\frac{A_1^{[m]}}{2A_0^{[m]}}\right],$$

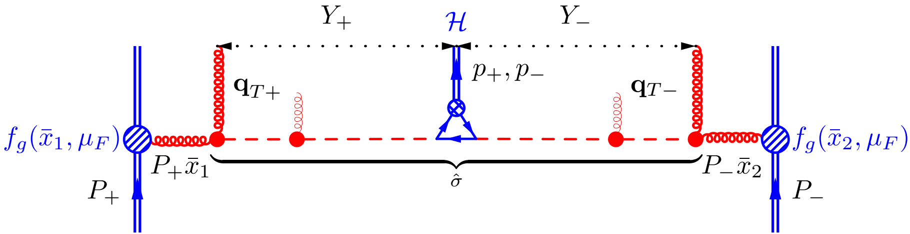
is equivalent to resummation of **some of the** terms $\propto \alpha_s^n \ln^{n-1}\frac{1}{z}$ (more on this later). The result (red curve):



Is the systematic resummation of $\propto \alpha_s^n \ln^{n-1}\frac{1}{z}$ possible?

High-Energy factorization in a nutshell

High-Energy Factorization [Collins, Ellis, 91'; Catani, Ciafaloni, Hautmann, 91',94']:



Small parameter $\boxed{z = \frac{M^2}{\hat{s}} = \frac{M^2}{M_T^2} z_+ z_-,}$ where $M_T^2 = M^2 + \mathbf{p}_T^2$ and

$$z_+ = \frac{p_+}{P_+\bar{x}_1},\ z_- = \frac{p_-}{P_-\bar{x}_2}$$

Using the BFKL formalism one resums corrections to $\hat{\sigma}$ enhanced by

$$Y_\pm = \ln\left(\frac{\mu_Y}{|\mathbf{q}_{T\pm}|}\frac{1-z_\pm}{z_\pm}\right) \simeq \cancel{\ln\frac{\mu_Y}{|\mathbf{q}_{T\pm}|}} + \ln\frac{1}{z_\pm},\ \text{in LP w.r.t. } \cancel{\frac{|\mathbf{q}_{T\pm}|}{\mu_Y}}\ \frac{z_\pm}{1-z_\pm}$$

where $\mu_Y = p_\pm e^{\mp y_\mathcal{H}} \sim M_T$, in inclusive observables.

Resummed coefficient function

$$\hat{\sigma}_{ij}^{[m],\ \mathrm{HEF}}(z,\mu_F,\mu_R) = \int\limits_{-\infty}^{\infty} d\eta \int\limits_{0}^{\infty} d\mathbf{q}_{T1}^2 d\mathbf{q}_{T2}^2\ \mathcal{C}_{gi}\left(\frac{M_T}{M}\sqrt{z}e^{\eta},\mathbf{q}_{T1}^2,\mu_F,\mu_R\right)$$

$$\times\mathcal{C}_{gj}\left(\frac{M_T}{M}\sqrt{z}e^{-\eta},\mathbf{q}_{T2}^2,\mu_F,\mu_R\right)\int\limits_{0}^{2\pi}\frac{d\phi}{2}\frac{H^{[m]}(\mathbf{q}_{T1}^2,\mathbf{q}_{T2}^2,\phi)}{M_T^4},$$

The coefficient functions $H^{[m]}$ are known at LO in α_s [Hagler *et.al*, 2000; Kniehl, Vasin, Saleev 2006] for $m = {}^1S_0^{(1,8)},\ {}^3P_J^{(1,8)},\ {}^3S_1^{(8)}$.

LLA evolution w.r.t. $\ln 1/z$

In the LL($\ln 1/z$)-approximation, the $Y = \ln 1/z$-evolution equation for *collinearly un-subtracted* $\tilde{\mathcal{C}}$-factor has the form:

$$\tilde{\mathcal{C}}(x, \mathbf{q}_T) = \delta(1-z)\delta(\mathbf{q}_T^2) + \hat{\alpha}_s \int\limits_x^1 \frac{dz}{z} \int d^{2-2\epsilon}\mathbf{k}_T K(\mathbf{k}_T^2, \mathbf{q}_T^2) \tilde{\mathcal{C}}\left(\frac{x}{z}, \mathbf{q}_T - \mathbf{k}_T\right)$$

with $\hat{\alpha}_s = \alpha_s C_A/\pi$ and

$$K(\mathbf{k}_T^2, \mathbf{p}_T^2) = \delta^{(2-2\epsilon)}(\mathbf{k}_T) \frac{(\mathbf{p}_T^2)^{-\epsilon}}{\epsilon} \frac{(4\pi)^\epsilon \Gamma(1+\epsilon)\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)} + \frac{1}{\pi(2\pi)^{-2\epsilon}\mathbf{k}_T^2}.$$

It is convenient to go from $(z, \mathbf{q}_T)$-space to $(N, \mathbf{x}_T)$-space:

$$\tilde{\mathcal{C}}(N, \mathbf{x}_T) = \int d^{2-2\epsilon}\mathbf{q}_T \, e^{i\mathbf{x}_T\mathbf{q}_T} \int\limits_0^1 dx \, x^{N-1} \, \tilde{\mathcal{C}}(x, \mathbf{q}_T),$$

because:

- Mellin convolutions over z turn into products: $\int \frac{dz}{z} \to \frac{1}{N}$
- Large logs map to poles at $N = 0$: $\boxed{\alpha_s^{k+1} \ln^k \frac{1}{z} \to \frac{\alpha_s^{k+1}}{N^{k+1}}}$
- All *collinear divergences* are contained inside $\mathcal{C}$ in $\mathbf{x}_T$-space.

Collinear divergences

Exact (up to terms $O(\epsilon)$) solution for $\tilde{\mathcal{C}}$ can be obtained [Catani, Hautmann, 94'] . It contains *collinear divergences*, which can be removed (absorbed into PDFs) in the $\overline{MS}$-scheme to all orders in α_s:

$$Z^{-1}_{\text{coll.}} = \exp\left[-\frac{1}{\epsilon}\int\limits_0^{\hat{\alpha}_s S_\epsilon \mu_F^{-2\epsilon}} \frac{d\alpha}{\alpha}\gamma_{gg}(\alpha, N)\right], \quad S_\epsilon = \exp\left[\epsilon\left(\ln 4\pi - \gamma_E\right)\right],$$

$$\tilde{\mathcal{C}}(N, \mathbf{x}_T) = Z^{-1}_{\text{coll.}}\mathcal{C}(N, \mathbf{x}_T, \mu_F)$$

Exact LL solution

In $(N, \mathbf{q}_T)$-space, subtracted $\mathcal{C}$, which resums all terms $\propto (\hat{\alpha}_s/N)^n$ has the form:

$$\mathcal{C}(N, \mathbf{q}_T, \mu_F) = R(\gamma_{gg}(N, \alpha_s)) \frac{\gamma_{gg}(N, \alpha_s)}{\mathbf{q}_T^2} \left(\frac{\mathbf{q}_T^2}{\mu_F^2} \right)^{\gamma_{gg}(N, \alpha_s)},$$

where $\gamma_{gg}(N, \alpha_s)$ is the solution of [Jaroszewicz, 82']:

$$\frac{\hat{\alpha}_s}{N} \chi(\gamma_{gg}(N, \alpha_s)) = 1, \text{ with } \chi(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1-\gamma),$$

where $\psi(\gamma) = d \ln \Gamma(\gamma)/d\gamma$ – Euler's ψ-function. The first few terms:

$$\gamma_{gg}(N, \alpha_s) = \underbrace{\underbrace{\frac{\hat{\alpha}_s}{N}}_{\text{DLA}} + 2\zeta(3) \frac{\hat{\alpha}_s^4}{N^4} + 2\zeta(5) \frac{\hat{\alpha}_s^6}{N^6} + \ldots}_{\text{LLA}}$$

The function $R(\gamma)$ is

$$R(\gamma_{gg}(N, \alpha_s)) = 1 + O(\alpha_s^3).$$

Doubly-logarithmic approximation

Taking the LO result for $\gamma_{gg}(N, \alpha_s) \to \boxed{\gamma_N = \frac{\hat{\alpha}_s}{N}}$ we obtain:

$$\boxed{\mathcal{C}_{\text{DL}}(N, \mathbf{q}_T, \mu_F) = \frac{\gamma_N}{\mathbf{q}_T^2} \left(\frac{\mathbf{q}_T^2}{\mu_F^2} \right)^{\gamma_N},}$$

which resums $\left(\frac{\hat{\alpha}_s}{N} \ln \frac{\mathbf{q}_T^2}{\mu_F^2} \right)^n \leftrightarrow \hat{\alpha}_s^n \ln^n \left(\frac{\mathbf{q}_T^2}{\mu_F^2} \right) \ln^{n-1} \left(\frac{1}{z} \right)$.

In $(z, \mathbf{q}_T)$-space it is [Blümlein, 94']:

$$\mathcal{C}_{\text{DL}}(z, \mathbf{q}_T, \mu_F) = \frac{\hat{\alpha}_s}{\mathbf{q}_T^2} \begin{cases} J_0 \left(2\sqrt{\hat{\alpha}_s \ln \frac{\mu_F^2}{\mathbf{q}_T^2} \ln \frac{1}{z}} \right), & |\mathbf{q}_T| < \mu_F, \\ I_0 \left(2\sqrt{\hat{\alpha}_s \ln \frac{\mathbf{q}_T^2}{\mu_F^2} \ln \frac{1}{z}} \right), & |\mathbf{q}_T| > \mu_F, \end{cases}$$

where J_0/I_0 is the Bessel function of the first/second kind.
This approximation should be used with standard LO, NLO, NNLO PDFs, because DGLAP evolution is taken at fixed order (LO, NLO, NNLO).

Does this work?

The resummation has to reporduce the $A_1^{[m]}$ NLO coefficient when expanded up to NLO in α_s. And it does. We have performed expansion up to NNLO:

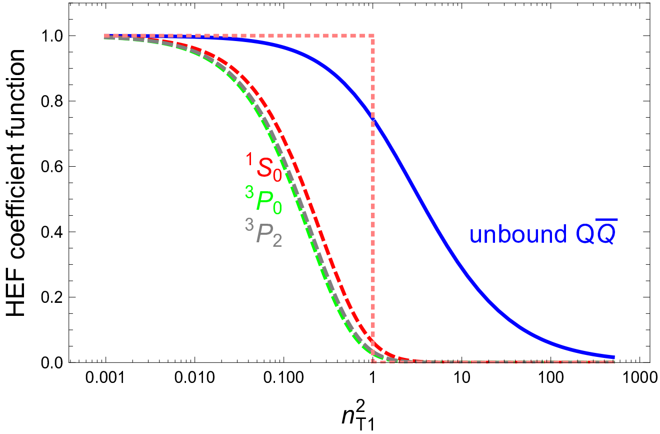
State	$A_0^{[m]}$	$A_1^{[m]}$	$A_2^{[m]}$	$B_2^{[m]}$
$^1S_0$	1	$\mathbf{-1}$	$\frac{\pi^2}{6}$	$\frac{\pi^2}{6}$
$^3S_1$	0	1	0	$\frac{\pi^2}{6}$
$^3P_0$	1	$\mathbf{-\frac{43}{27}}$	$\frac{\pi^2}{6}+\frac{2}{3}$	$\frac{\pi^2}{6}+\frac{40}{27}$
$^3P_1$	0	$\frac{5}{54}$	$-\frac{1}{9}$	$-\frac{2}{9}$
$^3P_2$	1	$\mathbf{-\frac{53}{36}}$	$\frac{\pi^2}{6}+\frac{1}{2}$	$\frac{\pi^2}{6}+\frac{11}{9}$

for e.g.

$$\hat{\sigma}_{gg}^{[m],\,\mathrm{HEF}}(z\to 0)=\sigma_{\mathrm{LO}}^{[m]}\left\{A_0^{[m]}\delta(1-z)+\frac{\alpha_s}{\pi}2C_A\left[A_1^{[m]}+A_0^{[m]}\ln\frac{M^2}{\mu_F^2}\right]\right.$$
$$\left.+\left(\frac{\alpha_s}{\pi}\right)^2 C_A^2\ln\frac{1}{z}\left[2A_2^{[m]}+B_2^{[m]}+4A_1^{[m]}\ln\frac{M^2}{\mu_F^2}+2A_0^{[m]}\ln^2\frac{M^2}{\mu_F^2}\right]+O(\alpha_s^3)\right\},$$

Connection with $\hat{\mu}_F$

The HEF resummation **would be equivalent** to the $\hat{\mu}_F$ prescription, if the HEF CF $H^{[m]}(\mathbf{q}_{T1}^2, \mathbf{q}_{T2}^2 = 0)/M_T^4$ was $\propto \theta(\hat{\mu}_F^2 - \mathbf{q}_{T1}^2)$. But it is not:



Matching with NLO of CF

The HEF works only at $z \ll 1$, misses power corrections $O(z)$, while NLO CF is exact in z, but only NLO in α_s. **We need to match them.**

- Simplest prescription: just subtract the overlap at $z \ll 1$:

$$\sigma^{[m]}_{\text{NLO+HEF}} = \sigma^{[m]}_{\text{LO CF}} + \int\limits_{z_{\min}}^{1} \frac{dz}{z} \left[\check{\sigma}^{[m],ij}_{\text{HEF}}(z) + \hat{\sigma}^{[m],ij}_{\text{NLO CF}}(z) - \hat{\sigma}^{[m],ij}_{\text{NLO CF}}(0) \right] \mathcal{L}_{ij}(z),$$

- Or introduce **smooth weights**:

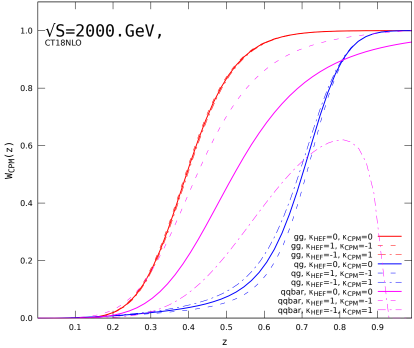
$$\sigma^{[m]}_{\text{NLO+HEF}} = \sigma^{[m]}_{\text{LO CF}} + \int\limits_{z_{\min}}^{1} dz \left\{ \left[\check{\sigma}^{[m],ij}_{\text{HEF}}(z) \frac{\mathcal{L}_{ij}(z)}{z} \right] w^{ij}_{\text{HEF}}(z) + \left[\hat{\sigma}^{[m],ij}_{\text{NLO CF}}(z) \frac{\mathcal{L}_{ij}(z)}{z} \right] (1 - w^{ij}_{\text{HEF}}(z)) \right\},$$

Inverse error weighting method

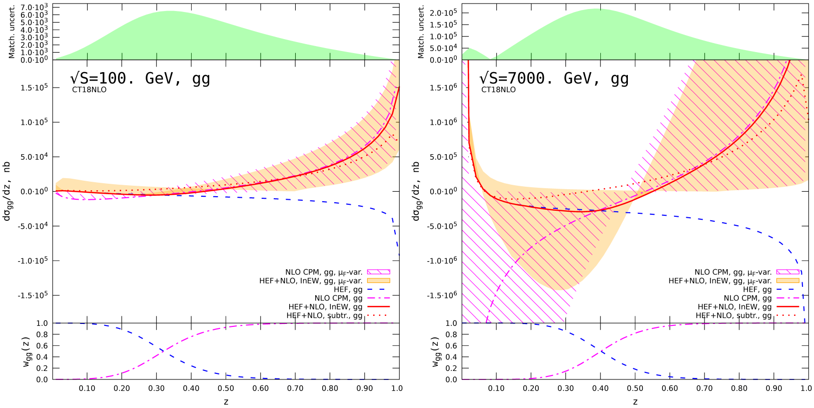
In the InEW method [Eichevarria, *et.al.*, 2018] the weights are calculated from **estimates of the error** of each contribution:

$$w^{ij}_{\rm HEF}(z) = \frac{[\Delta\sigma^{ij}_{\rm HEF}(z)]^{-2}}{[\Delta\sigma^{ij}_{\rm HEF}(z)]^{-2} + [\Delta\sigma^{ij}_{\rm CF}(z)]^{-2}},$$

- For $\Delta\sigma_{\rm CF}$ we take the $\alpha_s^2 \ln\frac{1}{z}$ term obtained from HEF $+O(\alpha_s^2)$-term which we vary.
- For $\Delta\sigma_{\rm HEF}$ we take the $\alpha_s O(z)$ part of the NLO CF result $+O(\alpha_s^2)$-term which we vary.

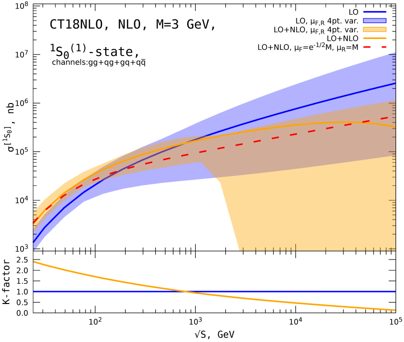


Matching plots

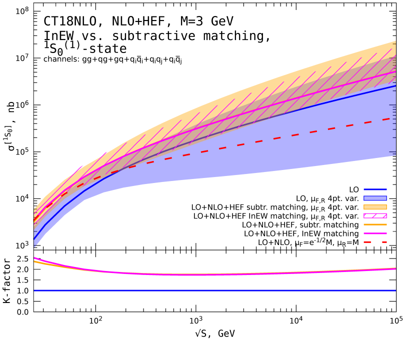


Matched results for η_c

NLO

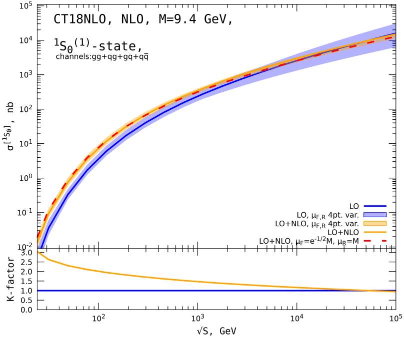


NLO+HEF DLA

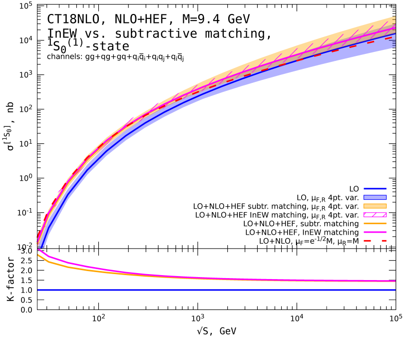


Matched results for η_b

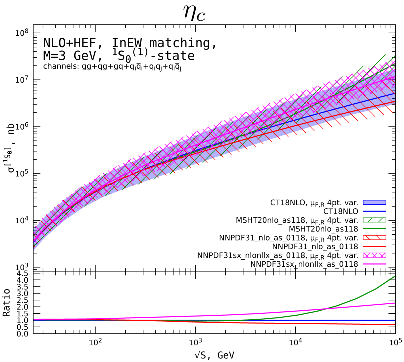
NLO

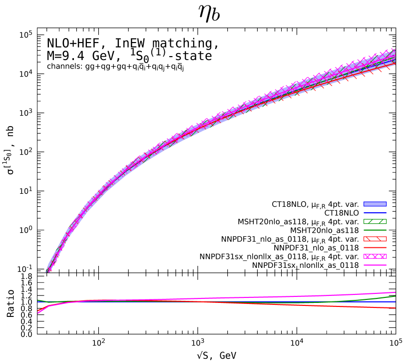


NLO+HEF DLA



The PDF dependence





Conclusions and outlook

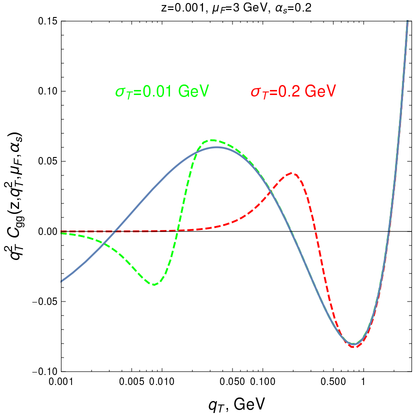
- The high-energy instability of the NLO cross section is related with lack of the $\alpha_s^n \ln^{n-1} \frac{1}{z}$ corrections in $\hat{\sigma}(z)$ at $z \ll 1$.
- The HEF at DLA is the formalism to solve this problem if the standard fixed-order PDFs are to be used.
- Matching between NLO CF (finite z) and HEF ($z \ll 1$) has to be performed, but there is no strong sensitivity to matching procedure.
- Scale-uncertainty is reduced, the K-factor is flat at high energy. But the uncertainty of NLO CF+DLA HEF calculation is still too large.
- NLO CF+NLL HEF calculation is in progress.
- Future plans: y-distributions, production of χ_{cJ}, photoproduction...

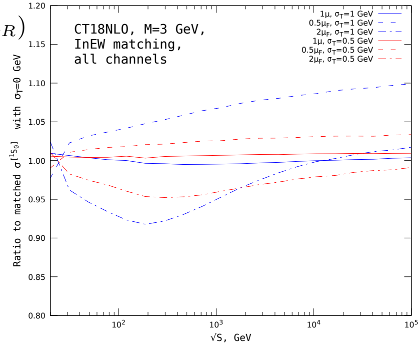
Thank you for your attention!

Backup: Higher-twist effects

Convolution of $\mathcal{C}$-factor with the Gaussian in $\mathbf{k}_T$:

$$\int \frac{d^2\mathbf{k}_T}{\pi\sigma_T^2} \exp\left[-\frac{\mathbf{k}_T^2}{\sigma_T^2}\right] \mathcal{C}_{gg}^{\mathrm{DLA}}(z, (\mathbf{q}_T + \mathbf{k}_T)^2, \mu_F, \mu_R)$$



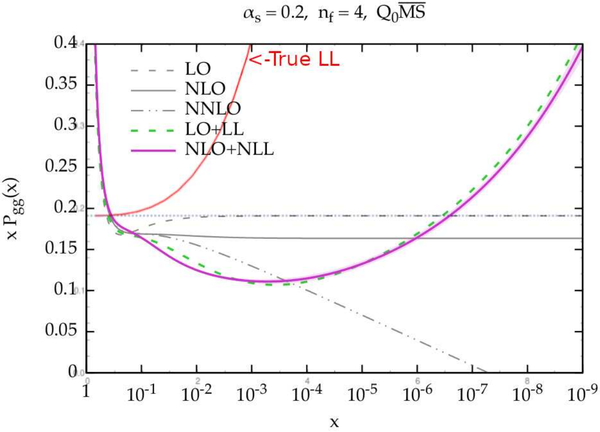


The correction is $O(\sigma_T^2/M^2)$, so it is **higher twist effect.**

Backup: DGLAP P_{gg} at small z

Plot from hep-ph/1607.02153 with my curve (in red) for LL

$\gamma_{gg}(N) = \frac{\hat{\alpha}_s}{N} + 2\zeta(3)\frac{\hat{\alpha}_s^4}{N^4} + 2\zeta(5)\frac{\hat{\alpha}_s^6}{N^6} + \ldots$



The "LO+LL" and "NLO+NLL" curves represent a form of matching between DGLAP and BFKL expansions, in a scheme by Altarelli, Ball and Forte.