Sum rules for triple parton distribution functions

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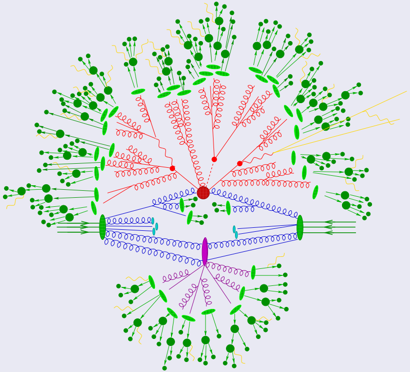
A composite nature of hadrons leads to very complicated event structure



Schematic representation of the event shape, 1411.4085

Introduction

- Composite nature of hadrons leads to multiple partonic interactions (MPI) in h-h collisions.
- Typically we consider double parton scattering (DPS) as a cleanest MPI process.
- However, with the increase of collision energy a probability to observe triple parton scattering (TPS) grows.
- For a long time TPS was a purely theoretical concept. However, the recent CMS measurements (CMS-PAS-BPH-21-004) suggest a non-negligible TPS contribution to triple J/ψ production![1]
- Therefore, in the future we will need all theoretical ingredients necessary for the TPS phenomenology.

[1] See also the talk of David d'Enterria.

Going from DPS to TPS

A master formula for DPS can be schematically written as

$$
\begin{aligned}
\sigma_{hh'}^{\mathrm{DPS}} &= \sum_{\text{partons}} \int \prod_{i=1}^{2} dx_i\, dx_i'\, d^2b \times \\
&\times \quad \Gamma_h\left(\{x_i\}, \mathbf{b}, \{Q_i\}\right) \Gamma_{h'}\left(\left\{x_i'\right\}, \mathbf{b}, \{Q_i\}\right) [\cdots],
\end{aligned}
$$

which for the TPS becomes

$$
\begin{aligned}
\sigma_{hh'}^{\mathrm{TPS}} &= \sum_{\text{partons}} \int \prod_{i=1}^{3} dx_i\, dx_i'\, d^2b_i\, d^2b \times \\
&\times \quad \Gamma_h\left(\{x_i\}, \mathbf{b}_i, \{Q_i\}\right) \Gamma_{h'}\left(\left\{x_i'\right\}, \{\mathbf{b}_i - \mathbf{b}\}, \{Q_i\}\right) [\cdots].
\end{aligned}
$$

A standard assumption on factorization into longitudinal and transverse parts reads
$\Gamma_h\left(\{x_i\}, \mathbf{b}_i, \{Q_i\}\right) \approx T_h\left(\{x_i\}, \{Q_i\}\right) \sum_i f(\mathbf{b}_i)$.

GS sum rules

The sum rules for dPDFs were proposed by Gaunt & Stirling some time ago

$$\sum_{j_2} \int_0^{1-x_1} dx_2\, x_2\, D_{j_1 j_2}(x_1, x_2, Q) = (1 - x_1)\, f_{j_1}(x_1, Q),$$

$$\int_0^{1-x_1} dx_2\, D_{j_1 j_{2v}}(x_1, x_2, Q) = \left(N_{j_{2v}} - \delta_{j_1 j_2} + \delta_{j_1 \bar{j}_2}\right) f_{j_1}(x_1, Q).$$

- The GS sum rules state conservation of momentum and describe changes in the number of valence partons after a DPS processes.
- Originally were proved using the Light-cone representation of dPDFs
- More rigorous proof was given later in 1811.00289

Let's start with momentum rule.

We get a chain of coupled equations:

$$\sum_{j_2} \int_0^{1-x_1} dx_2\, x_2\, D_{j_1 j_2}(x_1, x_2, Q) = (1 - x_1)\, f_{j_1}(x_1, Q),$$

$$\sum_{j_3} \int_0^{1-x_1-x_2} dx_3\ x_3\ T_{j_1 j_2 j_3}(x_1, x_2, x_3, Q) = (1 - x_1 - x_2) D_{j_1 j_2}(x_1, x_2, Q).$$

GS sum rules for tPDFs

The Light-cone formalism for "bare" PDFs implies

$$
\begin{aligned}
D_{j_1 j_2}(x_1, x_2) &= \sum_{N,\{\beta_i\}} \int [dz]_N \, [d^2 \ \boldsymbol{k}]_N \, |\Phi_N(\{\beta_i, z_i, \ \boldsymbol{k}_i\})|^2 \times \\
&\times \sum_i^N \delta(x_1 - z_i) \, \delta_{j_1 p_i} \sum_{k \neq i}^N \delta(x_2 - z_k) \, \delta_{j_2 p_k}, \\
T_{j_1 j_2 j_3}(x_1, x_2, x_3) &= \sum_{N,\{\beta_i\}} \int [dz]_N \, [d^2 \ \boldsymbol{k}]_N \, |\Phi_N(\{\beta_i, z_i, \ \boldsymbol{k}_i\})|^2 \times \\
&\times \sum_i^N \delta(x_1 - z_i) \, \delta_{j_1 p_i} \sum_{k \neq i}^N \delta(x_2 - z_k) \, \delta_{j_2 p_k} \sum_{l \neq i,k}^N \delta(x_3 - z_l) \, \delta_{j_3 p_l},
\end{aligned}
$$

where

$$
\begin{aligned}
[dz]_N &\equiv \prod_{i=1}^N dz_i \, \delta\left(1 - \sum_i^N z_1\right), \\
\left[d^2 \ \boldsymbol{k}\right]_N &\equiv \prod_{i=1}^N d^2 \ \boldsymbol{k}_i \, \delta^2\left(\sum_i^N \ \boldsymbol{k}_i\right).
\end{aligned}
$$

GS sum rules for tPDFs

Which can be written

$$\sum_{j_3} \int_0^{1-x_1-x_2} dx_3 \; x_3 \; T_{j_1 j_2 j_3}(x_1, x_2, x_3) = \sum_{N,\{\beta_i\}} \int [dz]_N \; [d^2 \; \boldsymbol{k}]_N \; |\Phi_N(\{\beta_i, z_i, \; \boldsymbol{k}_i\})|^2 \times$$

$$\times \sum_{j_3} \int_0^{1-x_1-x_2} dx_3 \sum_i^N \delta(x_1 - z_i) \; \delta_{j_1 p_i} \sum_{k \neq i}^N \delta(x_2 - z_k) \; \delta_{j_2 p_k} \sum_{l \neq i,k}^N \delta(x_3 - z_l) \; \delta_{j_3 p_l} =$$

$$= \sum_{N,\{\beta_i\}} \int [dz]_N \; [d^2 \; \boldsymbol{k}]_N \; |\Phi_N(\{\beta_i, z_i, \; \boldsymbol{k}_i\})|^2 \sum_i^N \delta(x_1 - z_i) \; \delta_{j_1 p_i} \times$$

$$\times \sum_{k \neq i}^N \delta(x_2 - z_k) \; \delta_{j_2 p_k} \sum_{j_3} \sum_{l \neq i,k}^N z_l \; \delta_{j_3 p_l}.$$

The last sum in equation above can be written as

$$\sum_{j_3} \sum_{l \neq i,k}^N z_l \; \delta_{j_3 p_l} = \sum_l^N z_l - z_i - z_k = 1 - x_1 - x_2,$$

which allows to recover expression for the momentum sum rule!

GS sum rules for tPDFs

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The number sum rule for "bare" tPDFs can be prooved in a similar way.

The sum rules for tPDFs are

$$
\begin{aligned}
\sum_{j_2} \int_0^{1-x_1} dx_2\, x_2\, D_{j_1 j_2}(x_1, x_2, Q) &= (1-x_1)\, f_{j_1}(x_1, Q), \\
\sum_{j_3} \int_0^{1-x_1-x_2} dx_3\;\, x_3\;\, T_{j_1 j_2 j_3}(x_1, x_2, x_3, Q) &= (1-x_1-x_2) D_{j_1 j_2}(x_1, x_2, Q), \\
\int_0^{1-x_1} dx_2\, D_{j_1 j_{2v}}(x_1, x_2, Q) &= \left(N_{j_{2v}} - \delta_{j_1 j_2} + \delta_{j_1 \bar{j}_2}\right) f_{j_1}(x_1, Q), \\
\int_0^{1-x_1-x_2} dx_3\;\, T_{j_1 j_2 j_{3v}}(x_1, x_2, x_3, Q) &= \left(N_{j_{3v}} - \delta_{j_3 j_1} - \delta_{j_3 j_2} + \delta_{\bar{j}_3 j_1} + \delta_{\bar{j}_3 j_2}\right) \times \\
&\times\;\; D_{j_1 j_2}(x_1, x_2, Q).
\end{aligned}
$$

How do we model tPDFs?

- The tPDFs are coupled to dPDFs which are unknown (initial conditions for the double DGALP evolution equations are unknown)
- One should use the sum rules as a guiding line to construct models of tPDFs
- One can construct tPDFs using PYTHIA8 model of MPI (essentially what PYTHIA8 does while generating underlying event!)

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According to the PYTHIA8 model

$$T_{j_1 j_2 j_3}(x_1, x_2, x_3, Q) = f^r_{j_1}(x_1, Q)\ f^{m \leftarrow j_1, x_1}_{j_2}(x_2, Q)\ f^{m \leftarrow j_1, x_1; j_2, x_2}_{j_3}(x_3, Q),$$
$$D_{j_1 j_2}(x_1, x_2, Q) = f^r_{j_1}(x_1, Q)\ f^{m \leftarrow j_1, x_1}_{j_2}(x_2, Q).$$

- To construct tPDFs one needs to access sPDFs used in PYTHIA8 at different generation stages.
- The tPDFs are constructed in a Monte Carlo way by taking an average over a large sample of “events”.
- The approach can be applied to construct nPDFs as well!

As a baseline we use “naive” approach to tPDFs and dPDFs

$$T_{j_1 j_2 j_3}(x_1, x_2, x_3, Q) = f^r_{j_1}(x_1, Q)\ f^r_{j_2}(x_2, Q)\ f^r_{j_3}(x_3, Q)\ \theta(1 - x_1 - x_2 - x_3).$$
$$D_{j_1 j_2}(x_1, x_2, Q) = f^r_{j_1}(x_1, Q)\ f^r_{j_2}(x_2, Q)\ \theta(1 - x_1 - x_2).$$

Phenomenology of tPDFs

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Let's check momentum rule first

x_1	x_2	j_1	j_2	PYTHIA tPDFs	"Naive" tPDFs
10^{-6}	10^{-4}	u	u	0.996	0.996
10^{-3}	10^{-4}	u	u	0.997	0.997
10^{-1}	10^{-4}	u	u	1.007	1.096
0.2	10^{-4}	u	u	1.008	1.195
0.4	10^{-4}	u	u	1.007	1.390
0.8	10^{-4}	u	u	1.002	1.626

Test of the momentum sum rule for the tPDFs.

Note that the factor $\theta(1 - x_1 - x_2 - x_3)$ in the definition of "naive" tPDFs does not imply that tPDFs obey the momentum sum rule!

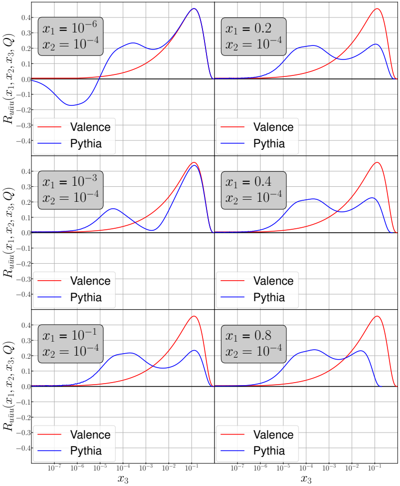
Now let's check number rule

We define

$$R_{j_1 j_2 j_3}(x_1, x_2, x_3, Q) \equiv x_3 \, \frac{T_{j_1 j_2 j_3}(x_1, x_2, x_3, Q) - T_{j_1 j_2 \bar{j}_3}(x_1, x_2, x_3, Q)}{D_{j_1 j_2}(x_1, x_2, Q)},$$

which can be seen as a response of the valence sPDF $f_{j_{3v}}(x_3, Q)$ to the first two interactions.

Check of the number rule



The responses of the valence u-quark sPDF $R_{u\bar{u}u}(x_1, x_2, x_3, Q)$ as function of x_3 for $x_1 \in [10^{-6}, 0.8]$ and $x_2 = 10^{-4}$. The response functions are averaged over 10^7 function calls.

Check of the number rule

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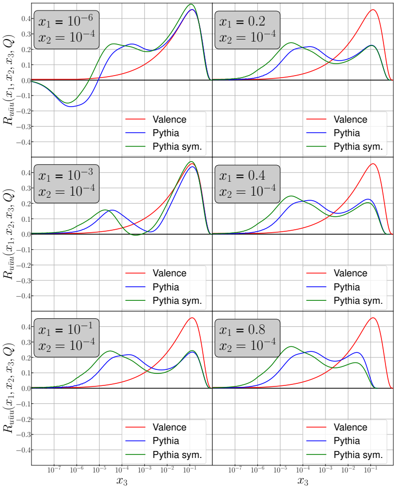
The numerical integration over the response function yields

x_1	x_2	N_{u_v} PYTHIA	N_{u_v} "Naive"
10^{-6}	10^{-4}	2.019	2.006
10^{-3}	10^{-4}	2.005	2.006
10^{-1}	10^{-4}	2.001	2.005
0.2	10^{-4}	2.000	2.005
0.4	10^{-4}	1.999	1.997
0.8	10^{-4}	1.995	1.708

Integration over $R_{u\bar{u}u}$ response function with respect to x_3 at fixed x_1, x_2.

- Similar checks can be made for other flavour combinations.
- PYTHIA8 tPDFs preserve the sum rules at about 1% accuracy level.
- PYTHIA8 tPDFs do not obey DGLAP evolution equation and are asymmetric.

Check of the number rule for symmetric tPDFs



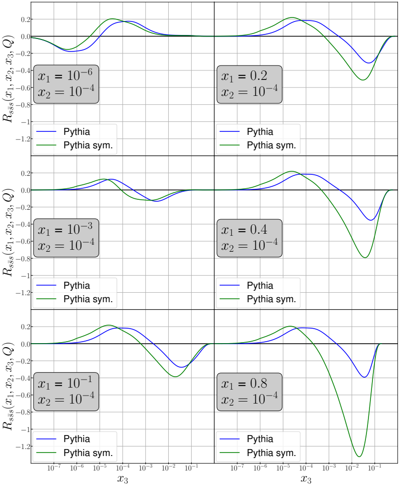
The responses of the valence u-quark sPDF $R_{u\bar{u}u}(x_1, x_2, x_3, Q)$ as functions of x_3 for $x_1 \in [10^{-6}, 0.8]$ and $x_2 = 10^{-4}$. The green lines are symmetrized PYTHIA8 tPDFs.

The numerical integration over the response function yields

x_1	x_2	N_{u_v} PYTHIA	N_{u_v} PYTHIA sym.	N_{u_v} "Naive"
10^{-6}	10^{-4}	2.019	2.542	2.006
10^{-3}	10^{-4}	2.005	2.154	2.006
10^{-1}	10^{-4}	2.001	2.188	2.005
0.2	10^{-4}	2.000	2.189	2.005
0.4	10^{-4}	1.999	2.161	1.997
0.8	10^{-4}	1.995	2.079	1.708

Integration over $R_{u\bar{u}u}$ response function with respect to x_3 at fixed x_1, x_2.

However, for some flavour combinations



The responses of the "valence" s-quark sPDF $R_{s\bar{s}s}(x_1, x_2, x_3, Q)$ as functions of x_3 for $x_1 \in [10^{-6}, 0.8]$ and $x_2 = 10^{-4}$. The green lines are symmetrized PYTHIA8 tPDFs.

Summary and possible next steps

- We generalized the GS sum rules for the case of tPDFs.
- The sketch of proof of sum rules for "bare" tPDFs is given within the Light-cone framework.
- We demonstrated how one can construct asymmetric tPDFs using PYTHIA8 code.
- Our attempt to construct symmetric tPDFs was not successful. However, the largest violations of the sum rules by symmetric tPDFs appear in the "deep valence region" ($x > 0.4$).
- Try to modify PYTHIA8 model to suppress large violations of the sum rules by symmetrized PYTHIA8 tPDFs.
- Give a rigorous proof as in 1811.00289.
- Make a phenomenological study with a tPDFs for different TPS production states. (e.g. to extend 4-jet DPS predictions of 2008.08347 to the TPS case using PYTHIA8 tPDFs).