Factorization for Quark and Gluon Lattice TMDs

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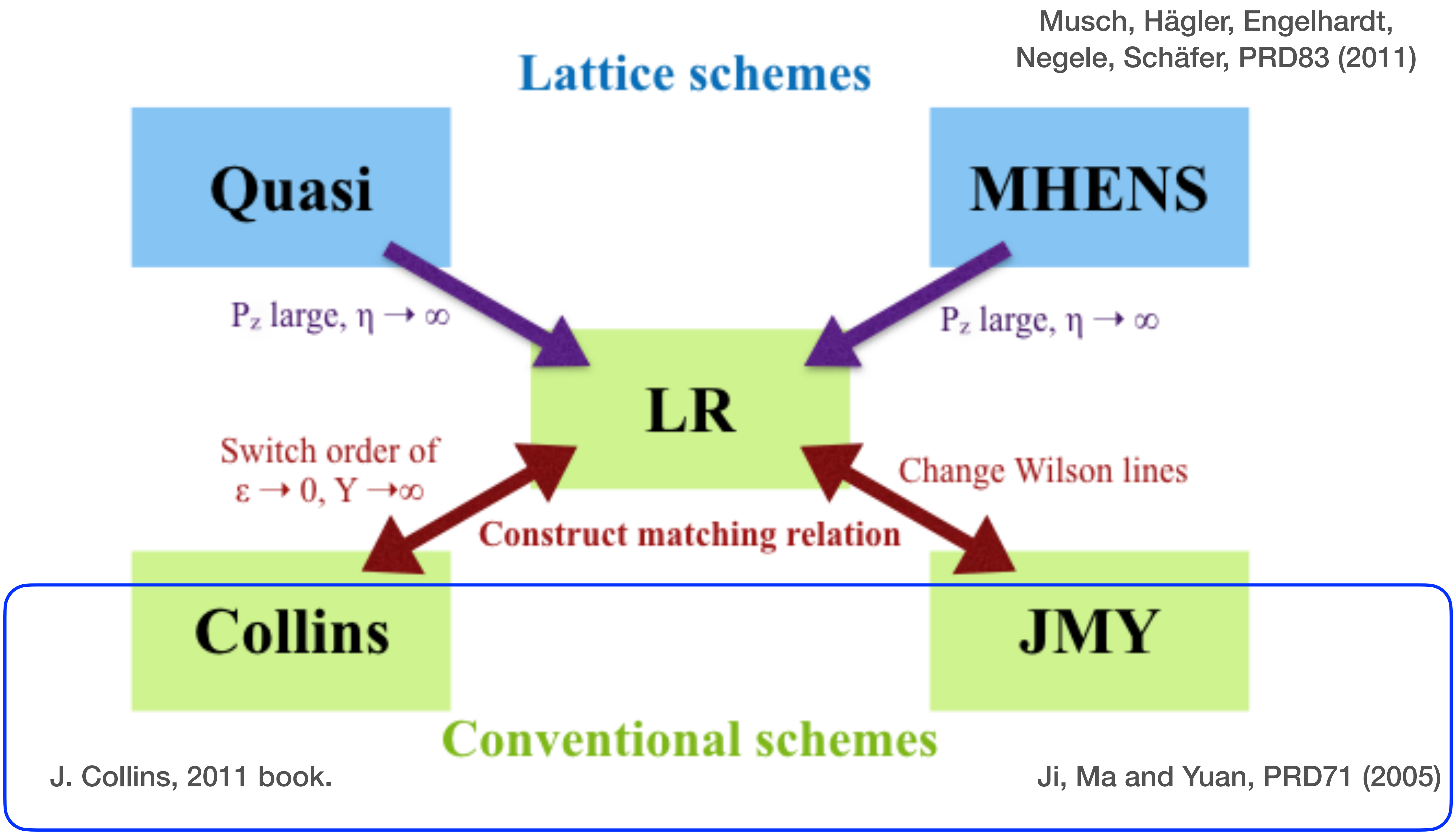
Argonne NATIONAL LABORATORY

In collaboration with M. Ebert, S. Schindler and I. Stewart, work in preparation.

Outline

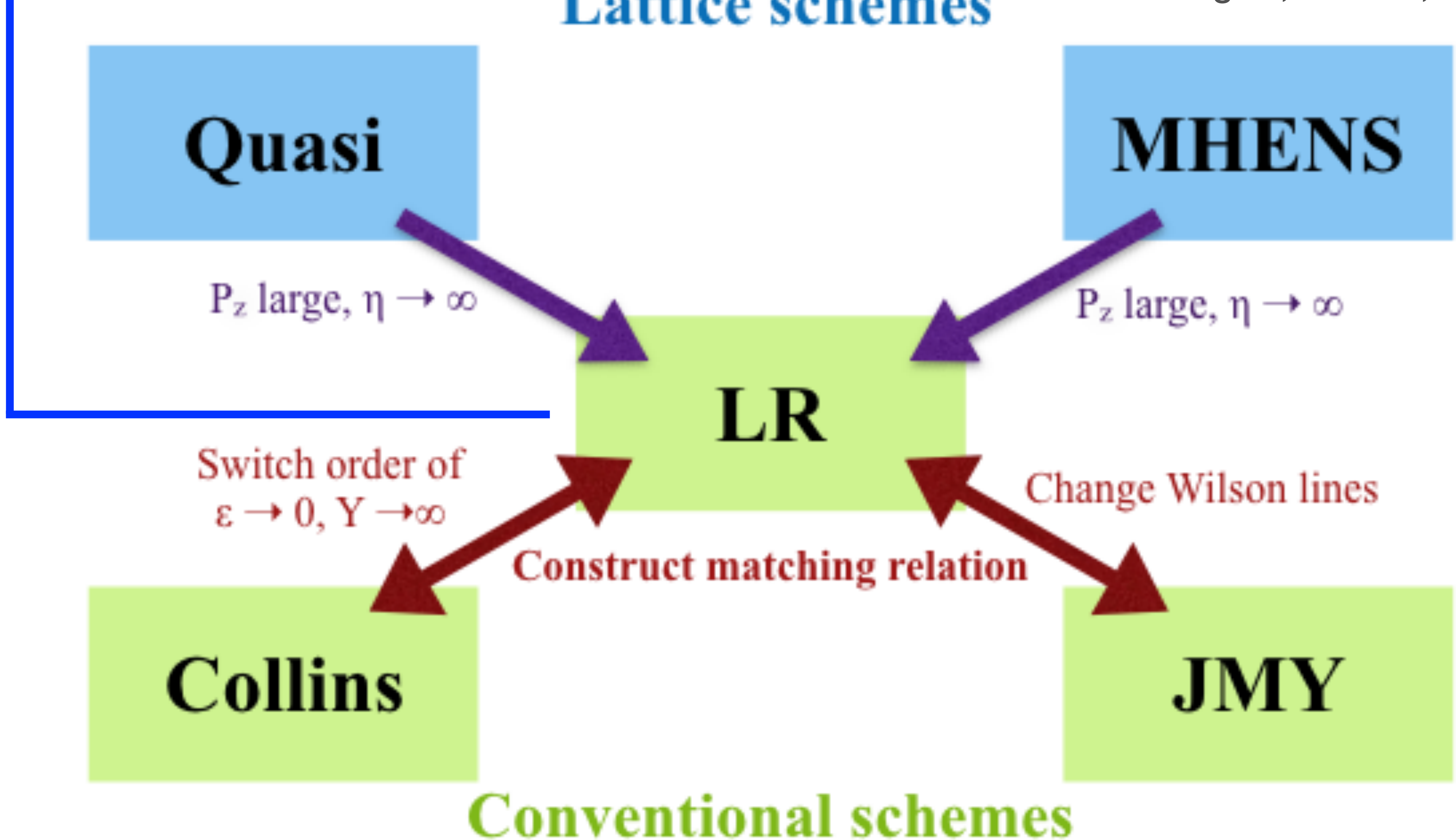


Off-the-light-cone schemes

Outline

Large-rapidity (LR) scheme, this work

Musch, Hägler, Engelhardt, Negele, Schäfer, PRD83 (2011)



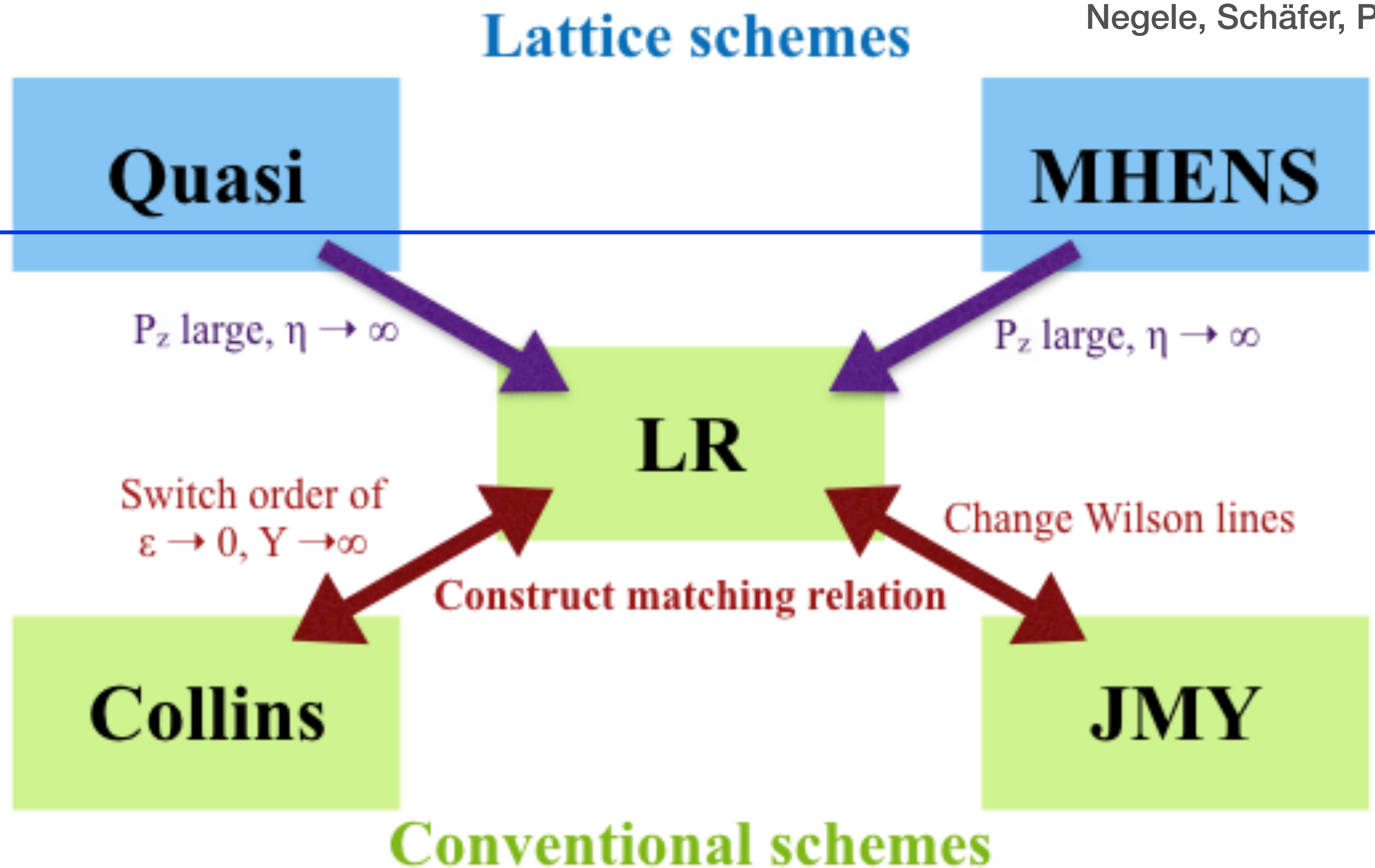
J. Collins, 2011 book.

Ji, Ma and Yuan, PRD71 (2005)

Outline

Lattice TMDs, in a large-momentum hadron state

Musch, Hägler, Engelhardt, Negele, Schäfer, PRD83 (2011)



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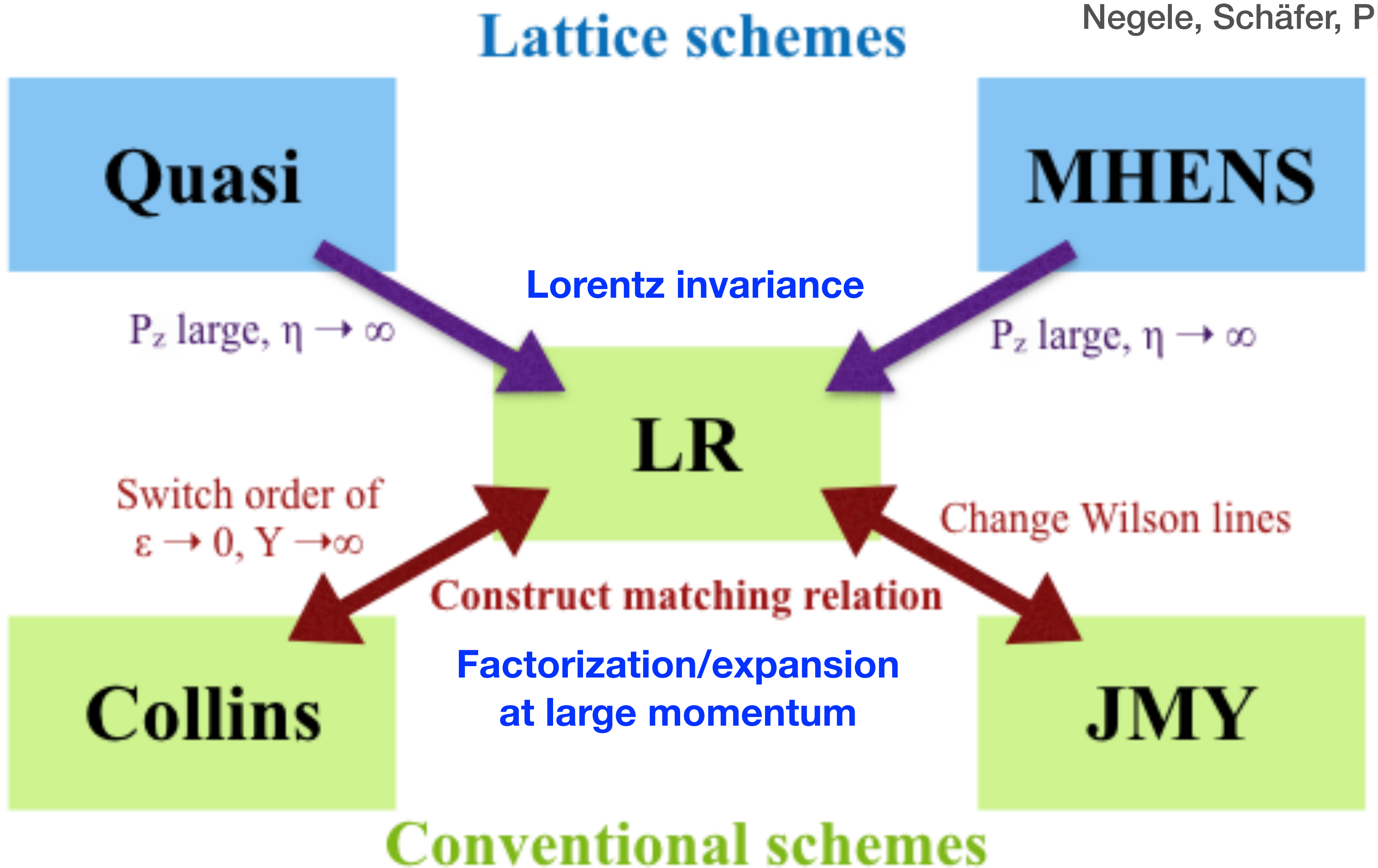
Outline

Musch, Hägler, Engelhardt, Negele, Schäfer, PRD83 (2011)



J. Collins, 2011 book.

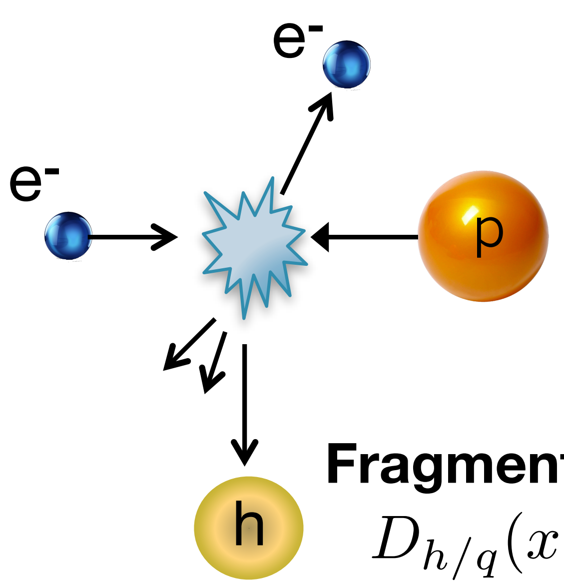
Ji, Ma and Yuan, PRD71 (2005)

Introduction to TMDs

- TMD processes:

Semi-Inclusive DIS

$$\sigma \sim f_{q/P}(x,k_T) D_{h/q}(x,k_T)$$

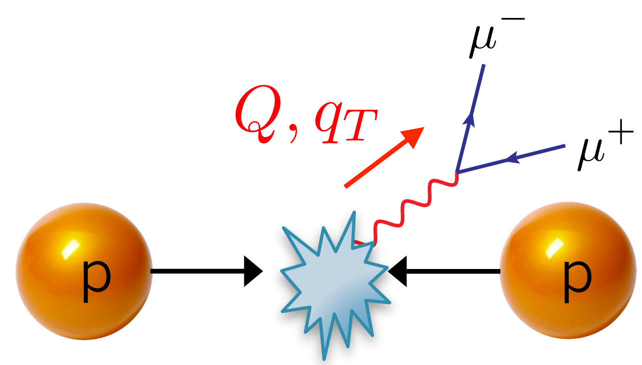


Fragmentation

$D_{h/q}(x,k_T)$

Drell-Yan

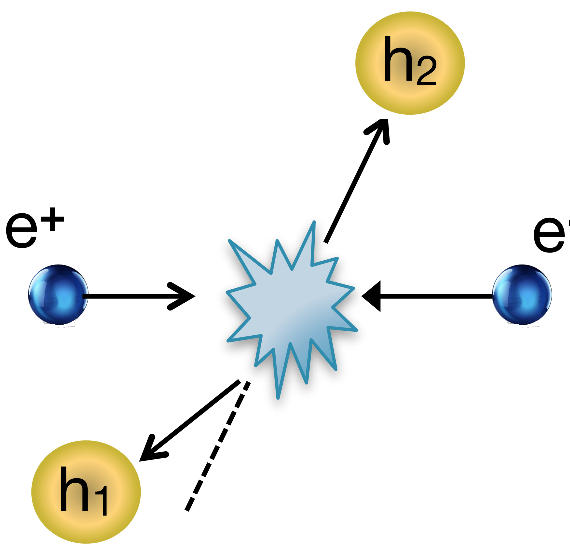
$$\sigma \sim f_{q/P}(x,k_T) f_{q/P}(x,k_T)$$



$$q_T \ll Q$$

Dihadron in e+e-

$$\sigma \sim D_{h_1/q}(x,k_T) D_{h_2/q}(x,k_T)$$

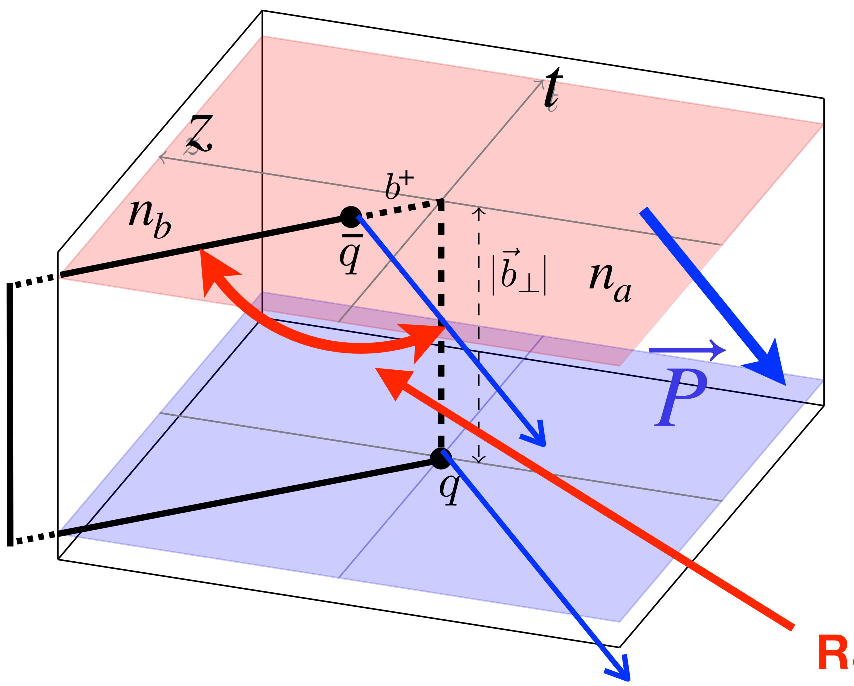


Factorization theorem (e.g., for Drell-Yan processes):

$$\frac{d\sigma_{\rm DY}}{dQdYd^2q_T} = \sum_{i,j} H_{ij}(Q,\cdots) \int \frac{d^2b_T}{(2\pi)^2} e^{i\vec{b}_T\cdot\vec{q}_T} f_{i/h_1}(x_1,\vec{b}_T,\cdots) f_{j/h_2}(x_2,\vec{b}_T,\cdots) \left[1+\mathcal{O}\left(\frac{q_T^2}{Q^2},\frac{\Lambda_{\rm QCD}^2}{Q^2}\right)\right]$$

Components of a TMD

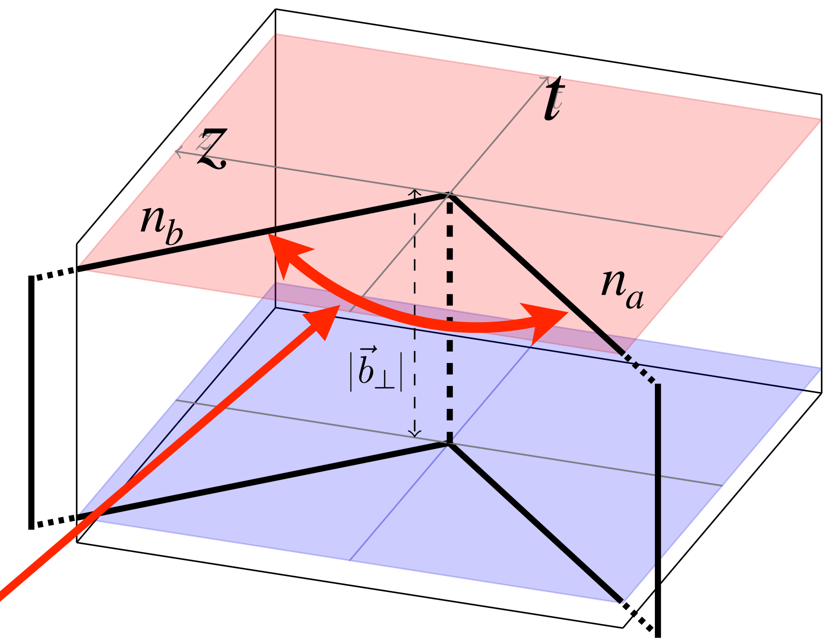
- Beam function :



$$n_a^\mu = \frac{1}{\sqrt{2}}(1,0,0,1)$$

$$n_b^\mu = \frac{1}{\sqrt{2}}(1,0,0,-1)$$

- Soft function :



Rapidity divergences

Scheme-independent

Rapidity divergence regulator

$$f_i^{\mathrm{TMD}}(x, \vec{b}_T, \mu, \zeta) = \lim_{\epsilon\to 0,\ \tau\to 0} Z_{\mathrm{UV}}(\epsilon, \mu, xP^+) B_i(x, \vec{b}_T, \epsilon, \tau, xP^+)\sqrt{S^i(b_T, \epsilon, \tau)}$$

UV divergence regulator

Collins-Soper scale: ζ

TMD definition

Many different schemes for TMD definition in the literature:

Wilson lines on the light-cone:

- Becher and Neubert, EPJC71 (2011);
- Echevarria, Idilbi and Scimemi, JHEP07 (2012), PLB726 (2013);
- Chiu, Jain, Neil and Rothstein, JHEP05 (2012), PRL108 (2012);
- Li, Neil and Zhu, Nucl.Phys.B 960 (2020) 115193;
- Ebert, Moult, Stewart, Tackman and Vita, JHEP 04 (2019) 123.

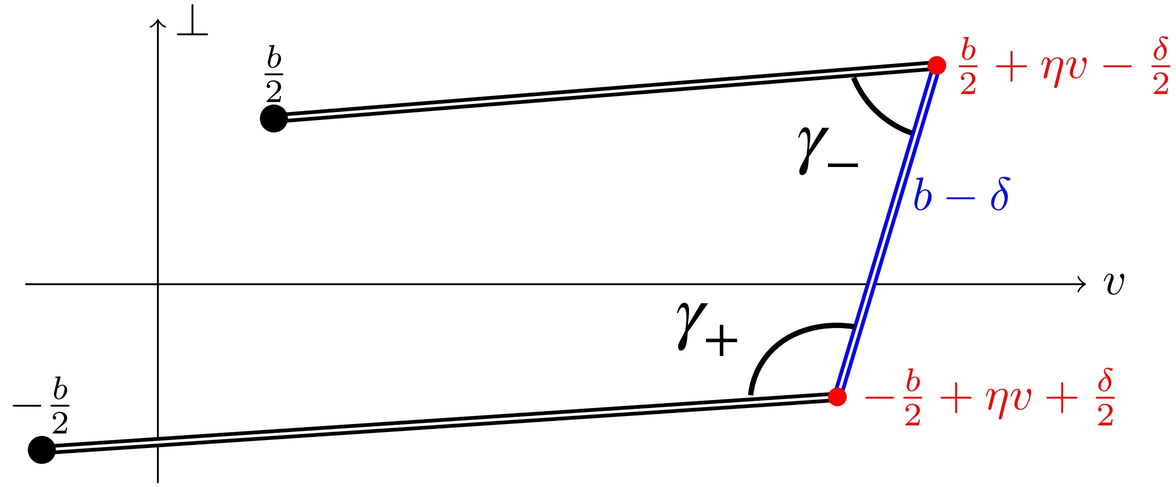
Wilson lines off the light-one:

- "Collins scheme", Collins, Soper and Sterman, NPB250 (1985); Collins, 2011;
- "JMY scheme", Ji, Ma and Yuan, PRD71 (2005) 034005.

TMD correlators

Generic staple-shaped Wilson line structures:

$$W^R_{\sqsupset}(b, \eta v, \delta) =$$



Cusp angle:

$$\cosh \gamma_{\pm} = \frac{(\eta v \pm \delta/2)\cdot(b-\delta)}{|\eta v \pm \delta/2||b-\delta|}$$

Beam functions:

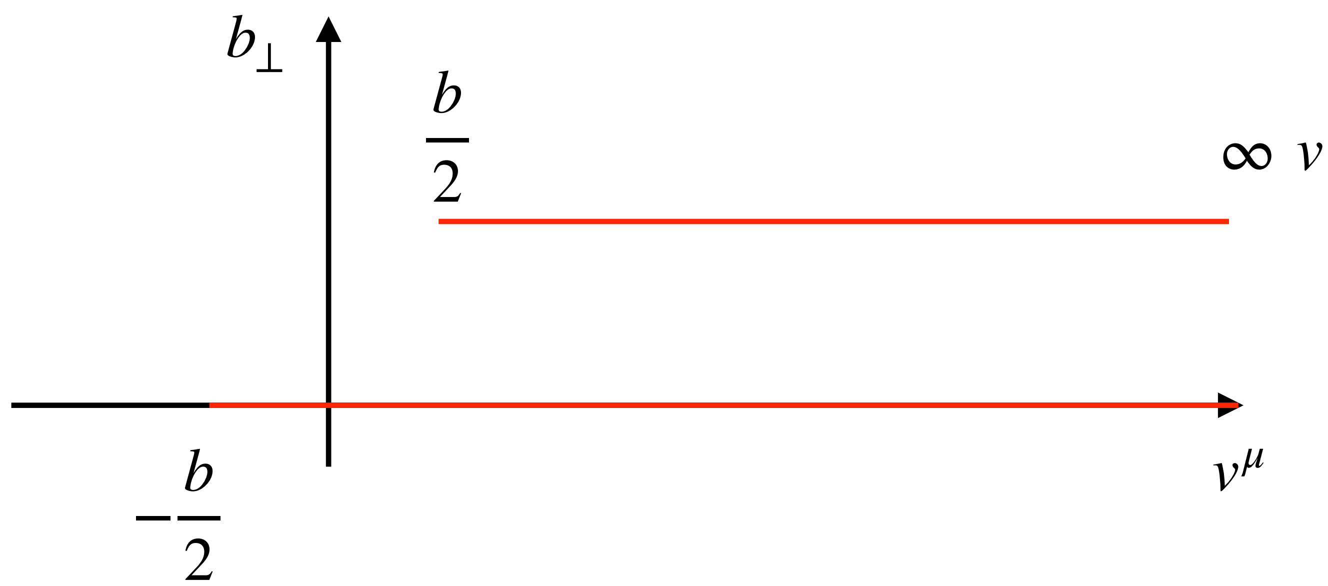
$$\Phi^{[\Gamma]}_{q/h}(b, P, \epsilon, \eta v, \delta) = \Big\langle h(P) \Big| \bar{q}\Big(\frac{b}{2}\Big)\frac{\Gamma}{2} W^F_{\sqsupset}(b, \eta v, \delta) q\Big(-\frac{b}{2}\Big) \Big| h(P) \Big\rangle \,,$$

$$\Phi^{\mu\nu\rho\sigma}_{g/h}(b, P, \epsilon, \eta v, \delta) = \Big\langle h(P) \Big| G^{\mu\nu}\Big(\frac{b}{2}\Big)\frac{\Gamma}{2} W^A_{\sqsupset}(b, \eta v, \delta) G^{\rho\sigma}\Big(-\frac{b}{2}\Big) \Big| h(P) \Big\rangle$$

Soft function:

$$S^R(b, \epsilon, \eta v, \bar{\eta}\bar{v}) = \frac{1}{d_R}\Big\langle 0 \Big| \mathrm{Tr}\Big[S^R_{\geqslant}(b, \eta v, \bar{\eta}\bar{v}) \Big] \Big| 0 \Big\rangle$$

Off-the-light-cone schemes (in continuum)



Schematic definition for all schemes: $b^\mu = b^- n_b^\mu + b_\perp^\mu = (0, b^-, b_\perp)$ $\delta^\mu = (0, b^-, 0_\perp)$

$$B_{q/h}(x, \vec{b}_T, \epsilon, \ldots) = \int \frac{db^-}{2\pi} e^{-ib^-(xP^+)} \Phi_{q/h}^{[\gamma^+]}\left[b, P, \epsilon, -\infty\ v, b^- n_b\right]$$

$$f_{i/h}(x, \vec{b}_T, \mu, \ldots) = \lim_{\substack{\epsilon \to 0 \\ v^2 \to 0}} Z_{\mathrm{UV}}^R(\epsilon, \mu, \ldots) \frac{B_{i/h}(x, \vec{b}_T, \epsilon, \ldots)}{\sqrt{S_C^R(b_T, \epsilon, \ldots)}}$$

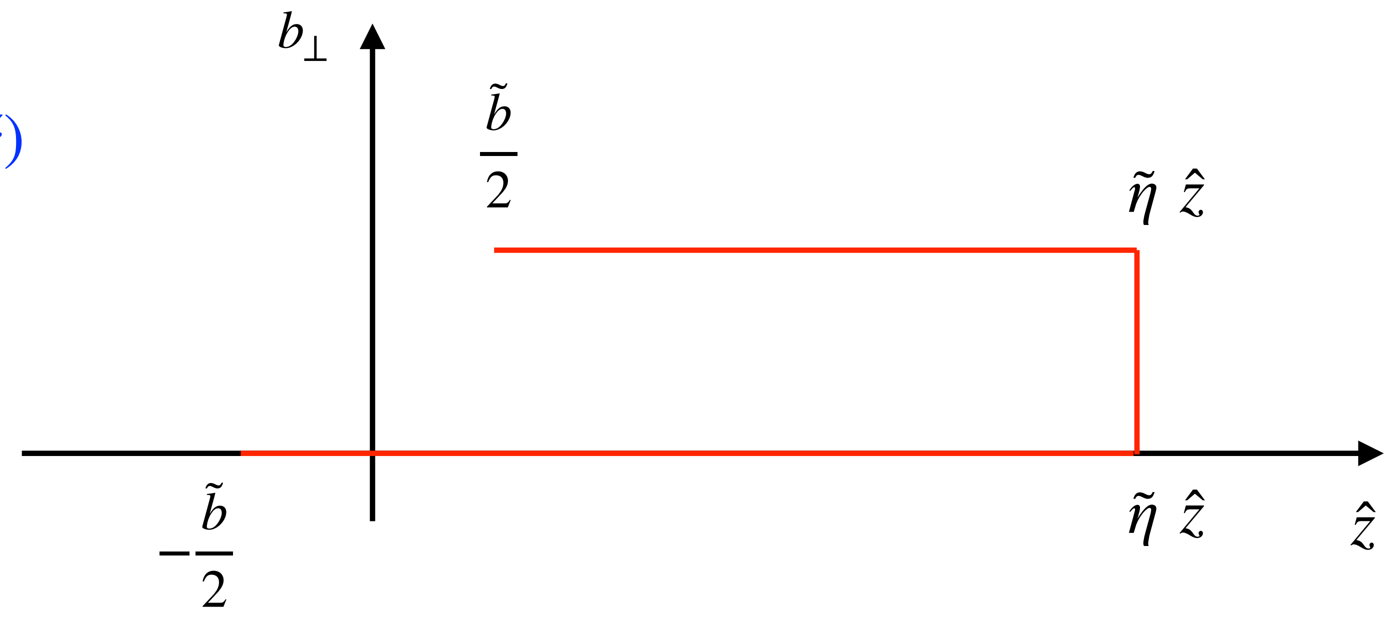
Off-the-light-cone schemes (in continuum)

	Collins	LR (new)	JMY
Beam function	$\Phi^{[\gamma^+]}_{q/h}\left[b, P, \epsilon, -\infty\ n_B(y_B), b^- n_b\right]$	$\Phi^{[\gamma^+]}_{q/h}\left[b, P, \epsilon, -\infty\ n_B(y_B), b^- n_b\right]$	$\Phi^{[\gamma^+]}_{q/h}[b, P, \mu, -\infty v, b^- n_b]$
Soft function	$S^R[b_\perp, \epsilon, -\infty n_A(y_A), -\infty n_B(y_B)]$	$S^R[b_\perp, \epsilon, -\infty n_A(y_A), -\infty n_B(y_B)]$	$S^R[b_\perp, \mu, -\infty v, -\infty \tilde{v}]$
Wilson line	$n_A^\mu(y_A) \equiv (1, e^{-2y_A}, 0_T)\ ,\ n_A^2 < 0$ $n_B^\mu(y_B) \equiv (e^{2y_B}, 1, 0_T), n_B^2 < 0$	$n_A^\mu(y_A) \equiv (1, e^{-2y_A}, 0_T)\ ,\ n_A^2 < 0$ $n_B^\mu(y_B) \equiv (e^{2y_B}, 1, 0_T), n_B^2 < 0$	$v^\mu = (v^+, v^-, 0_\perp)\ ,\ v^- \gg v^+ > 0$ $\tilde{v}^\mu = (\tilde{v}^+, \tilde{v}^-, 0_\perp)\ ,\ \tilde{v}^+ \gg \tilde{v}^- > 0$ $v^2 > 0\ ,\ \tilde{v}^2 > 0$
TMD	$\lim_{\epsilon\to 0} Z^R_{\rm UV} \lim_{y_B \to -\infty} \frac{B_{i/h}}{\sqrt{S^R}}$	$\lim_{-y_B \gg 1} \lim_{\epsilon\to 0} Z'^R_{\rm UV} \frac{B_{i/h}}{\sqrt{S^R}}$	$\lim_{\frac{v^-}{v^+} \gg 1} \lim_{\epsilon\to 0} Z'^R_{\rm UV} \frac{B_{i/h}}{\sqrt{S^R}}$

Quasi TMDs on the lattice

$$\tilde{b}^\mu = (0, b_T^x, b_T^y, \tilde{b}^z) \qquad \delta = \tilde{b}^z \hat{z} = (0,0,0,\tilde{b}^z)$$

No dependence on the real time, directly calculable in lattice QCD!



$$\tilde{B}_{q/h}^{[\tilde{\Gamma}]}(x, \vec{b}_T, a, \tilde{\eta}, x\tilde{P}^z) = N_{\tilde{\Gamma}} \int \frac{d\tilde{b}^z}{2\pi} e^{i\tilde{b}^z(x\tilde{P}^z)} \Phi_{q/h}^{[\tilde{\Gamma}]}(\tilde{b}, \tilde{P}, a, \tilde{\eta}\hat{z}, \tilde{b}^z\hat{z})$$

$$\tilde{S}^R(b_T, a, \tilde{\eta}, y_A, y_B) = S^R\left[b_\perp, a, -\tilde{\eta}\frac{n_A(y_A)}{|n_A(y_A)|}, -\tilde{\eta}\frac{n_B(y_B)}{|n_B(y_B)|}\right]$$

Quasi TMD:

$$\tilde{f}_{i/h}(x, \vec{b}_T, \mu, \tilde{\eta}, x\tilde{P}^z) = \lim_{a\to 0} Z_{\rm uv}(a,\mu) \frac{\tilde{B}_{i/h}(x, \vec{b}_T, a, \tilde{\eta}, x\tilde{P}^z)}{\sqrt{\tilde{S}^R(b_T, a, \tilde{\eta}, y_A, y_B)}}$$

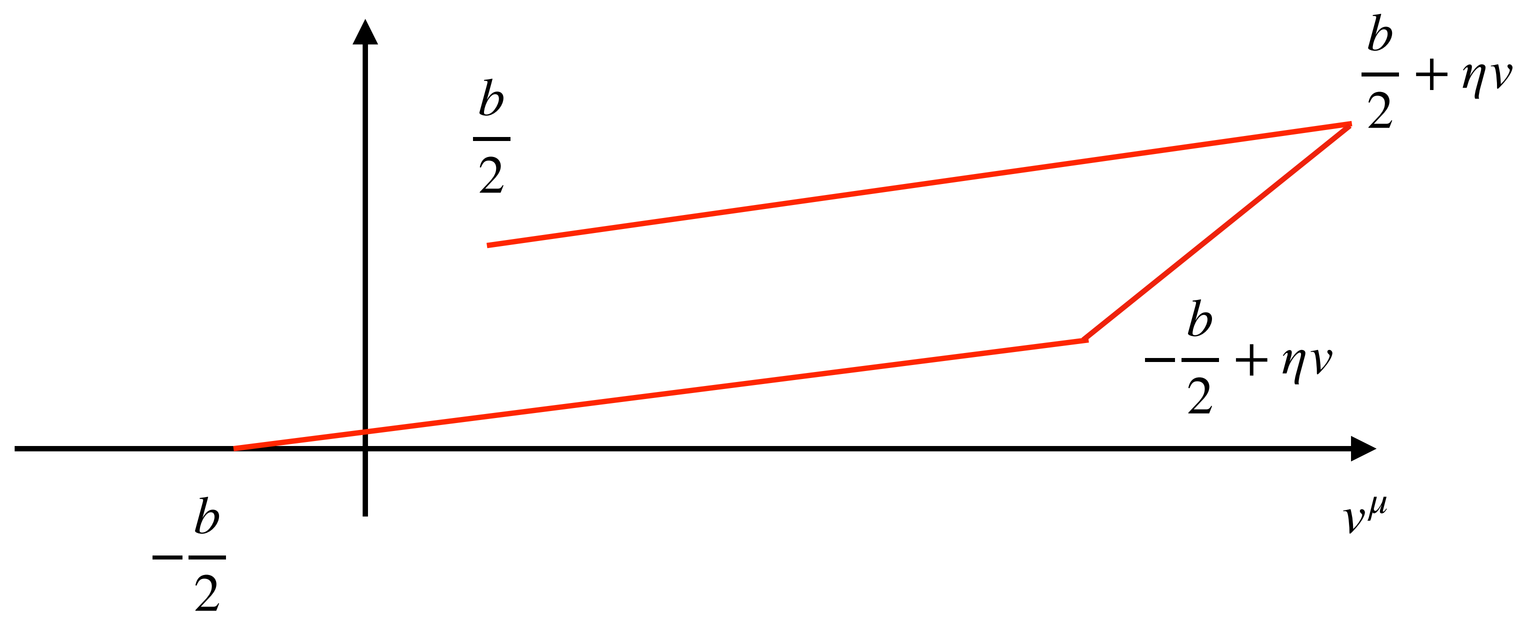
- Ji, Sun, Xiong and Yuan, PRD91 (2015);
- Ji, Jin, Yuan, Zhang and YZ, PRD99 (2019);
- M. Ebert, I. Stewart and YZ, PRD99 (2019), JHEP09 (2019) 037;
- Ji, Liu and Liu, Nucl.Phys.B 955 (2020), Phys.Lett.B 811 (2020);
- A. Vladimirov and A. Schäfer, Phys.Rev.D 101 (2020).

Lorentz-invariant approach: MHENS scheme

$b = (0, b^-, b_\perp)$ $\delta = 0$

Can always use Lorentz invariance of $P \cdot b$ to boost to a frame where

$\tilde{b}^\mu = (0, b_T^x, b_T^y, \tilde{b}^z)$



$$B_{q/h}^{\mathrm{MHENS}\,[\Gamma]}(x, \vec{b}_T, P, a, \eta, v) = \int \frac{db^-}{2\pi} e^{-ix(P\cdot b)} \Phi_{q/h}^{[\Gamma]}(b, P, a, \eta v, 0)\Big|_{b^+=0}$$

Ratios of beam functions can be calculated without the soft function.

Hägler, Musch, Engelhardt, Negele, Schäfer, et al., EPL88 (2009), PRD83 (2011), PRD85 (2012), PRD93 (2016), arXiv:1601.05717, PRD96 (2017)

Lorentz-invariant TMD variables

$$B_{q/h}(x, \overrightarrow{b}_T, \epsilon, \eta, \ldots) = \int \frac{db^-}{2\pi} e^{-ib^-(xP^+)} \Phi_{q/h}^{[\gamma^+]} = \int \frac{d(P\cdot b)}{2\pi} e^{-ix(P\cdot b)} \Phi_{q/h}(P\cdot b, b^2, \eta^2 v^2, \ldots)$$

$$\tilde{B}_{q/h}^{\tilde{\Gamma}}(x, \overrightarrow{b}_T, \epsilon, \eta, \ldots) = \int \frac{d\tilde{b}^z}{2\pi} e^{i\tilde{b}^z(x\tilde{P}^z)} \Phi_{q/h}^{[\tilde{\Gamma}]} = \int \frac{d(P\cdot b)}{2\pi} e^{-ix(P\cdot b)} \Phi_{q/h}(P\cdot b, b^2, \eta^2 v^2, \ldots)$$

- 10 Lorentz invariant scalars
- Reduces to 6 when $\delta = 0$ in the MHENS scheme

Lorentz-invariant TMD variables

	Collins	JMY	Quasi
b^2	$-b_T^2$	$-b_T^2$	$-b_T^2-(\tilde{b}^z)^2$
$(\eta v)^2$	$-2\eta^2 e^{2y_B}$	$2\eta^2(v^-)^2 e^{2y'_B}$	$-\tilde{\eta}^2$
$P\cdot b$	$\frac{M}{\sqrt{2}}b^- e^{y_P}$	$\frac{M}{\sqrt{2}}b^- e^{y_P}$	$-M\tilde{b}^z \sinh\tilde{y}_P$
$\frac{b\cdot(\eta v)}{\sqrt{\lvert(\eta v)^2 b^2\rvert}}$	$-\frac{b^- e^{y_B}}{\sqrt{2}\,b_T}\,\mathrm{sgn}(\eta)$	$\frac{b^- e^{y'_B}}{\sqrt{2}\,b_T}\,\mathrm{sgn}(\eta)$	$\frac{\tilde{b}^z}{\sqrt{(\tilde{b}^z)^2+b_T^2}}\,\mathrm{sgn}(\eta)$
$\frac{P\cdot(\eta v)}{\sqrt{P^2\lvert\eta v\rvert^2}}$	$\sinh(y_P-y_B)\,\mathrm{sgn}(\eta)$	$\cosh(y_P-y'_B)\,\mathrm{sgn}(\eta)$	$\sinh\tilde{y}_P\,\mathrm{sgn}(\eta)$
$\frac{\delta^2}{b^2}$	0	0	$\frac{(\tilde{b}^z)^2}{b_T^2+(\tilde{b}^z)^2}$
$\frac{b\cdot\delta}{b^2}$	0	0	$\frac{(\tilde{b}^z)^2}{b_T^2+(\tilde{b}^z)^2}$
$\frac{P\cdot\delta}{P\cdot b}$	1	1	1
$\frac{\delta\cdot(\eta v)}{b\cdot(\eta v)}$	1	1	1
P^2	M^2	M^2	M^2

Lorentz-invariant TMD variables

	Collins	JMY	Quasi
b^2	$-b_T^2$	$-b_T^2$	$-b_T^2 - (\tilde{b}^z)^2$
$(\eta v)^2$	$-2\eta^2 e^{2y_B}$	$2\eta^2 (v^-)^2 e^{2y'_B}$	$-\tilde{\eta}^2$
$P \cdot b$	$\frac{M}{\sqrt{2}} b^- e^{y_P}$	$\frac{M}{\sqrt{2}} b^- e^{y_P}$	$-M\tilde{b}^z \sinh \tilde{y}_P$
$\frac{b \cdot (\eta v)}{\sqrt{\lvert(\eta v)^2 b^2\rvert}}$	$-\frac{b^- e^{y_B}}{\sqrt{2}\, b_T} \operatorname{sgn}(\eta)$	$\frac{b^- e^{y'_B}}{\sqrt{2}\, b_T} \operatorname{sgn}(\eta)$	$\frac{\tilde{b}^z}{\sqrt{(\tilde{b}^z)^2 + b_T^2}} \operatorname{sgn}(\eta)$
$\frac{P \cdot (\eta v)}{\sqrt{P^2 \lvert\eta v\rvert^2}}$	$\sinh(y_P - y_B) \operatorname{sgn}(\eta)$	$\cosh(y_P - y'_B) \operatorname{sgn}(\eta)$	$\sinh \tilde{y}_P \operatorname{sgn}(\eta)$
$\frac{\delta^2}{b^2}$	0	0	$\frac{(\tilde{b}^z)^2}{b_T^2 + (\tilde{b}^z)^2}$
		0	$\frac{(\tilde{b}^z)^2}{b_T^2 + (\tilde{b}^z)^2}$
		1	1
		1	1
		M^2	M^2

$$\sinh(y_P - y_B) = \sinh \tilde{y}_P \Rightarrow \tilde{y}_P = y_P - y_B$$

$$P \cdot b = \frac{M}{\sqrt{2}} b^- e^{y_P} = -M\tilde{b}^z \sinh \tilde{y}_P$$

$$\Rightarrow \tilde{b}^z = -\sqrt{2} e^{y_B} b^- \xrightarrow{y_B \to -\infty} 0$$

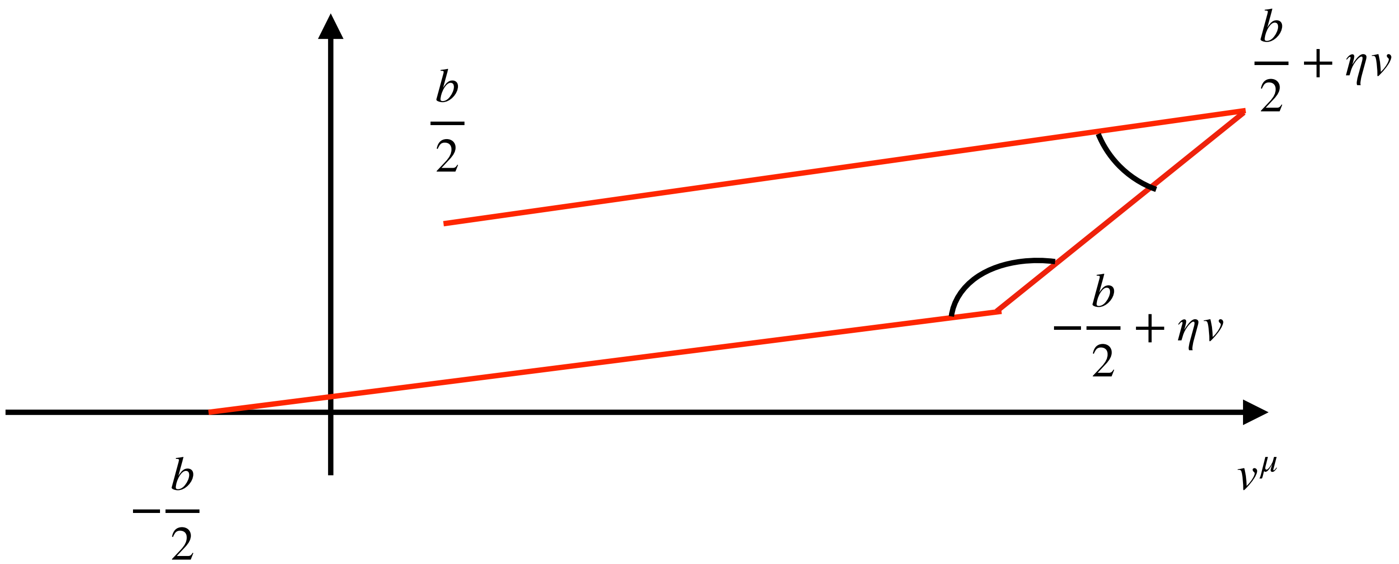
$$\tilde{b}^z \ll b_T$$

$$-2\eta^2 e^{2y_B} = -\tilde{\eta}^2 \Rightarrow \eta = \tilde{\eta} e^{-y_B}/\sqrt{2}$$

Lorentz-invariant TMD variables

	Collins	JMY	Quasi
b^2	$-b_T^2$	$-b_T^2$	$-b_T^2 - (\tilde{b}^z)^2$
$(\eta v)^2$	$-2\eta^2 e^{2y_B}$	$2\eta^2 (v^-)^2 e^{2y'_B}$	$-\tilde{\eta}^2$
$P \cdot b$	$\frac{M}{\sqrt{2}} b^- e^{y_P}$	$\frac{M}{\sqrt{2}} b^- e^{y_P}$	$-M\tilde{b}^z \sinh \tilde{y}_P$
$\frac{b \cdot (\eta v)}{\sqrt{\lvert(\eta v)^2 b^2\rvert}}$	$-\frac{b^- e^{y_B}}{\sqrt{2}\, b_T} \operatorname{sgn}(\eta)$	$\frac{b^- e^{y'_B}}{\sqrt{2}\, b_T} \operatorname{sgn}(\eta)$	$\frac{\tilde{b}^z}{\sqrt{(\tilde{b}^z)^2 + b_T^2}} \operatorname{sgn}(\eta)$
$\frac{P \cdot (\eta v)}{\sqrt{P^2 \lvert\eta v\rvert^2}}$	$\sinh(y_P - y_B) \operatorname{sgn}(\eta)$	$\cosh(y_P - y'_B) \operatorname{sgn}(\eta)$	$\sinh \tilde{y}_P \operatorname{sgn}(\eta)$
$\frac{\delta^2}{b^2}$	0	0	$\frac{(\tilde{b}^z)^2}{b_T^2 + (\tilde{b}^z)^2}$
$\frac{b \cdot \delta}{b^2}$	0	0	$\frac{(\tilde{b}^z)^2}{b_T^2 + (\tilde{b}^z)^2}$
$\frac{P \cdot \delta}{P \cdot b}$	1	1	1
$\frac{\delta \cdot (\eta v)}{b \cdot (\eta v)}$	1	1	1
P^2	M^2	M^2	M^2

The MHENS scheme



	Collins	JMY	Quasi	MHENS
$\frac{\delta^2}{b^2}$	0	0	$\frac{(\tilde{b}^z)^2}{b_T^2+(\tilde{b}^z)^2}$	1
$\frac{b\cdot\delta}{b^2}$	0	0	$\frac{(\tilde{b}^z)^2}{b_T^2+(\tilde{b}^z)^2}$	1

Additional challenges here beyond tree-level renormalization/matching:

- b^z-dependent renormalization

Linear divergence: $\propto \left(2|\eta v|+\sqrt{\tilde{b}_z^2+b_T^2}\right)/a$

Cusp divergence: $\propto \left[3-\frac{2\tilde{b}^z}{b_T}\tan^{-1}\frac{b_T}{\tilde{b}^z}\right]\ \ln(a)$

- b^z-dependent soft function?

Relating LR and quasi TMDs

$$\tilde{f}_{q/h}(x, \vec{b}_T, \mu, \tilde{\eta}, \zeta, x\tilde{P}^z) \equiv N_{\gamma^0} \int \frac{d\tilde{b}^z}{2\pi} e^{i\tilde{b}z(x\tilde{P}^z)} \lim_{\epsilon \to 0} Z_{\rm uv}(\epsilon, \ldots) \frac{\Phi_{q/h}^{[\gamma^0]}\left[\tilde{b}, \tilde{P}, \epsilon, \tilde{\eta}\hat{z}, \tilde{b}^z\hat{z}\right]}{\sqrt{S_C^R\left(b_T, \epsilon, \tilde{\eta}, 2y_n, 2y_B\right)}}$$

$$\zeta \equiv (2xP^+e^{-y_n})^2$$

$$= 4x^2M^2e^{2\tilde{y}_P+2y_B-2y_n}$$

$$= \int \frac{db^-}{2\pi} e^{-ib^-(xP^+)} \lim_{\epsilon \to 0} Z_{\rm uv}(\epsilon, \ldots) \frac{\Phi_{q/h}^{[\gamma^+]}\left[b, P, \epsilon, -\eta\ n_B(y_B), b^- n_b\right]}{\sqrt{S_C^R(b_T, \epsilon, \eta, 2y_n, 2y_B)}}$$

Therefore,

$$\lim_{\tilde{\eta} \to \infty} \tilde{f}_{q/h}(x, \vec{b}_T, \mu, \tilde{\eta}, \zeta, x\tilde{P}^z) = f_{i/h}^{\rm LR}(x, \vec{b}_T, \mu, \zeta, \tilde{y}_P = y_P - y_B)\ !$$

Relating LR and Collins TMDs

Collins scheme:

$$f^{C}_{i/h}(x, \vec{b}_T, \mu, \zeta) = \lim_{\epsilon\to 0} Z^R_{\rm UV} \lim_{y_B \to -\infty} \frac{B_{i/h}}{\sqrt{S^R}}$$

LR scheme:

$$f^{\rm LR}_{i/h}(x, \vec{b}_T, \mu, \zeta, y_P - y_B) = \lim_{-y_B \gg 1} \lim_{\epsilon\to 0} Z'^R_{\rm UV} \frac{B_{i/h}}{\sqrt{S^R}}$$

- Large $-y_B$ corresponds to a hard momentum scale $\zeta_C = 4x^2M^2 \sinh(y_P - y_B)$
- Exchange of $\epsilon \to 0$ and $\zeta_C \to \infty$ should not affect the infrared physics, so the difference between the orders of limits is compensated by perturbative matching !

$$f^{C}_{i/h}(x, \vec{b}_T, \mu, \zeta) = \mathscr{C}\left(\frac{\zeta_C}{\mu^2}\right) f^{\rm LR}_{i/h}(x, \vec{b}_T, \mu, \zeta, y_P - y_B) + \mathcal{O}(y_B^k e^{y_B})$$

Verified at 1-loop ✔

- Collins, 2011 book, Ch. 10;
- “Large Momentum Effective Theory”, Ji, PRL 110 (2013); SCPMA57 (2014); Ji, Liu, Liu, Zhang and YZ, RMP 93 (2021).

Matching between quasi and Collins TMDs

Since the quasi TMD is equivalent to the LR scheme, $C = \mathscr{C}^{-1}, \quad \tilde{\zeta}_z = (2x\tilde{P}^z)^2 = \zeta_C$

$$\lim_{\tilde{\eta}\to\infty} \tilde{f}_{q/h}(x, \vec{b}_T, \mu, \tilde{\eta}, \zeta, x\tilde{P}^z) = C\left(\frac{\tilde{\zeta}_z}{\mu^2}\right) f^C_{i/h}(x, \vec{b}_T, \mu, \zeta) + \mathcal{O}(\tilde{y}_P^{-k} e^{-\tilde{y}_P})$$

Moreover,

$$\tilde{f}_{q/h} = \frac{B_{q/h}(x, \vec{b}_T, \mu, \tilde{\eta}, x\tilde{P}^z)}{\sqrt{S_C^R(b_T, \mu, \tilde{\eta}, 2y_n, 2y_B)}}$$

Soft function not directly calculable on the lattice, but indirect methods may still be possible.

$$= \frac{B_{q/h}(x, \vec{b}_T, \mu, \tilde{\eta}, x\tilde{P}^z)}{\sqrt{S_C^R(b_T, \mu, \tilde{\eta}, 0, 0)}} \; \frac{\sqrt{S_C^R(b_T, \mu, \tilde{\eta}, 0, 0)}}{\sqrt{S_C^R(b_T, \mu, \tilde{\eta}, 2y_n, 2y_B)}}$$

Equal-time (naive) quasi soft function, lattice calculable!

$$\xrightarrow{\tilde{\eta}\to\infty, y_B\to-\infty} \frac{1}{g_S^q(b_T, \mu)} e^{-\frac{1}{2}\gamma_\zeta^q(b_T,\mu)\ln\frac{\tilde{\zeta}_z}{\zeta}}$$

Collins-Soper kernel

- **Reduced soft function:** $S_r(b_T, \mu) = [g_S^q(b_T, \mu)]^2$
- **Methods for calculation has been proposed and explored.**
  - Ji, Liu and Liu, Nucl.Phys.B 955 (2020);
  - Q.-A. Zhang, et al. (LP Collaboration), Phys.Rev.Lett. 125 (2020);
  - Y. Li et al., arXiv: 2106.13027.

Factorization formulas

① $$\lim_{\tilde{\eta}\to\infty} \tilde{f}_{q/h}(x, \vec{b}_T, \mu, \tilde{\eta}, \zeta, x\tilde{P}^z) = C\left(\frac{\tilde{\zeta}_z}{\mu^2}\right) f^C_{i/h}(x, \vec{b}_T, \mu, \zeta) + \mathcal{O}(\tilde{y}_P^{-k} e^{-\tilde{y}_P})$$

② $$\frac{1}{g_S^q(b_T,\mu)} \lim_{\tilde{\eta}\to 0} \frac{B_{q/h}\left(x, \vec{b}_T, \mu, \eta, x\tilde{P}^z\right)}{\sqrt{S_C^R\left(b_T, \mu, \tilde{\eta}, 0, 0\right)}}$$

Naive quasi TMD

$$= C\left(\frac{\tilde{\zeta}_z}{\mu^2}\right) e^{\frac{1}{2}\gamma_\zeta^q(b_T,\mu)\ln\frac{\tilde{\zeta}_z}{\zeta}} f^C_{q/h}(x, \vec{b}_T, \mu, \zeta) + \mathcal{O}(\tilde{y}_P^{-k} e^{-\tilde{y}_P})$$

↓

$$\mathcal{O}\left(\frac{b_T}{\tilde{\eta}}, \frac{1}{(xb_T\tilde{P}^z)^2}, \frac{1}{\tilde{P}^z\tilde{\eta}}, \frac{\Lambda^2_{\rm QCD}}{(x\tilde{P}^z)^2}\right)$$

- M. Ebert, I. Stewart and YZ, PRD99 (2019), JHEP09 (2019) 037;
- Ji, Liu and Liu, Nucl.Phys.B 955 (2020), Phys.Lett.B 811 (2020).
- A. Vladimirov and A. Schäfer, Phys.Rev.D 101 (2020).

Implications of factorization

$$\frac{1}{g_S^q(b_T,\mu)} \lim_{\tilde{\eta}\to 0} \frac{B_{q/h}\left(x, \overrightarrow{b}_T, \mu, \eta, x\tilde{P}^z\right)}{\sqrt{S_C^R\left(b_T, \mu, \tilde{\eta}, 0, 0\right)}}$$

$$= C\left(\frac{\tilde{\zeta}_z}{\mu^2}\right) e^{\frac{1}{2}\gamma_\zeta^q(b_T,\mu)\ln\frac{\tilde{\zeta}_z}{\zeta}} f_{q/h}^C(x, \overrightarrow{b}_T, \mu, \zeta) + \mathcal{O}(\tilde{y}_P^{-k} e^{-\tilde{y}_P})$$

- Same factorization formula can be derived for the gluon quasi TMD and for all spin-dependent quasi TMDs;
- The P^z-evolution provides information on the Collins-Soper kernel, which is diagonal in the parton flavor space;

See the talks by P. Shanahan and M. Schlemmer

- No mixing between quarks of different flavors, quark and gluon channels, or different spin structures.

Verified at 1-loop ✔

Ebert, Schindler, Stewart and YZ, JHEP 09 (2020).

Conclusion

- New large-rapidity (LR) scheme;
- The quasi TMD can be related to the LR scheme through Lorentz invariance;
- The LR and Collins schemes differ by the order of UV renormalization and light-cone limits, so we can perturbatively match them;
- We derive the factorization formula for both quark and gluon quasi TMDs using such relations
- There is no mixing between quarks of different flavors, quark and gluon channels, or different spin structures.

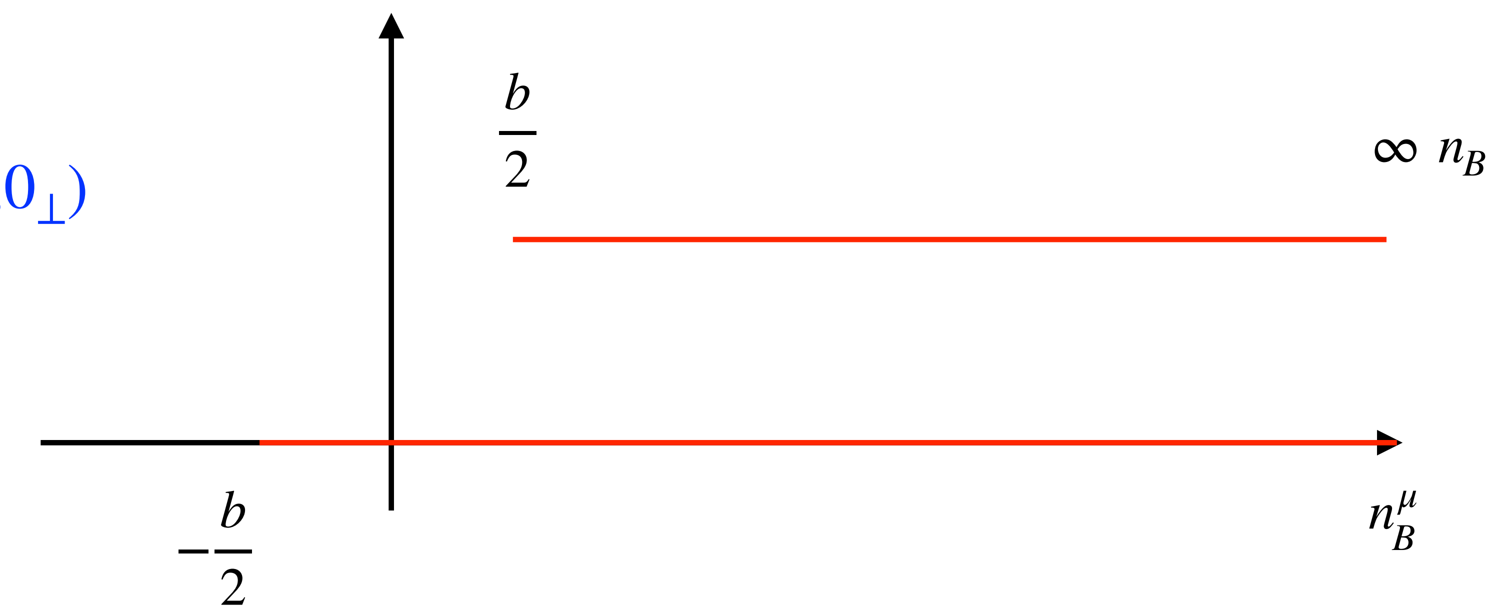
Collins scheme TMD

$b^\mu = (0, b^-, b_\perp)$ $\delta^\mu = (0, b^-, 0_\perp)$

$v^\mu = n_B^\mu(y_B)\,, \quad |\eta| \to \infty$

$n_A^\mu(y_A) \equiv n_a^\mu - e^{-2y_A} n_b^\mu = (1, e^{-2y_A}, 0_T)$

$n_B^\mu(y_B) \equiv n_b^\mu - e^{2y_B} n_a^\mu = (e^{2y_B}, 1, 0_T)$



$$B^C_{q/h}(x, \overrightarrow{b}_T, \epsilon, y_P - y_B) = \int \frac{db^-}{2\pi} e^{-ib^-(xP^+)} \Phi^{[\gamma^+]}_{q/h}\left[b, P, \epsilon, -\infty\ n_B(y_B), b^- n_b\right]$$

$$S^R_C(b_T, \epsilon, y_A, y_B) = S^R[b_\perp, \epsilon, -\infty n_A(y_A), -\infty n_B(y_B)]$$

TMDPDF:

$$f^C_{i/h}(x, \overrightarrow{b}_T, \mu, \zeta) = \lim_{\epsilon \to 0} Z^R_{\rm UV}(\epsilon, \mu, \zeta) \lim_{y_B \to -\infty} \frac{B^C_{i/h}(x, \overrightarrow{b}_T, \epsilon, y_P - y_B)}{\sqrt{S^R_C(b_T, \epsilon, 2y_n, 2y_B)}}$$

$\zeta = (2xP^+ e^{-y_n})^2$

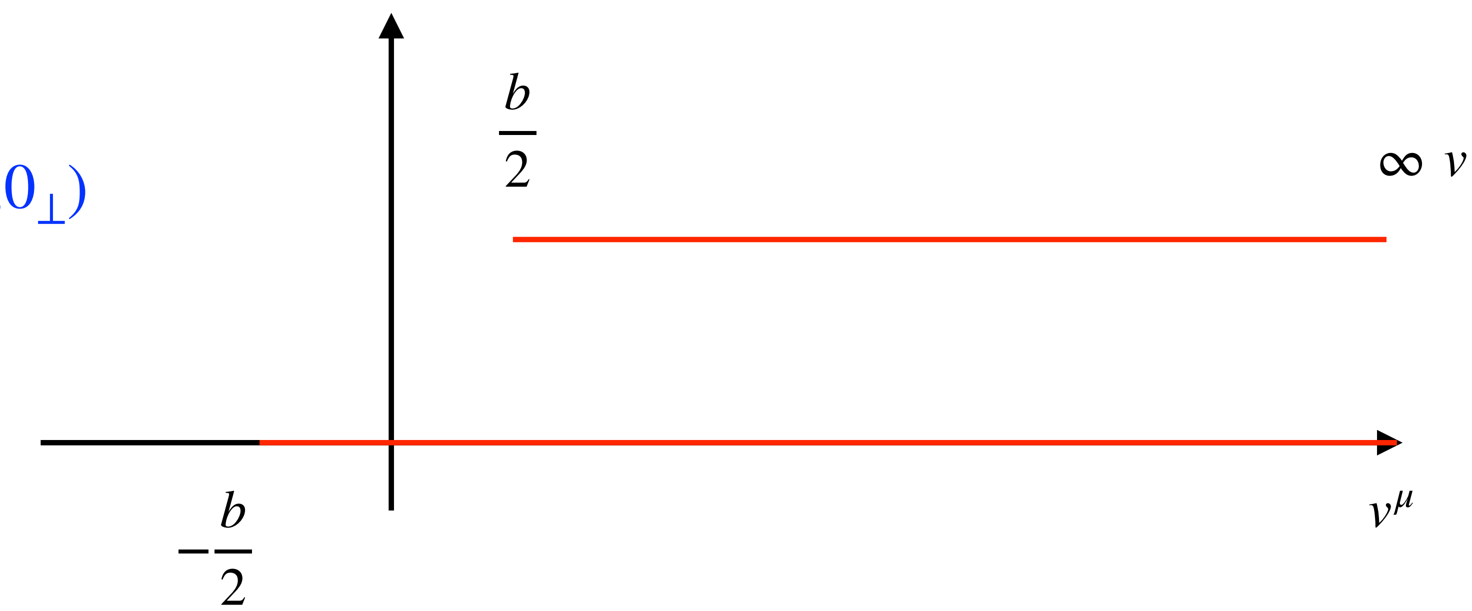
Ji-Ma-Yuan (JMY) scheme

$b^\mu = (0, b^-, b_\perp)$ $\delta^\mu = (0, b^-, 0_\perp)$

$v^\mu = (v^+, v^-, 0_\perp)\,,\quad v^- \gg v^+ > 0$

$\tilde{v}^\mu = (\tilde{v}^+, \tilde{v}^-, 0_\perp)\,,\quad \tilde{v}^+ \gg \tilde{v}^- > 0$

$|\eta| \to \infty$



$$B^{\mathrm{JMY}}_{q/h}(x, \vec{b}_T, \mu, \zeta_v) = \int \frac{db^-}{2\pi} e^{-ib^-(xP^+)} \Phi^{[\gamma^+]}_{q/h}[b, P, \mu, -\infty v, b^- n_b]$$

$$S^R_{\mathrm{JMY}}(b_T, \mu, y_A, y_B) = S^R[b_\perp, \mu, -\infty v, -\infty \tilde{v}]$$

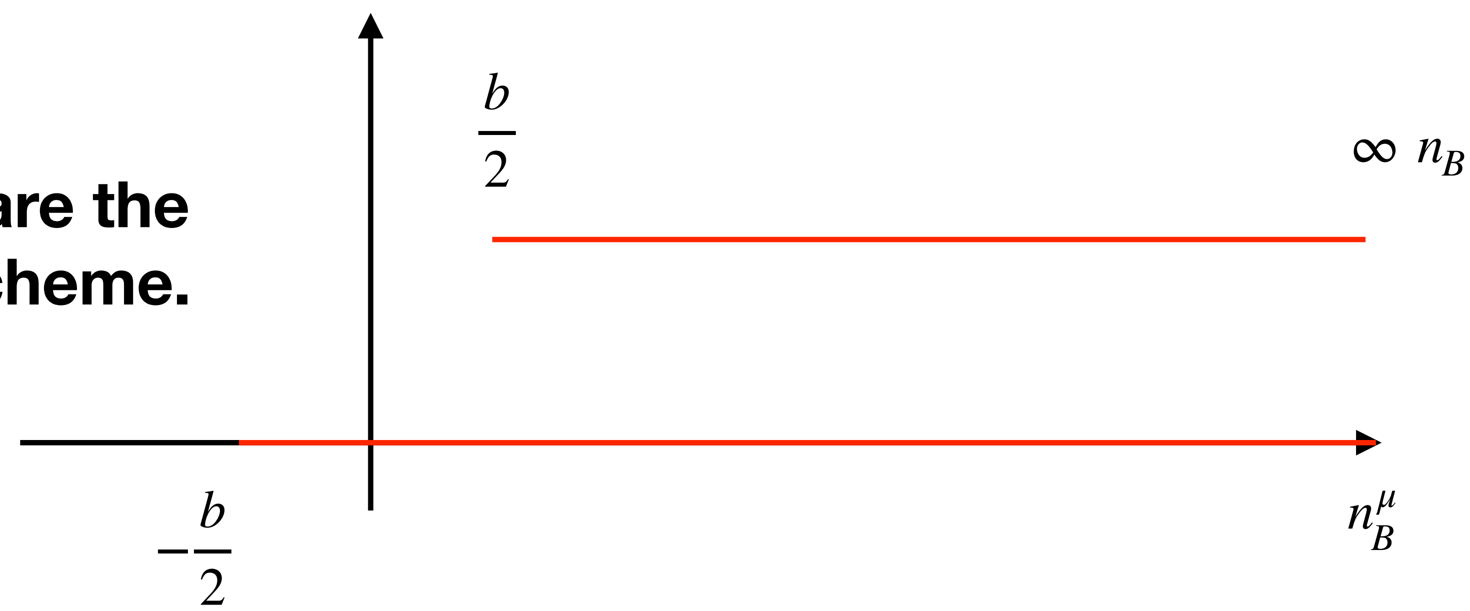
TMDPDF: $$f^{\mathrm{JMY}}_{i/h}(x, \vec{b}_T, \mu, \zeta_v, \rho) = \frac{B^{\mathrm{JMY}}_{i/h}(x, b_T, \mu, \zeta_v)}{\sqrt{S^R_{\mathrm{JMY}}(b_T, \mu, \rho)}}$$

$$\rho^2 = \frac{v^- \tilde{v}^+}{v^+ \tilde{v}^-}$$

$$\zeta_v^2 = \frac{(2P\cdot v)^2}{v^2} = 2(P^+)^2 \frac{v^-}{v^+}$$

New: Large-rapidity (LR) scheme TMD

Bare beam and soft functions are the same as those in the Collins scheme.



Collins scheme:

$$f^C_{i/h}(x, \vec{b}_T, \mu, \zeta) = \lim_{\epsilon\to 0} Z^R_{\rm UV}(\epsilon, \mu, \zeta) \lim_{y_B \to -\infty} \frac{B^C_{i/h}(x, \vec{b}_T, \epsilon, y_P - y_B)}{\sqrt{S^R_C(b_T, \epsilon, 2y_n, 2y_B)}}$$

$$\zeta = (2xP^+e^{-y_n})^2$$

LR scheme:

$$f^{\rm LR}_{i/h}(x, \vec{b}_T, \mu, \zeta, y_P - y_B) = \lim_{-y_B \gg 1} \lim_{\epsilon\to 0} Z^{\rm LR}_{\rm UV}(\epsilon, \mu, y_n - y_B) \frac{B^C_{i/h}(x, \vec{b}_T, \epsilon, y_P - y_B)}{\sqrt{S^R_C(b_T, \epsilon, 2y_n, 2y_B)}}$$

Analogous to the JMY scheme (y_P-y_B to ρ), except for the use of spacelike gauge links.