In-medium jet evolution via coherent medium induced radiation and scatterings

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based on:

[Blanco, Kutak, Płaczek, MR, Tywoniuk,arxiv: 2109.05918], [MR, arxiv: 2111.00323] (quark+gluon jets; Monte-Carlo)

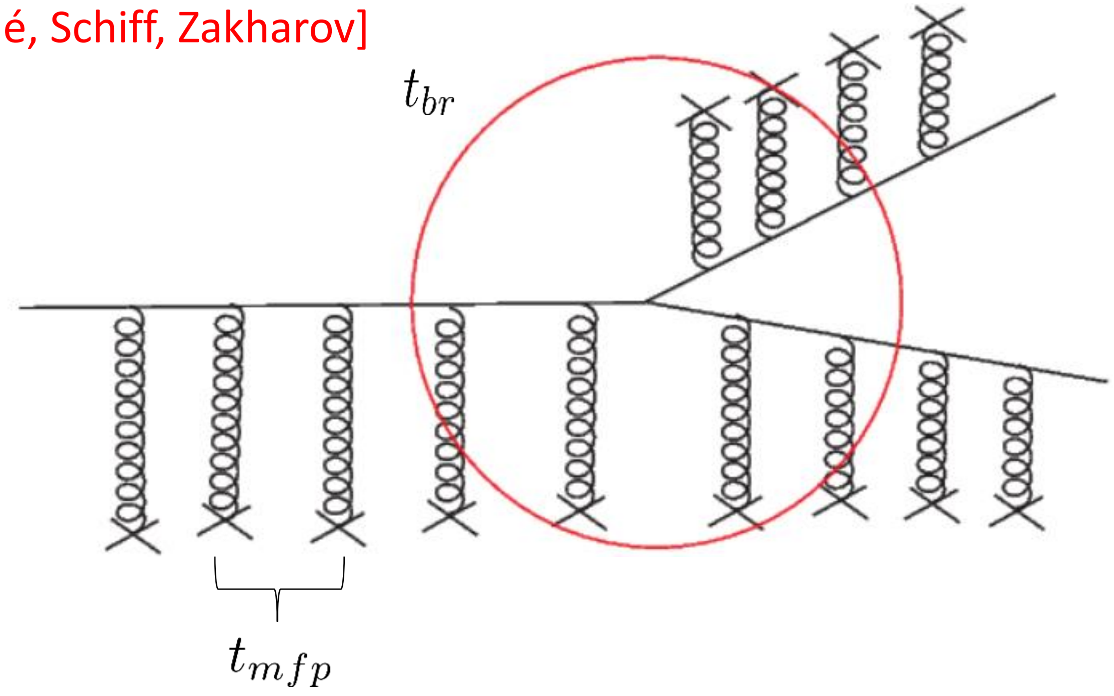
[Blanco, Kutak, Płaczek, MR, Straka, JHEP 04(2021)014] (k_T broadening in gluon jets)

[Kutak,Płaczek, Straka: Eur.Phys.J. C79 (2019) no.4, 317] (Monte Carlo for gluon fragmentation functions)

Coherent emission

...à la BDMPS-Z [Baier, Dokshitzer, Mueller, Peigné, Schiff, Zakharov]

$$t_{br} \sim \sqrt{\frac{2\omega}{\hat{q}}}$$



$t_{br} \sim t_{mfp}$: one scattering + radiation
...Bethe-Heitler spectrum

$t_{br} \gg t_{mfp}$: coherent radiation

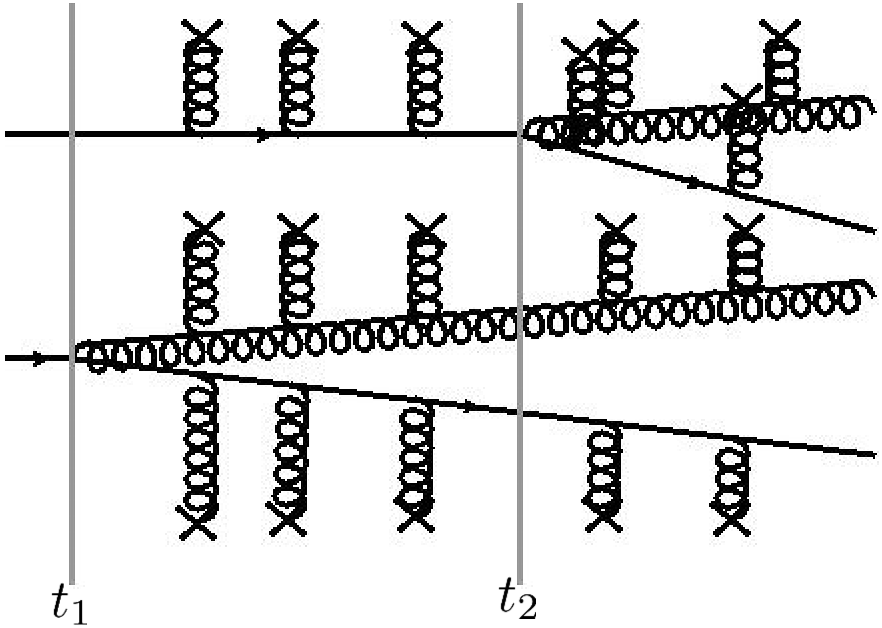
$$\omega \frac{dI}{d\omega} \sim \alpha_s \frac{L}{t_{br}} = \alpha_s \sqrt{\frac{\omega_c}{\omega}}$$

Look at range: $\omega_{BH} < \omega < \omega_c$

need effective kernel: $\mathcal{K}(z, k_T)$

cf. [Blaizot, Dominguez, Iancu, Mehtar-Tani: JHEP 1301 (2013) 143]

Effective Splitting Kernels



Assumptions:
Transverse momentum transfer only,
harmonic oscillator approximation,
static medium,
static scattering centers.

$$\int_{\mathbf{u}(t_1)=\mathbf{u}_1}^{\mathbf{u}(t_2)=\mathbf{u}_2} \mathcal{D}\boldsymbol{u}\, e^{i\frac{\omega_0}{2}\int_{t_1}^{t_2} ds\, \dot{\mathbf{u}}^2(s) - \int_{t_1}^{t_2} ds\, n(s)\sigma_{\mathrm{eff}}(\mathbf{u}(s),\boldsymbol{v})}$$

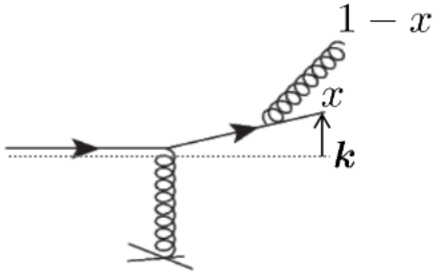
$$\boldsymbol{u} = \boldsymbol{r}_i - \boldsymbol{r}_k$$

$$\boldsymbol{v} = z\boldsymbol{r}_i + (1-z)\boldsymbol{r}_k - \boldsymbol{r}_j$$

$$\sigma_{\mathrm{eff}}(\boldsymbol{r}_0, \boldsymbol{r}_1, \boldsymbol{r}_2) = \frac{C_i + C_k - C_j}{2}\,\bar{\sigma}(\boldsymbol{r}_k - \boldsymbol{r}_i) + \frac{C_i + C_j - C_k}{2}\,\bar{\sigma}(\boldsymbol{r}_i - \boldsymbol{r}_j) + \frac{C_k + C_j - C_i}{2}\,\bar{\sigma}(\boldsymbol{r}_k - \boldsymbol{r}_j)$$

BDIM Equation for Gluons

[Blaizot, Dominguez, Iancu, Mehtar-Tani: JHEP 1406 (2014) 075]



Momentum distribution:
$p \to xp$
Momentum transfer:
$p \to p + \mathbf{k}$

→ Generalizes BDMPS-Z approach

→ Includes transverse momentum broadening

For gluon-jets:

$$\frac{\partial}{\partial t} D(x,\mathbf{k},t) = \alpha_s \int_0^1 dz \int \frac{d^2q}{(2\pi)^2} \left[2\mathcal{K}(\mathbf{Q}, z, \frac{x}{z} p_0^+) D\left(\frac{x}{z}, \mathbf{q}, t\right) - \mathcal{K}(\mathbf{q}, z, x p_0^+) D(x,\mathbf{k},t) \right] + \int \frac{d^2\mathbf{l}}{(2\pi)^2} C(\mathbf{l}) D(x, \mathbf{k}-\mathbf{l}, t).$$

Induced Radiation:

$$\mathcal{K}(\mathbf{Q}, z, p_0^+) = \frac{2}{p_0^+} \frac{P_{gg}(z)}{z(1-z)} \sin\left[\frac{\mathbf{Q}^2}{2k_{br}^2}\right] \exp\left[-\frac{\mathbf{Q}^2}{2k_{br}^2}\right]$$

$$\omega = x p_0^+, \; k_{br}^2 = \sqrt{\omega_0 \hat{q}_0}, \; \mathbf{Q} = \mathbf{k} - z\mathbf{q}, \; \omega_0 = z(1-z) p_0^+$$

$$\hat{q}_0 = \hat{q} f(z), \; f(z) = 1 - z(1-z), \; P_{gg}(z) = N_c \frac{[1 - z(1-z)]^2}{z(1-z)}$$

Scattering:

$$C(\boldsymbol{q}) = w(\boldsymbol{q}) - \delta(\boldsymbol{q}) \int d^2\boldsymbol{q}' w(\boldsymbol{q}')$$

BDIM Equation for Gluons

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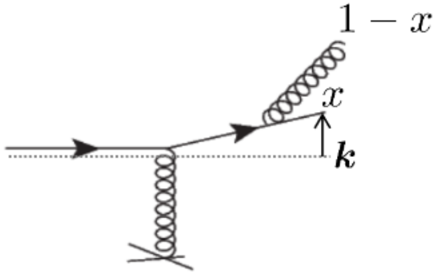
Generalizes BDMPS-Z approach



Momentum distribution:
$p \to xp$
Momentum transfer:
$p \to p + \mathbf{k}$

k_T averaged Kernel:

$$\int_0^\infty d^2\mathbf{Q}\mathcal{K}(z,\mathbf{Q},p_0^+) = 2\pi\sqrt{\frac{\hat{q}}{p_0^+}}N_c\mathcal{K}(z) \longrightarrow \mathcal{K}(z) = \frac{(1-z+z^2)^{\frac{5}{2}}}{[z(1-z)]^{\frac{3}{2}}}$$

$$\frac{\partial}{\partial t}D(x,\boldsymbol{k},t) = \frac{1}{t^*}\int_0^1 dz\mathcal{K}(z)\left[\frac{1}{z^2}\sqrt{\frac{z}{x}}D\left(\frac{x}{z},\frac{\boldsymbol{k}}{z},t\right)\theta(z-x) - \frac{z}{\sqrt{x}}D(x,\boldsymbol{k},t)\right] + \int\frac{d^2\boldsymbol{q}}{(2\pi)^2}C(\boldsymbol{q})D(x,\boldsymbol{k}-\boldsymbol{q},t)$$

Splitting:

$$\mathcal{K}(z) = \frac{(1-z+z^2)^{\frac{5}{2}}}{[z(1-z)]^{\frac{3}{2}}}$$

$$\frac{1}{t^*} = \frac{\alpha_s N_c}{\pi}\sqrt{\frac{\hat{q}}{p_0^+}}$$

$$\sqrt{x}t^* \propto t_{\mathrm{br}}$$

Integrate over $\mathbf{k}$

$$\frac{\partial}{\partial t}D(x,t) = \frac{1}{t^*}\int_0^1 dz\mathcal{K}(z)\left[\sqrt{\frac{z}{x}}D\left(\frac{x}{z},t\right) - \frac{z}{\sqrt{x}}D(x,t)\right]$$

$$D(x,t) = \int d^2\mathbf{k}D(x,\mathbf{k},t)$$

Scattering:

$$C(\boldsymbol{q}) = w(\boldsymbol{q}) - \delta(\boldsymbol{q})\int d^2\boldsymbol{q}'w(\boldsymbol{q}')$$

$$w(\boldsymbol{q}) = \frac{16\pi^2\alpha_s^2 N_c n}{\boldsymbol{q}^2(\boldsymbol{q}^2+m_D^2)}$$

Kernels for Quarks and Gluons

[Blanco, Kutak, Płaczek, MR, Tywoniuk,arxiv: 2109.05918]

$$\mathcal{K}_{ij}(\boldsymbol{Q}, z, p_0^+) = \frac{2P_{ij}(z)}{z(1-z)p_0^+} \sin\left(\frac{\boldsymbol{Q}^2}{2k_{\mathrm{br}}^2}\right) \exp\left(-\frac{\boldsymbol{Q}^2}{2k_{\mathrm{br}}^2}\right)$$

$$k_{\mathrm{br}}^2 = \sqrt{z(1-z)p_0^+ f_{ij}(z)\hat{\hat{q}}}$$

$$f_{gg}(z) = (1-z)C_A + z^2 C_A$$
$$f_{qg}(z) = C_F - z(1-z)C_A\,,$$
$$f_{gq}(z) = (1-z)C_A + z^2 C_F$$
$$f_{qq}(z) = zC_A + (1-z)^2 C_F$$

$$w_q(\mathbf{q}) = \frac{C_F}{C_A} w_g(\mathbf{q})$$

System of Equations for quarks and gluons

[Blanco, Kutak, Płaczek, MR, Tywoniuk,arxiv: 2109.05918]

$$\frac{\partial}{\partial t} D_g(x,\boldsymbol{k},t) = \int_0^1 \mathrm{d}z \int \frac{\mathrm{d}^2\boldsymbol{q}}{(2\pi)^2} \alpha_s \Big\{ 2\mathcal{K}_{gg}\left(\boldsymbol{Q}, z, \frac{x}{z}p_0^+\right) D_g\left(\frac{x}{z}, \boldsymbol{q}, t\right) + \mathcal{K}_{gq}\left(\boldsymbol{Q}, z, \frac{x}{z}p_0^+\right) \sum_i D_{q_i}\left(\frac{x}{z}, \boldsymbol{q}, t\right)$$
$$- \left[\mathcal{K}_{gg}(\boldsymbol{q}, z, xp_0^+) + \mathcal{K}_{qg}(\boldsymbol{q}, z, xp_0^+)\right] D_g(x,\boldsymbol{k},t) \Big\} + \int \frac{\mathrm{d}^2\boldsymbol{l}}{(2\pi)^2} C_g(\boldsymbol{l})\, D_g(x, \boldsymbol{k}-\boldsymbol{l}, t),$$

$$\frac{\partial}{\partial t} D_{q_i}(x,\boldsymbol{k},t) = \int_0^1 \mathrm{d}z \int \frac{\mathrm{d}^2\boldsymbol{q}}{(2\pi)^2} \alpha_s \Big\{ \mathcal{K}_{qq}\left(\boldsymbol{Q}, z, \frac{x}{z}p_0^+\right) D_{q_i}\left(\frac{x}{z}, \boldsymbol{q}, t\right) + \frac{1}{N_F}\mathcal{K}_{qg}\left(\boldsymbol{Q}, z, \frac{x}{z}p_0^+\right) D_g\left(\frac{x}{z}, \boldsymbol{q}, t\right)$$
$$- \mathcal{K}_{qq}(\boldsymbol{q}, z, xp_0^+)\, D_{q_i}(x,\boldsymbol{k},t) \Big\} + \int \frac{\mathrm{d}^2\boldsymbol{l}}{(2\pi)^2} C_q(\boldsymbol{l})\, D_{q_i}(x, \boldsymbol{k}-\boldsymbol{l}, t)\,,$$

$$C_{q(g)}(\boldsymbol{l}) = w_{q(g)}(\boldsymbol{l}) - \delta(\boldsymbol{l}) \int \mathrm{d}^2\boldsymbol{l}'\, w_{q(g)}(\boldsymbol{l}')$$

Evolution Equations as Integral Equations

[Kutak,Płaczek, Straka: Eur.Phys.J. C79 (2019) no.4, 317]

$$\frac{\partial}{\partial t}D(x,t) = \frac{1}{t^*}\int_0^1 dz\mathcal{K}(z)\left[\sqrt{\frac{z}{x}}D\left(\frac{x}{z},t\right) - \frac{z}{\sqrt{x}}D(x,t)\right]$$

Probabilities:

$$\Delta(x,\tau-\tau_0) = e^{-\phi(x)(\tau-\tau_0)}$$

$$\mathcal{K}(z)$$

Exist direct methods: Chebyshev method, Runge Kutta... [Blanco, Kutak, Płaczek, MR, Straka, JHEP 04(2021)014]

$$D(x,\tau) = e^{-\phi(x)(\tau-\tau_0)}D(x,\tau_0) + \int_{\tau_0}^{\tau} d\tau' \int_0^{1-\epsilon} dz \int_0^1 dy\delta(x-zy)\sqrt{\frac{z}{x}}z\mathcal{K}(z)e^{-\phi(x)(\tau-\tau')}D(y,\tau')$$

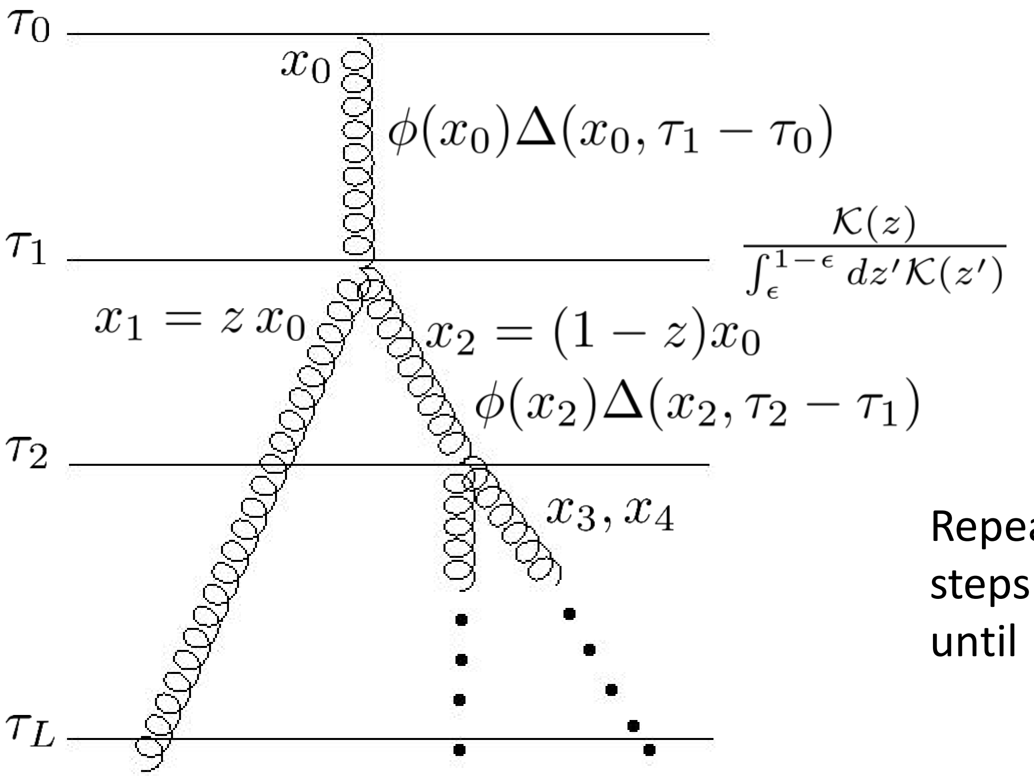
$$\tau = \frac{t}{t^*} \qquad \phi(x) = \frac{1}{\sqrt{x}}\int_0^{1-\epsilon} dzz\mathcal{K}(z)$$

Monte-Carlo algorithm TMDICE

[MR, arxiv: 2111.00323], [Blanco, Kutak, Płaczek, MR, Straka, JHEP 04(2021)014]

Other codes implementing BDMPS-Z spectra:

MARTINI, JEWEL, QPYTHIA, …



Analogous for the k_T dependent equation in x, k_T, and, τ and system of equations!

Repeat for all steps in τ and x until $\tau > \tau_L$

Other Monte-Carlo algorithm that solves the evolution equations:

MINCAS

[Kutak,Płaczek, Straka: Eur.Phys.J. C79 (2019) no.4, 317]

Different models

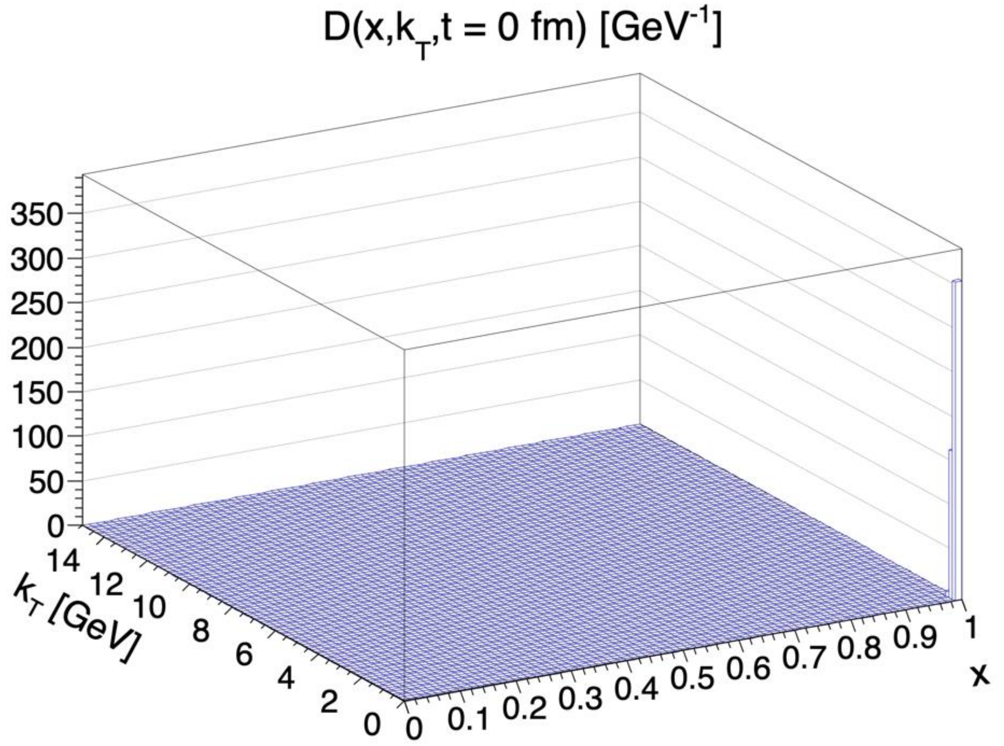
- Broadening in branching: $\mathcal{K}_{ij}(\mathbf{Q}, z, xp_0^+)$
  - No scattering
  - Scattering: $w_g(\mathbf{q}) = \frac{16\pi^2\alpha_s^2 N_c n}{\mathbf{q}^4}$
  - Scattering: $w_g(\boldsymbol{q}) = \frac{16\pi^2\alpha_s^2 N_c n}{\boldsymbol{q}^2(\boldsymbol{q}^2 + m_D^2)}$
- No broadening in branching: $\mathcal{K}_{ij}(z)$
  - Scattering: $w_g(\mathbf{q}) = \frac{16\pi^2\alpha_s^2 N_c n}{\mathbf{q}^4}$
  - Scattering: $w_g(\boldsymbol{q}) = \frac{16\pi^2\alpha_s^2 N_c n}{\boldsymbol{q}^2(\boldsymbol{q}^2 + m_D^2)}$
- Gaussian broadening:
  x given by collinear evolution without scattering via $\mathcal{K}_{ij}(z)$

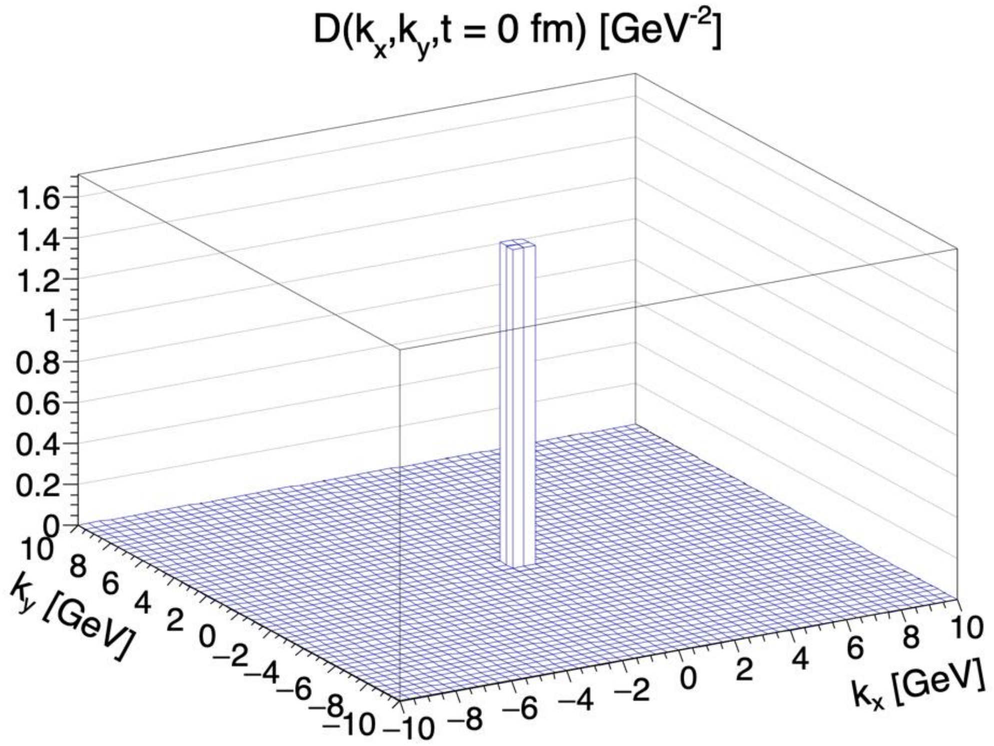
  $\boldsymbol{k}$ given by Gaussian distribution with variance $\sigma^2 \sim \hat{q}L$

All models yield the same k_T averaged splitting kernel $\mathcal{K}_{ij}(z)$!

Evolution of $D(x,k_T,t)$ (1/2)

$$\mathcal{K}(z) \qquad w(\boldsymbol{q}) = \frac{16\pi^2\alpha_s^2 N_c n}{\boldsymbol{q}^2(\boldsymbol{q}^2 + m_D^2)}$$

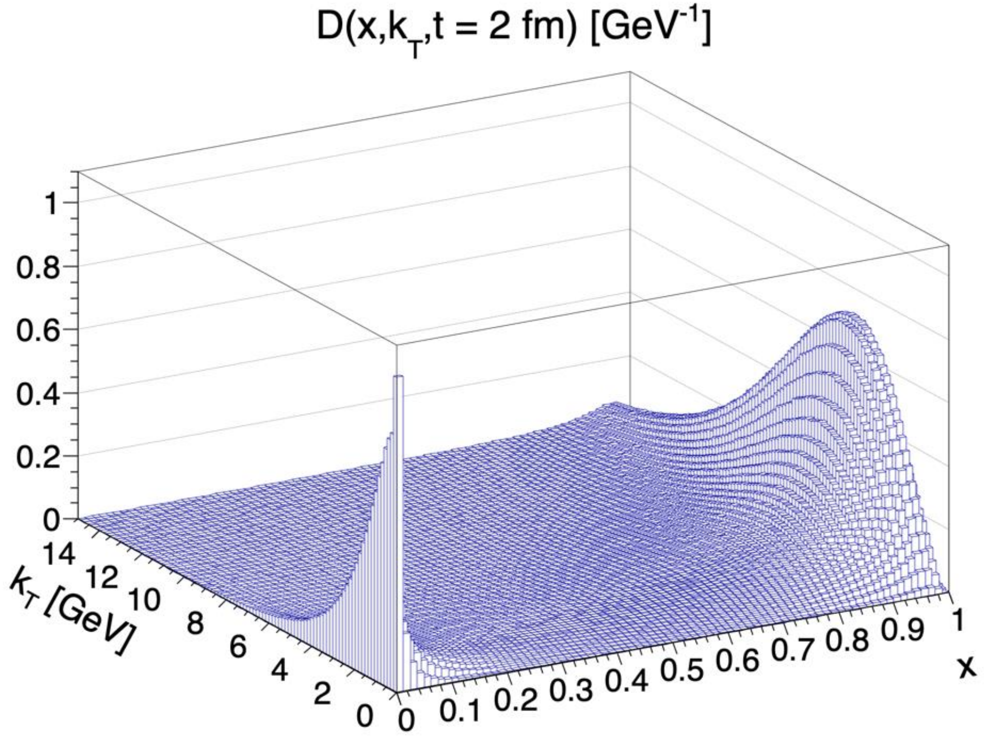


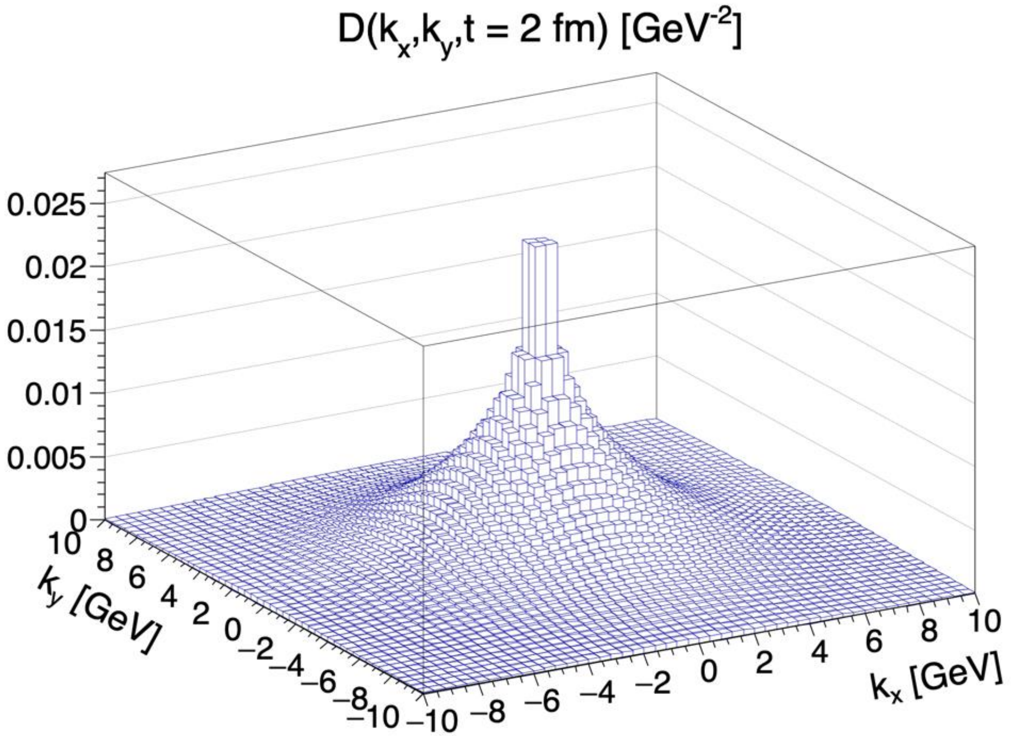


[Kutak,Płaczek, Straka: Eur.Phys.J. C79 (2019) no.4, 317]

Evolution of $D(x,k_T,t)$ (2/2)

$\mathcal{K}(z) \quad w(\boldsymbol{q}) = \frac{16\pi^2\alpha_s^2 N_c n}{\boldsymbol{q}^2(\boldsymbol{q}^2+m_D^2)}$





[Kutak,Płaczek, Straka: Eur.Phys.J. C79 (2019) no.4, 317]

Departure from Gaussian broadening

always same distribution for changes $p \to p + \boldsymbol{q}$
→central limit theorem

$$\frac{\partial}{\partial t} D(x, \boldsymbol{k}, t) = \int \frac{d^2\boldsymbol{q}}{(2\pi)^2} C(\boldsymbol{q}) D(x, \boldsymbol{k} - \boldsymbol{q}, t)$$
$$+ \frac{1}{t^*} \int_0^1 dz \mathcal{K}(z)$$
$$\left[\frac{1}{z^2} \sqrt{\frac{z}{x}} D\left(\frac{x}{z}, \frac{\boldsymbol{k}}{z}, t\right) \theta(z - x) - \frac{z}{\sqrt{x}} D(x, \boldsymbol{k}, t) \right]$$

Splitting à la $p \to zp$
→ perturbations of different sizes
→non Gaussian behavior

Virtual emissions

For example:

$$p \to z_1 p \to z_1 p + \boldsymbol{q}_1$$
$$\to z_1 p + \boldsymbol{q}_1 + \boldsymbol{q}_2$$
$$\to z_2(z_1 + \boldsymbol{q}_1 + \boldsymbol{q}_2) \to \ldots$$

$\mathcal{K}(z)$ $\quad w(\boldsymbol{q}) = \frac{16\pi^2 \alpha_s^2 N_c n}{\boldsymbol{q}^2(\boldsymbol{q}^2 + m_D^2)}$

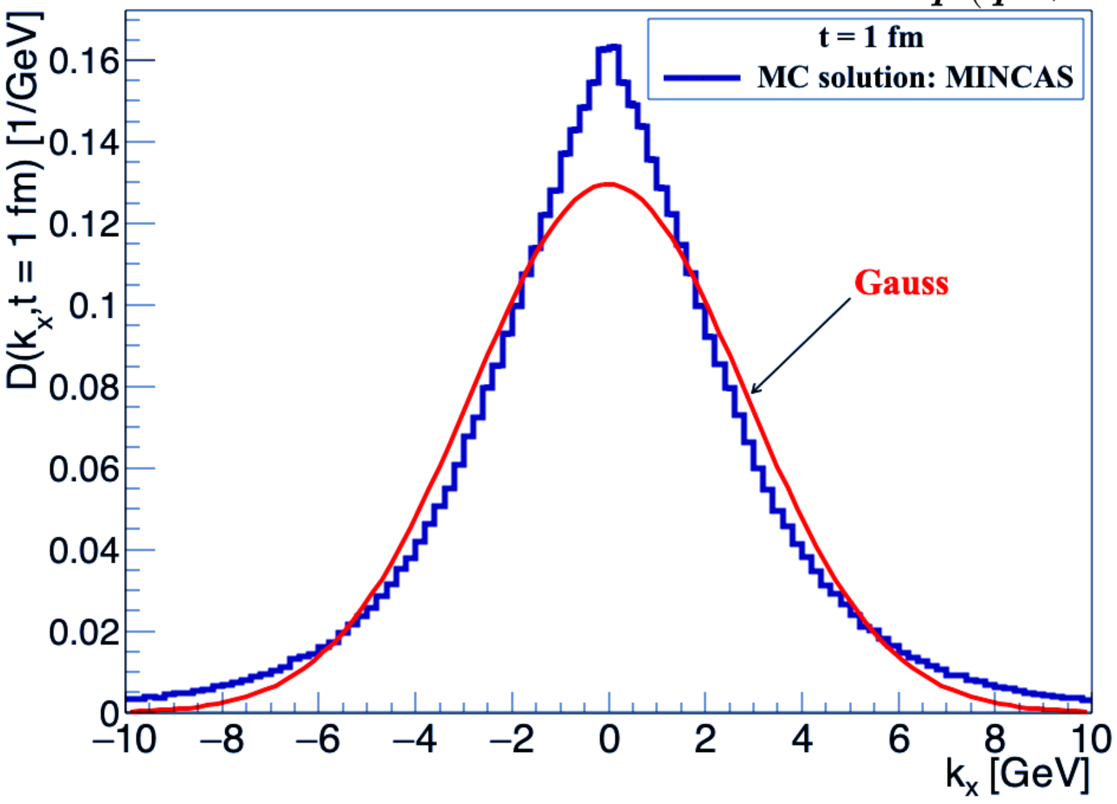
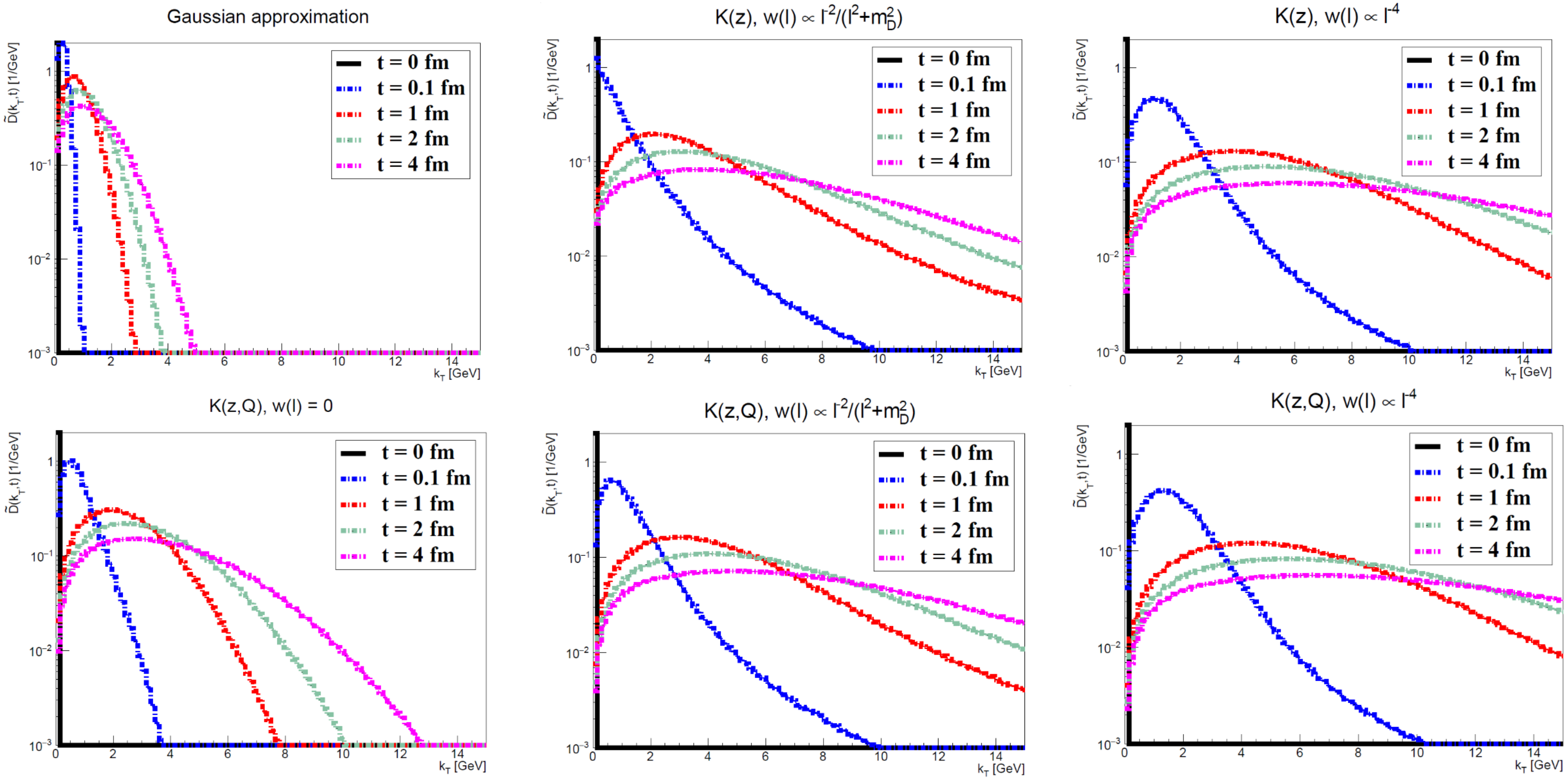


Figure: [Kutak, Płaczek, Straka: Eur.Phys.J. C79 (2019) no.4, 317]

k_T Broadening

$$\tilde{D}(x, k_T, t) = 2\pi k_T D(x, k_T, t)$$



[Blanco, Kutak, Płaczek, MR, Straka, JHEP 04(2021)014]

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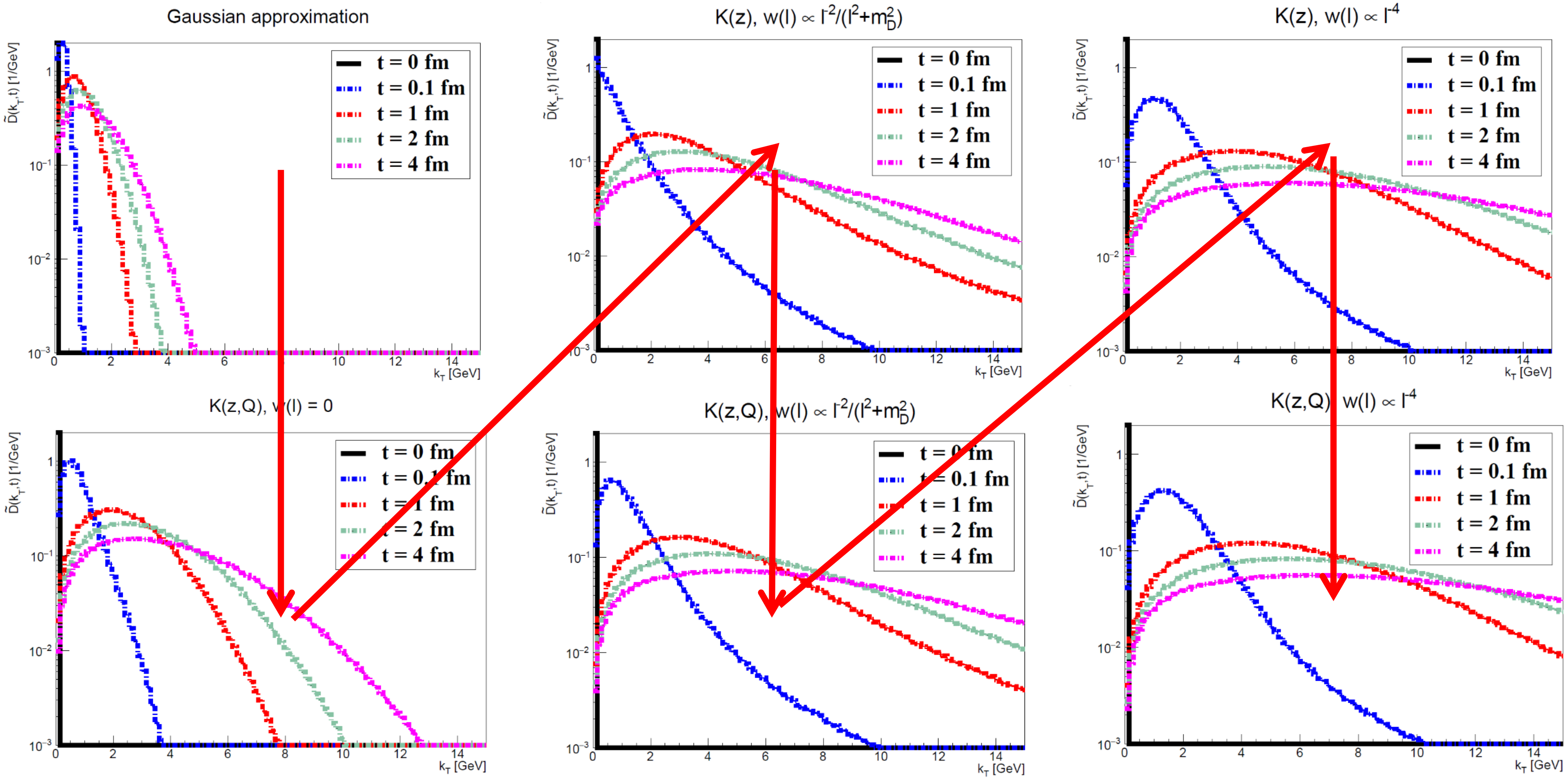
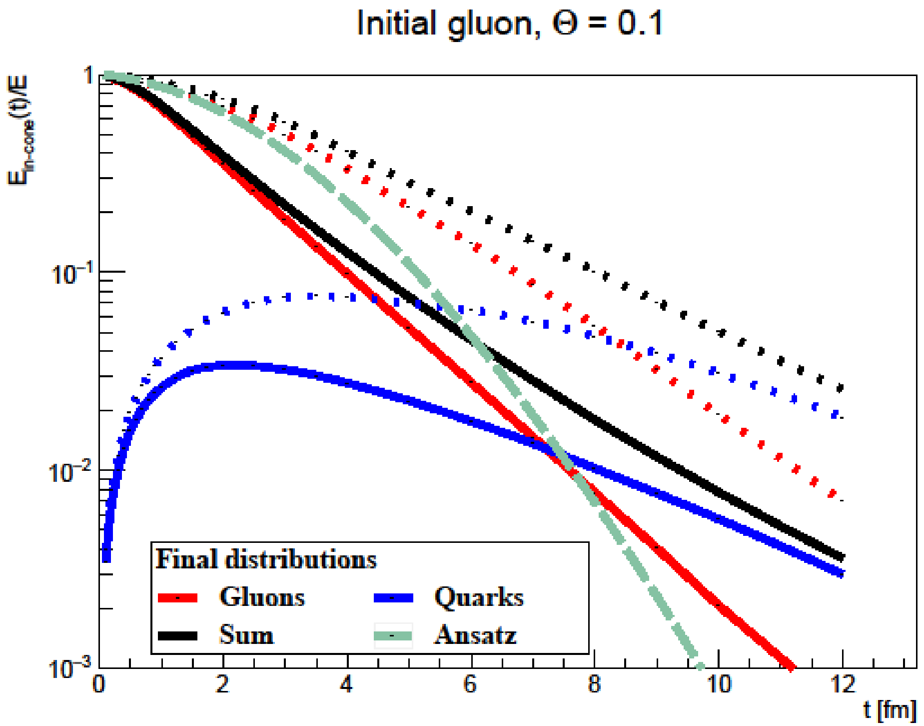
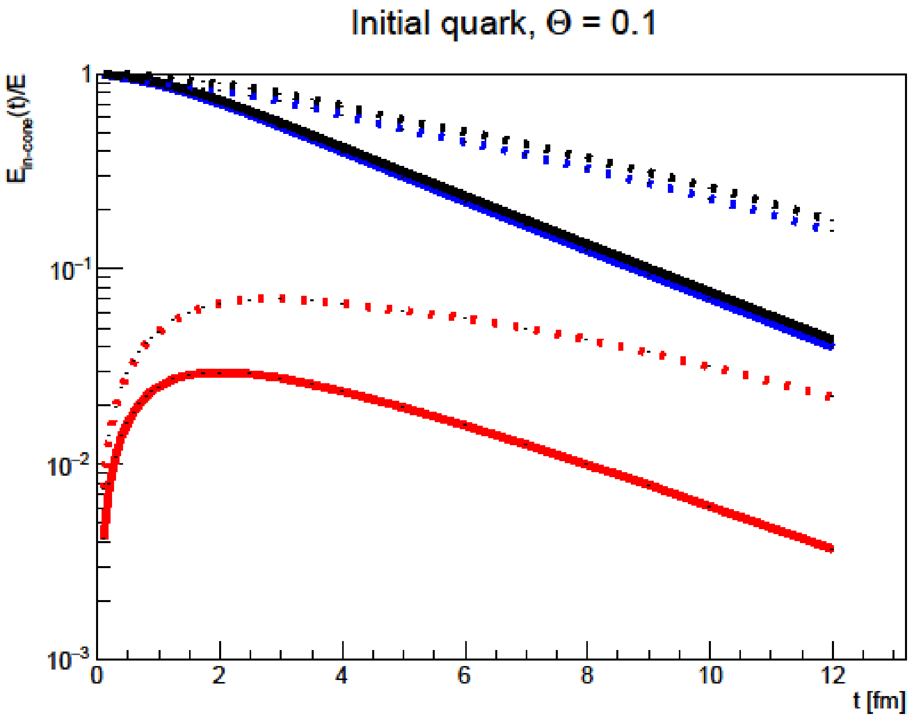


[Blanco, Kutak, Płaczek, MR, Straka, JHEP 04(2021)014]

In cone energy

$$E_{\text{in-cone}}(\Theta) = \int_0^1 \mathrm{d}x \int_0^{2\pi} d\varphi \int_0^{xE\sin\Theta} \mathrm{d}k_T k_T \, D(x, \mathbf{k}, t)$$





[Blanco, Kutak, Płaczek, MR, Tywoniuk,arxiv: 2109.05918]

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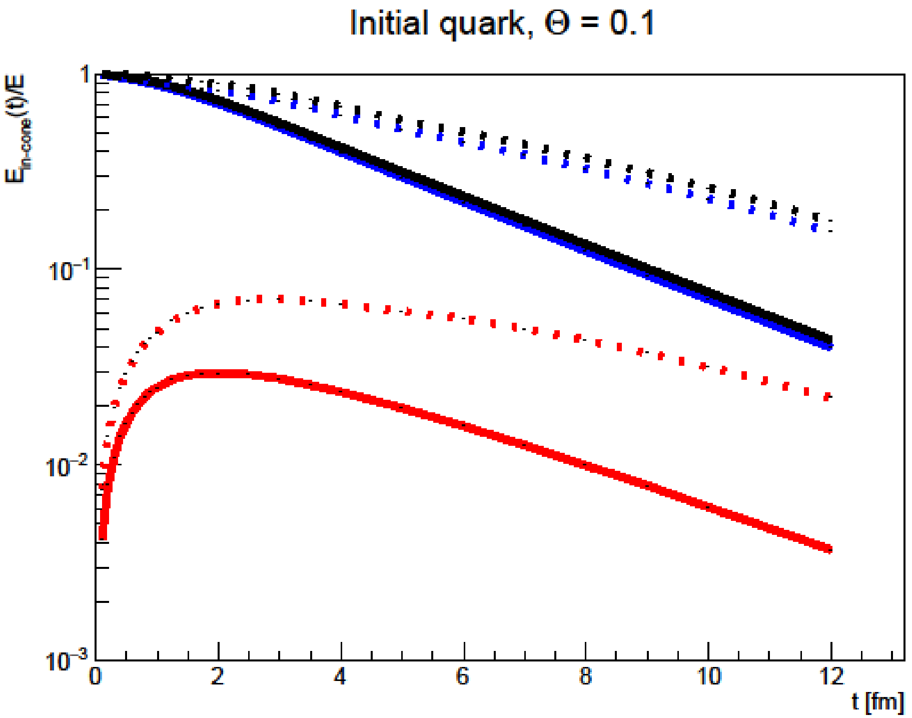

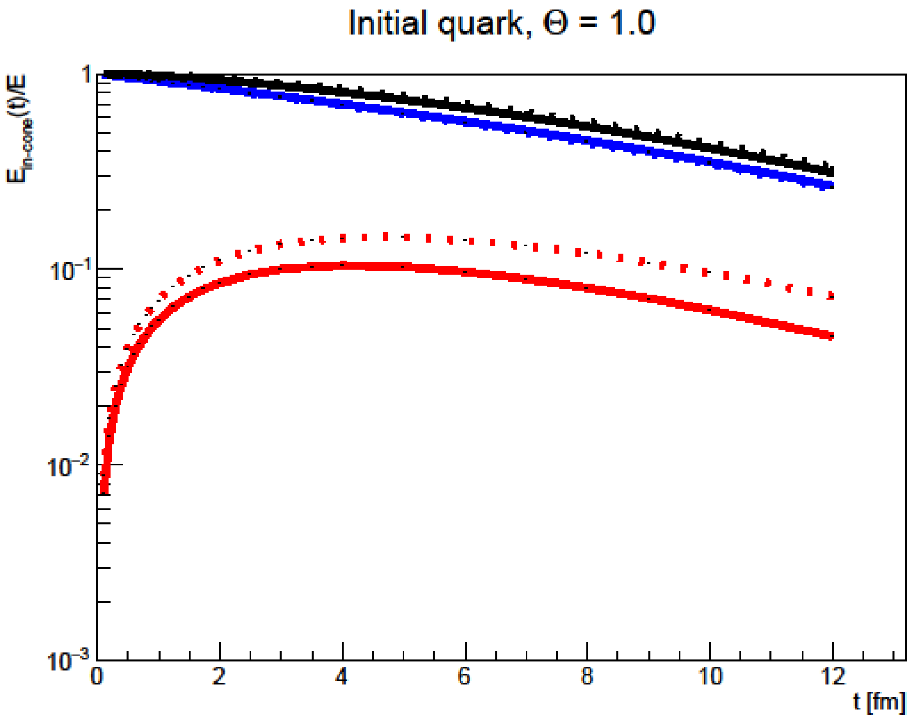

[Blanco, Kutak, Płaczek, MR, Tywoniuk,arxiv: 2109.05918]

Summary

- Monte-Carlo algorithms based on coherent emission and scattering for quarks and gluons
- Transverse momentum broadening differs from Gaussian distribution
- Gaussian distribution: smallest k_T broadening
- Clear ordering of broadening effects
- Quark jets keep more energy inside a jet cone.

Thank you for your attention!