Evidence for the maximally entangled low x proton in Deep Inelastic Scattering from H1 data

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Based on
2110.06156 Martin Hentschinski, Krzysztof Kutak

Motivation

Entropy in DIS (and hadronic collisions) attracts considerable theoretical interest since it

- constraints the growth of PDFs with energy through quantum bounds
- links to other areas (thermodynamics, gravity, quantum information, string theory)
- links of entropy to saturation K. Kutak ‘11

Nevertheless, phenomenological evidence for existing theory proposals is almost non-existent.

Boltzman and von Neuman entropy formulas

The entropy S of macrostate is given by the log of number W of distinct micro states that compose it

$$S = -\sum_{i=1}^{W} p(i) \ln p(i)$$

For uniform distribution $p(i) = \frac{1}{W}$ the entropy is maximal $S = \ln W$

Since partons are introduced as the microscopic constituents that compose the macroscopic state of the proton, it is natural to evaluate the corresponding entropy.

But proton as a whole is a pure state and the von Neuman entropy is 0.
Can we get any nontrivial result?

For pure state density matrix is

$$\rho = |\psi\rangle\langle\psi|$$

$$S_{VN} = -Tr[\rho \ln \rho] = -1 \ln 1 = 0$$

For mixed state i.e. classical statistical mixture

$$\rho = \sum p(i) |\psi_i\rangle\langle\psi_i|$$

$$S_{VN} \neq 0$$

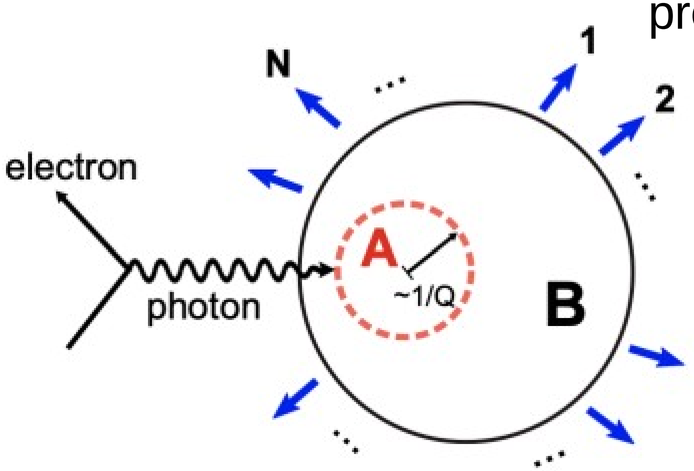
Entanglement entropy in DIS

The composite system is described by $\mathcal{H}_A \otimes \mathcal{H}_B$

$|\Psi_{AB}\rangle$ in $A \cap B$

physical state in A
physical state in B

$$|\Psi_{AB}\rangle = \sum_{i,j} c_{ij} |\varphi_i^A\rangle \otimes |\varphi_j^B\rangle$$



Entangled if the product can not be expressed as separable – product state

$$|\Psi_{AB}\rangle = |\varphi^A\rangle \otimes |\varphi^B\rangle$$

$\mathcal{H}_B$ of dimension n_B.

$\mathcal{H}_A$ of dimension n_A

related to matrix C

$$|\Psi_{AB}\rangle = \sum_n \alpha_n |\Psi_n^A\rangle |\Psi_n^B\rangle$$

(Schmidt decomposition)

Kharzeev, Levin '17

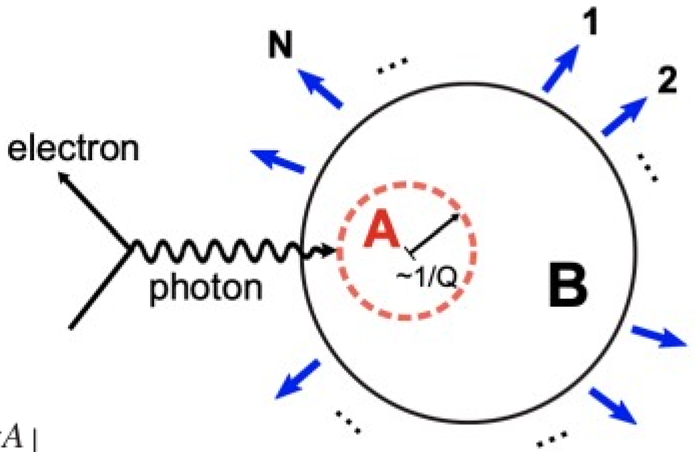
Density matrix is $\rho_{AB} = |\Psi_{AB}\rangle\langle\Psi_{AB}|$

The density matrix of the mixed state probed in region A can now be written down as

$$\rho_A = \mathrm{tr}_B \rho_{AB} = \sum_n \alpha_n^2 |\Psi_n^A\rangle\langle\Psi_n^A|$$

partial trace

Entanglement entropy in DIS



proton's rest frame

$$\rho_A = \mathrm{tr}_B \rho_{AB} = \sum_n \alpha_n^2 |\Psi_n^A\rangle\langle\Psi_n^A|$$

$\alpha_n^2 \equiv p_n$ probability of state with n partons

Kharzeev, Levin '17

$$S = -\sum_n p_n \ln p_n$$

entropy results from the entanglement between the regions A and B, and can thus be interpreted as the entanglement entropy.

Partonic cascade

$$p_n = P_n$$

set of partons is described by set of dipoles

Y is rapidity and is related to energy

$$\frac{dP_n(Y)}{dY} = -\lambda n P_n(Y) + (n-1)\lambda P_{n-1}(Y)$$

depletion of the probability to find n dipoles due to the splitting into (n + 1) dipoles.

the growth due to the splitting of (n − 1) dipoles into n dipoles.

$$P_n(Y) = e^{-\lambda Y}\left(1 - e^{-\lambda Y}\right)^{n-1}$$

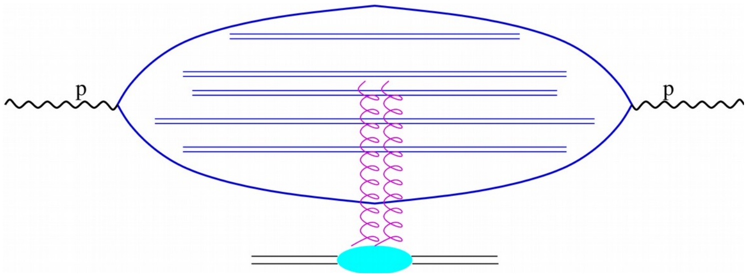
$$S = -\sum_n p_n \ln p_n$$

$$S(Y) = \ln\left(e^{\lambda Y} - 1\right) + e^{\lambda Y} \ln\left(\frac{1}{1 - e^{-\lambda Y}}\right)$$

$S(Y) \approx \lambda Y$ where $Y = \ln 1/x$

BFKL intercept

$$xg(x) = \langle n \rangle = \sum_n n P_n(Y) = \left(\frac{1}{x}\right)^{\lambda}$$



$$S(x) = \ln(xg(x))$$

$$S(x,Q) = \ln(xg(x,Q))$$

Kharzeev, Levin ‘17

KL entropy formula - interpretation

$$P_n(Y) = e^{-\lambda Y}\left(1 - e^{-\lambda Y}\right)^{n-1}$$

At low x partonic microstates have equal probabilities

In this equipartitioned state the entropy is maximal – the partonic state at small x is maximally entangled.

In terms of information theory as Shanon entropy:
- equipartitioning in the maximally entangled state means that all “signals” with different number of partons are equally likely
- it is impossible to predict how many partons will be detected in a give event.
- structure function at small x should become universal for all hadrons.

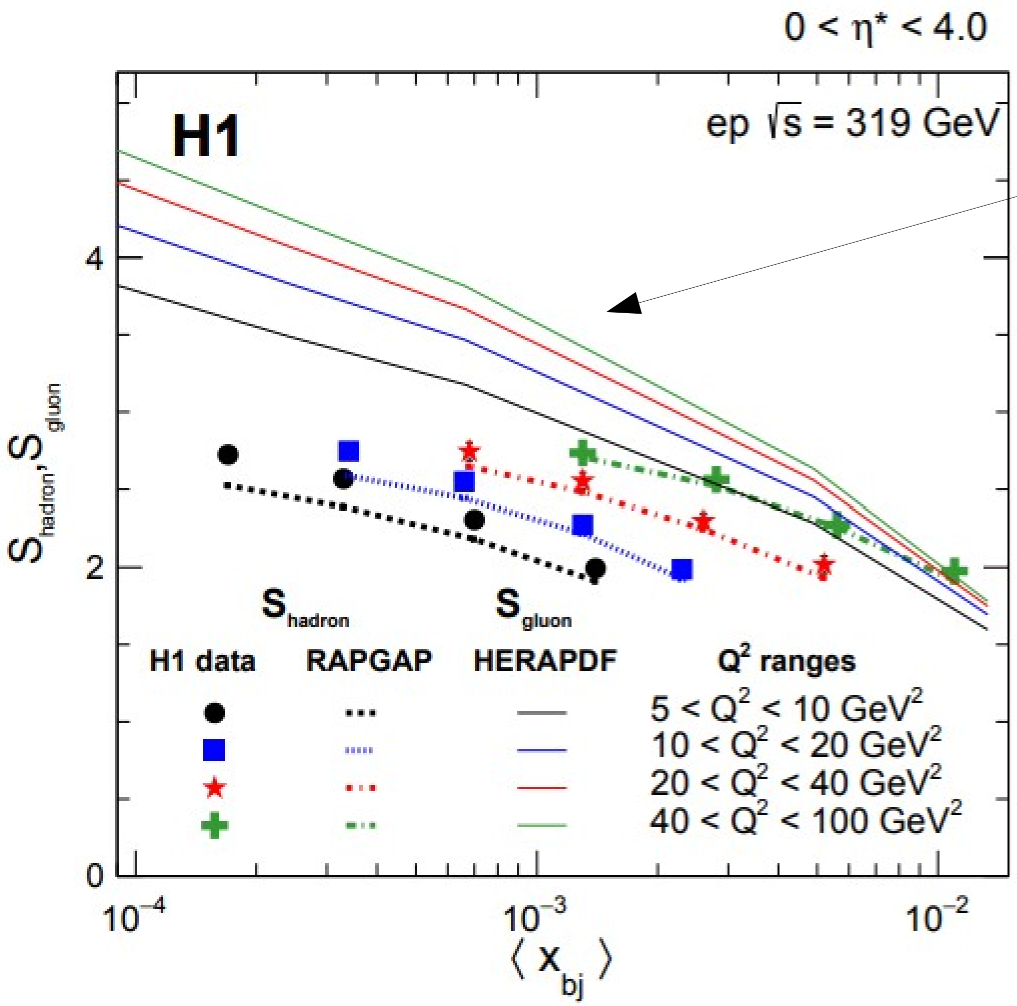
From strict bounds on entanglement entropy (from conformal field theory) one can obtain that at low x (in conformal regime) one has

$$xg(x) \leq \mathrm{const}\, x^{-1/3}$$

Kharzeev, Levin ‘17

Furthermore entropy of the final state hadrons can not be smaller than entropy of partons.

Monte Carlo KL formula and data



HERA pdf used

$$S(x, Q) = \ln(xg(x, Q))$$

Also attempt by Kharzeev and Levin to use quarks instead of gluons Phys. Rev. D 104, 031503 (2021)

$$S(x, Q) = \ln(x\Sigma(x, Q))$$

This argument is however based on incorrect formula...but it is a illuminating mistake

Eur.Phys.J.C 81 (2021) 3, 212

Extension of KL entropy formula

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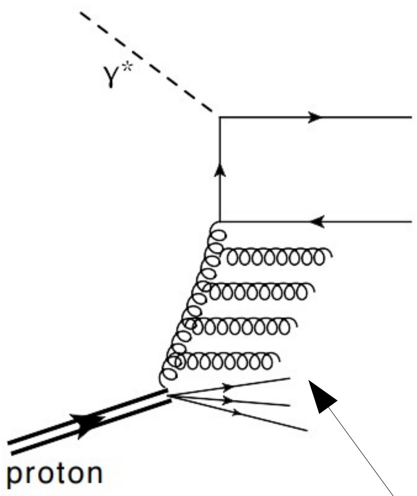
$$\left\langle n\left(\ln\frac{1}{x}, Q\right)\right\rangle = xg(x,Q) + x\Sigma(x,Q)$$

To get the entropy of system of partons one needs to account for both quarks and gluons. One can view this as a higher order correction to KL formula. Furthermore it is impossible to isolate quarks from gluons therefore the compete entropy formula should receive contributions from quarks and gluons

$$S(x,Q^2) = \ln\left\langle n\left(\ln\frac{1}{x}, Q\right)\right\rangle$$

Gluon and quark distribution



In the linear regime obeys BFKL Equation. In our calculations we use NLO BFKL with kinematical improvements and running coupling The gluon density has been fitted to F2 data (exact kinematics was used)

Hentschinski, Sabio-Vera, Salas.
Phys.Rev.D 87 (2013) 7, 076005
Phys.Rev.Lett. 110 (2013) 4, 041601

We calculate the sea quarks distribution using

$$x\Sigma(x,Q) = P_{qg}(Q,\mathbf{k}) \otimes \mathcal{F}(x,\mathbf{k}^2)$$

$$xg(x,Q) = \int_0^{Q^2} d\mathbf{k}^2 \mathcal{F}(x,\mathbf{k}^2)$$

Transverse momentum dependent splitting function
Catani, Hautmann
Nucl.Phys. B427 (1994) 475-524

Results

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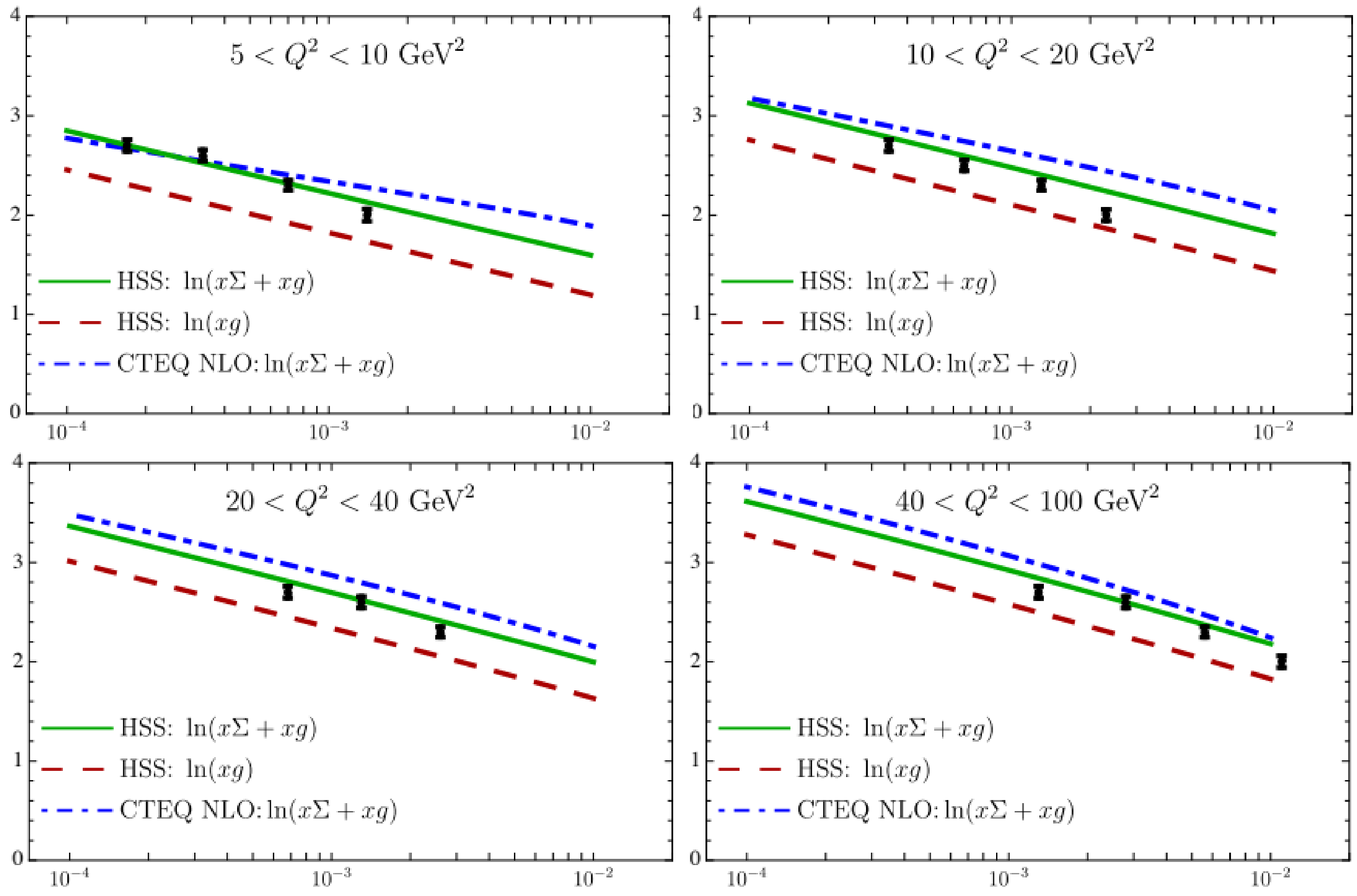

Good description of data. Hint that the general idea works. One definitely has to account both for quarks and gluons.

Open questions

One can get formula for entropy from momentum space entanglement by integrating out large x degrees of freedom as well as from thermodynamics considerations as well as from calculation so called Wehrl entropy.
Assuming saturation and using Golec-Biernat Wusthoff model one gets

$$S \sim \frac{S_\perp}{\alpha_s} Q_s^2$$

what is the relation to KL formula?

Kutak '11
R.Peschanski '12
Kovner, Lublinsky
Hatta, Xiao,Yuan'18
Kovner, Lublinsky, Serino '18

This formula looks similar to Hawking entropy formula and more generally to entropy formula studied in AdS/CFT

$$S_A = \frac{Area\, of\, \gamma_A}{4G}$$

Ryu – Takanayagi '07

Perhaps question is similar to the microstate counting (KL) and thermodynamics entropy in Black Hole physics

Conclusions

- We show that the Kharzeev and Levin proposal for low x entanglement entropy can be systematically improved (quark contributions, NLO BFKL) and can describe successfully H1 data.

- We therefore provide phenomenological evidence which is essential for the further development of the field.