Next-to soft virtual resummed Drell-Yan cross section beyond Leading-logarithm

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Joint work with Ajjath A.H, Pooja Mukherjee, V. Ravindran and Surabhi Tiwari

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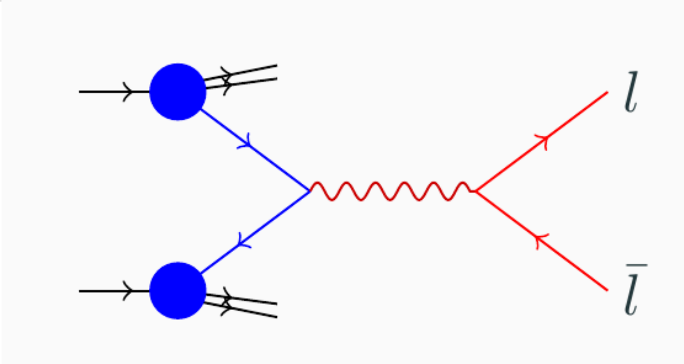
Outline

- Overview and Background
- Next-to soft virtual (NSV) formalism for inclusive cross sections
- NSV Resummation in Mellin space
- Phenomenology: Drell-Yan
- Summary and Outlook

Overview & Background

Drell-Yan



- **One of the standard candle processes**
  Large cross section and clean experimental signature - important for detector calibration and constraining parton distribution functions

- **Experimentally, one has a very clean environment for precise measurements**

- **Well-understood theoretically - known to N^3LO accuracy in QCD**
  Duhr, Dulat et.al ('20)

- **DY serves as an important process in collider experiments**

- **Higher order perturbative QCD corrections to DY provides ample opportunity to explore the structure of the perturbation series**

Overview & Background

- Large logarithms at kinematic threshold region spoil the reliability of fixed-order pertutrbative sries

- Resolution: Thresold resummation

  Sterman-Catani-Trentedue

Sterman (`87),Catani,Trentedue `89

SUMMATION OF LARGE CORRECTIONS TO SHORT-DISTANCE HADRONIC CROSS SECTIONS

George STERMAN*

Institute for Advanced Study, Princeton, NJ 08540, USA

RESUMMATION OF THE QCD PERTURBATIVE SERIES FOR HARD PROCESSES

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$$\Delta_N(Q^2) \underset{N\to\infty}{=} \exp\left[2\int_0^1 dx\, \frac{x^{N-1}-1}{1-x}\int_{Q^2}^{Q^2(1-x)}\frac{dk^2}{k^2}A\left(\alpha_s\left((1-x)Q^2\right)\right) + \frac{3}{2}\frac{C_F}{\pi}\int_0^1 dx\, \frac{x^{N-1}-1}{1-x}\alpha_s\left((1-x)Q^2\right)\right] + O\left(\alpha_s(\alpha_s \ln N)^n\right). \qquad (3.25)$$

- Resummation is necessary to provide reliable theoretical predictions

- Threshold resummation : known to N^3LL accuracy

Resummed Drell-Yan cross-section at N^3LL

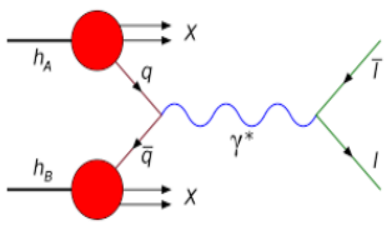
Ajjath A H,[a] Goutam Das,[b,c] M. C. Kumar,[d] Pooja Mukherjee,[a] V. Ravindran,[a] Kajal Samanta [d]

Incusive Reactions – QCD Improved Parton Model

Drell-Yan (DY) / Higgs boson production in Hadron collisions

$$\sigma(q^2,\tau) = \sigma_0(\mu_R^2)\int \frac{dz}{z}\Phi_{ab}\left(\frac{\tau}{z},\mu_F^2\right)\Delta_{ab}(q^2,\mu_F^2,z)$$

Partonic Coeff. function



Partonic flux

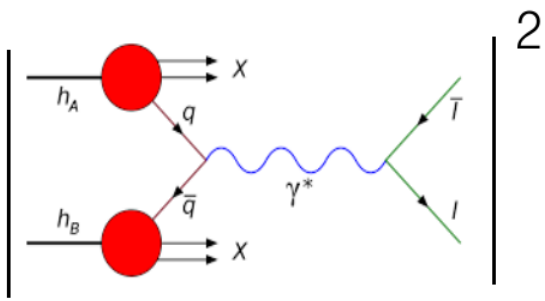
$$\Phi_{ab}(\mu_F^2,z) = \int \frac{dy}{y} f_a(y,\mu_F^2) f_b\left(\frac{z}{y},\mu_F^2\right)$$

Parton distribution fns (PDFs)

- τ Hadronic scaling variable
- q^2 Invariant mass sq
- z Partonic scaling variable
- μ_R^2 Renormalisation scale
- μ_F^2 Factorisation scale

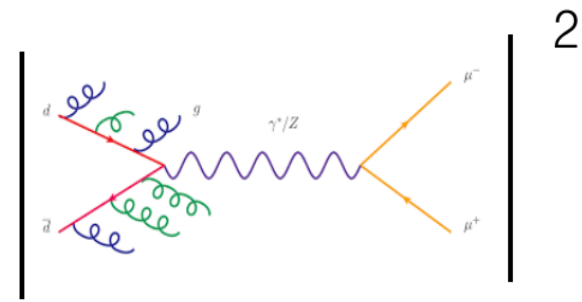
Threshold Expansion

Threshold Region : z is closer to 1



Hadron level

$$= \sum_{ab} \Phi_{ab}(z) \otimes$$



Parton level $\Delta_{ab}(z)$

$$z = \frac{q^2}{\hat{s}} \to 1$$

$\hat{s}$: square of partonic c.m energy

Perturbatibe Structure

$$\Delta_c^{\mathrm{SV+NSV,i}}(z,q^2) = \sum_{k=0}^{2i-1} c_{ik}^{\mathcal{D}}\ \mathcal{D}_k + c_i^{\delta}\ \delta(1-z) + \sum_{k=0}^{2i-1} c_{ik}^{L}\ \log^k(1-z)$$

$$\mathcal{D}_k = \left(\frac{\log^k(1-z)}{(1-z)}\right)_+$$

Plus distribution

Soft-virtual corrections

Most Singular when z→ 1
Corrections from diagonal Channels
Resummation to N3LL accuracy
Well-understood

Next-to SV corrections

Next-to-dominant singular
Collinear logarithms
Corrections from both diagonal & off-diagonal
Resummation to LL accuracy
Not much studied

NSV in History

The problem of NSV/NLP(next-to-leading power) logarithms has been of interest for a long time, and several different approaches have been proposed.

* The earliest evidence that IR effects can be studied at NLP
  [Low, Burnett, Kroll]

* Early attempts :
  [Kraemer, Laenen, Spira (98)]
  [Akhoury, Sotiropoulos & Sterman (98)]

* Important Results & Predictions using Physical Kernel Approach & explicit computation:
  [Moch , Vogt et al. (09-20)]
  [Anastasiou, Duhr, Dulat et al.(14)]

NSV in History

* **Universality of NLP effects and LL Resummation:**
  [Laenen, Magnea, et al. (08-19)]
  [Grunberg & Ravindran (09)]
  [Ball, Bonvini, Forte, Marzani, Ridolfi (13)]
  [Del Duca et al. (17)]

* **Subleading Factorisation and LL Resummation at NLP using SCET:**
  [Larkoski, Nelli , Stewart et al. (14)]
  [Kolodrubetz, Moult, Neill ,Stewart et al. (17)]
  [Beneke et al. (19-20)]

And many other works…

Our Works

- ⭐ **Factorisation and RG invariance approach to study NSV resummation effects**
  [Ajjath, Pooja, Ravindran , hep-ph/ 2006.06726]

- ⭐ **On next to soft threshold corrections to DIS and SIA processes**
  [Ajjath, Pooja, Ravindran, A.Sankar, S.Tiwari, JHEP 04 (2021) 131]

- ⭐ **Next-to SV resummed Drell-Yan cross section beyond Leading-logarithm** ✔ Today's talk
  [Ajjath, Pooja, Ravindran, A.Sankar, S.Tiwari, hep-ph/2107.09717]

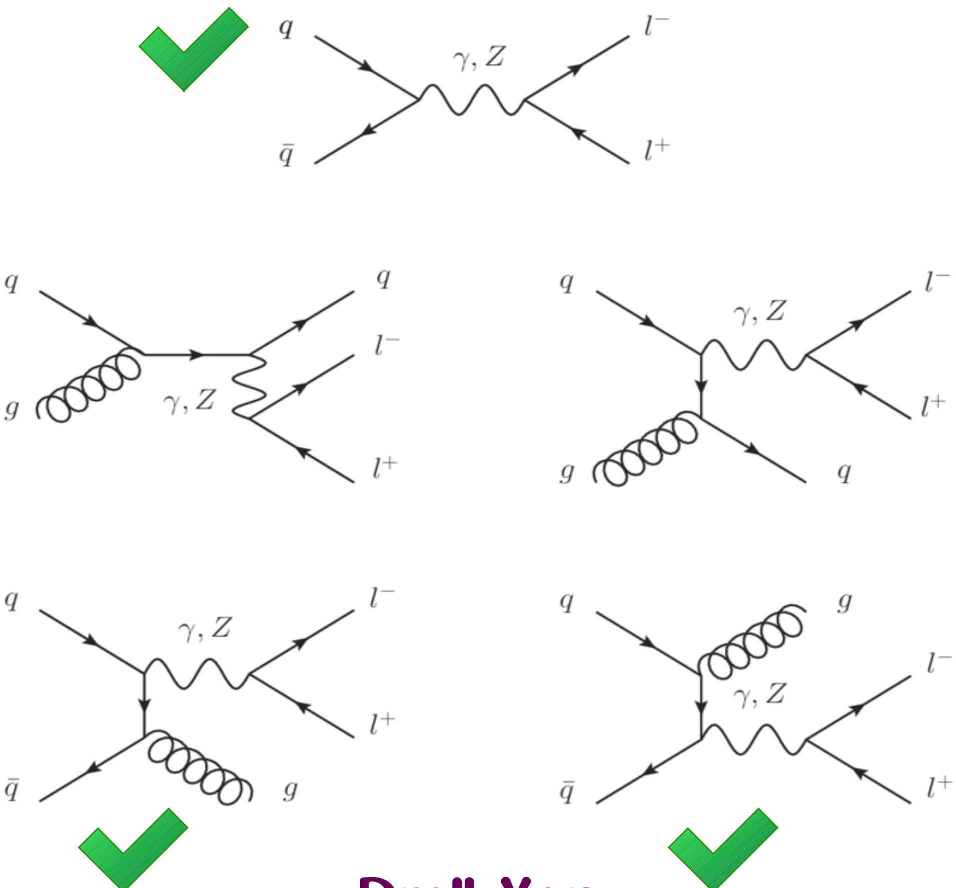
- ⭐ **Resummed Higgs boson cross section at next-to SV to NNLO + $\overline{\text{NNLL}}$**
  [Ajjath, Pooja, Ravindran, A.Sankar, S.Tiwari, hep-ph/2109.12657]

- ⭐ **Rapidity distribution at soft-virtual and beyond for n-colorless particles to N4LO in QCD**
  [Taushif, Ajjath, Pooja, Ravindran, A.Sankar, Eur. Phys. J. C 81, 943 (2021)]

- ⭐ **Next-to-soft corrections for Drell-Yan and Higgs boson rapidity distributions beyond N3LO**
  [Ajjath, Pooja, Ravindran, A.Sankar, S.Tiwari, Phys.Rev.D 103 (2021) L111502]

Our Approach

Considered only diagonal channels :



Drell-Yan

Keypoints

- Collinear Factorisation
- Renormalisation Group (RG) Invariance
- Logarithmic structure of higher order perturbative results

The Theory - Formalism

Factoring out the pure virtual contributions

Soft+Next-to soft corrections

$$\hat{\sigma}_{c\bar{c}}(z,\epsilon) = \left(Z_{c,UV}\right)^2 |\hat{F}_c(\epsilon)|^2 S_c(z,\epsilon)$$

Partonic cross-section

UV Renormalisation constant

Unrenormalised Form Factor (FF)
(pure virtual corrections)

Mass Factorisation

Altarelli-Parisi (AP) kernel

$$\frac{1}{z}\hat{\sigma}_{ab}(z,\epsilon) = \sigma_0 \sum_{a'b'} \Gamma_{aa'}(\mu_F^2, z, \epsilon) \otimes \left(\frac{1}{z}\Delta_{a'b'}(\mu_F^2, z, \epsilon)\right) \otimes \Gamma_{b'b}(\mu_F^2, z, \epsilon)$$

Partonic cross-section containing only
Initial state collinear singularities

Collinear Finite

Collinear Singular

Coefficient function – Diagonal channel

UV finite mass-factorised partonic coefficient function for the diagonal channels:

$$\Delta_{c\bar{c}}(z, \epsilon, q^2 \mu_R^2, \mu_F^2) = \left(\Gamma^T\right)^{-1} \otimes \left\{ \left(Z_{c,UV}\right)^2 |\hat{F}_c(Q^2, \epsilon)|^2 S_c(q^2, z, \epsilon) \right\} \otimes \left(\Gamma\right)^{-1}$$

Now, let us study these each building block separately

Set of governing differential eqns

$$\Delta_{c\bar{c}}(z,\epsilon,q^2\mu_R^2,\mu_F^2) = \left(\Gamma^T\right)^{-1} \otimes \left\{ \left(Z_{c,UV}\right)^2 |\hat{F}_c(Q^2,\epsilon)|^2 S_c(q^2,z,\epsilon) \right\} \otimes \left(\Gamma\right)^{-1}$$

$$Q^2 \frac{d}{dQ^2} \log \hat{F}^c = \frac{1}{2}\left[K^c\left(\hat{a}_s, \frac{\mu_R^2}{\mu^2}, \varepsilon\right) + G^c\left(\hat{a}_s, \frac{Q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, \varepsilon\right)\right]$$

K+G

$$\mu_R^2 \frac{d}{d\mu_R^2} \log Z_{c,UV}(\hat{a}_s, \mu_R^2, \mu^2, \varepsilon) = \sum_{i=1}^{\infty} a_s^i(\mu_R^2) \;\; \gamma_{i-1}^c$$

RGE

$$\mu_F^2 \frac{d}{d\mu_F^2} \Gamma_{ab}(z,\mu_F^2,\epsilon) = \frac{1}{2} \sum_{a'=q,\bar{q},g} P_{aa'}(z, a_s(\mu_F^2)) \otimes \Gamma_{a'b}(z,\mu_F^2,\epsilon)$$

AP evolution eqn

Altarelli-Parisi kernels

[Moch,Vogt,Vermaseren]

Required to remove the initial state collinear singularities

AP kernels which satisfy renormalisation group equations

$$\mu_F^2 \frac{d}{d\mu_F^2} \Gamma_{ab}(z,\mu_F^2,\epsilon) = \frac{1}{2} \sum_{a'=q,\bar{q},g} P_{aa'}\left(z, a_s(\mu_F^2)\right) \otimes \Gamma_{a'b}(z,\mu_F^2,\epsilon)\,, \qquad a,b = q,\bar{q},g$$

AP Splitting function

Collinear ano dim (contributes to NSV)

$$P_{cc}(z,\mu_F^2) = 2\left[\frac{A^c}{(1-z)_+} + B^c\,\delta(1-z) + C^c\,\log(1-z) + D^c + \mathcal{O}(1-z)\right]$$

SV **NSV** **beyond NSV**

We consider only diagonal parts of splitting functions

RGE - Summary

Building blocks

$Z_{c,UV}$	Renormalisation const
$\hat{F}_c$	Form Factor (FF)
Γ_c	AP Kernels
$\mathcal{S}_c$	Soft + Next-to soft factor

Guiding factors

- Finiteness of the partonic coefficient function Δ_c
- Sudakov differential eqn of FFs (K+G eqn)
- RG Eqns of AP kernels and $Z_{c,UV}$

Differential Eqn – Soft + Next-to Soft

$$q^2 \frac{d}{dq^2} \mathcal{S}_c = \frac{1}{2} \left[\overline{K}_c \left(\hat{a}_s, \frac{\mu_R^2}{\mu^2}, \epsilon, z \right) + \overline{G}_c \left(\hat{a}_s, \frac{q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, \epsilon, z \right) \right] \otimes \mathcal{S}_c$$

IR singular **IR finite**

Soft-collinear contributions exhibits exponential behaviour

$$\mathcal{S}_c = \mathcal{C} \exp\left(2\Phi_c\right)$$

Soft- collinear function (will be discussed in detail)

$$\mathcal{C} \exp\left(2\Phi_c(z)\right) = \frac{\hat{\sigma}_{c\bar{c}}(z)}{Z_{c,UV}^2 \left|\hat{F}_c\right|^2}, \qquad c = q, b, g$$

No pure virtual , Only Real-Virtual (RV), Real-Real (RR) etc

SV+NSV CF in Nutshell

Restricting to diagonal channels and around z=1

↓

Mass Factorization Formula

↓

All-order factorisation formula for $\Delta_{c\bar{c}}$

↓

Sudakov type differential Equation & RG Eq.

↓

$$\Delta_c(q^2, \mu_R^2, \mu_F^2, z) = C \exp\left(\Psi^c\left(q^2, \mu_R^2, \mu_F^2, z, \varepsilon\right)\right)\Big|_{\varepsilon=0}$$

$$\mathcal{C}e^{f(z)} = \delta(1-z) + \frac{1}{1!}f(z) + \frac{1}{2!}f(z) \otimes f(z) + \cdots .$$

The Master Formula

$$\Psi_c = \left(\ln\left(Z_{c,UV}\left(\hat{a}_s, \mu^2, \mu_R^2, \epsilon\right)\right)^2 + \ln\left|\hat{F}_c\left(\hat{a}_s, \mu^2, q^2, \epsilon\right)\right|^2 \right)\delta(1-z)$$
$$+2\Phi_c\left(\hat{a}_s, \mu^2, q^2, z, \epsilon\right) - 2\mathcal{C}\ln\Gamma_{cc}\left(\hat{a}_s, \mu^2, \mu_F^2, z, \epsilon\right)$$

SV Ditributions → $\delta(1-z), \left(\frac{\ln^k(1-z)}{(1-z)}\right)_+$

NSV Logarithms → $\ln^k(1-z)$

Soft-Collinear Function

[Ravindran]

$$q^2 \frac{d}{dq^2} \Phi_c = \frac{1}{2} \left[\overline{K}_c \left(\hat{a}_s, \frac{\mu_R^2}{\mu^2}, \epsilon, z \right) + \overline{G}_c \left(\hat{a}_s, \frac{q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, \epsilon, z \right) \right]$$

IR singular

IR finite, needs to be determined

RG invariance implies

$$\mu_R^2 \frac{d}{d\mu_R^2} \overline{K}_c = -\mu_R^2 \frac{d}{d\mu_R^2} \overline{G}_c = A_c \delta(1-z)$$

identical to the cusp anomalous dimension that appears in the FFs confirming the universality of IR structure

Soft-Collinear Function -- Solution

$$\Phi_c(\hat{a}_s, q^2, \mu^2, \epsilon, z) = \sum_i \hat{a}_s^i \left(\frac{q^2(1-z)^2}{\mu^2 z}\right)^{i\frac{\epsilon}{2}} S_\epsilon^i \left(\frac{i\epsilon}{1-z}\right)\left[\hat{\phi}_c^{A,(i)}(\epsilon) + (1-z)\,\hat{\phi}_c^{B,(i)}(z,\epsilon)\right]$$

Phase-space factor

From matrix elements

Solution verified up to 3rd order

- **Expanding the ansatz:**

$$\frac{1}{(1-z)}\left[(1-z)^2\right]^{i\frac{\epsilon}{2}} = \frac{\delta(1-z)}{i\epsilon} + \sum_{k=0}^{\infty} [i\epsilon]^k \frac{\mathcal{D}_k}{k!}$$

→ Contributes to SV

$$z^{-i\frac{\epsilon}{2}} = \sum_{n=0}^{\infty} \frac{\left[\frac{-i\epsilon}{2}\log(z)\right]^n}{n!}$$

→ Combining with SV, contributes to NSV

$$\left[(1-z)^2\right]^{i\frac{\epsilon}{2}} = \sum_{n=0}^{\infty} \frac{\left[i\epsilon\ \log(1-z)\right]^n}{n!}$$

→ Contributes to pure NSV

All order structure – Predictive power

[Ajjath, Pooja,Ravindran]

All order exponentiation can predict to all orders from lower orders:

$$\Delta_c(z) = \mathcal{C} \exp\left(\Psi^c(q^2, \mu_R^2, \mu_F^2, z, \varepsilon)\right)\Big|_{\varepsilon=0}$$
$$= \sum_{i=0}^{\infty} a_s^i \, \Delta_c^{(i)}(z)$$

$$\mathcal{D}_k = \left(\frac{\log^k(1-z)}{1-z}\right)_+$$

$$L_z = \log(1-z)$$

GIVEN				PREDICTIONS		
$\Psi_c^{(1)}$	$\Psi_c^{(2)}$	$\Psi_c^{(3)}$	$\Psi_c^{(n)}$	$\Delta_c^{(2)}$	$\Delta_c^{(3)}$	$\Delta_c^{(i)}$
$\mathcal{D}_0, \mathcal{D}_1, \delta$ L_z^1, L_z^0				$\mathcal{D}_3, \mathcal{D}_2$ L_z^3	$\mathcal{D}_5, \mathcal{D}_4$ L_z^5	$\mathcal{D}_{(2i-1)}, \mathcal{D}_{(2i-2)}$ $L_z^{(2i-1)}$
	$\mathcal{D}_0, \mathcal{D}_1, \delta$ L_z^2, L_z^1, L_z^0				$\mathcal{D}_3, \mathcal{D}_2$ L_z^4	$\mathcal{D}_{(2i-3)}, \mathcal{D}_{(2i-4)}$ $L_z^{(2i-2)}$
		$\mathcal{D}_0, \mathcal{D}_1, \delta$ $L_z^3, \cdots, L_z^0$				$\mathcal{D}_{(2i-5)}, \mathcal{D}_{(2i-6)}$ $L_z^{(2i-3)}$
			$\mathcal{D}_0, \mathcal{D}_1, \delta$ $L_z^n, \cdots, L_z^0$			$\mathcal{D}_{(2i-(2n-1))}, \mathcal{D}_{(2i-2n)}$ $L_z^{(2i-n)}$

using n^{th} order info at every order in a_s^i for all i

Our predictions agree with the those obtained by explicit computation

Integral representation in z-space

Knowing the functional form of each building blocks one can derive the integral form as:

Integral representation:

captures the delta contribution from FF and S_c

$$\Delta_c(q^2,z) = C_0^c(q^2)\ \mathcal{C}\exp\left(2\Psi_{\mathcal{D}}^c(q^2,z)\right),$$

Exponent:

$$\Psi_{\mathcal{D}}^c(q^2,z) = \frac{1}{2}\int_{\mu_F^2}^{q^2(1-z)^2}\frac{d\lambda^2}{\lambda^2}P'_{cc}(a_s(\lambda^2),z) + \mathcal{Q}^c(a_s(q^2(1-z)^2),z)$$

Finite contributions from cancellation between Γ_{cc} & S_c

$$P'_{cc} = 2\Big[A^c\mathcal{D}_0(z) + C^c\ln(1-z) + D^c\Big]$$

$$\mathcal{Q}^c(a_s(q^2(1-z)^2),z) = \left(\frac{1}{1-z}\overline{G}_{SV}^c(a_s(q^2(1-z)^2))\right)_+ + \varphi_{f,c}(a_s(q^2(1-z)^2),z).$$

Finite contribution coming from S_c

In the Mellin N space

Mellin moment of CFs

$$\Delta^c_N = \int_0^1 dz \; z^{N-1} \Delta_c(z)$$

Threshold limit $z \to 1$ in z-Space translates to

$N \to \infty$ in N-Space

$N \to \infty$ Taking into account SV and NSV terms

$$\left(\frac{\log(1-z)}{1-z}\right)_+ = \frac{\log^2 N}{N} - \frac{\log N}{2N} + \mathcal{O}\left(\frac{1}{N^2}\right)$$

$$\log^k(1-z) = \frac{\log^k N}{N} + \mathcal{O}\left(\frac{1}{N^2}\right)$$

Tower of NSV logarithms – Can we resum ?

Structure of Next to SV terms

$$\Delta_N^c = 1 + a_s\Big[c_1^2 \log^2 N + c_1^1 \log N + c_1^0 + d_1^1 \frac{\log N}{N} + \mathcal{O}(1/N)\Big]$$

$$+a_s^2\Big[c_2^4 \log^4 N + \cdots + c_2^0 + d_2^3 \frac{\log^3 N}{N} + \cdots + \mathcal{O}(1/N)\Big]$$

$$+\cdots$$

$$+a_s^n\Big[c_n^{2n} \log^{2n} N + \cdots + d_n^{2n-1} \frac{\log^{2n-1} N}{N} + \cdots + \mathcal{O}(1/N)\Big]$$

$a_s \log N$ is of order `one` when a_s is very small at every order 1/N

Resummed Coefficient function

Inclusion of the NSV logarithms modifies the existing resumed expression as :

$$\omega = 2\beta_0 a_s(\mu_R^2) \log N$$

$$\Delta_{c,N}(q^2, \mu_R^2, \mu_F^2) = C_0^c(q^2, \mu_R^2, \mu_F^2) \exp\left(\Psi_N^c(q^2, \mu_F^2)\right)$$

$$\Psi^c_{\mathrm{SV,N}} = \log(g_0^c(a_s(\mu_R^2))) + g_1^c(\omega)\log N + \sum_{i=0}^{\infty} a_s^i(\mu_R^2) g_{i+2}^c(\omega)$$

Known since 1989

[Sterman et.al]
[Catani et.al]

$$\Psi^c_{\mathrm{NSV},N} = \frac{1}{N}\sum_{i=0}^{\infty} a_s^i(\mu_R^2)\Big(\bar{g}_{i+1}^c(\omega) + h_i^c(\omega, N)\Big)$$

$$h_i^c(\omega, N) = \sum_{k=0}^{i} h_{ik}^c(\omega) \;\log^k N.$$

New Result!!

Predictive power – N Space

Logarithmic Accuracy	Resummed Exponents
$\overline{\text{LL}}$	$\tilde{g}^q_{0,0}, g^q_1, \overline{g}^q_1, h^q_0$
$\overline{\text{NLL}}$	$\tilde{g}^q_{0,1}, g^q_2, \overline{g}^q_2, h^q_1$
$\overline{\text{NNLL}}$	$\tilde{g}^q_{0,2}, g^q_3, \overline{g}^q_3, h^q_2$

$\rightarrow a_s^i \frac{1}{N} \log^{2i-1} N$

$\rightarrow a_s^i \frac{1}{N} \log^{2i-2} N$

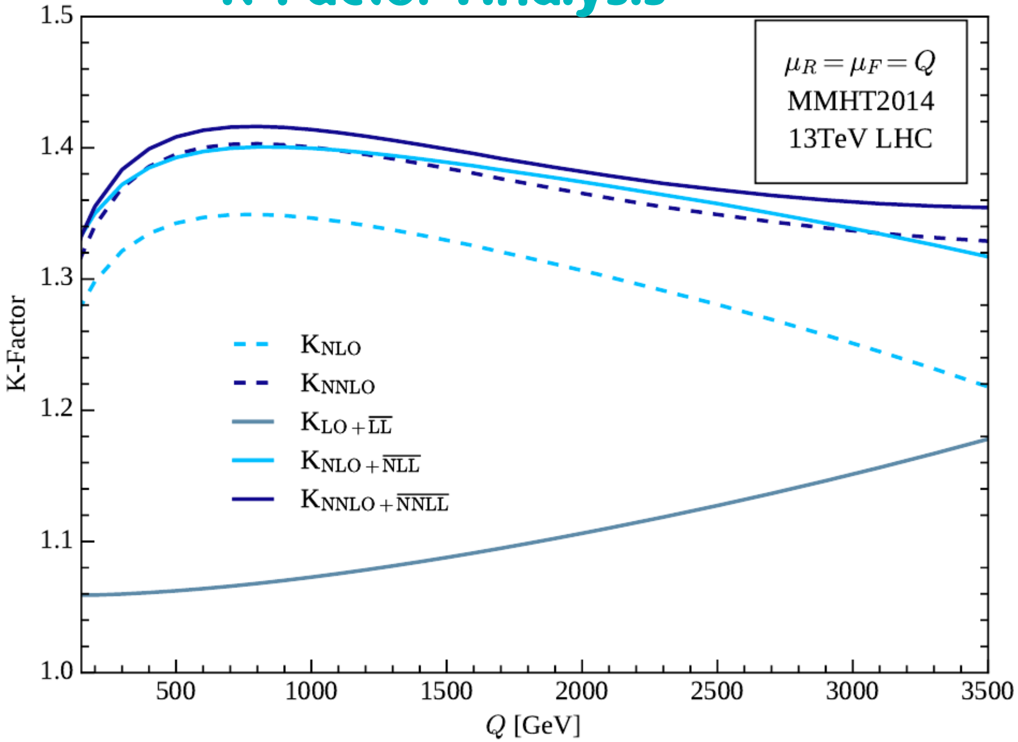
$\rightarrow a_s^i \frac{1}{N} \log^{2i-n} N$

$\overline{\text{LL}}$	$\overline{\text{NLL}}$	…	$\overline{\text{N}^{\text{n}}\text{LL}}$
$a_s \frac{1}{N} \log N$	$a_s^2 \frac{1}{N} \log^2 N$		$a_s^n \frac{1}{N} \log^n N$
$a_s^3 \frac{1}{N} \log^3 N$	$a_s^4 \frac{1}{N} \log^4 N$		
$a_s^5 \frac{1}{N} \log^5 N$	$a_s^6 \frac{1}{N} \log^6 N$		
⋮	⋮	…	⋮
$a_s^i \frac{1}{N} \log^{2i-1} N$	$a_s^i \frac{1}{N} \log^{2i-2} N$		$a_s^i \frac{1}{N} \log^{2i-n} N$

Tower of NSV logarithms

Phenomenology - Drell-Yan

K-Factor Analysis



resummed curves lie above their corresponding fixed order ones - enhancement due to the resummed corrections

resummed curves are closer - resummed effect improves the reliability of perturbative Predictions

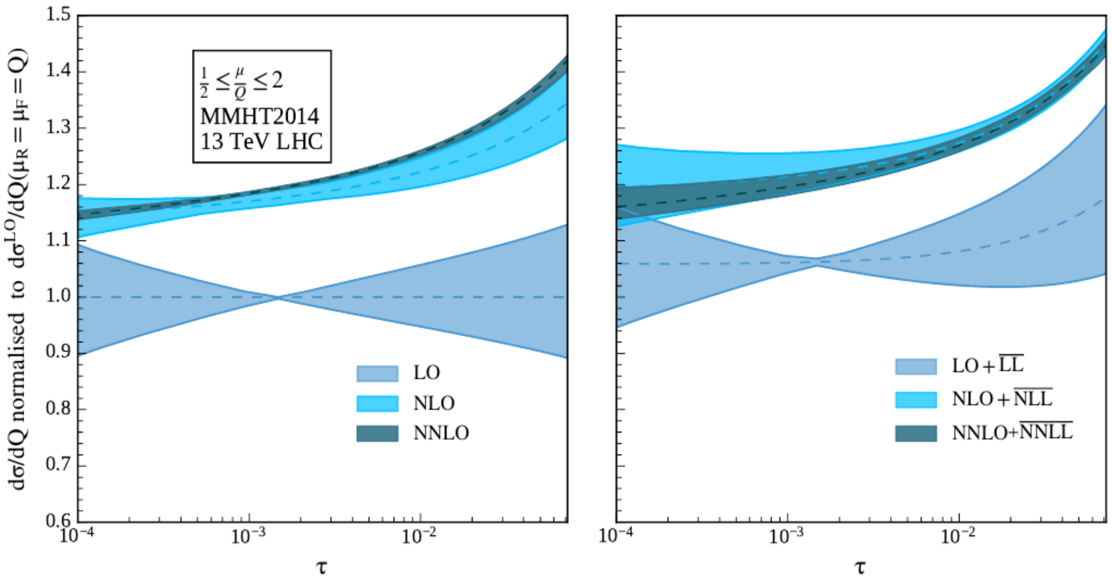
resummed correction decreases as we go for higher order resummed contributions

$\mu_R = \mu_F = Q$(GeV)	$\mathrm{LO} + \overline{\mathrm{LL}}$	NLO	$\mathrm{NLO} + \overline{\mathrm{NLL}}$	NNLO	$\mathrm{NNLO} + \overline{\mathrm{NNLL}}$
500	1.0624	1.3425	1.3925	1.3950	1.4082
1000	1.0728	1.3464	1.3995	1.4004	1.4138
2000	1.1062	1.3064	1.3739	1.3652	1.3818

$$\mathrm{K}\,(Q) = \frac{\dfrac{d\sigma}{dQ}(\mu_R = \mu_F = Q)}{\dfrac{d\sigma^{\mathrm{LO}}}{dQ}(\mu_R = \mu_F = Q)}$$

Phenomenology - Drell-Yan

7-point scale uncertainities of the resummed results



resummed result shows a systematic reduction of the uncertainties with the inclusion of each logarithmic corrections

improvement at the NLO+$\overline{\text{NLL}}$ than at the NNLO+$\overline{\text{NNLL}}$
Why ?

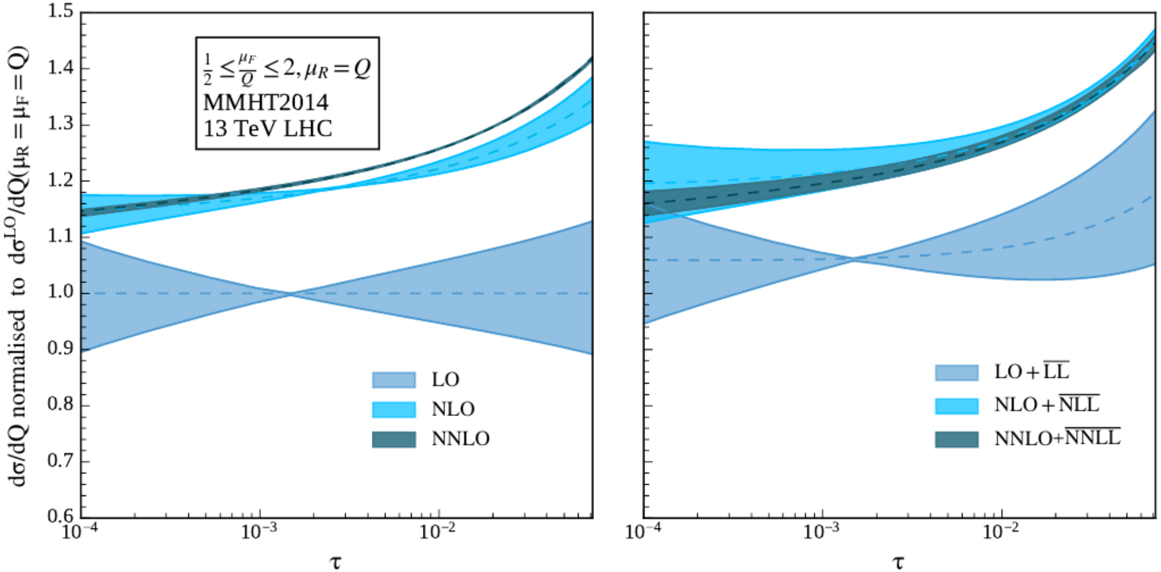
Let us analyze the effect of each scale individually on the resummed result

Q	LO	LO+$\overline{\text{LL}}$	NLO	NLO+$\overline{\text{NLL}}$	NNLO	NNLO+$\overline{\text{NNLL}}$
1000	$2.3476^{+4.10\%}_{-3.92\%}$	$2.5184^{+4.49\%}_{-4.25\%}$	$3.1609^{+1.79\%}_{-1.69\%}$	$3.2857^{+2.08\%}_{-1.18\%}$	$3.2876^{+0.20\%}_{-0.31\%}$	$3.3191^{+1.13\%}_{-0.86\%}$
2000	$0.0501^{+8.50\%}_{-7.46\%}$	$0.0554^{+9.10\%}_{-7.91\%}$	$0.0654^{+2.83\%}_{-2.98\%}$	$0.0688^{+1.43\%}_{-1.23\%}$	$0.0684^{+0.37\%}_{-0.62\%}$	$0.0692^{+0.89\%}_{-0.78\%}$

$\mu = \{\mu F , \mu R\}$ is varied in the range [1/2Q, 2Q] keeping the ratio $\mu R / \mu F$ not larger than 2 and smaller than 1/2.

Phenomenology - Drell-Yan

Uncertainities w.r.t μ_F scale variation



resummed bands look similar to that of 7-point bands

width of NLO+$\overline{NLL}$ and NNLO+$\overline{NNLL}$ bands become slightly thinner

contribution to the width of the 7-poiint bands mainly comes from the uncertainties from the μF variations

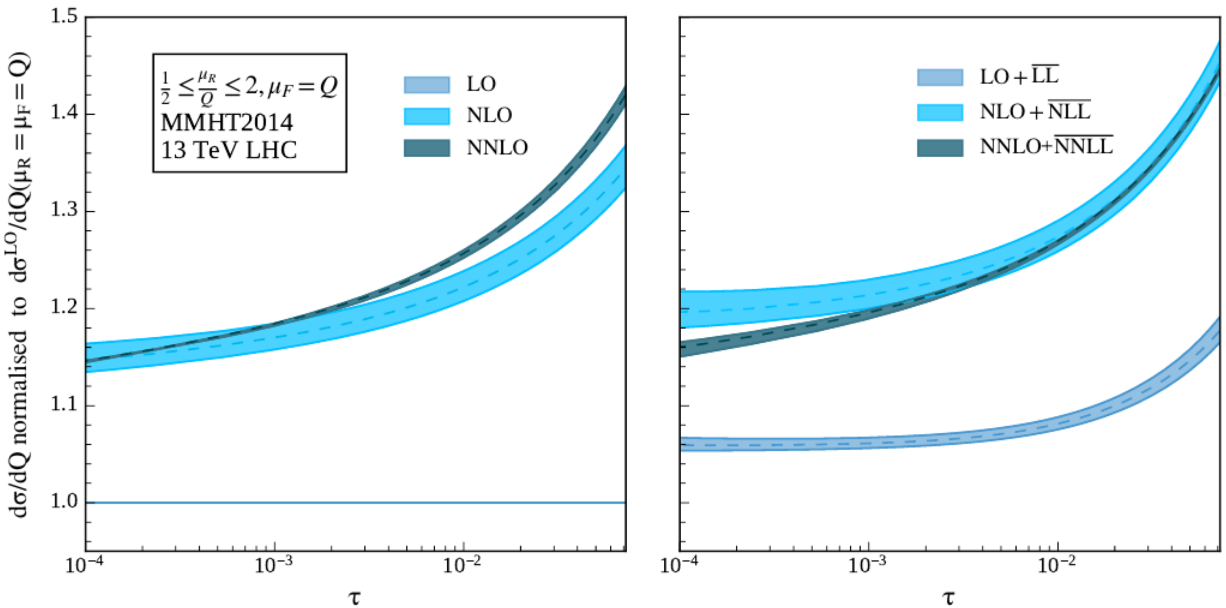
Missing qg-channel resummed contribution !
Missing resummed PDFs !

NLO :	$q\bar{q}$	->	22 %
	qg	->	-5 %
NNLO :	$q\bar{q}$	->	4.9 %
	qg	->	-2.8 %

unavailability of the qg resummed collinear logarithms leads to more uncertainty at NNLO+$\overline{NNLL}$

Phenomenology - Drell-Yan

Uncertainities w.r.t μ_R scale variation



NNLO+$\overline{\text{NNLL}}$ the error band becomes substantially thinner

each partonic channel is invariant under μ_R variation and hence inclusion of more corrections within a channel is expected to reduce the uncertainity

Inclusion of resummed result reduces the μ_R uncertainly remarkably as compared to the fixed order ones

Summary & Outlook

Using collinear factorisation and RG invariance and exploiting fixed order results, we propose an all order formula.

We propose an integral representation which can resume both SV and NSV logarithms to all orders.

Hence we have extended the Resummation of the NSV logarithms till NNLL accuracy.

We find the SV + NSV resummed results give significant contributions owing to the large coefficients of the NSV terms.

Summary & Outlook

The inclusion of resummed NSV terms improves perturbative convergence and reduces the uncertainty from the choice of renormalisation scale.

The absence of quark gluon initiated contributions to NSV part in the resummed terms leaves large factorisation scale dependence indicating their importance at NSV level for DY.

Summary & Outlook

What more to do ?

Modify the existing formalism for off-Diagonal Channels.

THANK YOU

Additional Slides ...

Soft-Virtual (SV)

$$\Delta_{ab}^{SV,(i)}(z) = \delta_{a\bar{a}} \left(\Delta_{\bar{a}b,\delta}\ \delta(1-z) + \sum_{k=0}^{2i-1} \Delta_{\bar{a}b,\mathcal{D}_k}^{(i)} \mathcal{D}_k(z) \right)$$

$$\mathcal{D}_k = \left(\frac{\log^k(1-z)}{(1-z)} \right)_+$$

Plus distribution

Regular part

$$\Delta_{ab}^{reg,(i)}(z) = \sum_{k=0}^{2i-1} \sum_{l=0}^{\infty} \Delta_{ab,l,k}^{reg,(i)} (1-z)^l \log^k(1-z)$$

Next-to Soft-Virtual (NSV)

$$\Delta_{ab}^{NSV,(i)}(z) = \sum_{k=0}^{2i-1} \Delta_{ab,0,k}^{reg,(i)} \log^k(1-z)$$

Form Factor – K+G Eqn

IR singularities factorise

[Sen,sterman,Magnea]

[Moch,Vogt,Vermasern; Ravindran]

$$\hat{F}^c(Q^2,\mu^2,\epsilon) = Z_{IR}(Q^2,\mu^2,\mu_R^2,\epsilon)\hat{F}_c^{fin}(Q^2,\mu^2,\mu_R^2,\epsilon)$$

universal IR counter term contains poles

Finite part

Differentiating both sides with respect to Q^2, we obtain **K+G equation** for the FFs

$$Q^2\frac{d}{dQ^2}\log\hat{F}^c = \frac{1}{2}\left[K^c\left(\hat{a}_s,\frac{\mu_R^2}{\mu^2},\varepsilon\right) + G^c\left(\hat{a}_s,\frac{Q^2}{\mu_R^2},\frac{\mu_R^2}{\mu^2},\varepsilon\right)\right]$$

Poles — No Poles

RG Invariance

$$\mu_R^2\frac{d}{d\mu_R^2}K^c(a_s(\mu_R^2)) = -\mu_R^2\frac{d}{d\mu_R^2}G^c(a_s(\mu_R^2)) = -\overline{A}^c(a_s(\mu_R^2))$$

$$A_q = \frac{C_F}{C_A}A_g$$

Maximally non-abelian, verified up to 4 loops

Factorisation – Diagonal channel

For Drell-Yan process:

Diagonal Channel:

$$\frac{\hat{\sigma}_{q\bar{q}}}{z\sigma_0} = \Gamma^T_{qq} \otimes \frac{\Delta_{qq}}{z} \otimes \Gamma_{q\bar{q}} + \Gamma^T_{qq} \otimes \frac{\Delta_{qg}}{z} \otimes \Gamma_{g\bar{q}} + \cdots$$

In the threshold limit z -> 1 , keeping only $\left(\frac{\ln(1-z_i)}{(1-z_i)}\right)_+$ $\delta(1-z_i)$ SV

$\log^k(1-z_i), \quad k = 0, \cdots \infty$ next to SV

dropping $(1-z_i)^k, \quad k = 1, \cdots \infty$

$$\frac{\hat{\sigma}^{\rm sv+nsv}_{q\bar{q}}}{z\sigma_0} = \Gamma^T_{qq} \otimes \Delta^{\rm sv+nsv}_{q\bar{q}} \otimes \Gamma_{\bar{q}\bar{q}} \,.$$

Remarkably Simple form !

Factorisation – off-diagonal channel

Off-diagonal Channel: $$\frac{\hat{\sigma}_{qg}}{z\sigma_0} = \Gamma^T_{qq} \otimes \Delta_{qq} \otimes \Gamma_{qg} + \Gamma^T_{qq} \otimes \Delta_{qg} \otimes \Gamma_{gg} + \cdots$$

In the threshold limit z -> 1 , keeping only $\log^k(1 - z_i), \quad k = 0, \cdots \infty$ next to SV

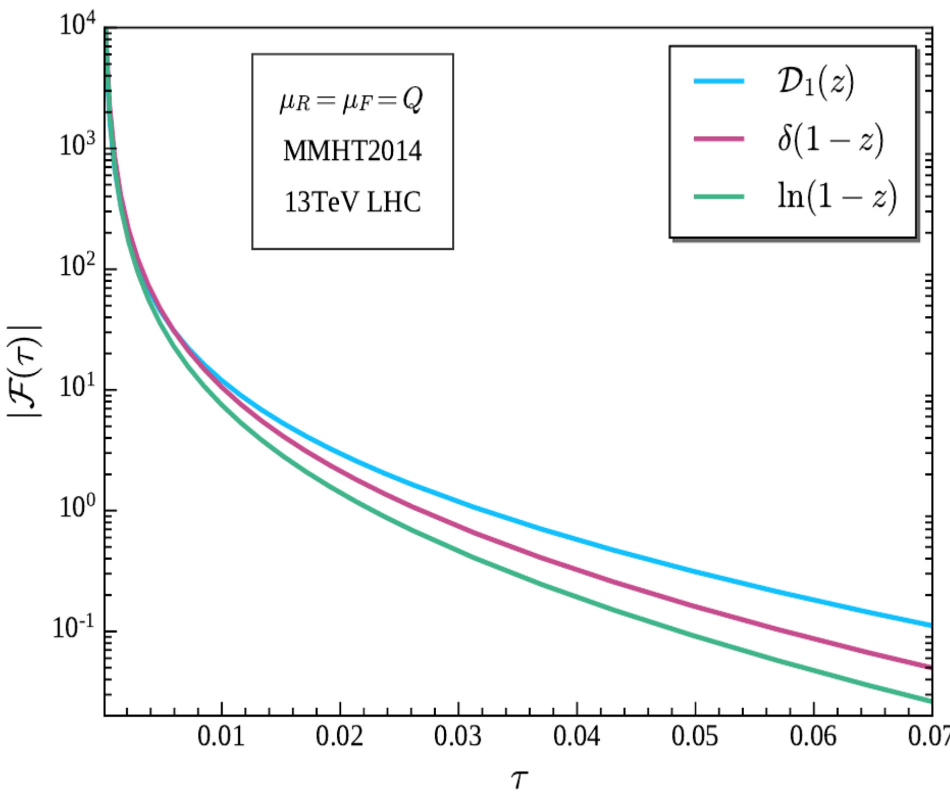
$$\frac{\hat{\sigma}^{\mathrm{sv+nsv}}_{qg}}{z\sigma_0} = \Gamma^T_{qq} \otimes \Delta^{\mathrm{sv+nsv}}_{q\bar{q}} \otimes \Gamma_{\bar{q}g} + \Gamma^T_{qq} \otimes \Delta^{\mathrm{nsv}}_{qg} \otimes \Gamma_{gg} \,.$$

NNSV terms

dropping $(1 - z_i)^k, \quad k = 1, \cdots \infty$

Getting complicated due to Mixing of channels

NSV contributions



$Q = \mu_R = \mu_F$ (GeV)	SV		NSV	
200	$\mathcal{D}_3$	6.13%	$\ln^3(1-z)$	12.4%
	$\mathcal{D}_2$	1.49%	$\ln^2(1-z)$	7.83%
	$\mathcal{D}_1$	-3.24%	$\ln^1(1-z)$	-2.82%
	$\mathcal{D}_0$	-4.74%	$\ln^0(1-z)$	-6.57%
	$\delta(1-z)$	0.003%		
TOTAL		-0.035%		10.8%

$Q = \mu_R = \mu_F$ (GeV)	SV		NSV	
200	$\mathcal{D}_5$	5.44%	$\ln^5(1-z)$	8.60%
	$\mathcal{D}_4$	2.62%	$\ln^4(1-z)$	9.82%
	$\mathcal{D}_3$	-2.73%	$\ln^3(1-z)$	-1.54%
	$\mathcal{D}_2$	-4.25%	$\ln^2(1-z)$	-8.98%
	$\mathcal{D}_1$	-1.94%	$\ln^1(1-z)$	-6.14%
	$\mathcal{D}_0$	-0.146%	$\ln^0(1-z)$	-1.28%
	$\delta(1-z)$	1.03%		
TOTAL		0.026%		0.47%

$$\mathcal{F}(\tau) = \int_\tau^1 \frac{dz}{z} \tilde{\Phi}_{q\bar{q}}\left(\frac{\tau}{z}\right) \mathcal{G}(z)$$

$$\mathcal{G}(z) = \{\delta(1-z), \mathcal{D}_1(z), \ln(1-z)\}$$

[AAH,PM,VR,AS,ST, hep-ph/2107.09717]

The Matched Result

Now we perform Mellin Inversion of the resummed result to study the numerical impact.

$$\sigma_N^{\mathrm{N^nLO}+\overline{\mathrm{N^nLL}}} = \sigma_N^{\mathrm{N^nLO}} + \sigma^{(0)} \sum_{ab\in\{q,\bar{q}\}} \int_{c-i\infty}^{c+i\infty} \frac{dN}{2\pi i} (\tau)^{-N} \delta_{a\bar{b}} f_{a,N}(\mu_F^2) f_{b,N}(\mu_F^2) \times \left(\Delta_{q,N}\Big|_{\overline{\mathrm{N^nLL}}} - \Delta_{q,N}\Big|_{tr\ \mathrm{N^nLO}} \right).$$

The resummed results are matched to the fixed order result in order to avoid any double counting of threshold logarithms

The contour c in the Mellin inversion is chosen according to Minimal prescription

Used for phenomenological studies