

# Soft Gluon Rsummation for Dijet Production in pp Collisions

Reference:

Peng Sun, C.-P. Yuan, Feng Yuan, arXiv: 1405.1105; arXiv: 1506.06170

Hatta, Xiao, Yuan, Zhou, arXiv: 2010.10774; arXiv: 2106.05307

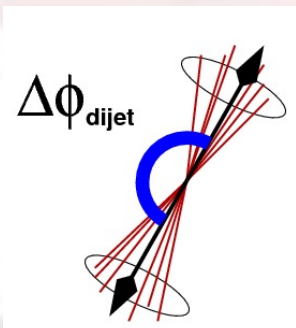
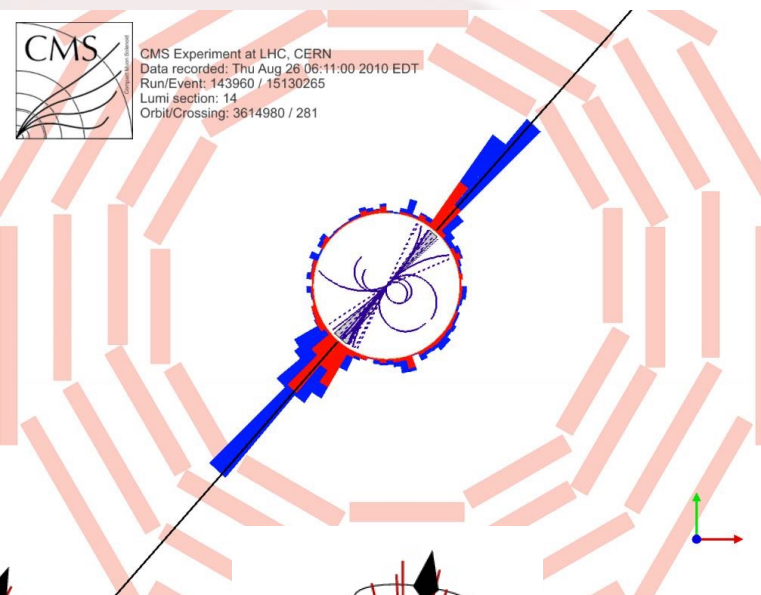


# Outlines

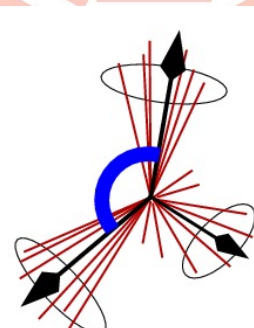
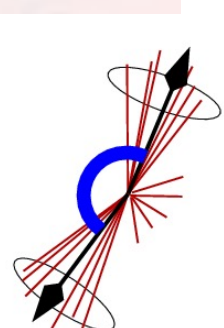
- Experiments at Tevatron and LHC
- Soft gluon resummation in dijet production
  - Collins-Soper-Sterman
- Azimuthal angular asymmetry due to soft gluon radiation

# Dijet production at the hadron colliders

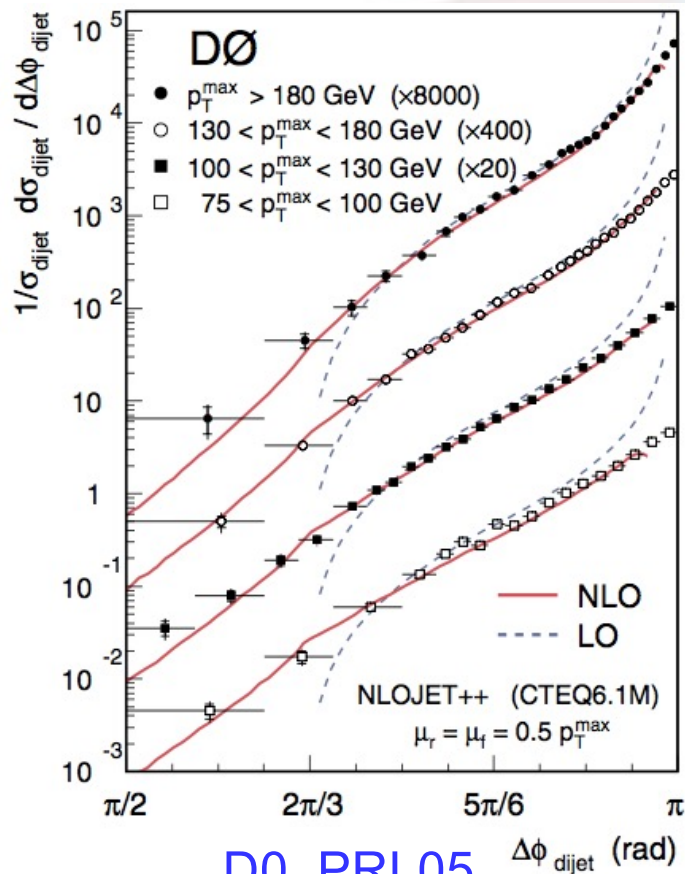
- Most abundant events
- Almost back-to-back
- De-correlation comes
  - Hard gluon jet
  - Soft gluon radiation



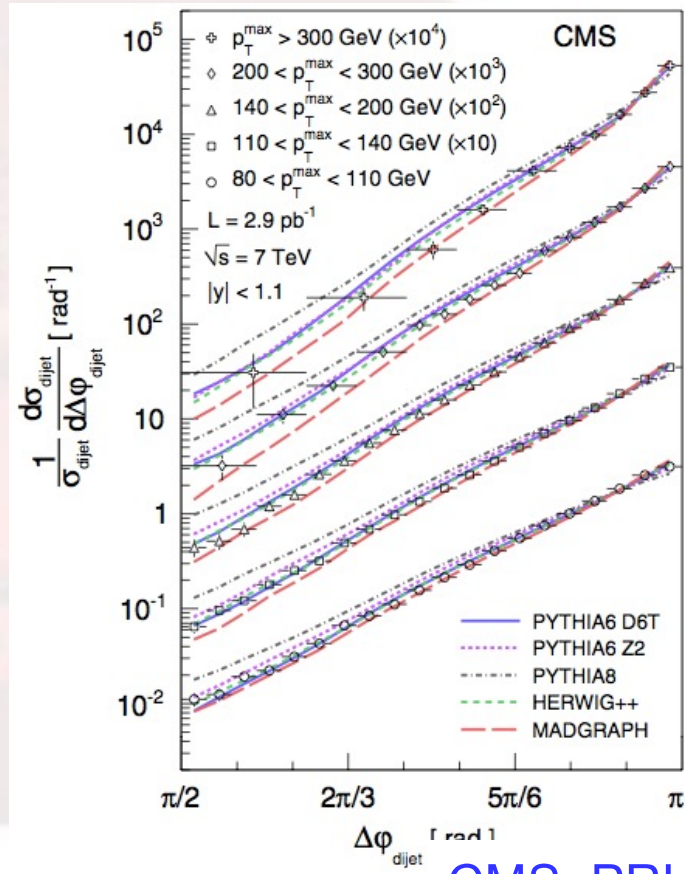
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# Beautiful data from Tevatron/LHC

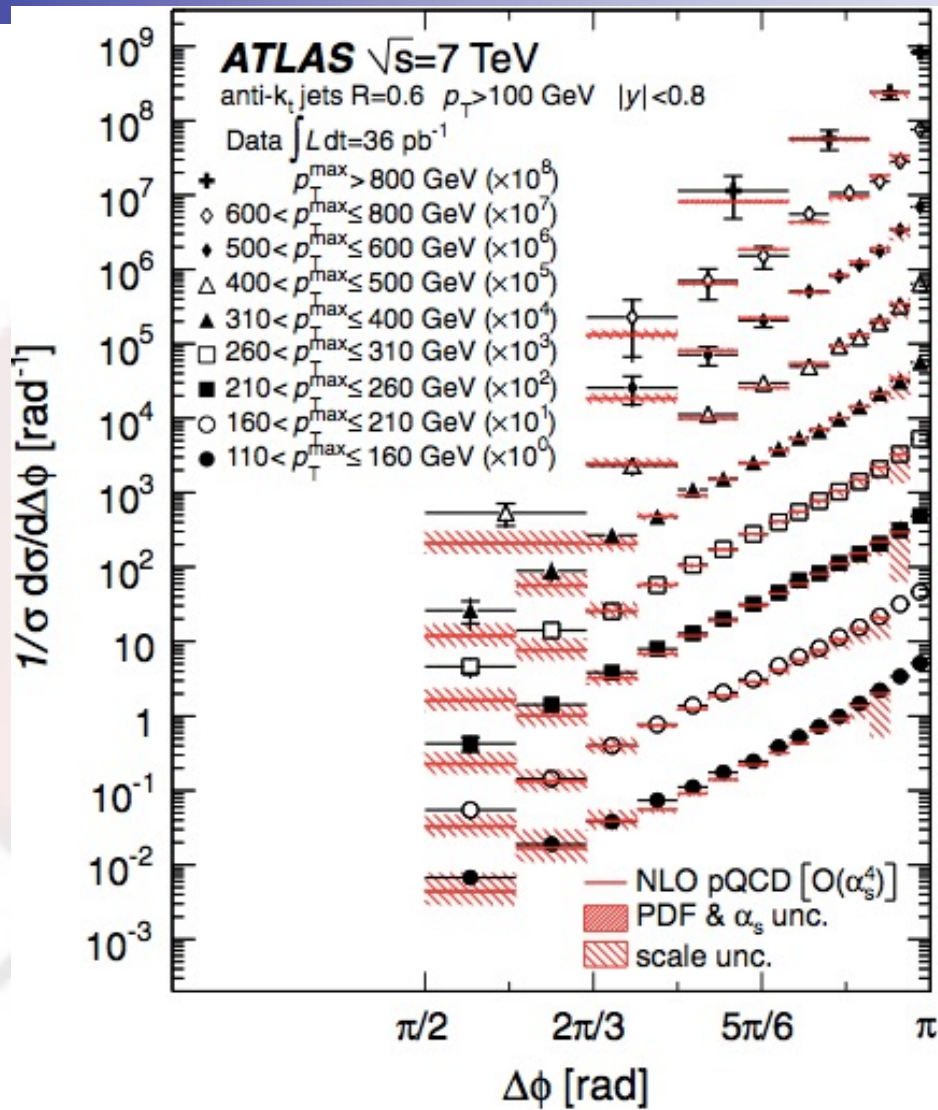


D0, PRL05



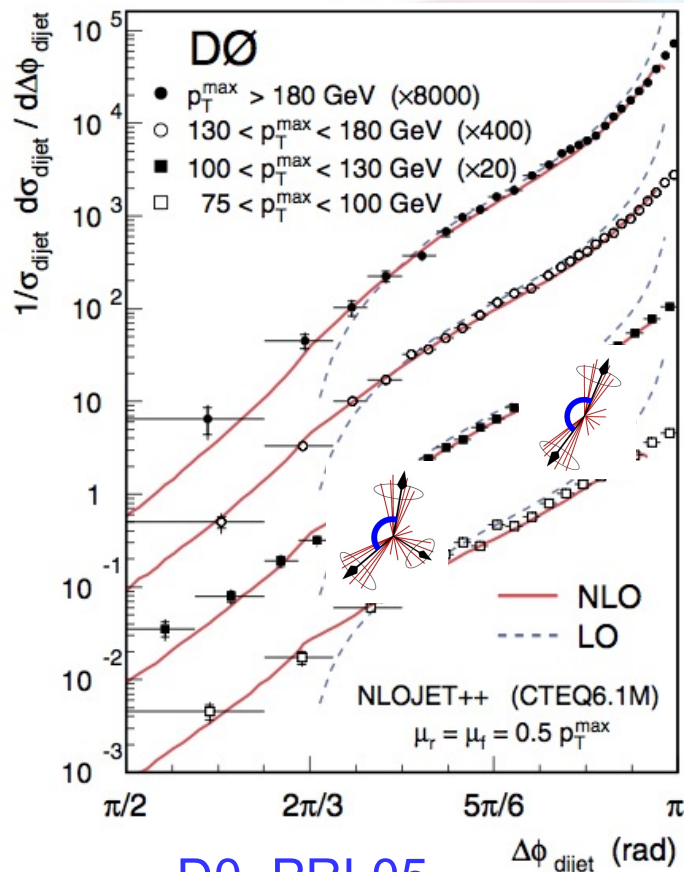
CMS, PRL11 4





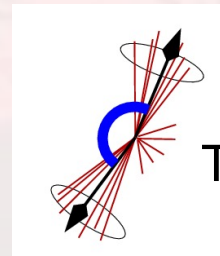
ATLAS, PRL11

# QCD calculations



D0, PRL05

- Fixed order calculations divergent around  $\pi$ , where soft gluon radiation dominates
- All order resummation is needed to understand the physics around here
  - Two separate scales  $P_T \gg q_T$



Leading  $P_T$

Total  $q_T \approx P_T \sin(\Delta\phi)$

# Soft gluon radiation leads to Sudakov Logarithms

Sudakov, 1956; Collins-Soper-Sterman 1985

- Differential cross section depends on  $Q_1=q_T$ , where  $Q^2 \gg Q_1^2 \gg \Lambda_{\text{QCD}}^2$

$$\frac{d\sigma}{dQ_1^2} = \frac{1}{Q_1^2} f_1 \otimes f_2 \otimes \sum_i \alpha_s^i \ln^{2i-1} \frac{Q^2}{Q_1^2} + \dots$$

- Resummation of these large logs
  - In terms of transverse momentum dependent parton distributions and fragmentation functions and apply to
  - Semi-inclusive hadron production in DIS, Drell-Yan type of hard processes in pp collisions, e.g., Higgs, Z/W boson, ...

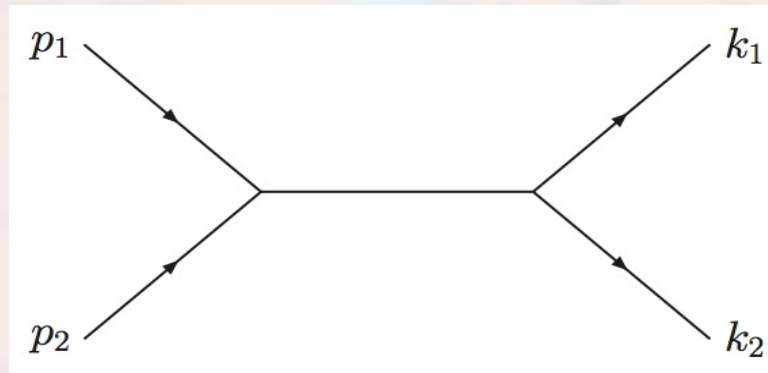
## Hard process with jet is different

- Final states carry color
  - Soft gluon radiation associated with the jet will contribute
  - Additional soft factor
- Jet algorithm will enter into the calculations as well
  - Only out of cone radiation contributes to the imbalance between the two jets



# Leading double logs in dijet case

- Power counting: each **incoming** parton contributes to a half of the associated color factor



**DL coefficient:**  
 $A^{(1)} = (C_{p_1} + C_{p_2})/2$

Banfi-Dasgupta-Delenda, PLB 2008  
Mueller-Xiao-Yuan, PRD 2013

# Beyond the leading double logs

- Jet size-dependence is computed by averaging the azimuthal angle between the soft gluon and leading jet
- Matrix form due to colored final state [Kidonakis-Sterman 1997](#)

$$x_1 f_a(x_1, \mu = b_0/b_\perp) x_2 f_b(x_2, \mu = b_0/b_\perp) e^{-S_{\text{Sud}}(Q^2, b_\perp)} \\ \text{Tr} \left[ \mathbf{H}_{ab \rightarrow cd} \exp\left[-\int_{b_0/b_\perp}^Q \frac{d\mu}{\mu} \gamma^{s^\dagger}\right] \mathbf{S}_{ab \rightarrow cd} \exp\left[-\int_{b_0/b_\perp}^Q \frac{d\mu}{\mu} \gamma^s\right] \right]$$

([Sun, C.-P. Yuan, F. Yuan, PRL 2014](#))

$$S_{\text{Sud}}(Q^2, b_\perp) = \int_{b_0^2/b_\perp^2}^{Q^2} \frac{d\mu^2}{\mu^2} \left[ \ln\left(\frac{Q^2}{\mu^2}\right) A + B + D_1 \ln \frac{Q^2}{P_T^2 R_1^2} + D_2 \ln \frac{Q^2}{P_T^2 R_2^2} \right]$$

**D:** color-factor for the jet

**R:** jet size

see also, heavy quark pair resummation:

[Zhu-Li-Li-Shao-Yang 2012](#)

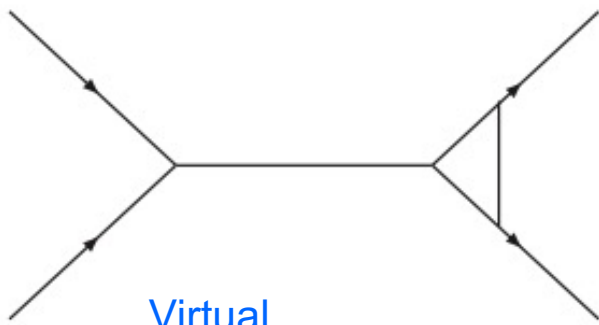
[Catani-Grazzini-Torre 2014](#)



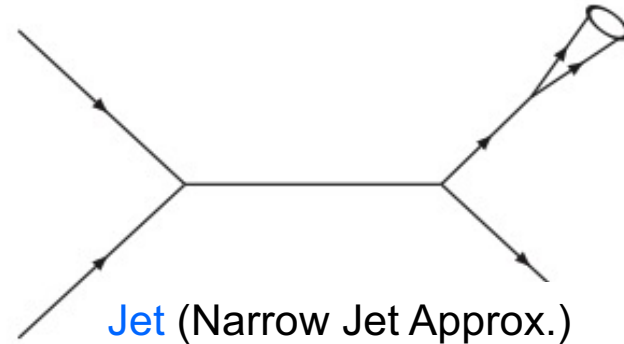
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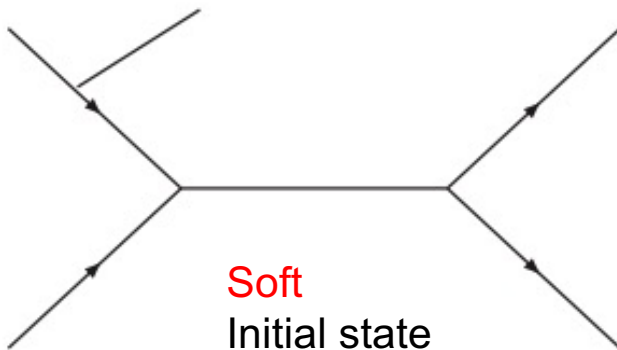
# Soft and collinear gluon at one-loop



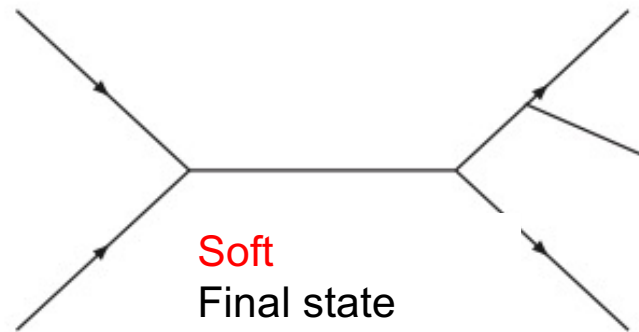
Virtual  
Ellis-Sexton 86



Jet (Narrow Jet Approx.)  
Jager-Stratmann-Vogelsang  
2004



Soft  
Initial state



Soft  
Final state  
(out of jet cone)

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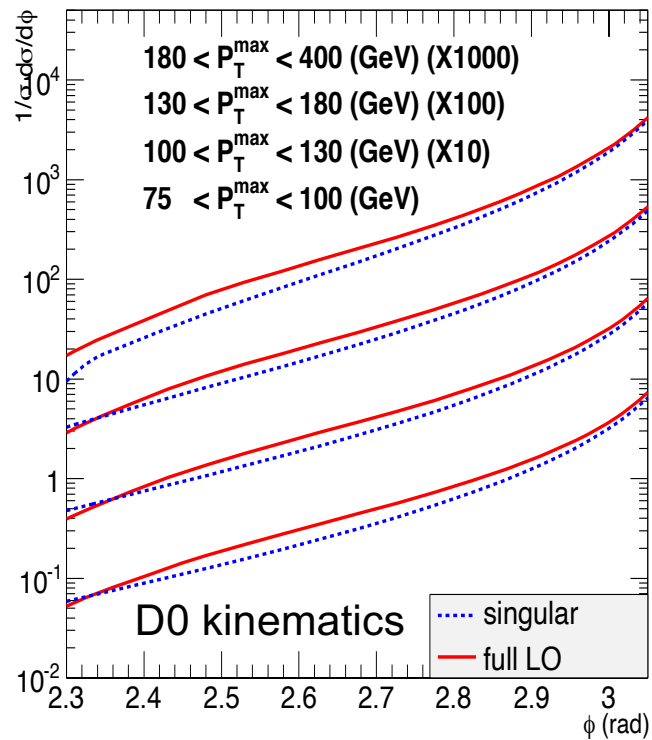
## Cross checks

- Divergences cancelled out between virtual, jet, soft contributions (dimensional regulation applied)
- Final results : **double logs**, **single logs**, ..

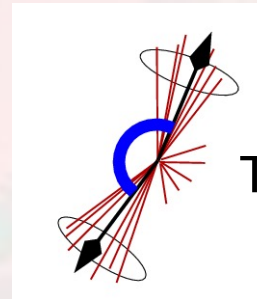
$$\begin{aligned}
 W^{(1)}(b_{\perp})|_{logs.} = & \frac{\alpha_s}{2\pi} \left\{ h_{q_i q_j \rightarrow q_i q_j}^{(0)} \left[ \underbrace{-\ln\left(\frac{\mu^2 b_{\perp}^2}{b_0^2}\right)}_{\text{red}} (\mathcal{P}_{qq}(\xi)\delta(1-\xi') + \mathcal{P}_{qq}(\xi')\delta(1-\xi)) - \delta(1-\xi) \right. \right. \\
 & \times \delta(1-\xi') \left( \underbrace{C_F \ln^2\left(\frac{Q^2 b_{\perp}^2}{b_0^2}\right)}_{\text{blue}} + \underbrace{\ln\left(\frac{Q^2 b_{\perp}^2}{b_0^2}\right)}_{\text{red}} \left( -3C_F + C_F \ln\frac{1}{R_1^2} + C_F \ln\frac{1}{R_2^2} \right) \right) \left. \right] \\
 & \left. - \delta(1-\xi)\delta(1-\xi') \ln\left(\frac{Q^2 b_{\perp}^2}{b_0^2}\right) \Gamma_{sn}^{(qq')} \right\}, \quad (71)
 \end{aligned}$$

Quark channel:  $q_i q_j \rightarrow q_i q_j$

# Compare to the full calculations



$$\begin{aligned}
 & \frac{\alpha_s}{2\pi^2} \frac{1}{q_{\perp}^2} \sum_{ab,a'b'} \sigma_0 \int \frac{dx'_1}{x'_1} \frac{dx'_2}{x'_2} x'_1 f_a(x'_1, \mu) x'_2 f_b(x'_2, \mu) \\
 & \times \left\{ h_{a'b' \rightarrow cd}^{(0)} \left[ \xi_1 \mathcal{P}_{a'/a}(\xi_1) \delta(1 - \xi_2) + \xi_2 \mathcal{P}_{b'/b}(\xi_2) \delta(1 - \xi_1) \right. \right. \\
 & \left. \left. + \delta(1 - \xi_1) \delta(1 - \xi_2) \delta_{aa'} \delta_{bb'} \left( (C_a + C_b) \ln \frac{Q^2}{q_{\perp}^2} + C_c \ln \frac{1}{R_1^2} + C_d \ln \frac{1}{R_2^2} \right) \right] \right. \\
 & \left. + \delta(1 - \xi_1) \delta(1 - \xi_2) \delta_{aa'} \delta_{bb'} \Gamma_{sn}^{ab \rightarrow cd} \right\} , \tag{10}
 \end{aligned}$$

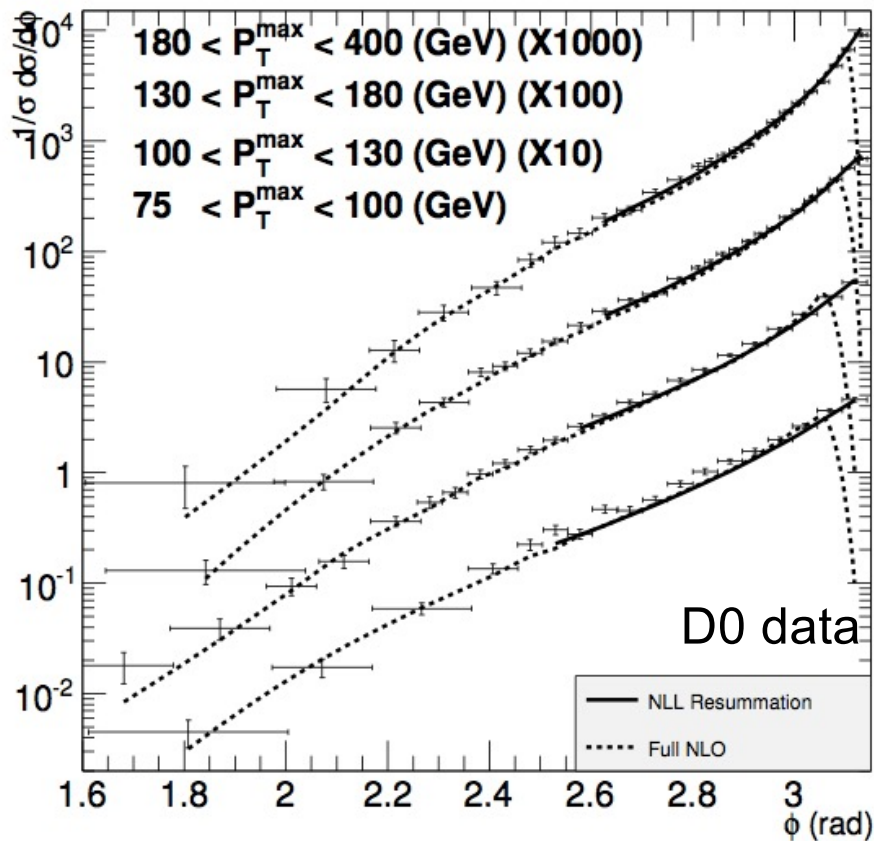


Leading  $P_T$

Total  $q_T \approx P_T \sin(\Delta\phi)$

full LO: Nagy 2002, NLOJET++

# Compare to the data

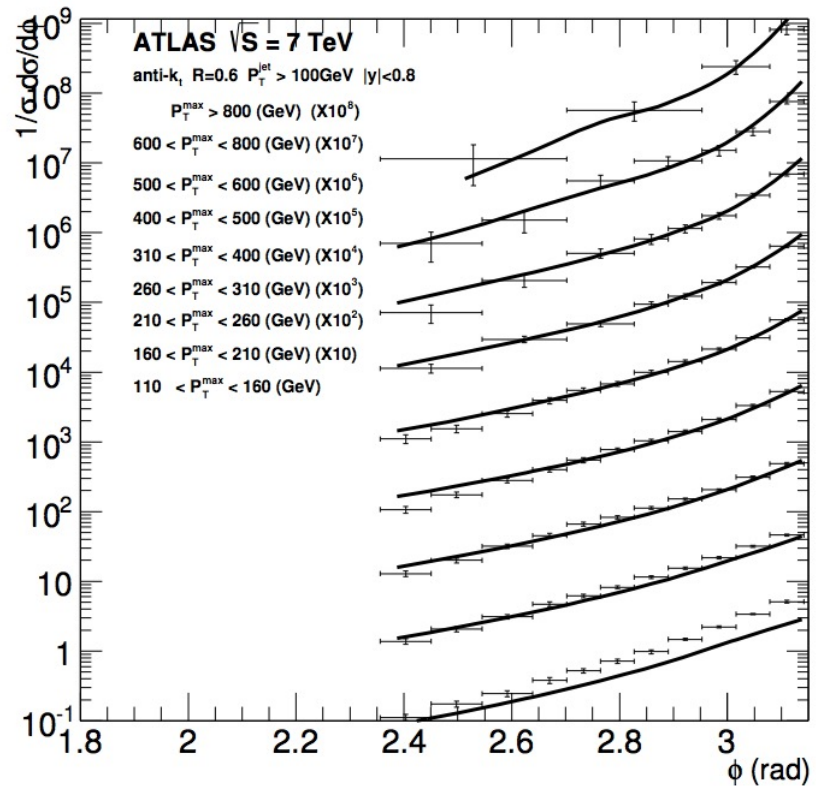
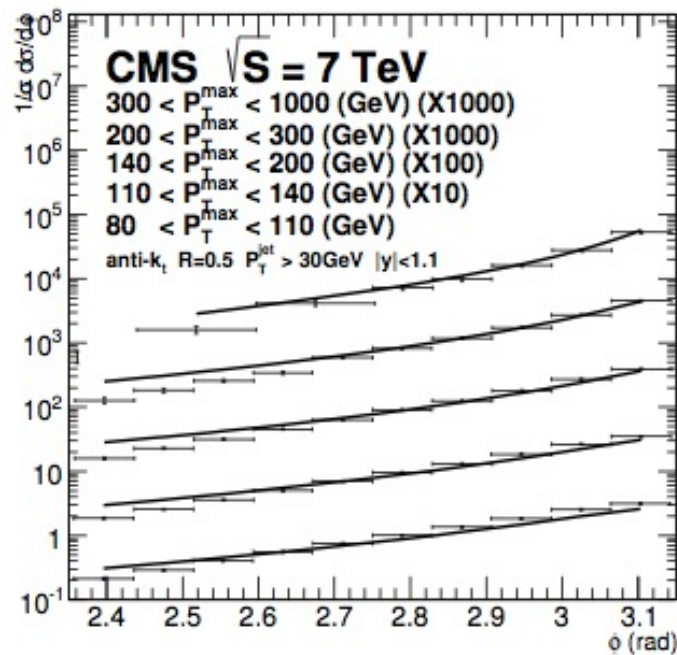


NLL Resummation:  
Sun, C.P. Yuan, F. Yuan, PRL2014

$$x_1 f_a(x_1, \mu = b_0/b_\perp) x_2 f_b(x_2, \mu = b_0/b_\perp) e^{-S_{\text{Sud}}(Q^2, b_\perp)} \text{Tr} \left[ \mathbf{H}_{ab \rightarrow cd} \exp \left[ - \int_{b_0/b_\perp}^Q \frac{d\mu}{\mu} \gamma^{s\dagger} \right] \mathbf{S}_{ab \rightarrow cd} \exp \left[ - \int_{b_0/b_\perp}^Q \frac{d\mu}{\mu} \gamma^s \right] \right]$$

Full NLO: Nagy 2002, NLOJET++

# At the LHC



# More simpler processes

- Higgs+Jet

- Sun, C.P. Yuan, F. Yuan, [1409.4121](#)

- Z(photon)+Jet

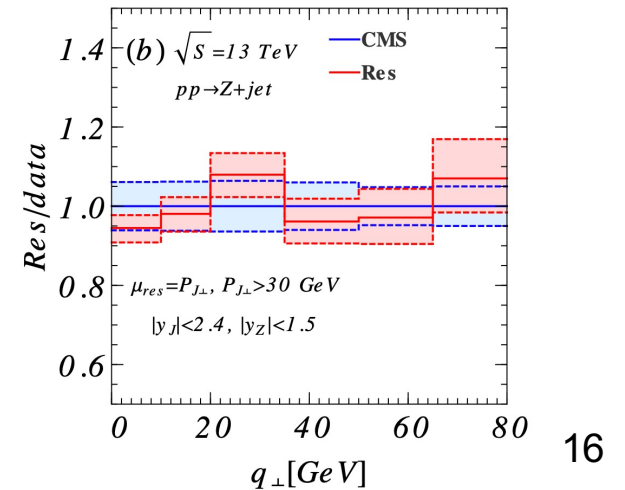
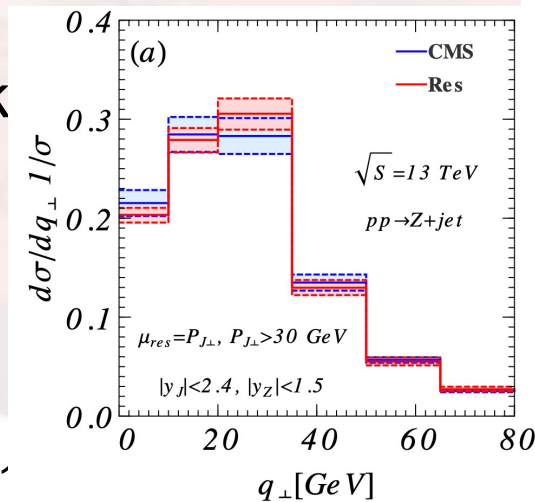
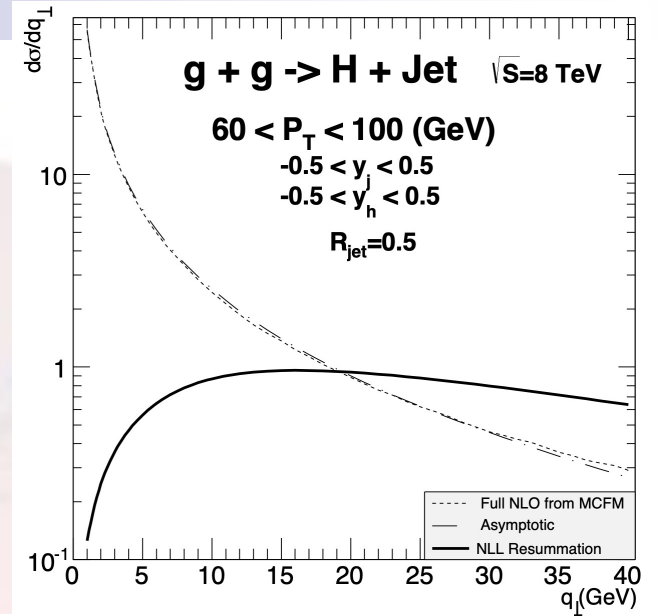
- Sun, Yan, C.P. Yuan, F. Yuan, [1810.03804](#)

- Dijet in DIS

- Rafael and Farid's talk

- Lepton+Jet

- Arratia's talk



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# Azimuthal angular asymmetries

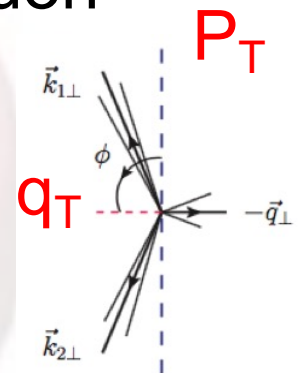
Catani-Grazzini-Sargsyan 2017

- Azimuthal angular asymmetries arise from soft gluon radiations

- $\phi$  is defined as angle between total and different transverse momenta of the two final state particles

- Infrared safe but divergent

- $\langle \cos(\phi) \rangle$ ,  $\langle \cos(2\phi) \rangle$ , ... divergent,  $\sim 1/q_T^2$
  - Integral is finite for small  $q_T$ -cutoff, resummation can be carried out for the harmonics,  $\langle \cos(n\phi) \rangle \sim q_T^n$
  - Examples discussed include  $V_j$ , top quark pair production





# Azimuthal angular correlations in jet production processes

Hatta, Xiao, Yuan, Zhou, arXiv: 2010.10774; 2106.05307

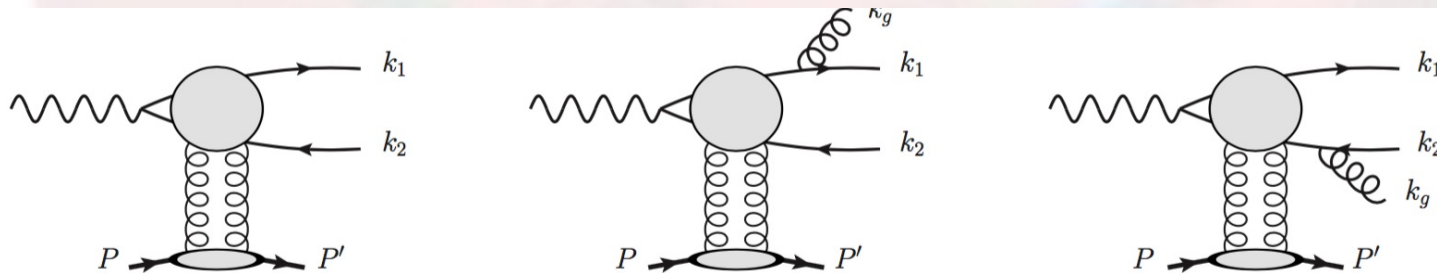
- Diffractive photoproduction of dijet
- Lepton plus jet production at the EIC
- Inclusive dijet in DIS

# Example: Diffractive dijet production

- Gluon radiation tends to be aligned with the jet direction

$$S_J(q_\perp) = \delta(q_\perp) + \frac{\alpha_s}{2\pi^2} \int dy_g \left( \frac{k_1 \cdot k_2}{k_1 \cdot k_g k_2 \cdot k_g} \right)_{\vec{q}_\perp = -\vec{k}_{g\perp}}$$

$$S_{J0}(|q_\perp|) + 2 \cos(2\phi) S_{J2}(|q_\perp|) + \dots$$



Hatta-Xiao-Yuan-Zhou, 2010.10774

anisotropy was neglected in an earlier paper:

Hatta-Mueller-Ueda-Yuan, 1907.09491

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## Leading power contributions, explicit result at $\alpha_s$

$$S_J(q_\perp) = S_{J0}(|q_\perp|) + 2 \cos(2\phi) S_{J2}(|q_\perp|)$$

$$S_{J0}(q_\perp) = \delta(q_\perp) + \frac{\alpha_0}{\pi} \frac{1}{q_\perp^2}, \quad S_{J2}(q_\perp) = \frac{\alpha_2}{\pi} \frac{1}{q_\perp^2},$$

where

$$\alpha_0 = \frac{\alpha_s C_F}{2\pi} 2 \ln \frac{a_0}{R^2}, \quad \alpha_2 = \frac{\alpha_s C_F}{2\pi} 2 \ln \frac{a_2}{R^2}.$$

$a_0, a_2$  are order 1 constants, so,

in the small-R limit,  $\langle \cos(2\phi) \rangle$  goes to 1

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# Leading logarithmic resummation

$$\tilde{S}_{J0}(b_{\perp}) = e^{-\Gamma_0(b_{\perp})}, \quad \tilde{S}_{J2}(b_{\perp}) = \alpha_2 e^{-\Gamma_0(b_{\perp})} \quad \Gamma_0(b_{\perp}) = \int_{\mu_b^2}^{P_{\perp}^2} \frac{d\mu^2}{\mu^2} \alpha_0$$

EIC

Kinematics:

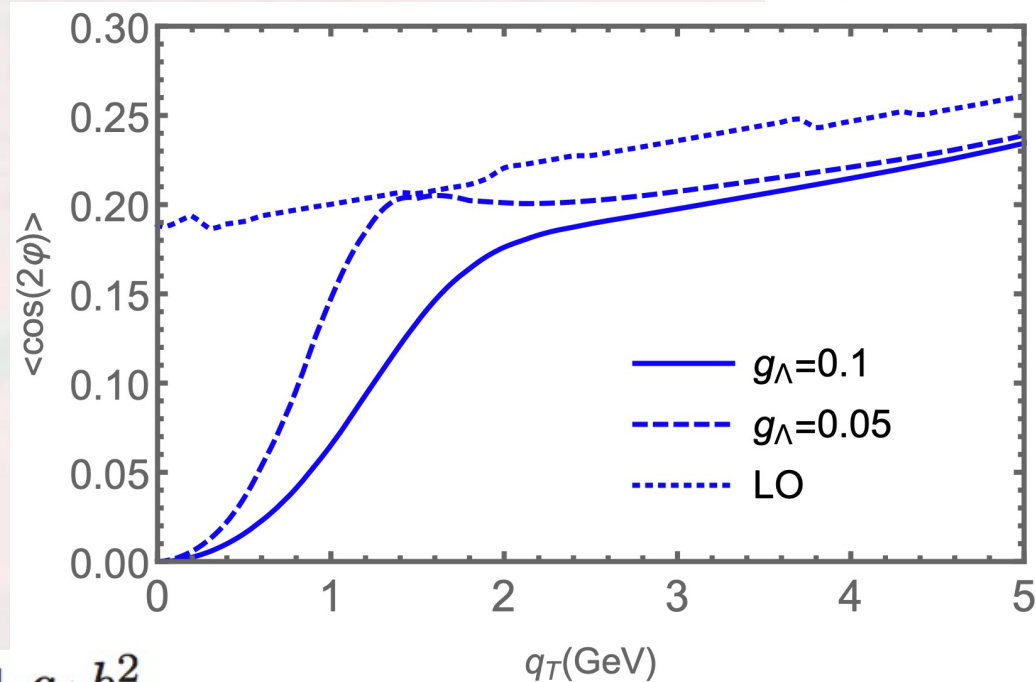
$P_T \sim 15 \text{ GeV}$

$R=0.4$

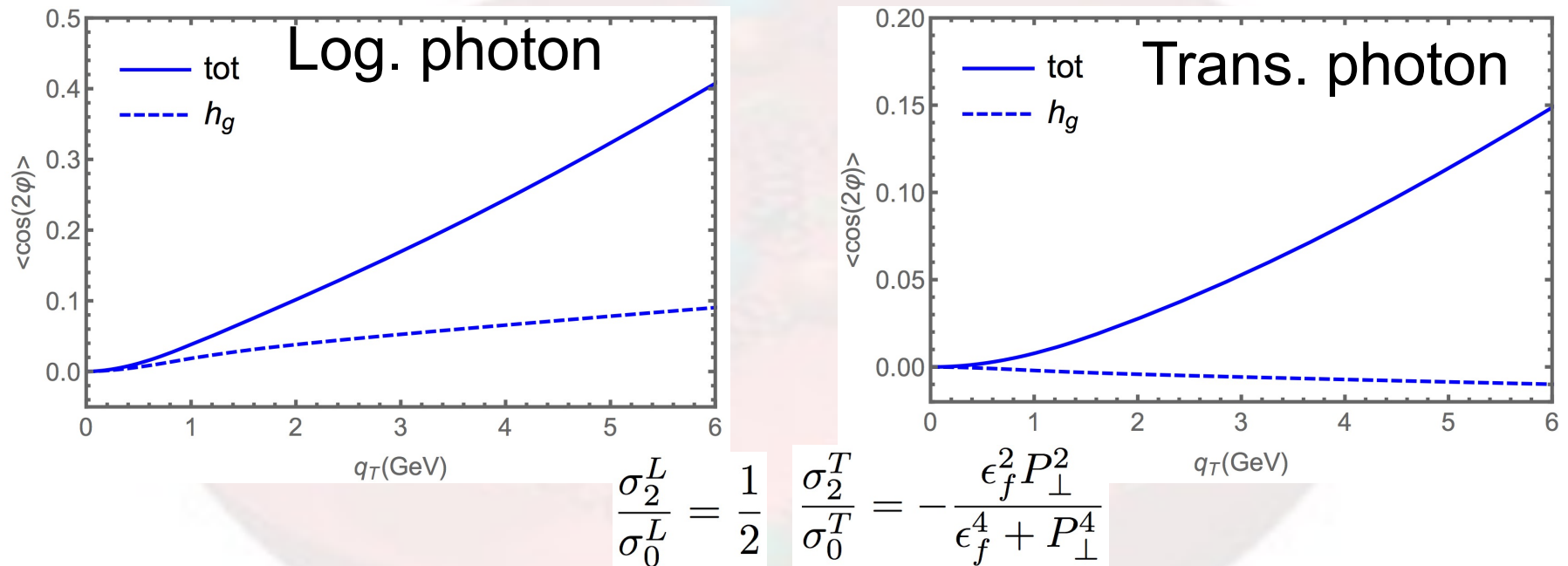
$y_1=y_2$

Non-pert. input:

$$\Gamma_0(b_{\perp}) \implies \Gamma_0(b_*) + g_{\Lambda} b_{\perp}^2$$



## Inclusive dijet at EIC: $Q=10\text{GeV}$ , $P_T=15\text{GeV}$ , $R=0.4$



- The difference between the above two purely comes from the linearly polarized gluon distribution



# Summary

- Soft gluon resummation is needed to describe the dijet correlation from Tevatron/LHC
- Soft gluon radiation contributes to azimuthal angular asymmetry as well
  - LLA resummation derived
  - More sophistic approach, see talk by Jürg Haag (Tue)
- Implement all these in a parton shower?
  - See previous talk