PB TMD fits at NLO with dynamical resolution scale

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REF RESUMMATION, EVOLUTION, FACTORIZATION WORKSHOP

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https://indico.desy.de/event/28334/

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Outline

- Recap of Parton Branching method
- Fixed and Dynamical soft-gluon resolution scale z_M
- Fits with fixed z_M at NLO
- Fits with dynamical z_M at NLO

Recap of PB TMDs

TMD evolution in the PB formalism:

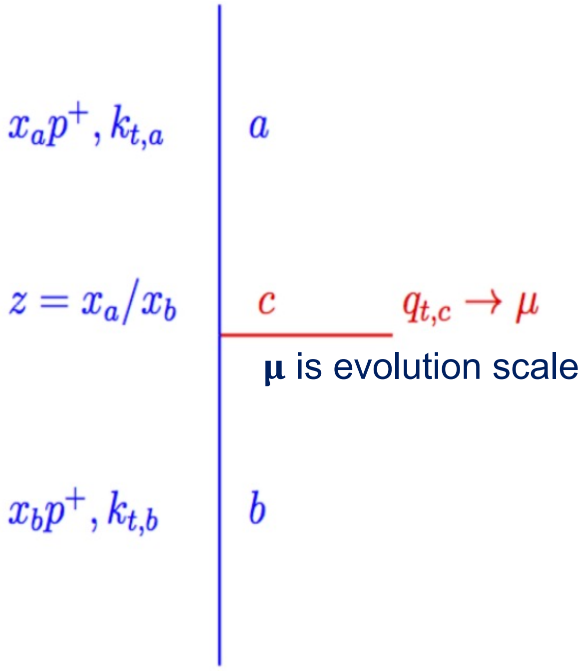
$$\widetilde{A}_a(x, k_\perp, \mu^2) =$$

$$\widetilde{A}_a(x, k_\perp, \mu_0^2)\,\Delta_a(\mu^2) + \sum_b \int \frac{d^2\mu'_\perp}{\pi\mu'^2}\,\Theta(\mu^2 - \mu'^2)\,\Theta(\mu'^2 - \mu_0^2) \times \frac{\Delta_a(\mu^2)}{\Delta_a(\mu'^2)} \int_x^{z_M} dz\, P^R_{ab}(z, \alpha_s(q_\perp)) \times \widetilde{A}_b\left(\frac{x}{z}, k_\perp + (1-z)\mu', \mu'^2\right)$$

- z_M **: Resolution scale** : → Resolvable branching : z < z_M
  → Non-resolvable branching : z > z_M

- **Splitting functions: P^R_{ab}(z):** The real emission parts of the DGLAP splitting function:
  Probability that a branching will happen

- **Sudakov form factor:** Δ_a= exp(- $\int_{\ln \mu_0^2}^{\ln \mu^2} d\,(ln\mu'^2) \sum_b \int_0^{z_M} dz\; z\; P^R_{ba}(\alpha_s, z)$)
  The probability of an evolution without any resolvable branching



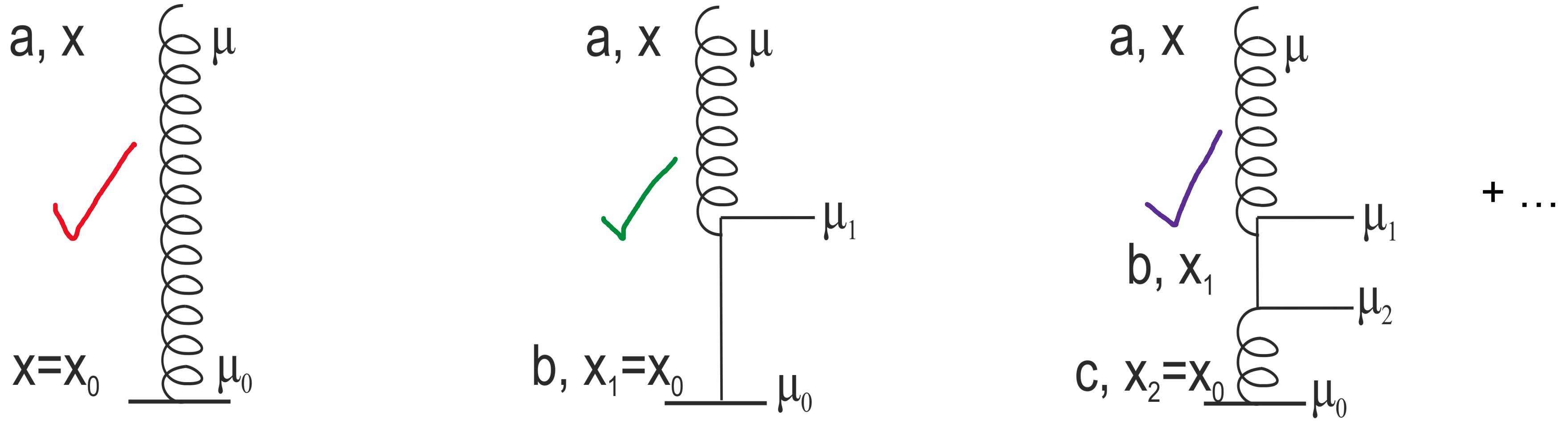
[Hautmann et all., JHEP 01 (2018) 070, 1708.03279]

At every step kinematics can be calculated!

Recap of PB TMDs

Iterative form of the PB evolution equation:

$$\widetilde{\mathbf{A}_a}(x, k_\perp, \mu^2) = \widetilde{\mathbf{A}_a}(x, k_\perp, \mu_0^2)\,\Delta_a(\mu^2) + \sum_b \int_{ln\mu_0^2}^{ln\mu^2} d\ln{\mu_1}^2 \times \frac{\Delta_a(\mu^2)}{\Delta_a({\mu_1}^2)} \int_x^{z_M} dz\, P_{ab}^R(z, \alpha_s(q_\perp))\Delta_b({\mu_1}^2)\times\widetilde{\mathbf{A}_b}\left(\frac{x}{z}, k_\perp + (1-z)\mu_1, {\mu_0}^2\right) + \ldots$$



Solvable by MC iterative technique:

- generated μ_1^2 : if $\mu_1^2 > \mu^2$ stop, otherwise splitting,
- generated the next scale μ_2^2: if $\mu_2^2 > \mu^2$ stop, otherwise splitting,
- …

Angular Ordering:

Color coherence phenomena:

- Angular ordering of the soft gluon emissions

$$\Theta_{i+1} > \Theta_i$$
$$|q_{\perp,i}| = (1 - z_i)\ |E_i| \sin\Theta_i$$

Associating "$|E_i| \sin\Theta_i$" with μ'

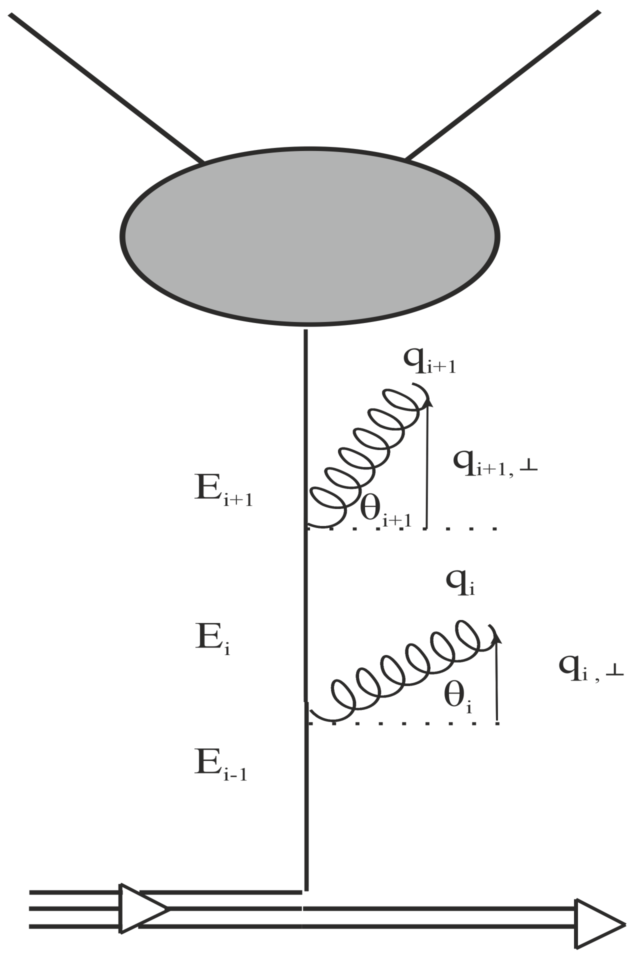
$$q_{\perp,i}^2 = (1 - z_i)^2\ \mu_i'^2$$

- The **argument of $\boldsymbol{\alpha_s}$** should be $\boldsymbol{q_\perp^2}$

$$\alpha_s(q_\perp^2) = \alpha_s((1 - z)^2 \mu'^2)$$

- resolvable & non-resolvable → condition on **min** $q_{\perp,i}^2 \rightarrow z_M$

$$z_M = 1 - \left(\frac{q_0}{\mu'}\right)$$



Fixed and dynamical resolution scale

- **Fixed z_M:**
- μ independent

$$z_M = 1 - \epsilon$$

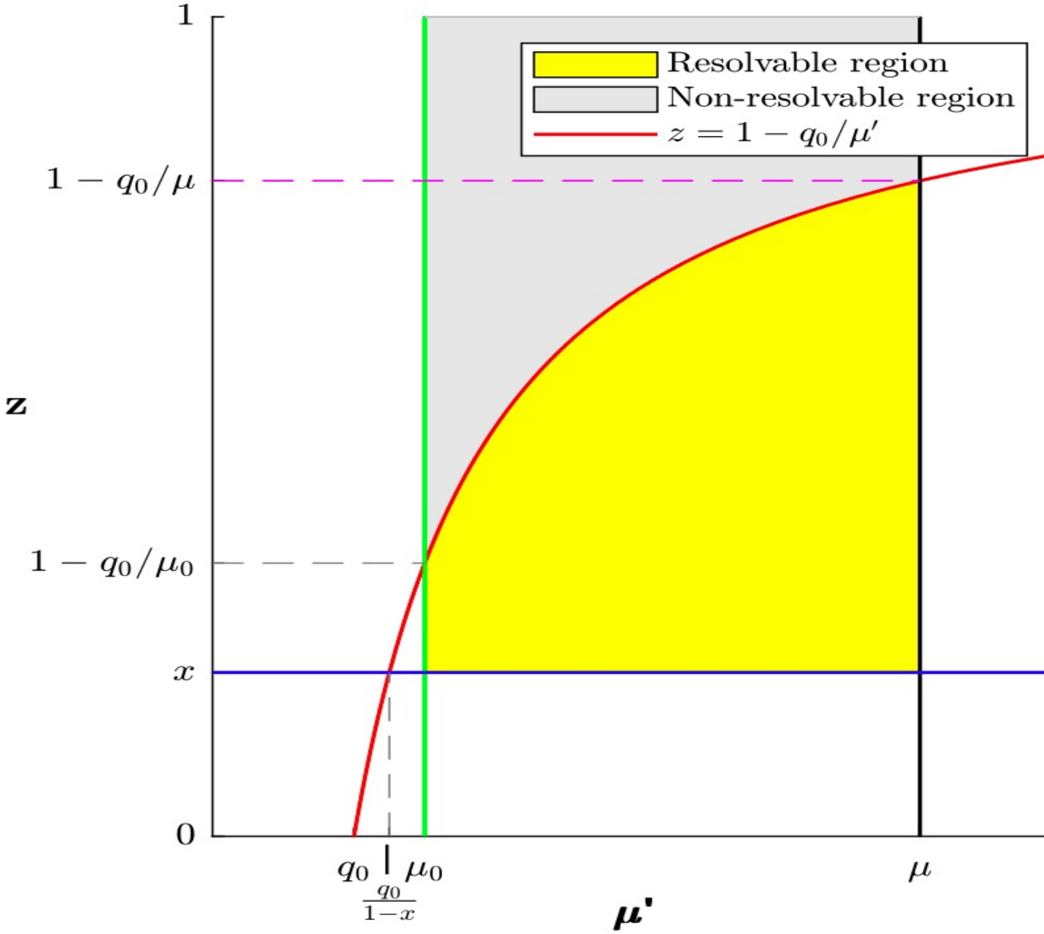
where ϵ is small: 10^{-3}, 10^{-4}, 10^{-5},...

- **Dynamical Resolution scale in Angular Ordering:**

$$z_M = 1 - \left(\frac{q_0}{\mu'}\right)$$

where q_0 is smallest emitted transverse momentum for resolvable partons

- Sudakov form factor Δ_a: non- resolvable region
- Splitting functions P^R_{ab}: resolvable region



[Hautmann, Keersmaekers, Lelek, van Kampen NuclPhysB (2019) 114795,1908.08524]

Dynamical resolution scale

The Condition on q_0 of $\mathbf{z_M = 1 - \left(\frac{q_0}{\mu'}\right)}$

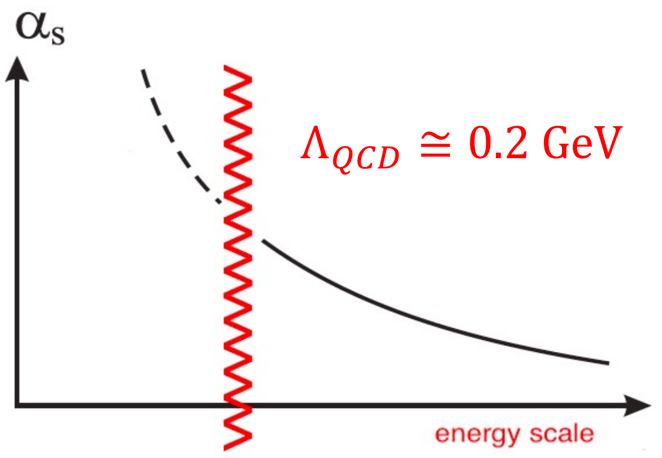
- PB equation for integrated distribution $\widetilde{f}_a(x,\mu^2) = \int d^2\boldsymbol{k}\, \widetilde{A}_a(x,\boldsymbol{k}_\perp,\mu^2)$

Collinear equation:

$$\widetilde{f}_a(x,\mu^2) = \widetilde{f}_a(x,\mu_0^2)\,\Delta_a(\mu^2) + \int_{\ln\mu_0^2}^{\ln\mu^2} d\ln\mu_1^2\, \frac{\Delta_a(\mu^2)}{\Delta_a(\mu_1^2)} \sum_b \int_x^{z_M} dz_1 P_{ab}^R(\alpha_s((1-z)^2\mu'^2), z_1)\, \widetilde{f}_b\left(\frac{x}{z_1},\mu_0^2\right) \Delta_b(\mu_1^2) + \cdots$$

- The integrated equation coincides with CMW (Catani-Marchesini-Webber 1991) coherent branching

- Scale of strong coupling: $\alpha_s(q_\perp^2) = \alpha_s((1-z)^2\mu'^2)$
- Lowest scale in α_s corresponds to minimal $q_\perp$
- $q_{\perp,min} = q_0$ & $q_0 > \Lambda_{QCD}$ => we stay in the weak coupling region!



PB TMD fits at NLO with fixed zmax

The Past PB TMD fits at NLO calculation using angular ordering : fixed z_M

"NLO DIS Matrix Element (ME) and NLO evolution kernel"

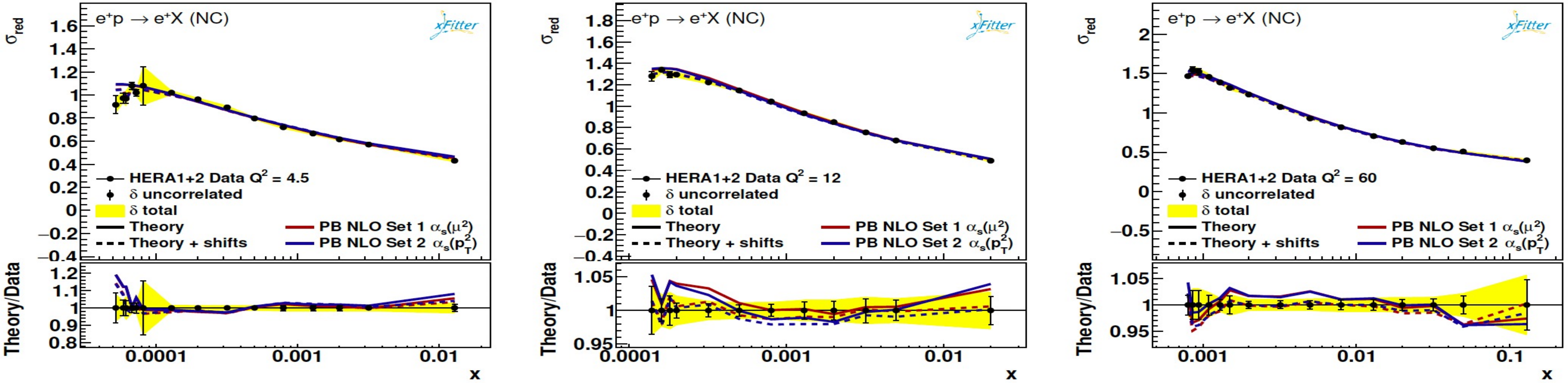
Phys. Rev. D 99, 074008 (2019),

- Associating the evolution scale with some physical interpretation:

Two scenarios
- Set 1 → $\alpha_s(\mu'^2)$
- Set 2 → $\alpha_s(q_\perp^2) = \alpha_s((1-z)^2\mu'^2)$

- Data set: HERA 1+ 2 inclusive DIS data

PB NLO Set1 $\alpha_s(\mu_i^2)$			
	χ^2	d.o.f	χ^2/d.o.f
$\mu_0^2 = 1.9\ \mathrm{GeV}^2$	1363.37	1131	1.21
PB NLO Set 2 $\alpha_s(q_{t\,i}^2)$			
	χ^2	d.o.f	χ^2/d.o.f
$\mu_0^2 = 1.4\ \mathrm{GeV}^2$	1369.80	1131	1.21



✓ Measurement of the inclusive DIS cross section obtained at HERA compared to predictions using Set 1 and Set 2

PB TMD fits at NLO with dynamical zmax

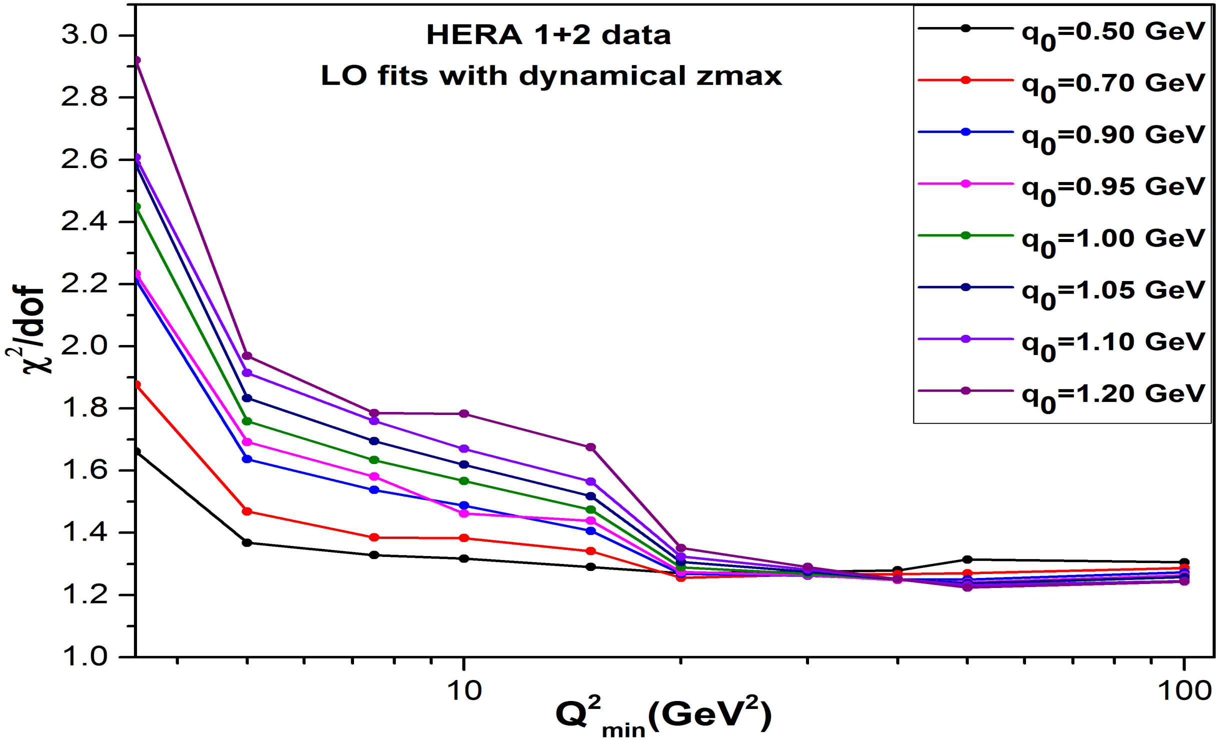
New study

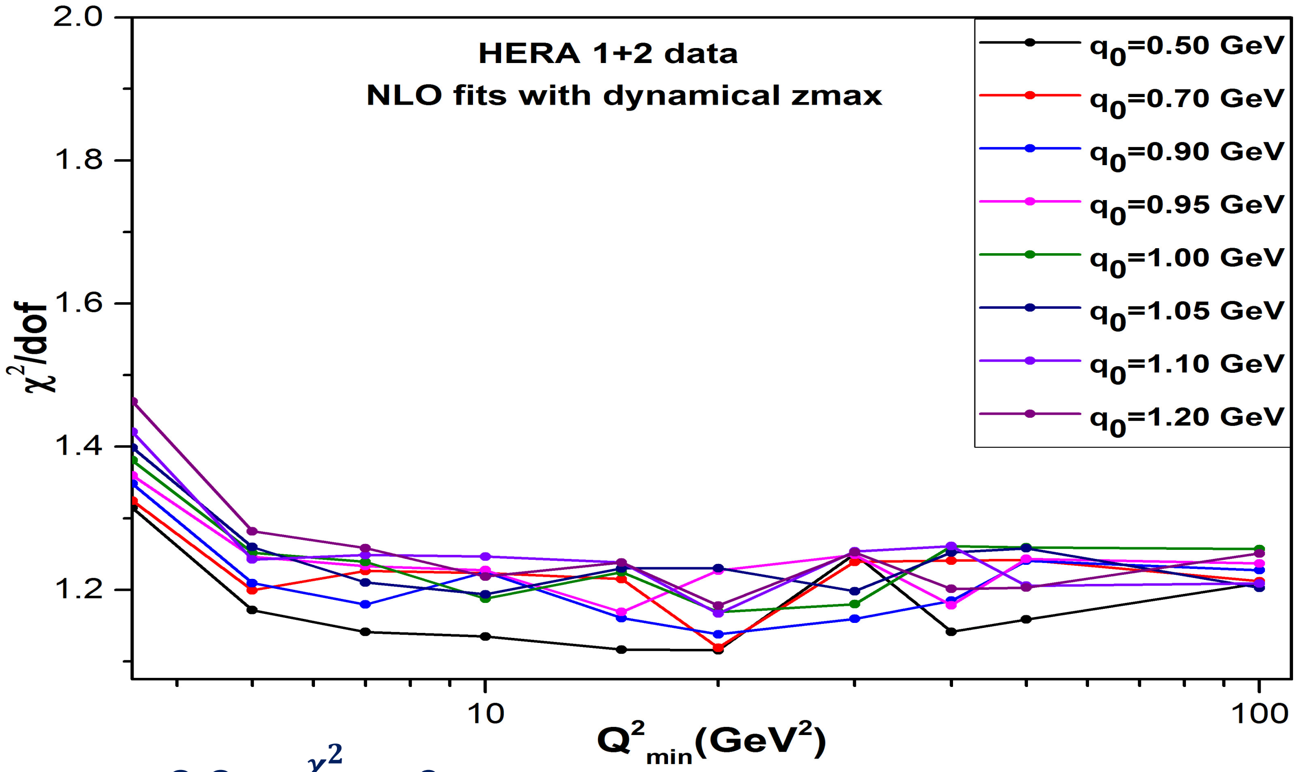
From fixed resolution scale to dynamical resolution scale

PB TMD fits at NLO with dynamical zmax: $z_M = 1 - \left(\frac{q_0}{\mu'}\right)$

New fits with dynamical zmax at LO and NLO with HERA 1 + 2 Data set:

✓ Performing different fits, each time by varying Q^2_{min} and on top of that with different $\mathbf{q_0}$ values





- **At LO , for small Q^2_{min} and 0.9 GeV $< q_0 <$ 1.2 GeV** ⟶ **2.2 $< \frac{\chi^2}{\mathrm{dof}} <$ 3**
- **AT NLO , for small Q^2_{min} and all values of q_0, we have better fits with good $\frac{\chi^2}{dof}$!**

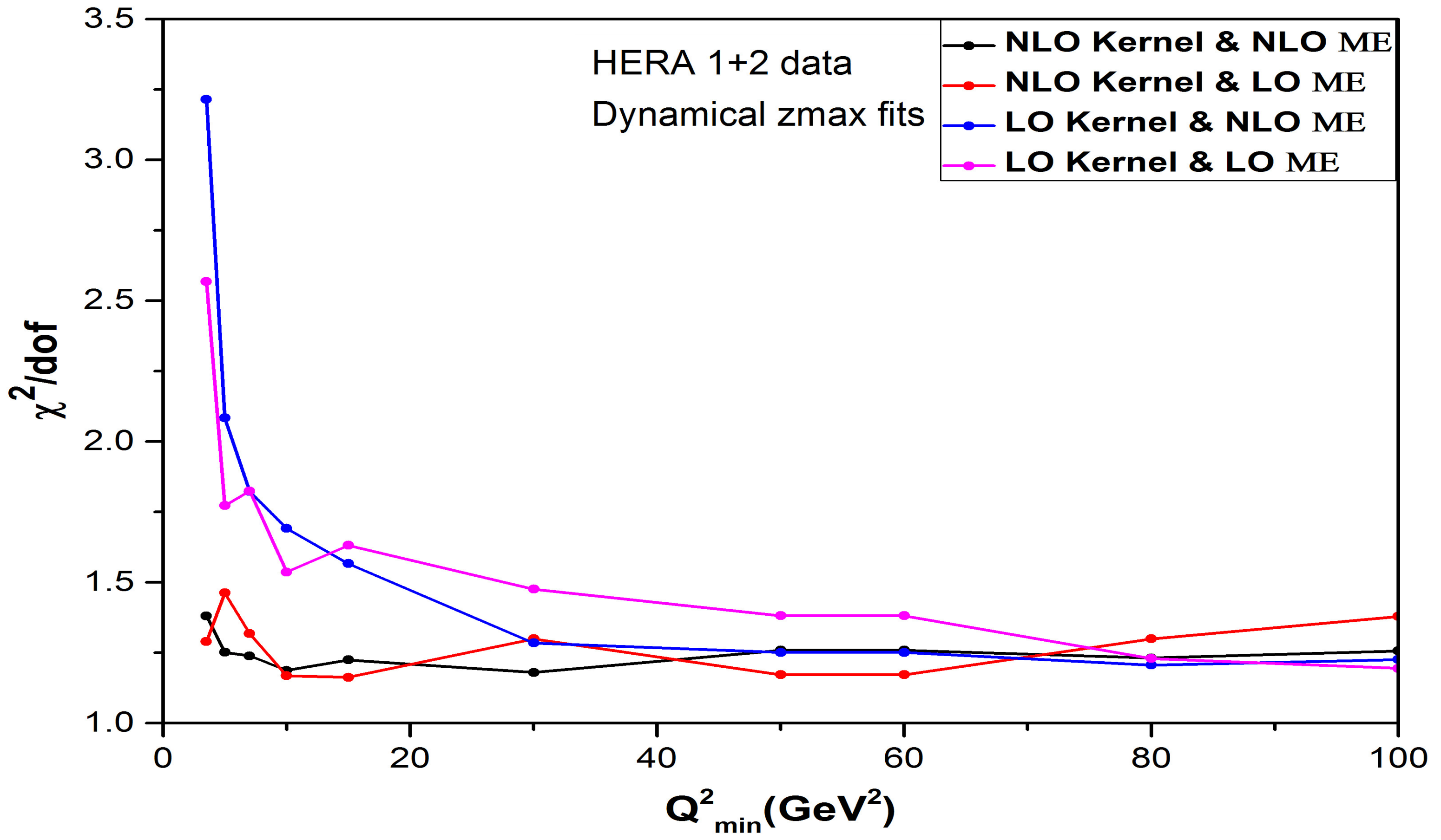
The difference between LO and NLO

- Does the difference between LO and NLO come from the kernels? or ME?!..

4 states for this purpose:

1. Fitting with NLO kernel & NLO ME
2. Fitting with NLO kernel & LO ME
3. Fitting with LO kernel & LO ME
4. Fitting with LO kernel & NLO ME

For q_0=1.0 GeV



The difference is dominated by the kernel not ME..!

The difference between LO and NLO

- Which part of the kernel is responsible?

$P_{ab}(z,\mu^2)$? or α_s?

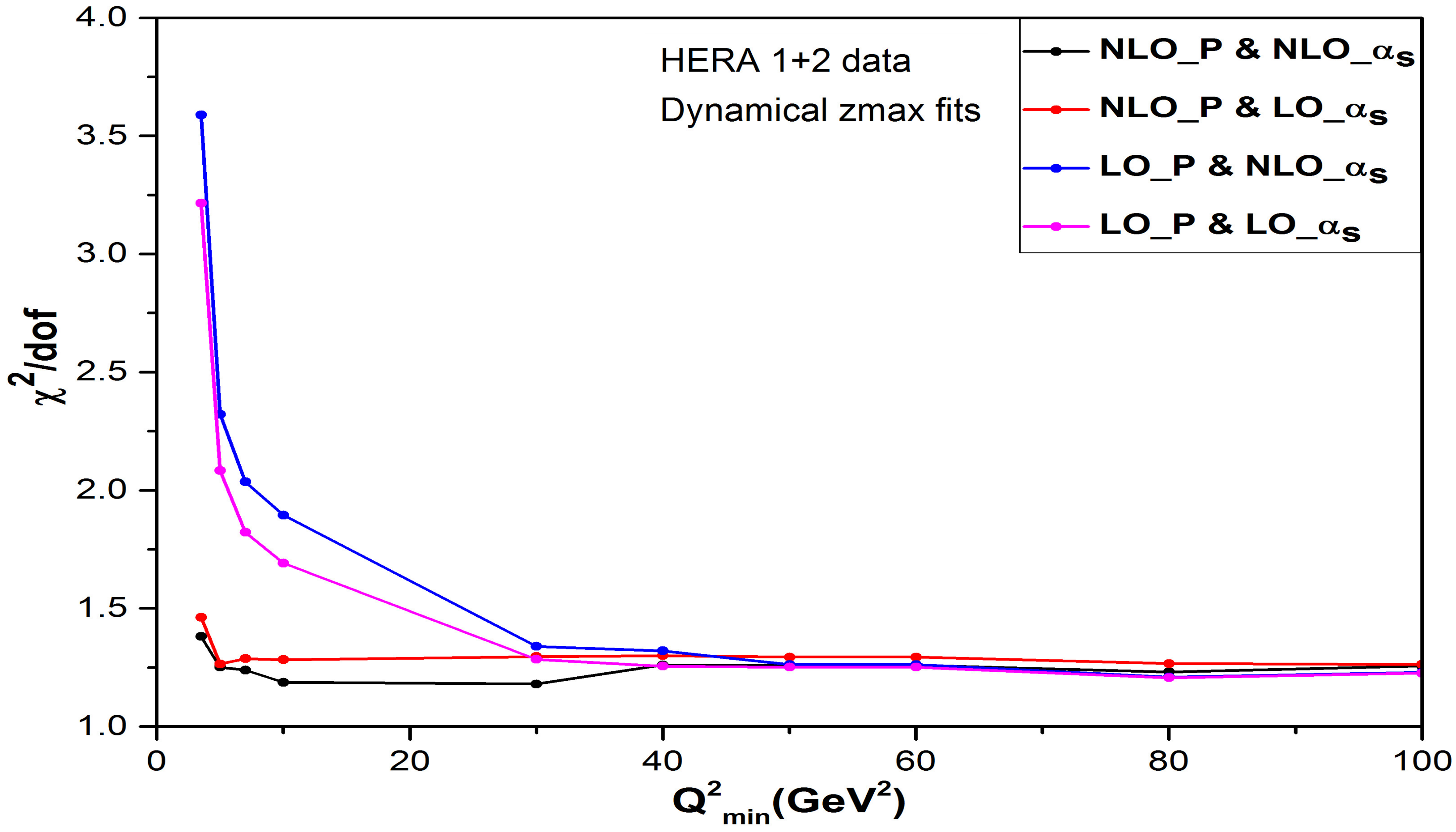
4 states for this purpose:

1. Fitting with NLO P_{ab} & NLO α_s
2. Fitting with NLO P_{ab} & LO α_s
3. Fitting with LO P_{ab} & LO α_s
4. Fitting with LO P_{ab} & NLO α_s

The difference is dominated by the splitting functions not α_s..!

For NLO ME

For q_0=1.0 GeV



Which part of the splitting functions is responsible for the difference between LO and NLO?

- **For high values of q_0(e.g, $[1.0\ \mathrm{Gev}, 1.2\ \mathrm{Gev}]$) or low values of $z_M = 1 - \left(\frac{q_0}{\mu'}\right)$, LO and NLO have different behavior.**

The first piece for checking is → $\frac{1}{z}$

- In the NLO, all the splitting functions have pieces with (1/z) term :

  $P_{ab}(z,\mu^2) \sim P_{qq}(1/z,\mu^2), P_{qg}(1/z,\mu^2), P_{gg}(1/z,\mu^2), P_{gq}(1/z,\mu^2)$

- In the LO, just the splitting functions with "gluon" in the final state have (1/z) piece:

  $P_{gg}(z,\mu^2) = \frac{1}{1-z} + \frac{1}{z} - 2 + z(1-z),$

  $P_{gq}(z,\mu^2) = \frac{1+(1-z)^2}{z}$

- And the splitting functions with "quark" in the final state don't have (1/z) piece:

  $P_{qq}(z,\mu^2) = \frac{2}{1-z} - 1 - z,$

  $P_{qg}(z,\mu^2) = z^2 + (1-z)^2$

- Is the lack of (1/z) piece in LO splitting function with quark in the final state responsible for this difference?

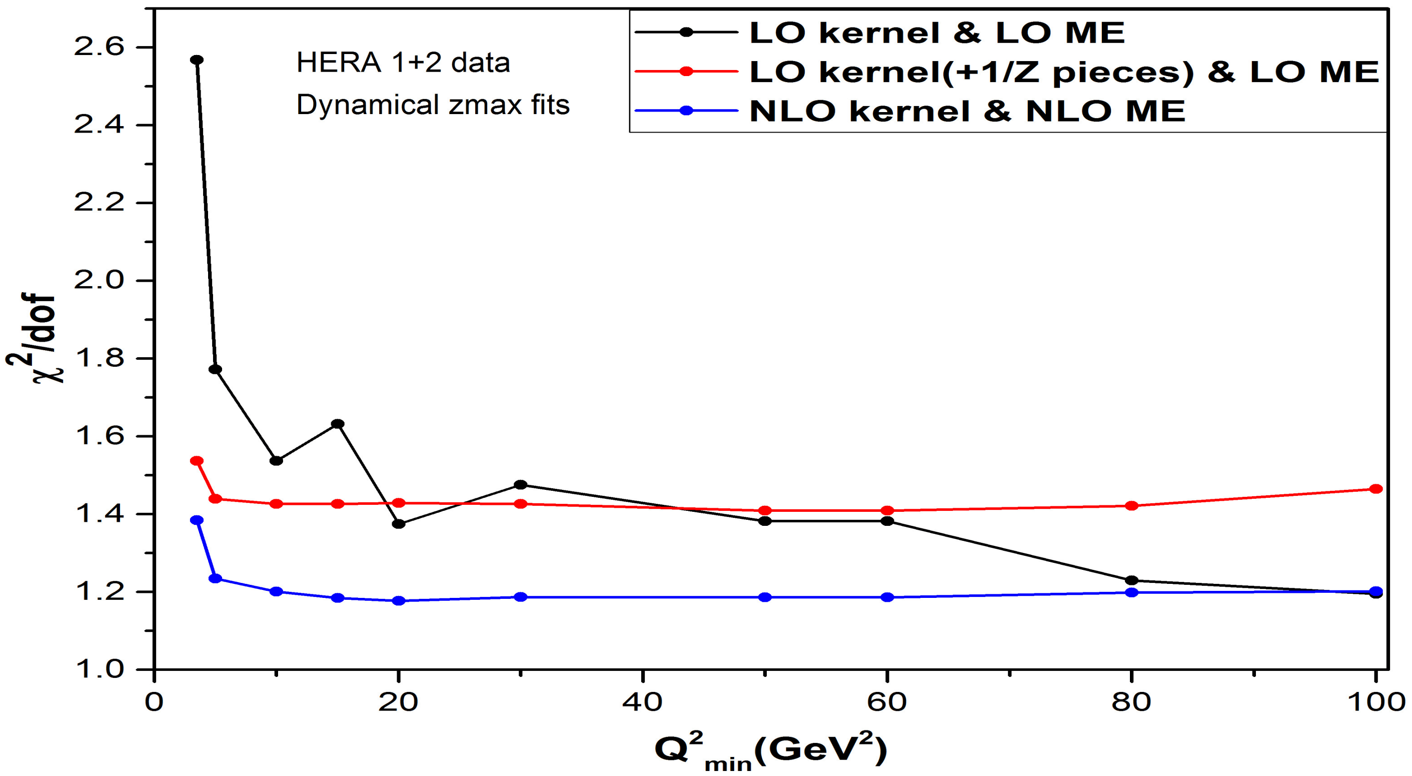
Let's check it!

Does the difference come from 1/z piece of NLO splitting function?

For better understanding: **“We added to the LO splitting functions(P_{qg}, P_{qq}) the 1/z pieces of NLO”**

- ✓ $P_{qq}\ (z,\mu^2) = \frac{2}{1-z} - 1 - z + \left(\frac{1}{z}\right)$ **pieces of P_{qq} NLO**
- ✓ $P_{qg}\ (z,\mu^2) = z^2 + (1-z)^2 + \left(\frac{1}{z}\right)$ **pieces of P_{qg} NLO**

For q_0=1.0 GeV

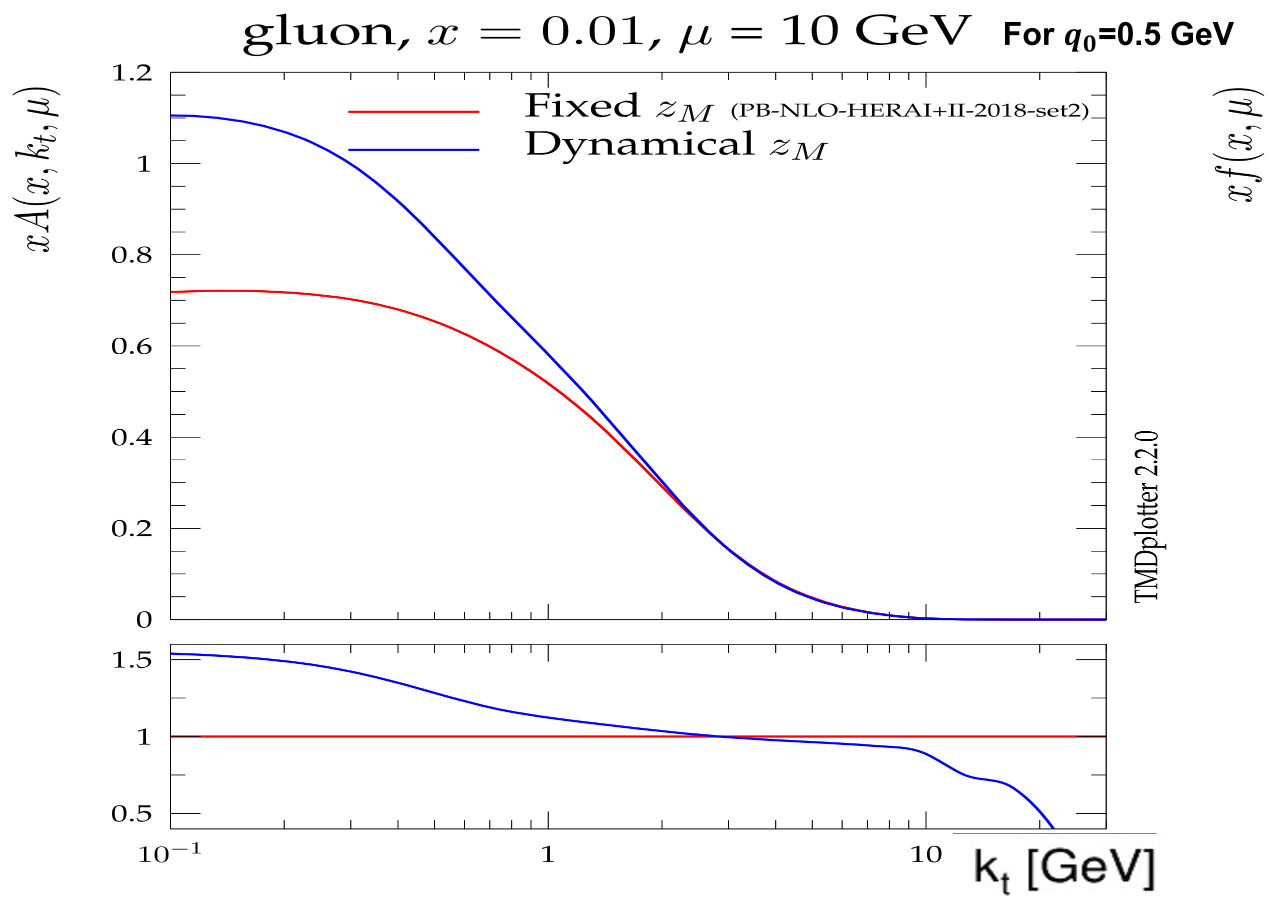


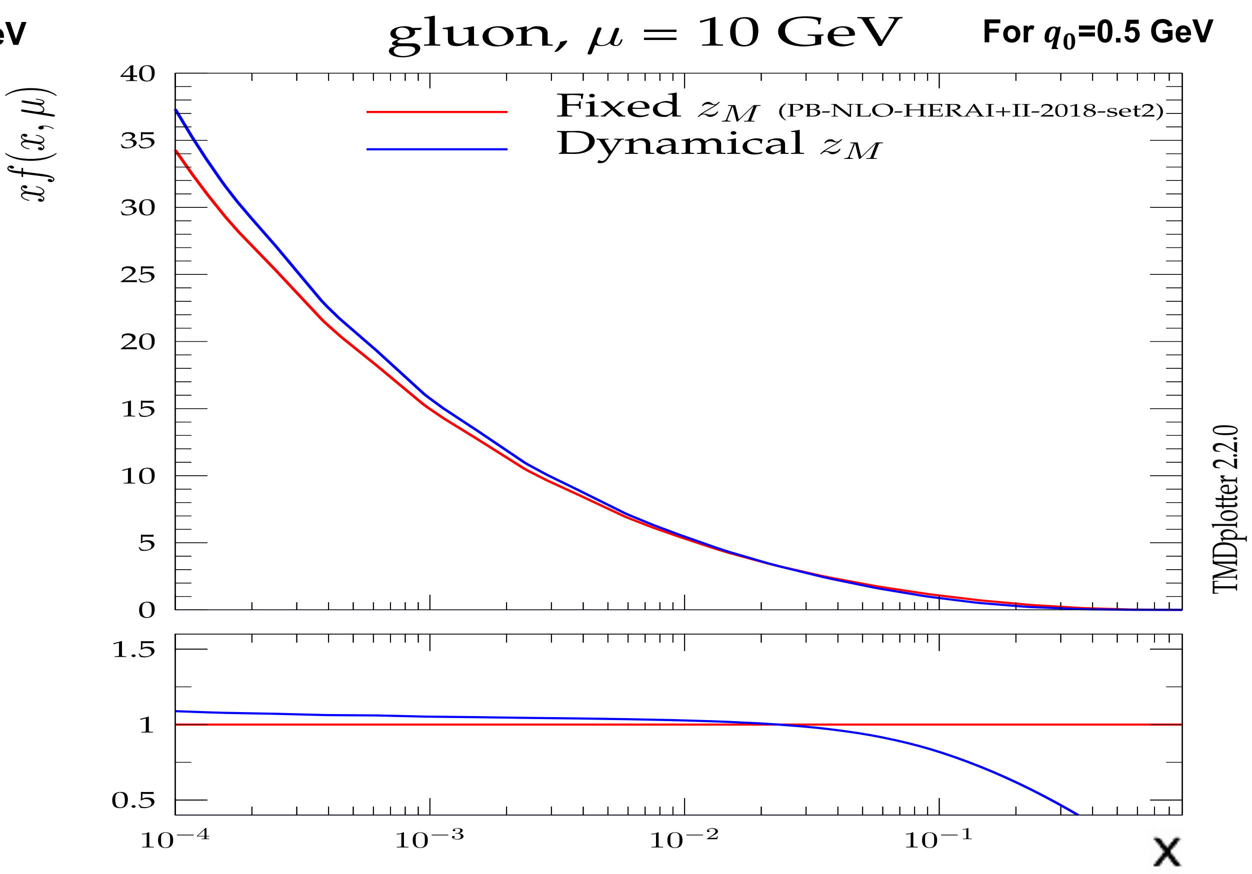
- ✓ In NLO we have an extra (1/z) pieces in the quark channels compared with LO which is responsible for this difference!
- ✓ With this piece we are describing data well! Amount of $\frac{\chi^2}{\text{dof}}$ is reasonably good!

**** For PB-TMD fit with dynamical zmax we obtain a reasonably good $\frac{\chi^2}{dof}$ at NLO! ****

How does dynamical zmax affect the fitted TMD (iTMD)?

Set 2: fixed zmax & $\alpha_s(q_\perp^2) = \alpha_s((1-z)^2\mu'^2)$





✓ The dynamical zmax fit implies an effect not only in the kT dependence but also in the x dependence!

Summary

- **PB TMD fits at NLO with dynamical zmax for the first time!**
- **For PB-TMD fit to HERA data with dynamical zmax, we obtain a reasonably good $\frac{\chi^2}{dof}$ at NLO!**
- **The difference between LO and NLO fits is mostly due to (1/z) pieces in quark channel in NLO splitting functions!**
- **The dynamical zmax impacts both the kT dependence and the x dependence of the fitted parton distribution!**
- **The next step: Using the PB TMD with dynamical zmax in phenomenology of LHC and lower energy colliders!**

Thank you …

BACK UP …

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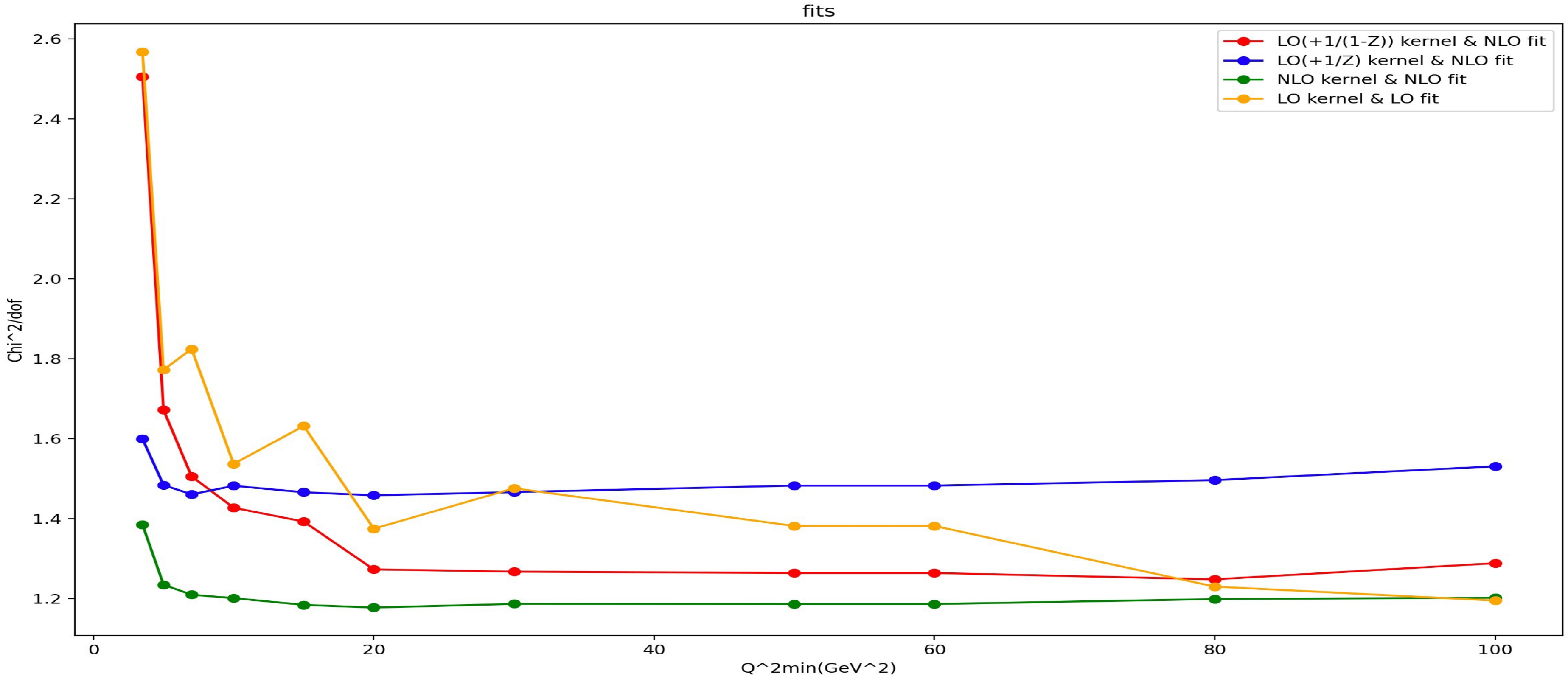
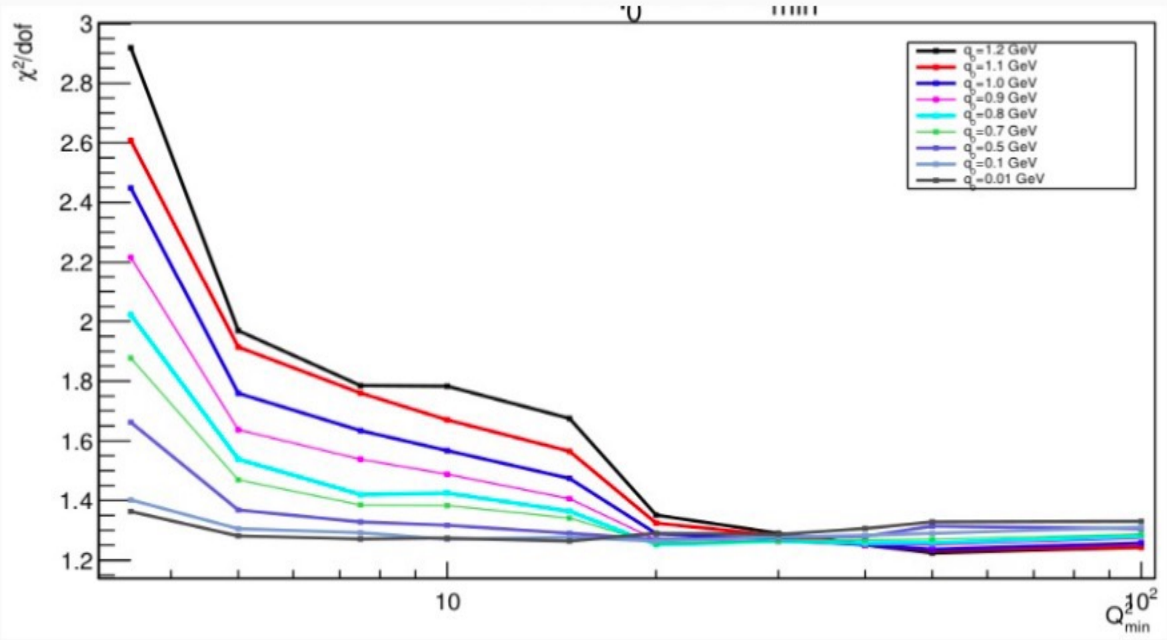
Back up…

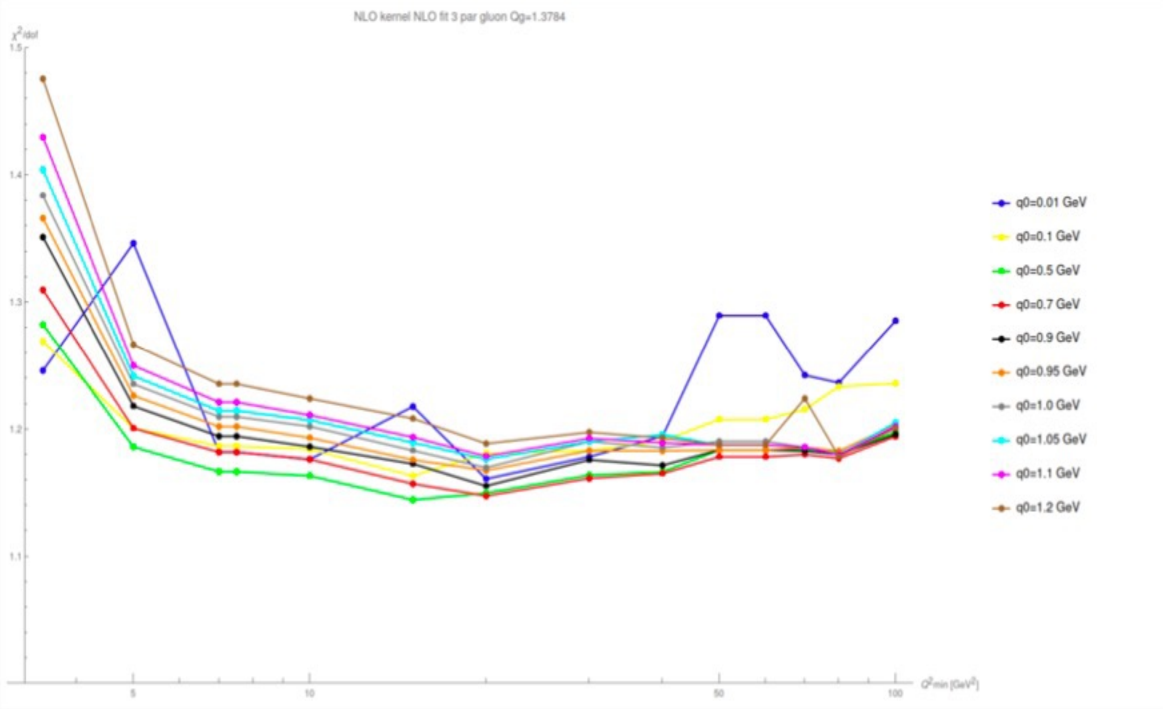
- Is this piece responsible for the difference? "We modified the LO splitting functions(P_{qg}, P_{qq}) with (z ->1) pieces of NLO"



Back up…



LO setting with 3 parameters for gluon.



NLO setting with 3 parameters for gluon

PB TMD fits at NLO with dynamical zmax: