Power corrections in TMD factorization

based on [2109.09771]

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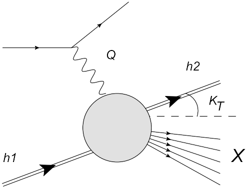
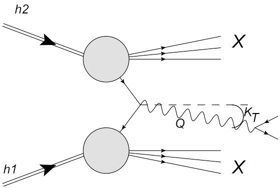
Transverse momentum dependent factorization

$$\frac{d\sigma}{dq_T} \simeq \sigma_0 \int \frac{d^2b}{(2\pi)^2} e^{-i(bq_T)} |C_V(Q)|^2 F_1(x_1, b; Q, Q^2) F_2(x_2, b; Q, Q^2)$$

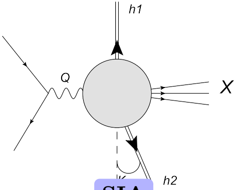
LP term is studied VERY WELL!



SIDIS



Drell-Yan



SIA

q is momentum of initiating EW-boson
$q^2 = \pm Q^2$
q_T^μ transverse component

$$\begin{cases} Q^2 \gg \Lambda^2_{QCD} \\ Q^2 \gg q_T^2 \end{cases}$$

Transverse momentum dependent factorization

$$\frac{d\sigma}{dq_T} \simeq \sigma_0 \int \frac{d^2b}{(2\pi)^2} e^{-i(bq_T)} \Big\{ |C_V(Q)|^2 F_1(x_1, b; Q, Q^2) F_2(x_2, b; Q, Q^2) \quad \longleftarrow \text{LP}$$

$$+ \frac{q_T}{Q} [C_2(Q) \otimes F_3(x, b; Q, Q^2) F_4(x, b; Q, Q^2)](x_1, x_2) \quad \longleftarrow \text{NLP}$$

$$+ \frac{q_T^2}{Q^2} [C_3(Q) \otimes F_5(x, b; Q, Q^2) F_6(x, b; Q, Q^2)](x_1, x_2) \quad \longleftarrow \text{NNLP}$$

$$+ \ldots$$

Outline

- General approach to TMD factorization
- Systematics of power-suppressed TMD operators (distributions)
- TMD factorization at NLP/NLO

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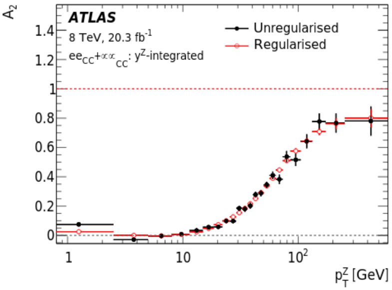
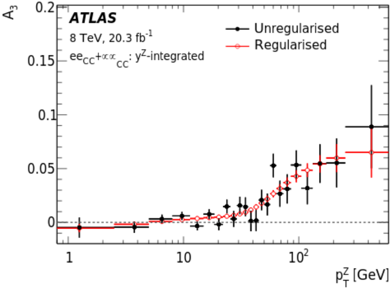
- **Sub-leading power observables**

$$\frac{d\sigma}{dp_T^Z\, dy^Z\, dm^Z\, d\cos\theta\, d\phi} = \frac{3}{16\pi}\frac{d\sigma^{U+L}}{dp_T^Z\, dy^Z\, dm^Z}$$
$$\Big\{(1+\cos^2\theta) + \frac{1}{2}A_0(1-3\cos^2\theta) + A_1\ \sin 2\theta\ \cos\phi$$
$$+\frac{1}{2}A_2\ \sin^2\theta\ \cos 2\phi + A_3\ \sin\theta\ \cos\phi + A_4\ \cos\theta$$
$$+A_5\ \sin^2\theta\ \sin 2\phi + A_6\ \sin 2\theta\ \sin\phi + A_7\ \sin\theta\ \sin\phi\Big\}.$$

To describe it, one needs TMD factorization at NNLP.

- JLab
- LHC





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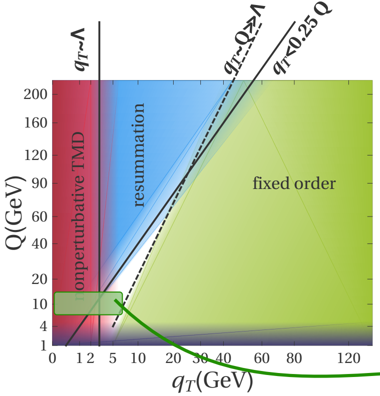
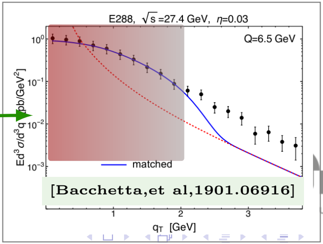
- Sub-leading power observables
- **Increase of applicability domain**



LP TMD factorization has limited region of application.

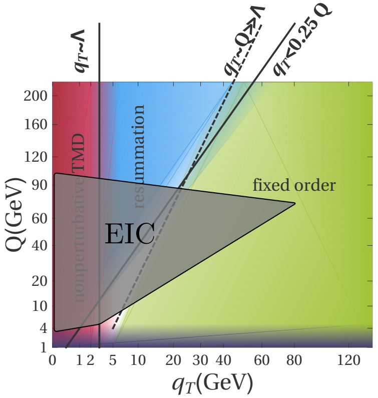
For SIDIS it cuts **the most part** of the data



[Bacchetta,et al,1901.06916]

Motivation

- Sub-leading power observables
- **Increase of applicability domain**



Phase space of EIC is centered directly in the transition region

COMPASS, JLab have large contribution of power corrections

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Motivation

- Sub-leading power observables
- Increase of applicability domain
- **Restoration of broken properties**

LP TMD factorization breaks EM-gauge invariance

$$W^{\mu\nu} = \int dy e^{iqy} \langle J^\mu(y) J^\nu(0) \rangle \qquad W^{\mu\nu}_{\text{LP}} = g_T^{\mu\nu} |C_V|^2 \mathcal{F}(F_1 F_2)$$

$$q_\mu W^{\mu\nu} = 0 \qquad q_\mu W^{\mu\nu}_{\text{LP}} \sim q_T^\nu$$

- The violation is of the NLP
- Similar problem with frame-dependence (GTMD case)

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There are already computations of TMD factorization at NLP/NNLP

- Small-x-like
  - Balitksy [1712.09389],[2012.01588],...
  - Nefedov, Saleev, [1810.04061],[1906.08681]
- SCET
  - Ebert, et al [1812.08189] *resummation*
  - Inglis-Whalen, et al [2105.09277]
  - Beneke, et al, [1712.04416],[1808.04742],... *not TMD, but closely related*
- Boer, Mulders, Pijlman [hep-ph/0303034]
- ...

I suggest another method to derive TMD factorization

TMD operator expansion

- Based on the experience of higher-twist, and higher power computations in collinear factorization
  - Systematicness of OPE
  - Operator level
  - Position space [a lot of simplification for beyond leading twist]
- Has common parts with small-x and SCET computations

Sources of power corrections

*(exact)=known at all powers

$$\frac{d\sigma}{dP.S.} = \sigma_{PS} L_{\mu\nu} W^{\mu\nu}$$

Phase space PC (exact)

e.g. SIDIS $\sigma_{PS} = \frac{\pi}{\sqrt{1+\gamma^2 \frac{\mathbf{p}_{h\perp}^2}{z^2 Q^2}}}$

Hadronic tensor (e.g. DY)

$$W^{\mu\nu} = \int \frac{d^4 y e^{i(yq)}}{(2\pi)^4} \langle p_1 p_2 | J^\mu(y) | X \rangle \langle X | J^\nu | p_1 p_2 \rangle$$

Factorized in powers of

$\frac{q_T}{q^+}$, $\frac{q_T}{q^-}$ $\left(\text{not } \frac{q_T}{Q}\right)$

because

$\{y^+, y^-, y_T\} \sim Q^{-1}\{1, 1, \lambda^{-1}\}$

Leptonic tensor (exact)

e.g. un.DY with fid.cuts

$L^{\mu\nu} \sim (l^\mu l'^\nu + l^\nu l'^\mu - g^{\mu\nu}(ll'))\mathcal{P}$

- l, l' with transverse parts
- $\mathcal{P}$ fiducial part

Power corrections due to frame choice (exact)

$p_1^+ \gg p_1^-$, $p_2^- \gg p_2^+$

e.g. SIDIS $q_T^2 = \frac{p_\perp^2}{z^2} \frac{1+\gamma^2}{1-\gamma^2 \frac{p_\perp^2}{z^2 Q^2}}$

Sources of power corrections

*(exact)=known at all powers

$$\frac{d\sigma}{dP.S.} = \sigma_{PS} L_{\mu\nu} W^{\mu\nu}$$

Phase space PC (exact)

e.g. SIDIS $\sigma_{PS} = \frac{\pi}{\sqrt{1 + \ldots}}$

QCD Factorization
(this talk)

Leptonic tensor (exact)

e.g. un.DY with fid.cuts

$$L^{\mu\nu} \sim (l^\mu l'^\nu + l^\nu l'^\mu - g^{\mu\nu}(ll'))\mathcal{P}$$

- l, l' with transverse parts
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Hadronic tensor (e.g. DY)

$$W^{\mu\nu} = \int \frac{d^4y e^{i(yq)}}{(2\pi)^4} \langle p_1 p_2 | J^\mu(y) | X\rangle \langle X | J^\nu | p_1 p_2\rangle$$

Factorized in powers of

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because

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Power corrections due to frame choice (exact)

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e.g. SIDIS $q_T^2 = \frac{p_\perp^2}{z^2} \frac{1+\gamma^2}{1 - \gamma^2 \frac{p_\perp^2}{z^2 Q^2}}$

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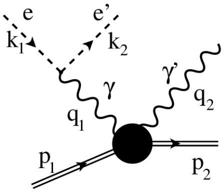
The most efficient way to study power corrections: **OPE + background formalism**

- Many (so far) results unreachable by other methods
  - Twist-3, twist-4 evolution kernels [Braun,Manashov,08-09]
  - Coefficient function for various observables (e.g. quasi-PDFs at twist-3 [Braun,Ji,AV,20-21])
  - All-Power corrections (DVCS [Braun,Manashov,17-21], target-mass corrections to TMDs [Moos,AV,20])
- Clear and strict formulation ⇒ Simple computation
- Twist-decomposition

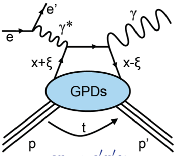


DVCS

$$J^\mu(z)J^\nu(0) \xrightarrow{\text{OPE}} \sum_{n=0}^{\infty} z^n [C_n^{\mu\nu} \otimes O_n](z^+)$$

Leading power ⇒ GPDs

violates EM Ward identities and translation invariance



$ep \to e'p'\gamma$

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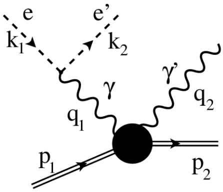

DVCS

$$J^\mu(z)J^\nu(0) \xrightarrow{\text{OPE}} \sum_{n=0}^{\infty} z^n [C_n^{\mu\nu} \otimes O_n](z^+)$$

power	operators		
0	$\bar{q}[..]q$		
1	$\bar{q}[..]q$	$\bar{q}F_{\mu+}[..]q$	
2	$\bar{q}[..]q$	$\bar{q}F_{\mu+}[..]q$	$\bar{q}F_{\mu+}F_{\nu+}[..]q$ $\bar{q}[..]q\bar{q}[..]q$ $\bar{q}F_{\mu+}[..]\gamma^- q$
...	tw2	tw3	tw4

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The most efficient way to study power corrections: **OPE + background formalism**

- Many (so far) results unreachable by other methods
  - Twist-3, twist-4 evolution kernels [Braun,Manashov,08-09]
  - Coefficient function for various observables (e.g. quasi-PDFs at twist-3 [Braun,Ji,AV,20-21])
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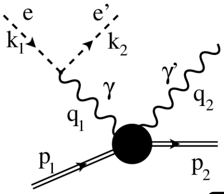

DVCS

TMD factorization has same structure

All properties restored

$$\xrightarrow{\text{OPE}} \sum^{\infty} z^n [C_n^{\mu\nu} \otimes O_n](z^+)$$

All properties restored

Independent
Do not mix

power	operators		
0	$\bar q[..]q$		...
1	$\bar q[..]q$	$\bar q F_{\mu+}[..]q$	
2	$\bar q[..]q$	$\bar q F_{\mu+}[..]q$	$\bar q F_{\mu+}F_{\nu+}[..]q$ $\bar q[..]q\bar q[..]q$ $\bar q F_{\mu+}[..]\gamma^- q$
...	tw2	tw3	tw4

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Background QCD with 2-component background

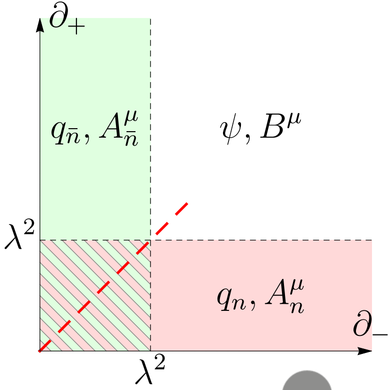
$$q \to q_n + q_{\bar n} + \psi \qquad A^\mu \to A^\mu_n + A^\mu_{\bar n} + B^\mu$$

collinear-fields
(associated with hadron 1)

$$\{\partial_+, \partial_-, \partial_T\}\, q_{\bar n} \lesssim Q\{1, \lambda^2, \lambda\}\, q_{\bar n},$$
$$\{\partial_+, \partial_-, \partial_T\}\, A^\mu_{\bar n} \lesssim Q\{1, \lambda^2, \lambda\}\, A^\mu_{\bar n},$$

anti-collinear-fields
(associated with hadron 2)

$$\{\partial_+, \partial_-, \partial_T\}\, q_n \lesssim Q\{\lambda^2, 1, \lambda\}\, q_n,$$
$$\{\partial_+, \partial_-, \partial_T\}\, A^\mu_n \lesssim Q\{\lambda^2, 1, \lambda\}\, A^\mu_n.$$



Keldysh thechnique to deal with causality structure → $J^{(+)\mu}(y)J^{(-)\nu}(0)$

Details & examples
in [2109.09711]

$$J^{(+)\mu}(y)J^{(-)\nu}(0)$$

Details & examples in [2109.09711]

(power) Expand in background fields
sort operators

$$\bar{q}_{\bar{n}}(y^- n + y_T)\gamma_T^\mu q_n(y^+\bar{n} + y_T)\bar{q}_n(0)\gamma_T^\nu q_{\bar{n}}(0) + \bar{\psi}_{\bar{n}}(y)\gamma_T^\mu q_n(y^+\bar{n} + y_T)\bar{q}_n(0)\gamma_T^\nu q_{\bar{n}}(0) + ...$$
$$+n^\mu \bar{q}_{\bar{n}}(y^- n + y_T)\gamma^- q_n(y^+\bar{n} + y_T)\bar{q}_n(0)\gamma_T^\nu q_{\bar{n}}(0) + ...$$
$$+y^+ \bar{q}_{\bar{n}}(y^- n + y_T)\overleftarrow{\partial}_- \gamma^- q_n(y^+\bar{n} + y_T)\bar{q}_n(0)\gamma_T^\nu q_{\bar{n}}(0) + ...$$

$$J^{(+)\mu}(y)J^{(-)\nu}(0)$$

Details & examples in [2109.09711]

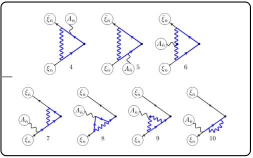
(power) Expand in background fields
sort operators

$$\bar q_{\bar n}(y^- n + y_T)\gamma_T^\mu q_n(y^+\bar n + y_T)\bar q_n(0)\gamma_T^\nu q_{\bar n}(0) + \bar\psi_{\bar n}(y)\gamma_T^\mu q_n(y^+\bar n + y_T)\bar q_n(0)\gamma_T^\nu q_{\bar n}(0) + \dots$$
$$+n^\mu \bar q_{\bar n}(y^- n + y_T)\gamma^- q_n(y^+\bar n + y_T)\bar q_n(0)\gamma_T^\nu q_{\bar n}(0) + \dots$$
$$+y^+ \bar q_{\bar n}(y^- n + y_T)\overleftarrow{\partial}_-\gamma^- q_n(y^+\bar n + y_T)\bar q_n(0)\gamma_T^\nu q_{\bar n}(0) + \dots$$

(loop) Integrate over fast components
with 2-bcg.QCD action

at least NLO is needed to confirm factorization (WL direction, pole-cancelation)

$$\mathcal{J}^{\mu\nu}_{\rm NLP} = -\frac{n^\mu\gamma^\rho_{T,ij}\gamma^\nu_{T,kl} + n^\nu\gamma^\mu_{T,ij}\gamma^\rho_{T,kl}}{N_c}\left(\frac{\partial_\rho}{\partial_+}\mathcal{O}^{li}_{11,\bar n}\overline{\mathcal{O}}^{jk}_{11,n} + \frac{\partial_\rho}{\partial_+}\overline{\mathcal{O}}^{jk}_{11,\bar n}\mathcal{O}^{li}_{11,n}\right)$$
$$-\frac{\bar n^\mu\gamma^\rho_{T,ij}\gamma^\nu_{T,kl} + \bar n^\nu\gamma^\mu_{T,ij}\gamma^\rho_{T,kl}}{N_c}\left(\mathcal{O}^{li}_{11,\bar n}\frac{\partial_\rho}{\partial_-}\overline{\mathcal{O}}^{jk}_{11,n} + \overline{\mathcal{O}}^{jk}_{11,\bar n}\frac{\partial_\rho}{\partial_-}\mathcal{O}^{li}_{11,n}\right)$$
$$+ig\frac{\delta_{ij}\gamma^\nu_{T,kl}}{N_c}\left\{\mathbb{O}^{li}_{21,\bar n}\left(\frac{\bar n^\mu}{\overrightarrow{\partial}_-} - \frac{n^\mu}{\overleftarrow{\partial}_+}\right)\overline{\mathcal{O}}^{jk}_{11,n} - \overline{\mathbb{O}}^{jk}_{21,\bar n}\left(\frac{\bar n^\mu}{\overrightarrow{\partial}_-} - \frac{n^\mu}{\overleftarrow{\partial}_+}\right)\mathcal{O}^{li}_{11,n}\right.$$



Process ⇔ boundary conditions

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$$J^{(+)\mu}(y)J^{(-)\nu}(0)$$

Details & examples in [2109.09711]

(power) Expand in background fields
sort operators

$$\bar{q}_{\bar{n}}(y^-n+y_T)\gamma_T^\mu q_n(y^+\bar{n}+y_T)\bar{q}_n(0)\gamma_T^\nu q_{\bar{n}}(0)+\bar{\psi}_{\bar{n}}(y)\gamma_T^\mu q_n(y^+\bar{n}+y_T)\bar{q}_n(0)\gamma_T^\nu q_{\bar{n}}(0)+\ldots$$
$$+n^\mu\bar{q}_{\bar{n}}(y^-n+y_T)\gamma^- q_n(y^+\bar{n}+y_T)\bar{q}_n(0)\gamma_T^\nu q_{\bar{n}}(0)+\ldots$$
$$+y^+\bar{q}_{\bar{n}}(y^-n+y_T)\overleftarrow{\partial_-}\gamma^- q_n(y^+\bar{n}+y_T)\bar{q}_n(0)\gamma_T^\nu q_{\bar{n}}(0)+\ldots$$

(loop) Integrate over fast components
with 2-bcg.QCD action

$$\mathcal{J}^{\mu\nu}_{\text{NLP}}=-\frac{n^\mu\gamma^\rho_{T,ij}\gamma^\nu_{T,kl}+n^\nu\gamma^\mu_{T,ij}\gamma^\rho_{T,kl}}{N_c}\Big(\frac{\partial_\rho}{\partial_+}\mathcal{O}^{li}_{11,\bar{n}}\overline{\mathcal{O}}^{jk}_{11,n}+\frac{\partial_\rho}{\partial_+}\overline{\mathcal{O}}^{jk}_{11,\bar{n}}\mathcal{O}^{li}_{11,n}\Big)$$
$$-\frac{\bar{n}^\mu\gamma^\rho_{T,ij}\gamma^\nu_{T,kl}+\bar{n}^\nu\gamma^\mu_{T,ij}\gamma^\rho_{T,kl}}{N_c}\Big(\mathcal{O}^{li}_{11,\bar{n}}\frac{\partial_\rho}{\partial_-}\overline{\mathcal{O}}^{jk}_{11,n}+\overline{\mathcal{O}}^{jk}_{11,\bar{n}}\frac{\partial_\rho}{\partial_-}\mathcal{O}^{li}_{11,n}\Big)$$
$$+ig\frac{\delta_{ij}\gamma^\nu_{T,kl}}{N_c}\Big\{\mathbb{O}^{li}_{21,\bar{n}}\left(\frac{\bar{n}^\mu}{\overrightarrow{\partial_-}}-\frac{n^\mu}{\overleftarrow{\partial_+}}\right)\overline{\mathcal{O}}^{jk}_{11,n}-\overline{\mathbb{O}}^{jk}_{21,\bar{n}}\left(\frac{\bar{n}^\mu}{\overrightarrow{\partial_-}}-\frac{n^\mu}{\overleftarrow{\partial_+}}\right)\mathcal{O}^{li}_{11,n}$$

Take matrix element,
parametrize TMDs,
cross-section

"Just" algebra

Details & examples in [2109.09711]

$$J^{(+)\mu}(y)J^{(-)\nu}(0)$$

(power) Expand in background fields
sort operators

$$\bar{q}_{\bar{n}}(y^-n+y_T)\gamma_T^\mu q_n(y^+\bar{n}+y_T)\bar{q}_n(0)\gamma_T^\nu q_{\bar{n}}(0)+\bar{\psi}_{\bar{n}}(y)\gamma_T^\mu q_n(y^+\bar{n}+y_T)\bar{q}_n(0)\gamma_T^\nu q_{\bar{n}}(0)+\ldots$$
$$+n^\mu\bar{q}_{\bar{n}}(y^-n+y_T)\gamma^- q_n(y^+\bar{n}+y_T)\bar{q}_n(0)\gamma_T^\nu q_{\bar{n}}(0)+\ldots$$
$$+y^+\bar{q}_{\bar{n}}(y^-n+y_T)\overleftarrow{\partial}_-\gamma^- q_n(y^+\bar{n}+y_T)\bar{q}_n(0)\gamma_T^\nu q_{\bar{n}}(0)+\ldots$$

(loop) Integrate over fast components with 2-bcg.QCD action

$$\ldots\frac{\ldots\gamma^\rho_{T,ij}\gamma^\nu_{T,kl}+n^\nu\gamma^\mu_{T,ij}\gamma^\rho_{T,kl}}{N_c}\left(\frac{\partial_\rho}{\partial_+}\mathcal{O}^{li}_{11,\bar{n}}\overline{\mathcal{O}}^{jk}_{11,n}+\frac{\partial_\rho}{\partial_+}\overline{\mathcal{O}}^{jk}_{11,\bar{n}}\mathcal{O}^{li}_{11,n}\right)$$
$$\ldots\frac{\ldots\gamma^\rho_{T,ij}\gamma^\nu_{T,kl}+\bar{n}^\nu\gamma^\mu_{T,ij}\gamma^\rho_{T,kl}}{N_c}\left(\mathcal{O}^{li}_{11,\bar{n}}\frac{\partial_\rho}{\partial_-}\overline{\mathcal{O}}^{jk}_{11,n}+\overline{\mathcal{O}}^{jk}_{11,\bar{n}}\frac{\partial_\rho}{\partial_-}\mathcal{O}^{li}_{11,n}\right)$$
$$\ldots\frac{\ldots\gamma^\nu_{T,kl}}{N_c}\left\{\mathbb{O}^{li}_{21,\bar{n}}\left(\frac{\bar{n}^\mu}{\overrightarrow{\partial}_-}-\frac{n^\mu}{\overleftarrow{\partial}_+}\right)\overline{\mathcal{O}}^{jk}_{11,n}-\overline{\mathbb{O}}^{jk}_{21,\bar{n}}\left(\frac{\bar{n}^\mu}{\overrightarrow{\partial}_-}-\frac{n^\mu}{\overleftarrow{\partial}_+}\right)\mathcal{O}^{li}_{11,n}\right.$$

Just like ordinary derivation of factorization with OPE

the "only" difference is here
$$y_T^\mu\partial_\mu\sim 1$$

Take matrix element, parametrize TMDs, cross-section

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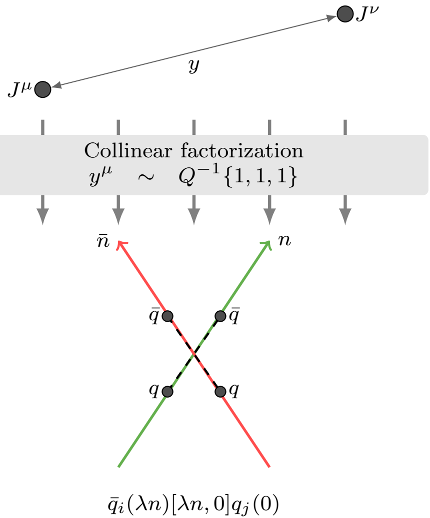
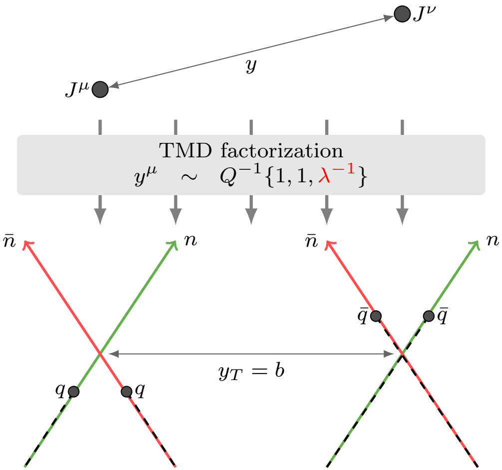


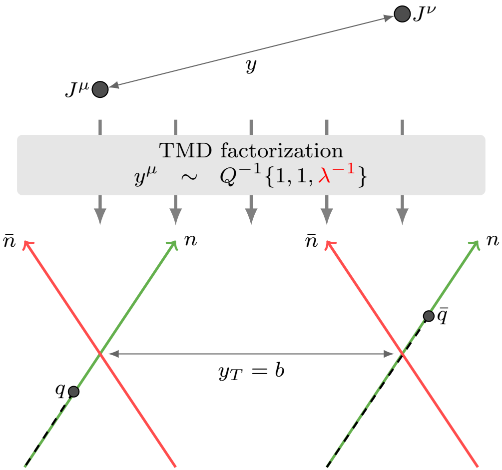
TMD operator expansion has different geometry

Two
light-cone operators
$\Downarrow$
Two
parton distribution function
PDFs & FFs

TMD operator expansion has different geometry



Four
light-cone operators
$\Downarrow$
Two
TMD distributions
TMDPDFs & TMDFFs



$\bar q_i(\lambda n + b)[\lambda n + b, \pm\infty n + b]\quad [\pm\infty n, 0] q_j(0)$

TMD operator expansion has different geometry

Four
light-cone operators
$\Downarrow$
Two
TMD distributions
TMDPDFs & TMDFFs

TMD-twist

Each light-cone operator must be twist-decomposed

- Geometrical twist = dimension - spin (projected to light-cone)
- Half-integer spin operators
  - $(\bar q \gamma^+ \gamma^-)_i$ = twist-1 $(\frac{3}{2}-\frac{1}{2})$
  - $(\bar q \gamma^- \gamma^+)_i$ = twist-indefinite $\Rightarrow$ EOM $\Rightarrow \underbrace{\left(\bar q \gamma^+ \frac{\overleftarrow{\not\partial}_T}{\overleftarrow{\partial}_+}\right)_i}_{\substack{\text{tot.der.}\\ \text{twist-1}}} + \underbrace{\int (\bar q \gamma^\mu F_{\mu+}\gamma^+)_i}_{\text{twist-2}}$

Twist of the TMD operator is enumerated by twists of each light-cone components (N,M) =TMD-twist

e.g. usual TMD operator

$$\underbrace{\overbrace{\bar q(\lambda n + b)[\lambda n + b, \pm\infty n + b]}^{\text{twist-1}}\ \gamma^+\ \overbrace{[\pm\infty n, 0] q(0)}^{\text{twist-1}}}_{\text{TMD-twist=(1,1)}}$$

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Operators with different TMD-twists do not mix
renormalization/evolution is independent
independent TMD distributions

Evolution of TMD operator with TMD-twist=(N,M)

$$O_{NM}(\{z_1,...,z_k\},b)=\overline{U}_N(\{z_1,...\},b)U_M(\{...,z_k\},0_T)$$

- Each light-cone operator U renormalizes independently (because there is a finite y_T between them)

$$\mu\frac{d}{d\mu}U_N(\{z_1,...\},b)=\gamma_N\otimes U_N(\{z_1,...\},b)$$

- Light-cone operators with different N do not mix (Lorentz invariance!)
- Evolution of TMD operator

$$\mu\frac{d}{d\mu}O_{NM}(\{z_1,...\},b)=(\overline{\gamma}_N+\gamma_M)\otimes O_{NM}(\{z_1,...\},b)$$

- (Note) operators with TMD twist (N,M) do not mix with (M,N)
  - **4 independent structures at NLP**: $(2,1)\times(1,1)$, $(1,2)\times(1,1)$, $(1,1)\times(2,1)$, $(1,1)\times(1,2)$
  - **10 independent structures at NNLP**: $(3,1)\times(1,1)$,... $(2,2)\times(1,1)$,... $(1,2)\times(2,1)$, ...
  - **+ total.derivatives!**

Rapidity divergences
appears due to overlap of the fields in the soft region

collinear-fields & anti-collinear
are the same at

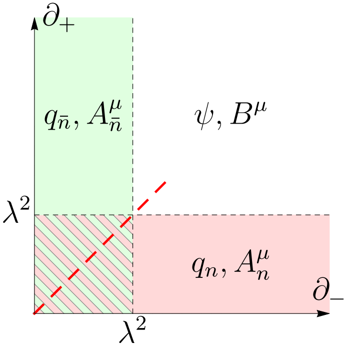
$$\{\partial_+, \partial_-, \partial_T\}\, q \lesssim Q\{\lambda^2, \lambda^2, \lambda\}\, q,$$

$$\{\partial_+, \partial_-, \partial_T\}\, A^\mu \lesssim Q\{\lambda^2, \lambda^2, \lambda\}\, A^\mu,$$

- (or) Introduce separating-scale
- (or) Subtract by soft-factor
- (or) ...

⇒ multiplicative renormalization
[AV,1707.07606] ⇒ evolution equation with ζ

$$\zeta \frac{d}{d\zeta} O_{NM}(\{z_1, ...\}, b) = -\mathcal{D}(b) O_{NM}(\{z_1, ...\}, b)$$



TMD factorization at NLP

Effective operator for any process (DY, SIDIS, SIA)

$$\mathcal{J}^{\mu\nu}_{\text{eff}}(q) = \int \frac{d^2b}{(2\pi)^2} e^{-i(qb)} \Bigg\{ \int dx d\tilde{x} \delta\left(x - \frac{q^+}{p_1^+}\right) \delta\left(\tilde{x} - \frac{q^-}{p_2^-}\right) |C_1|^2 \mathcal{J}^{\mu\nu}_{1111}(x,\tilde{x},b) \tag{6.17}$$

$$+ \int [dx] d\tilde{x} \delta\left(\tilde{x} - \frac{q^-}{p_2^-}\right)$$

$$\times \left(\delta\left(x_1 - \frac{q_1^+}{p_1^+}\right) C_1^* C_2(x_{2,3}) \mathcal{J}^{\mu\nu}_{1211}(x,\tilde{x},b) + \delta\left(x_3 + \frac{q_1^+}{p_1^+}\right) C_2^*(x_{1,2}) C_1 \mathcal{J}^{\mu\nu}_{2111}(x,\tilde{x},b) \right)$$

$$+ \int dx [d\tilde{x}] \delta\left(x - \frac{q^+}{p_1^+}\right)$$

$$\times \left(C_1^* C_2(\tilde{x}_{2,3}) \delta\left(\tilde{x}_1 - \frac{q^-}{p_2^-}\right) \mathcal{J}^{\mu\nu}_{1112}(x,\tilde{x},b) + C_2^*(\tilde{x}_{1,2}) C_1 \delta\left(\tilde{x}_3 + \frac{q^-}{p_2^-}\right) \mathcal{J}^{\mu\nu}_{1121}(x,\tilde{x},b) \right)$$

$$+ \ldots \Bigg\}$$

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TMD factorization at NLP

Effective operator for any process (DY, SIDIS, SIA)

$$\mathcal{J}^{\mu\nu}_{\text{eff}}(q) = \int \frac{d^2b}{(2\pi)^2} e^{-i(qb)} \Bigg\{ \int dx d\tilde{x} \delta\left(x - \frac{q^+}{p_1^+}\right) \delta\left(\tilde{x} - \frac{q^-}{p_2^-}\right) |C_1|^2 \mathcal{J}^{\mu\nu}_{1111}(x, \tilde{x}, b) \qquad (6.17)$$

$$+ \int [dx] d\tilde{x} \delta\left(\tilde{x} - \frac{q^-}{p_2^-}\right) \ldots (x, \tilde{x}, b)\Big) \ldots (x, \tilde{x}, b)\Big)$$

$$\mathcal{J}^{\mu\nu}_{1111}(x, \tilde{x}, b) = \frac{\gamma^{\mu}_{T,ij}\gamma^{\nu}_{T,kl}}{N_c} \Big(\mathcal{O}^{li}_{11,\bar{n}}(x, b) \overline{\mathcal{O}}^{jk}_{11,n}(\tilde{x}, b) + \overline{\mathcal{O}}^{jk}_{11,\bar{n}}(x, b) \mathcal{O}^{li}_{11,n}(\tilde{x}, b) \Big)$$
$$+ i \frac{n^\mu \gamma^{\rho}_{T,ij}\gamma^{\nu}_{T,kl} + n^\nu \gamma^{\mu}_{T,ij}\gamma^{\rho}_{T,kl}}{q^+ N_c} \Big(\partial_\rho \mathcal{O}^{li}_{11,\bar{n}}(x, b) \overline{\mathcal{O}}^{jk}_{11,n}(\tilde{x}, b) + \partial_\rho \overline{\mathcal{O}}^{jk}_{11,\bar{n}}(x, b) \mathcal{O}^{li}_{11,n}(\tilde{x}, b) \Big)$$
$$+ i \frac{\bar{n}^\mu \gamma^{\rho}_{T,ij}\gamma^{\nu}_{T,kl} + \bar{n}^\nu \gamma^{\mu}_{T,ij}\gamma^{\rho}_{T,kl}}{q^- N_c} \Big(\mathcal{O}^{li}_{11,\bar{n}}(x, b) \partial_\rho \overline{\mathcal{O}}^{jk}_{11,n}(\tilde{x}, b) + \overline{\mathcal{O}}^{jk}_{11,\bar{n}}(x, b) \partial_\rho \mathcal{O}^{li}_{11,n}(\tilde{x}, b) \Big),$$

- Operators of $(1,1) \times (1,1)$ (ordinary TMDs)

$$\mathcal{O}^{ij}_{11}(x, b) = p_+ \int \frac{d\lambda}{2\pi} e^{-ix\lambda p_+} \bar{q}_j[\lambda n + b, \pm\infty n + b][\pm\infty n, 0] q_i$$

- Contains LP and NLP (total derivatives)
- Restores EM gauge invariance up to λ^3

$$q_\mu J^{\mu\nu}_{1111} \quad \sim \quad (p_1^- q_T \; + \; p_2^+ q_T) J_{1111}$$

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TMD factorization at NLP

Effective operator for any process (DY, SIDIS, SIA)

$$\mathcal{J}^{\mu\nu}_{\text{eff}}(q) = \int \frac{d^2b}{(2\pi)^2} e^{-i(qb)} \Big\{ \int dx d\tilde{x} \delta\left(x - \frac{q^+}{p_1^+}\right) \delta\left(\tilde{x} - \frac{q^-}{p_2^-}\right) |C_1|^2 \mathcal{J}^{\mu\nu}_{1111}(x,\tilde{x},b) \tag{6.17}$$

$$+ \int [dx] d\tilde{x} \delta\left(\tilde{x} - \frac{q^-}{p_2^-}\right)$$

$$\times \left(\delta\left(x_1 - \frac{q_1^+}{p_1^+}\right) C_1^* C_2(x_{2,3}) \mathcal{J}^{\mu\nu}_{1211}(x,\tilde{x},b) + \delta\left(x_3 + \frac{q_1^+}{p_1^+}\right) C_2^*(x_{1,2}) C_1 \mathcal{J}^{\mu\nu}_{2111}(x,\tilde{x},b) \right)$$

$$+ \int dx [d\tilde{x}] \delta\left(x - \frac{q^+}{p_1^+}\right)$$

[partly hidden] $\ldots (x,\tilde{x},b) \Big)$

$$\mathcal{J}^{\mu\nu}_{1211}(x,\tilde{x},b) =$$
$$\frac{ig}{x_2}\left(\frac{\bar{n}^\nu}{q^-} - \frac{n^\nu}{q^+}\right) \frac{\gamma^\mu_{T,ij}\delta_{kl}}{N_c} \Big(\mathcal{O}^{jk}_{12,\bar{n}}(x,b) \overline{\mathcal{O}}^{jk}_{11,n}(\tilde{x},b) - \overline{\mathcal{O}}^{jk}_{12,\bar{n}}(x,b) \mathcal{O}^{li}_{11,n}(\tilde{x},b) \Big)$$

- Operators of $(1,2)\times(1,1)$
  $$\mathcal{O}^{ij}_{12}(x_{1,2,3},b) = p_+^2 \int \frac{dz_{1,2,3}}{2\pi} e^{-ix^i z_i p_+} \bar{q}_j [z_1 n + b, \pm\infty n + b][\pm\infty n, z_2 n]\gamma^\mu F_{\mu+}[z_2 n, z_3 n] q_i$$
- EM gauge invarint only up to NNLP
  $$q_\mu J^{\mu\nu}_{1211} \quad \sim \quad (p_1^- \; + \; p_2^+) J_{1211}$$

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TMD factorization at NLP

Effective operator for any process (DY, SIDIS, SIA)

$$\mathcal{J}^{\mu\nu}_{\text{eff}}(q) = \int \frac{d^2b}{(2\pi)^2} e^{-i(qb)} \Bigg\{ \int dx d\tilde{x} \delta\left(x - \frac{q^+}{p_1^+}\right) \delta\left(\tilde{x} - \frac{q^-}{p_2^-}\right) |C_1|^2 \mathcal{J}^{\mu\nu}_{1111}(x,\tilde{x},b) \tag{6.17}$$

$$+ \int [dx] d\tilde{x} \delta\left(\tilde{x} - \frac{q^-}{p_2^-}\right)$$

$$\times \left(\delta\left(x_1 - \frac{q_1^+}{p_1^+}\right) C_1^* C_2(x_{2,3}) \mathcal{J}^{\mu\nu}_{1211}(x,\tilde{x},b) + \delta\left(x_3 + \frac{q_1^+}{p_1^+}\right) C_2^*(x_{1,2}) C_1 \mathcal{J}^{\mu\nu}_{2111}(x,\tilde{x},b) \right)$$

$$+ \int dx [d\tilde{x}] \delta\left(x - \frac{q^+}{p_1^+}\right)$$

$$(x,\tilde{x},b)\Big)$$

$$C_1 = 1 + a_s C_F\left(-\mathbf{L}_Q^2 + 3\mathbf{L}_Q - 8 + \frac{\pi^2}{6}\right) + O(a_s),$$

$$C_2(x_{1,2}) = 1 + a_s\Bigg[C_F\left(-\mathbf{L}_Q^2 + \mathbf{L}_Q - 3 + \frac{\pi^2}{6}\right) + C_A \frac{x_1+x_2}{x_1} \ln\left(\frac{x_1+x_2}{x_2}\right)$$
$$+ \left(C_F - \frac{C_A}{2}\right) \frac{x_1+x_2}{x_2} \ln\left(\frac{x_1+x_2}{x_1}\right)\left(2\mathbf{L}_Q - \ln\left(\frac{x_1+x_2}{x_1}\right) - 4\right)\Bigg]$$

- Coefficient functions up to NLO
- C_1 is know up to N^{3}LO
- C_1 is same for LP, NLP, ... parts of operator $J^{\mu\nu}_{1111}$

Evolution for NLP TMD operators (distributions)

$$\mu^2 \frac{dO_{NM}}{d\mu^2}(\mu,\zeta) = (\gamma_N(\mu,\zeta)+\gamma_M(\mu,\zeta))\otimes O_{NM}(\mu,\zeta)$$
$$\zeta \frac{dO_{NM}}{d\zeta}(\mu,\zeta) = -\mathcal{D}(b,\mu)\otimes O_{NM}(\mu,\zeta)$$

- $\gamma_1 = a_s C_F\left(\frac{3}{2}+\ln(\mu^2/\zeta)\right)+...$ (known up to N^{3}LO)
- $\gamma_2 = a_s\{2\mathbb{H}_1+\gamma_1+\frac{\ln x_1}{N_c}-N_c\ln x_2\}+a_s^2...$
  - $\mathbb{H}_1$ is the Bukhvostov-Frolov-Lipatov-Kuraev kernel for qF
- $\mathcal{D}$ is CS-kernel (non-perturbative)
  - Same for LP and NLP operators!
- **For higher power operators evolution has same structure**
  - UV AD at NLO can be easily reconstructed from [Braun,Manashov,Rohrwild,09] (+ cusps)
  - CS-kernel is identical for all quasi-partonic operators

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Conclusion

TMD operator expansion – an efficient approach to TMD factorization beyond LP

- Operator level
- Position space
- Strict & intuitive rules for operator sorting (TMD-twist)
- All processes

TMD factorization at NLP is derived

- Coefficient function at NLO
- Evolution at NLO
- Rapidity evolution of NLP is the same as for LP
- **Some results are simple to generalize beyond NLP**