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Universality of quantum corrections to transverse momentum broadening

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@ Resummation, Evolution, Factorization
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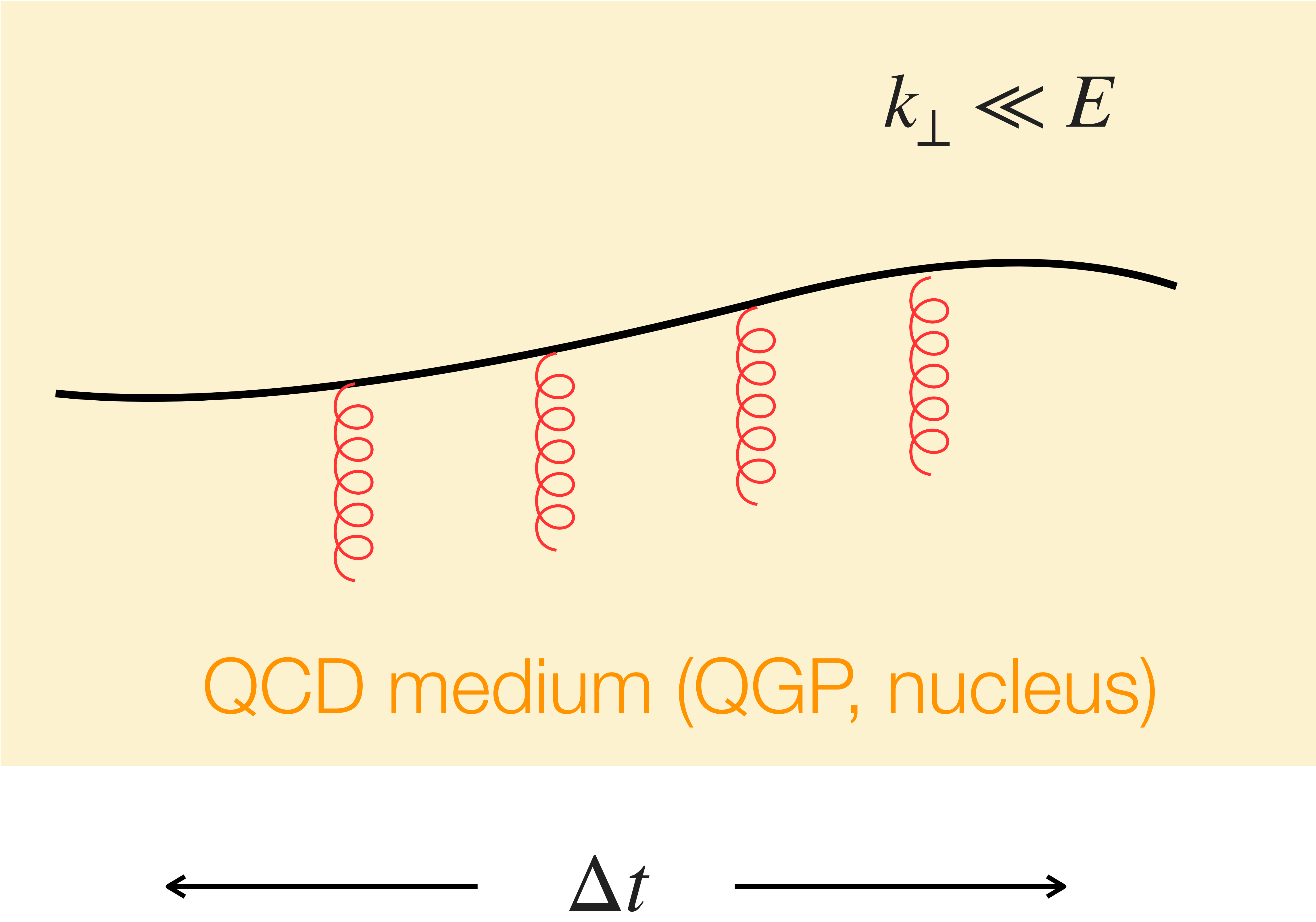
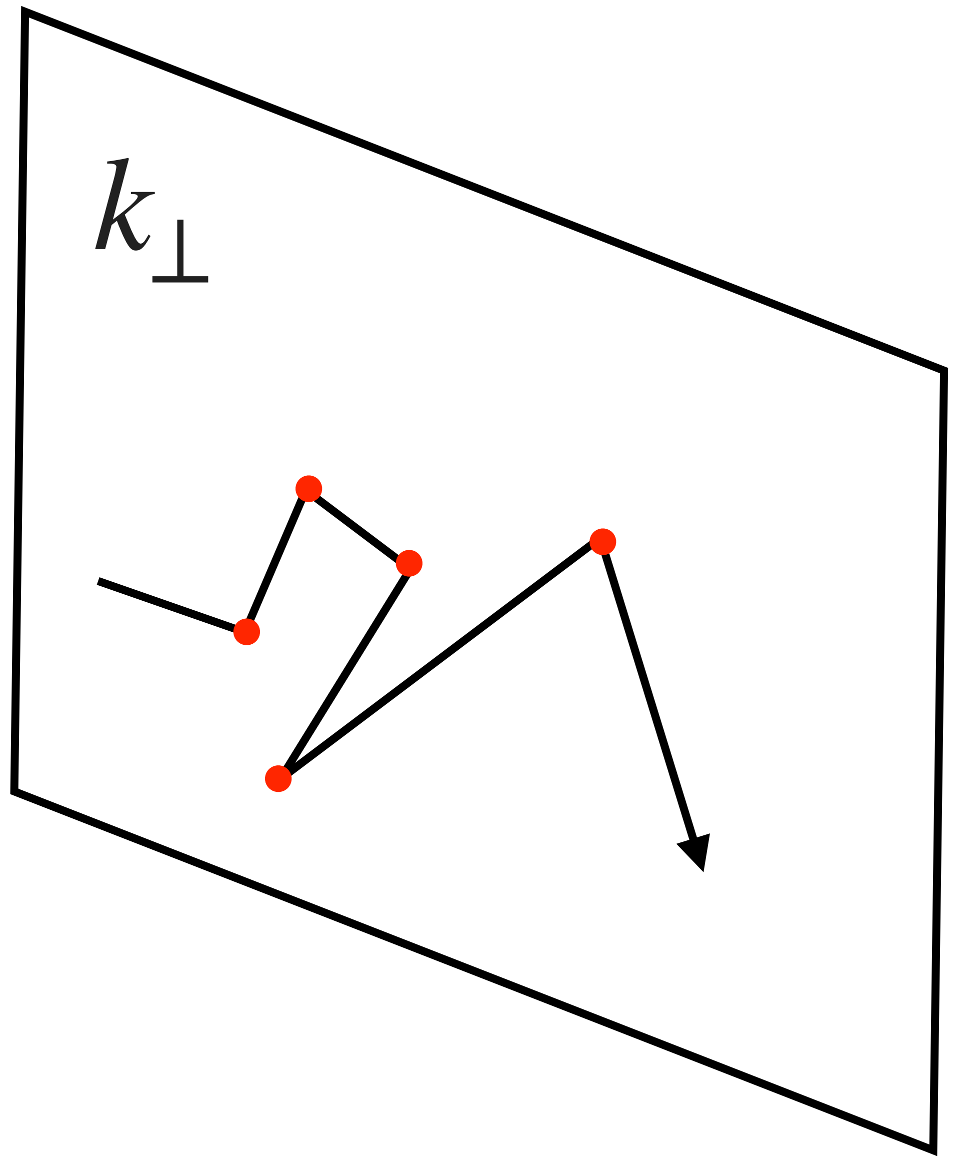
Transverse momentum broadening in QCD media

High energy partons experience random kicks in hot or cold nuclear matter that cause their transverse momentum to increase over time

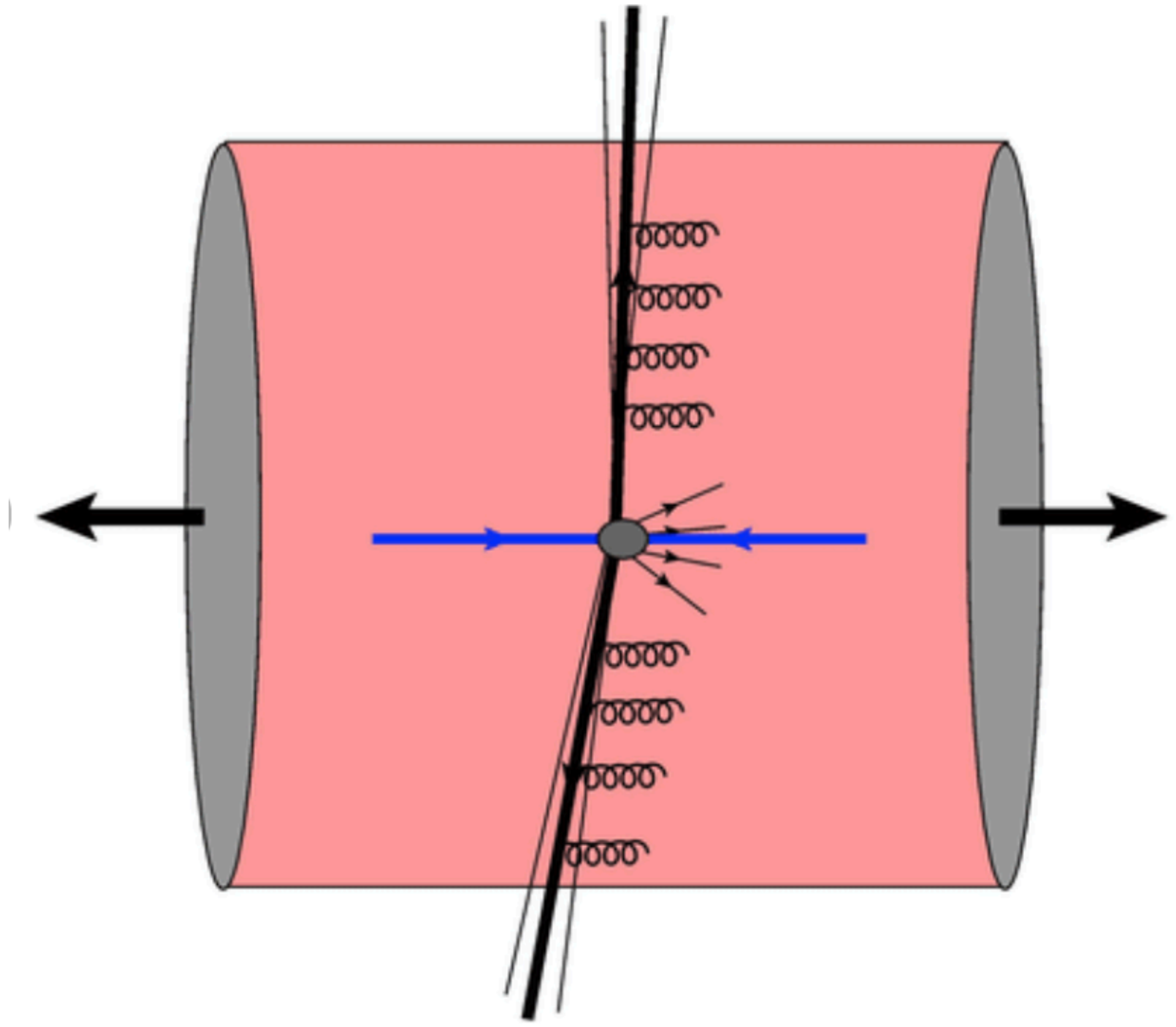




Transverse plane

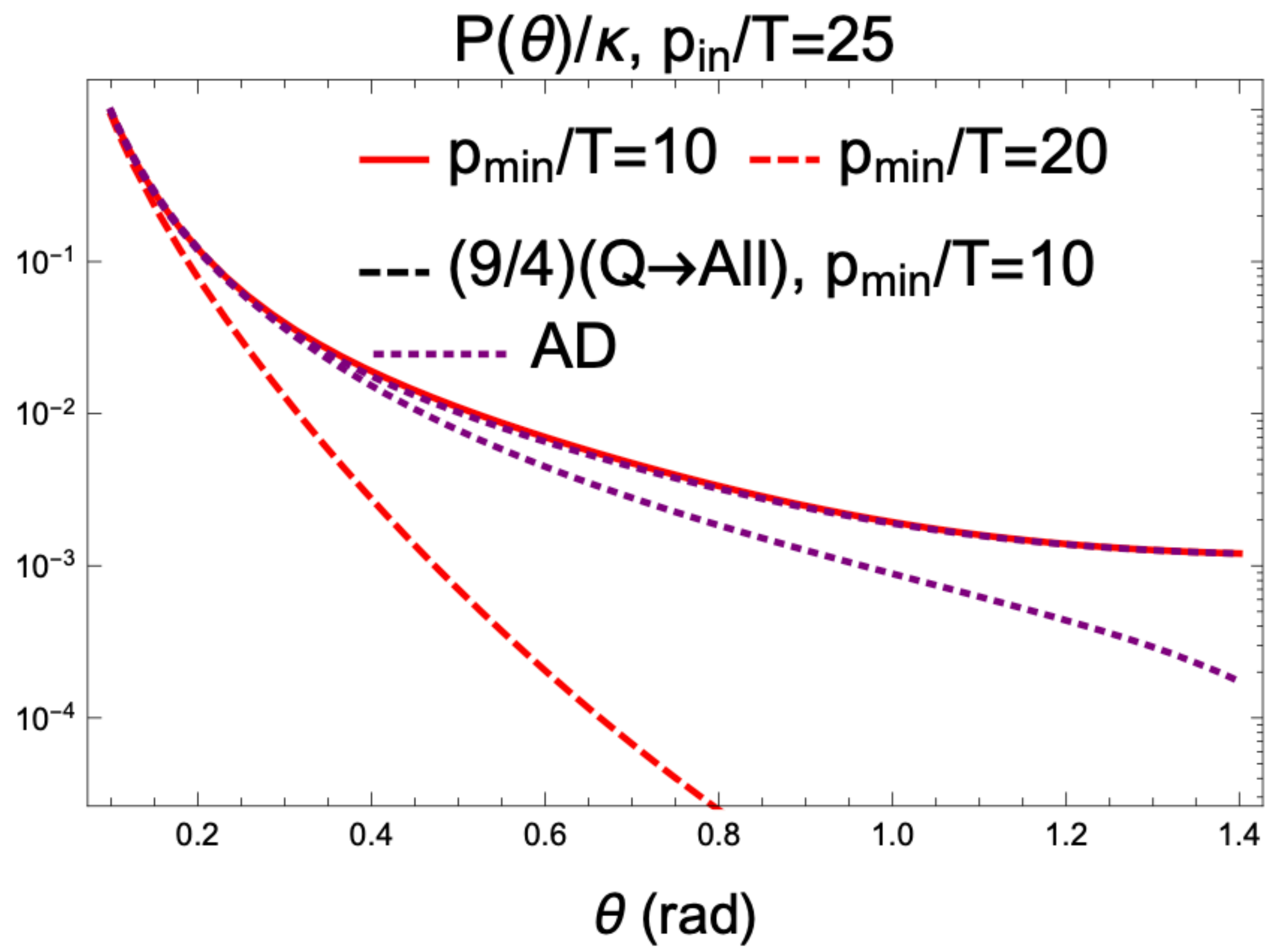
TMB in various processes

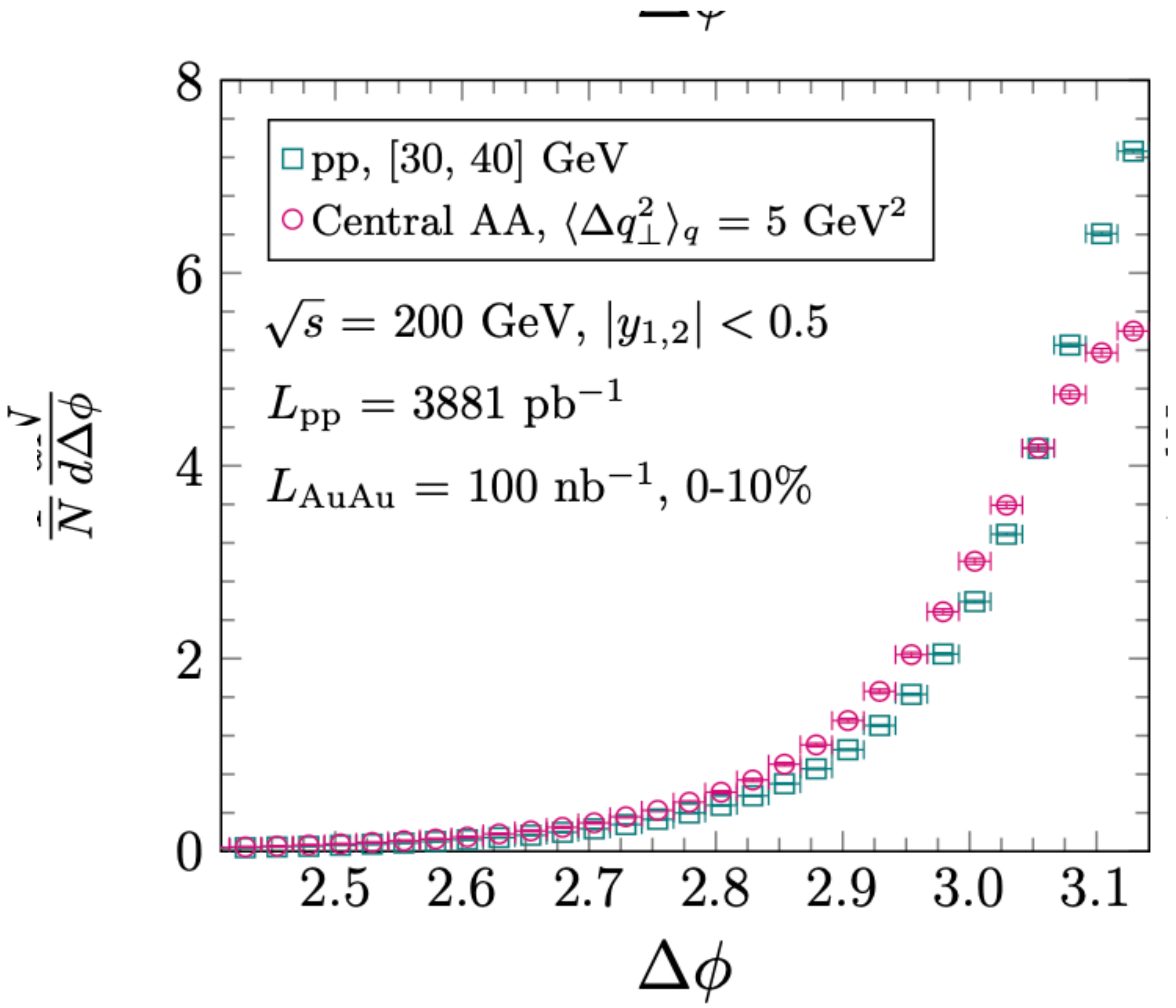
Probe the QGP in Heavy Ion Collisions: dijet azimuthal de-correlation, Rutherford scattering, …



Mueller, Wu, Xiao, Yuan (2016)

D'Eramo, Rajagopal, Yi (2019)



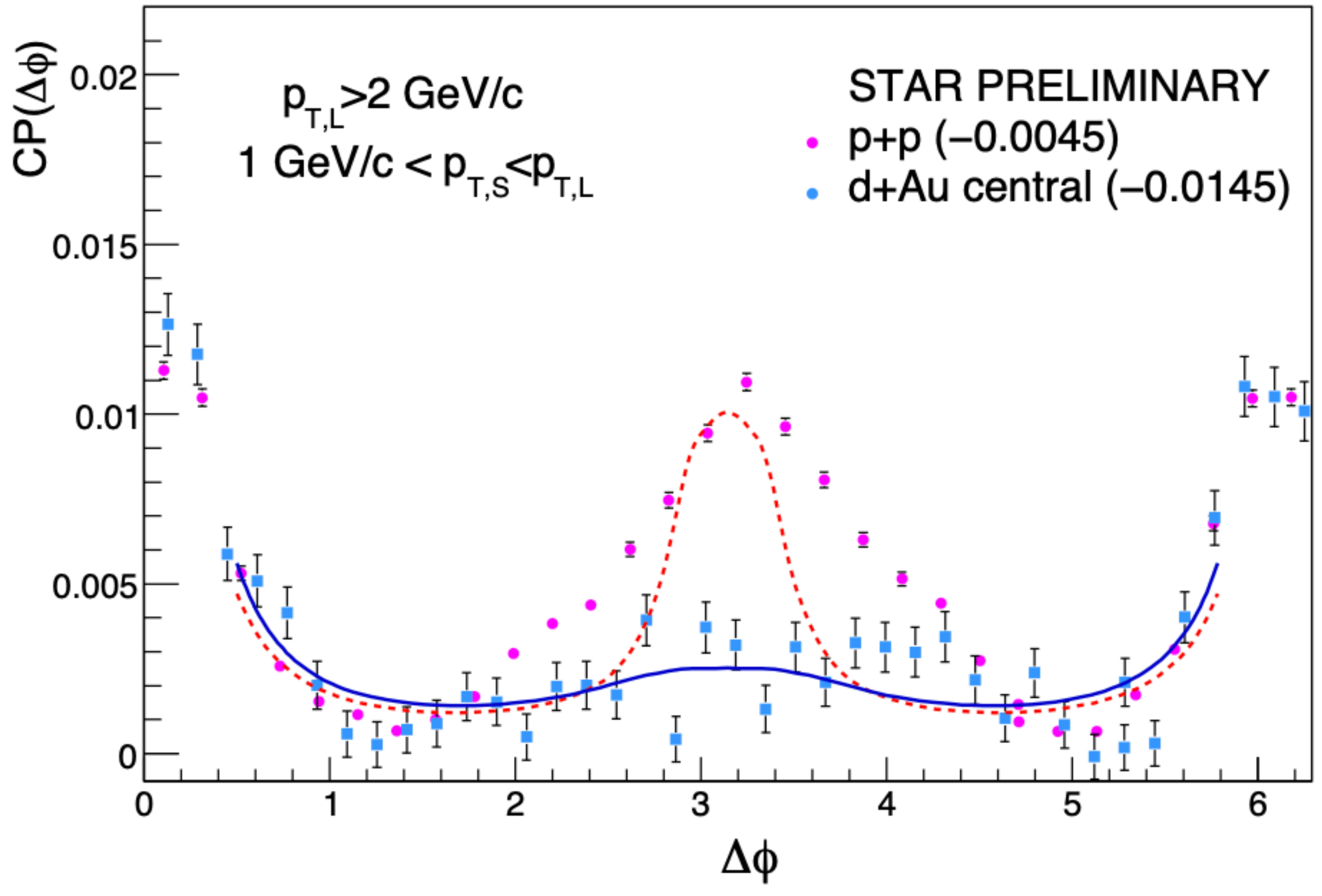


Jia, Xiao, Yuan (2019)

TMB in various processes

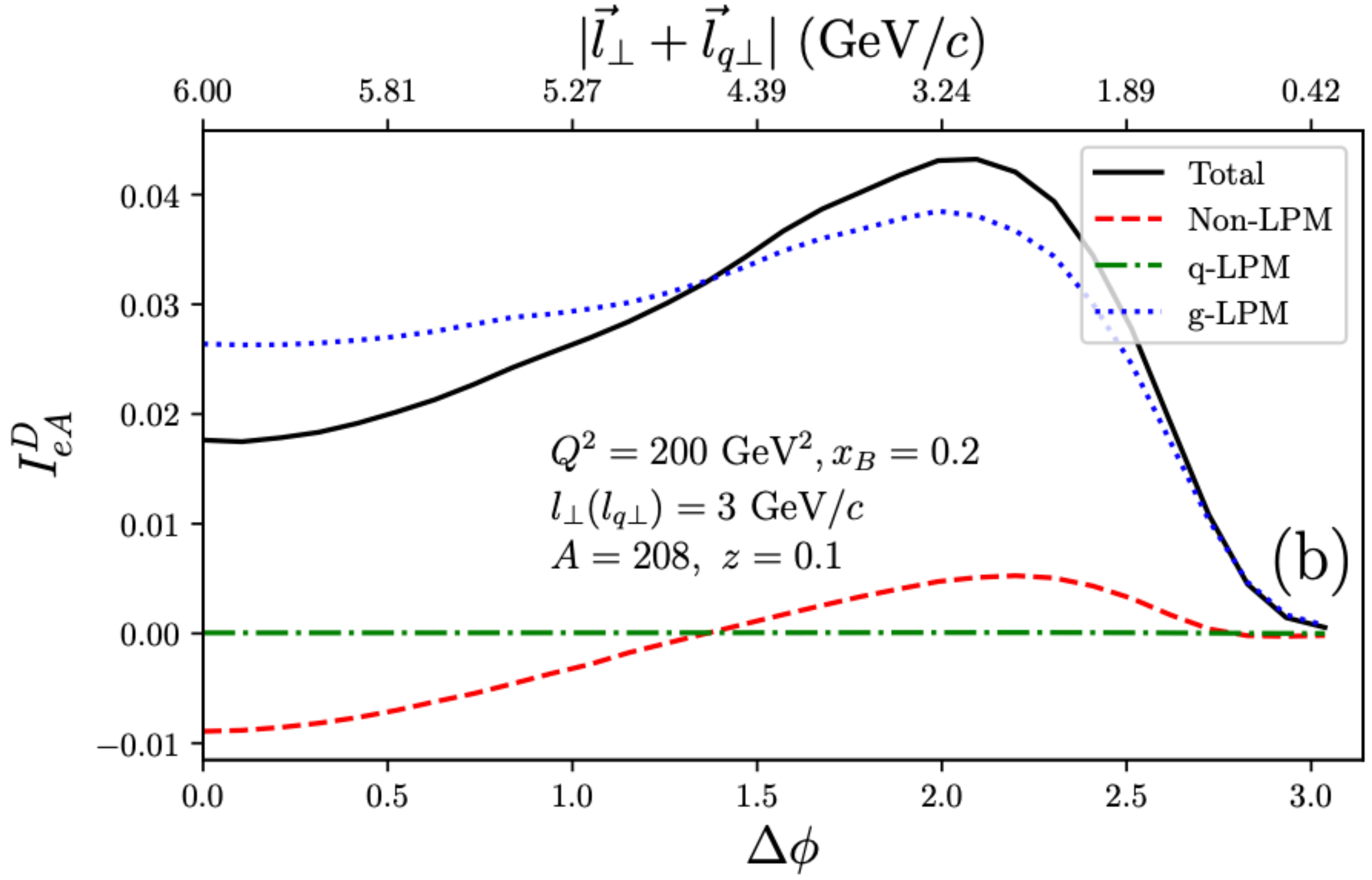
Probe cold nuclear matter electron-Nucleus collisions: Semi-Inclusive DIS, (forward) dihadron/dijet

di-hadron



Albacete, Marquet (2010)

di-jet



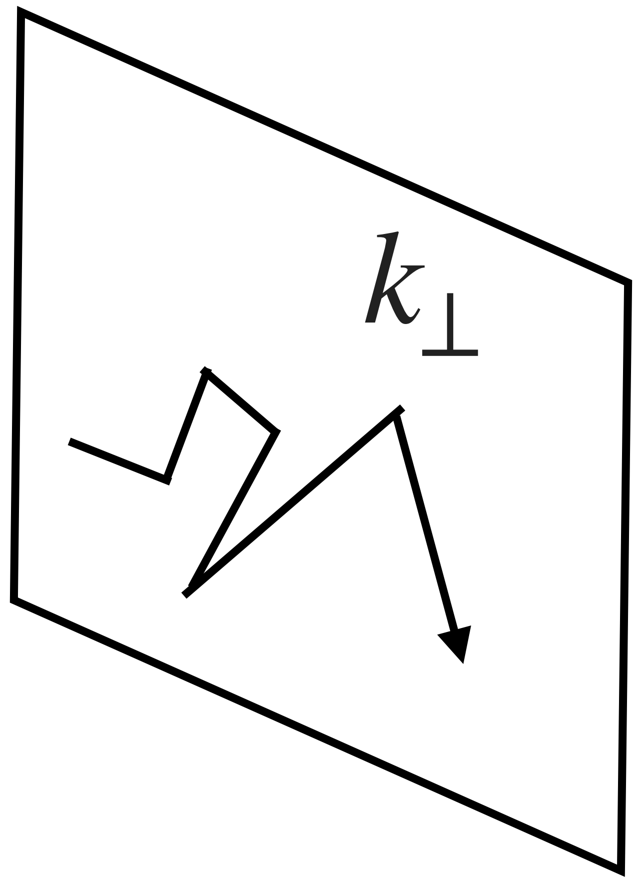
Wang, Zhang (2021)

- Probability distribution for a high energy particle to acquire a transverse momentum $k_\perp$ after a time t in the plasma obeys a Fokker-Planck equation:

$$\frac{\partial}{\partial t} P(k_\perp) = \frac{1}{2}\,\hat{q}\,\nabla_\perp^2 P(k_\perp)$$

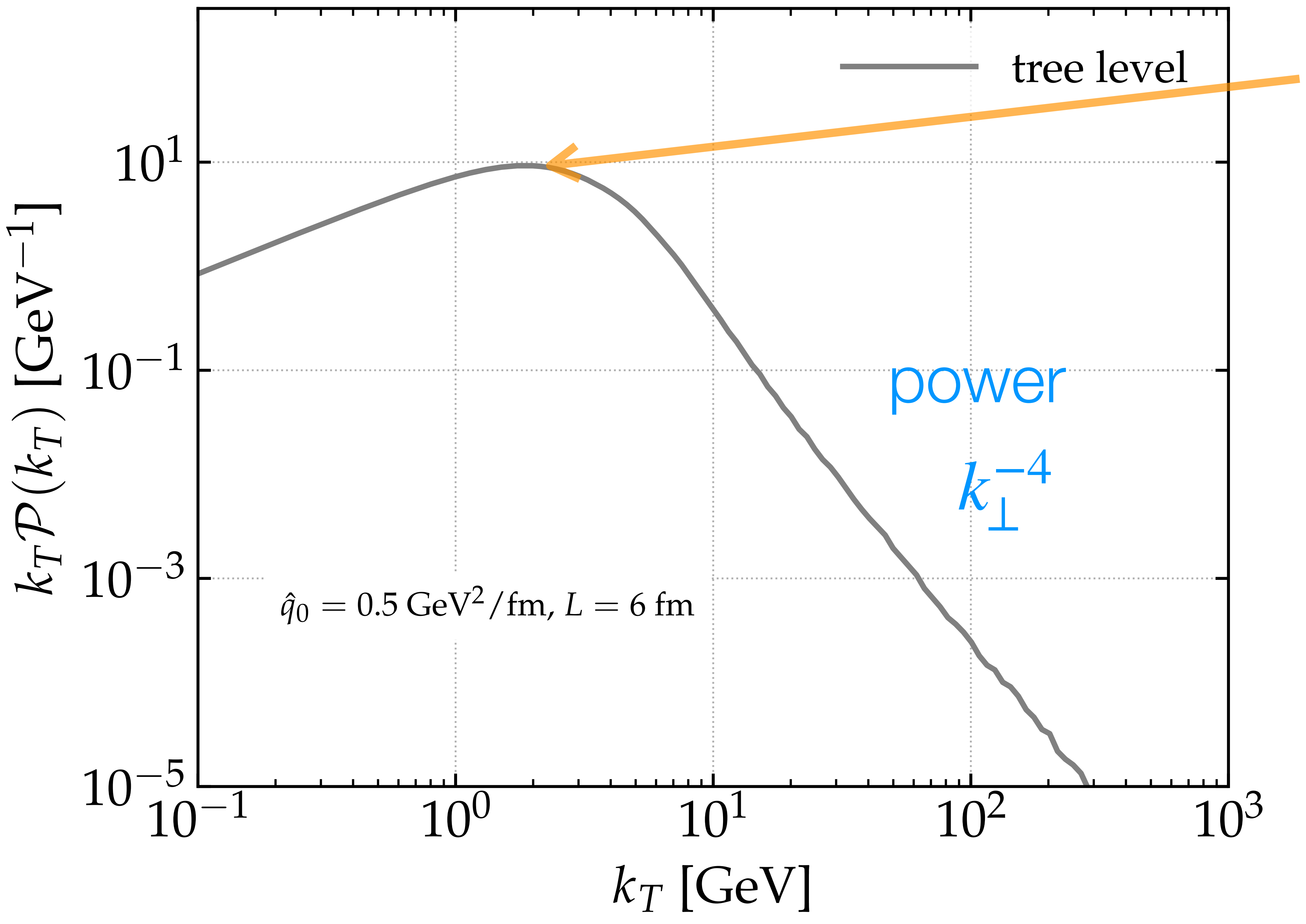
- Transverse momentum broadening: normal diffusion

$$\langle k_\perp^2 \rangle_{\rm typ} \propto t$$



Tree level

Transverse momentum broadening distribution at leading order: gaussian for $k_\perp < Q_s \sim \hat{q}_0 L$ and exhibits the power law tail $k_\perp^{-4}$ for $k_\perp > Q_s$



$$P(k_\perp) = \frac{4\pi}{\hat{q}L} e^{-\frac{k_\perp^2}{\hat{q}L}}$$

Standard diffusion

$$\langle k_\perp^2 \rangle_{\rm typ} \propto L$$

Jet quenching parameter at tree level

$$\hat{q} = C_R \int_{q_\perp} q_\perp^2 \frac{\mathrm{d}^2\sigma}{\mathrm{d}^2 q_\perp} \simeq 4\pi\alpha_s^2 n \log\frac{q_{\max}}{\mu^2}$$

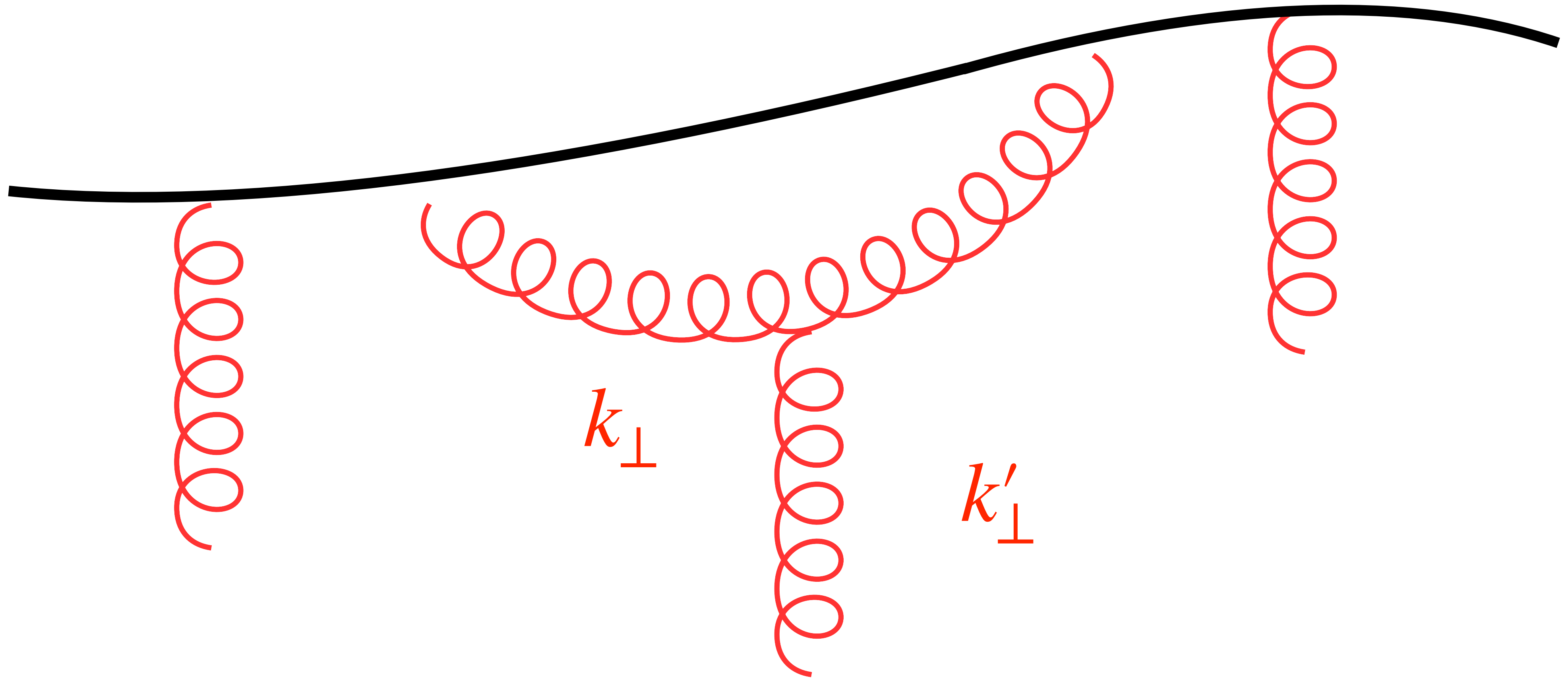
Q: What are the effects of quantum corrections on transverse momentum broadening?

Quantum corrections to $\hat{q}$

- Potentially large double logs (DL) in transverse momentum broadening at NLO

$$\langle k_\perp^2 \rangle = \hat{q}_0 L \left(1 + \frac{\bar{\alpha}}{2} \log^2 \frac{L}{\tau_0} \right)$$

$$\tau_0 \sim \ell_{\rm mfp} \ll L$$



[Liou, Mueller, Wu (2013)]

- Not the standard DGLAP double log: the factor 1/2 reflects the presence of multiple scattering constraint $k_\perp > \hat{q}\,\tau \equiv Q_s^2$

Nonlinear evolution of $\hat{q}$

- All orders resummed and absorbed in a redefinition of the jet quenching parameter

$$\frac{\partial}{\partial \tau}\hat{q}(k_\perp,\tau) = \bar{\alpha}\int_{Q_s^2(\tau)}^{k_\perp^2} \frac{\mathrm{d}k_\perp'^2}{k_\perp'^2}\,\hat{q}(k_\perp',\tau)$$

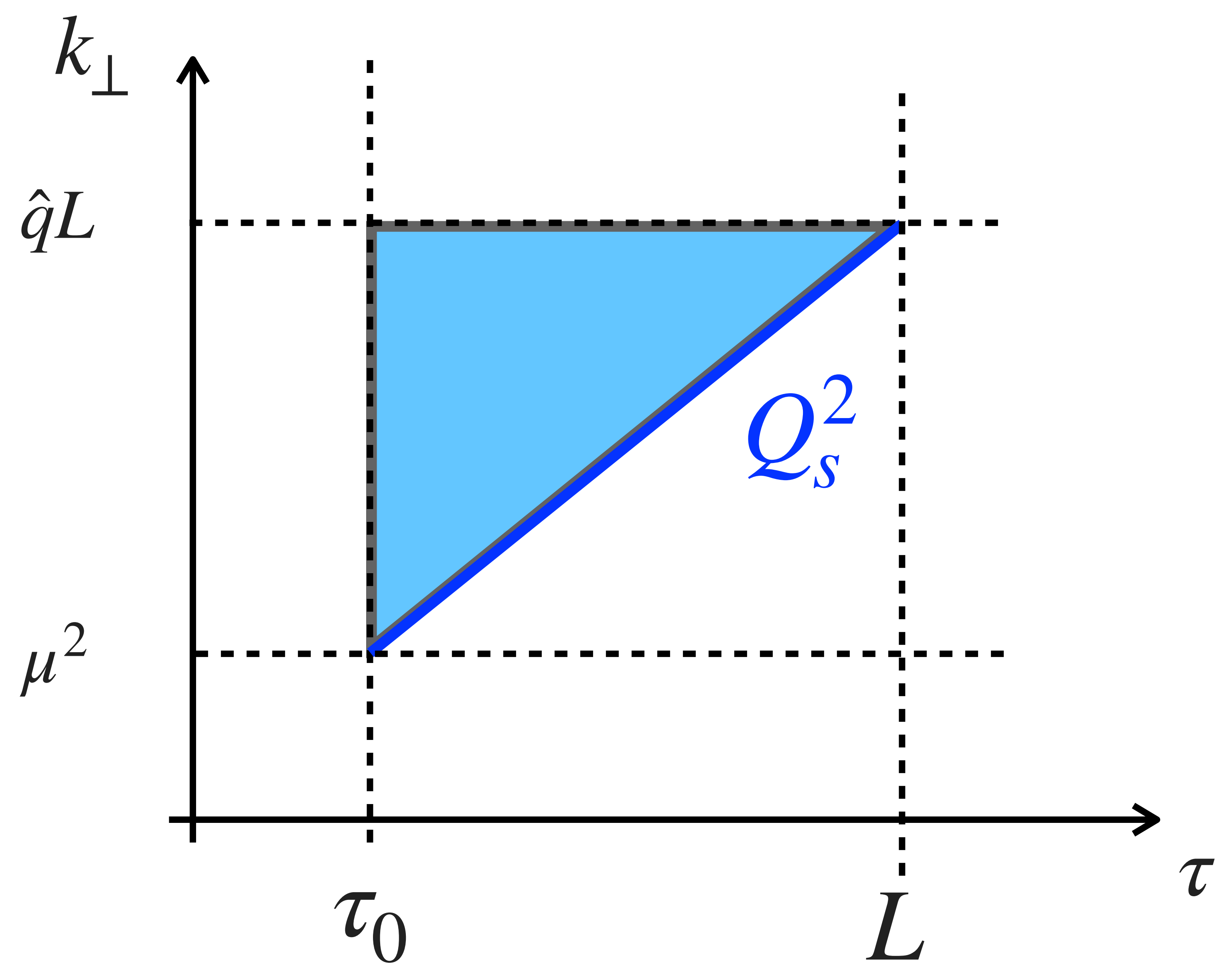
[Blaizot, MT (2014), Iancu (2014)]

$\rightsquigarrow$ reminiscent of Color-Glass-Condensate

[Mueller, Qiu, (1986), Venugopalan, McLerran (1993), Balitsky, Kovchegov, Jallilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner (1996-2001)]

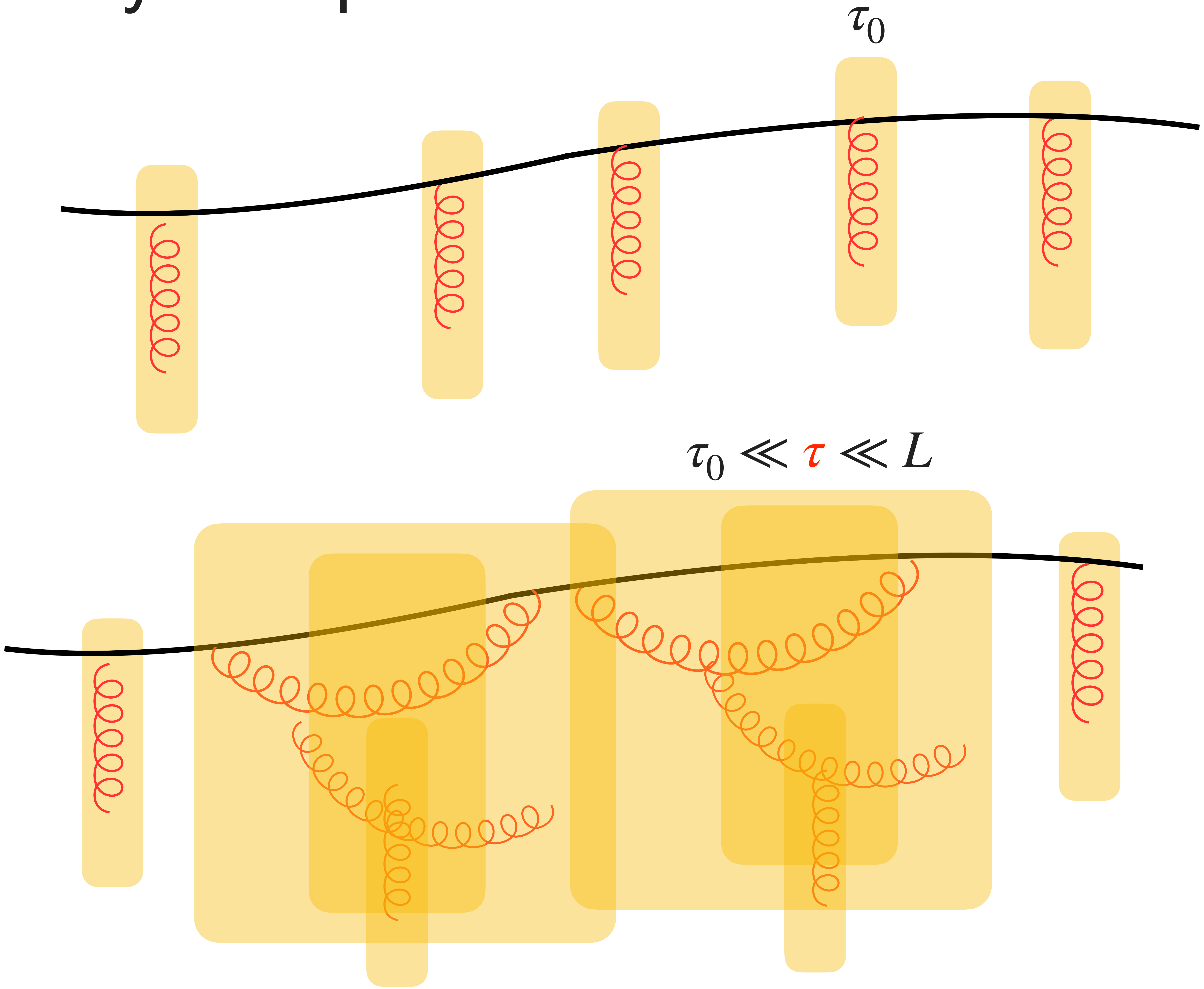
Multiple-scattering boundary:
saturation line

$$Q_s^2 \equiv \hat{q}(Q_s,\tau)\,\tau$$



Physical picture

- LO: local/instantaneous interactions
- DLA + saturation: non-locality of interactions
- Exponentiation of the double logs with adequate phase space constraints



Large momentum transfer (all orders DLA)

- Modification of the power law $k_\perp^{-4}$ (Coulomb scattering) due to higher orders

$$P(k_\perp) \equiv -\frac{1}{4} L \int \mathrm{d}^2 x_\perp \, x_\perp^2 \, \hat{q}(1/x_\perp, L) \, \mathrm{e}^{-i x_\perp \cdot k_\perp} \simeq \frac{L}{k_\perp^4} \frac{\partial}{\partial \log k_\perp^2} \hat{q}(k_\perp^2, L)$$

- Hard scattering regime: gluon PDF of the plasma to DLA:

$$\hat{q}(k_\perp^2) \equiv x g(x=0, k_\perp^2) \simeq \exp\left(\sqrt{\log \frac{k_\perp^2}{\mu^2} \log \frac{L}{\tau_0}} \right)$$

Expect energy dependence at high x:

Wang, Casalderrey-Solana (2007)
Kumar, Majumder, Chen (2019)

$\rightsquigarrow$ Non-linear effects due to multiple scattering may change this picture

observed in the context of saturation physics in DIS: [Mueller, Triantafyllopoulos, Iancu, McLerran, Itakura, Munier, Peschanski (2002-2004)]

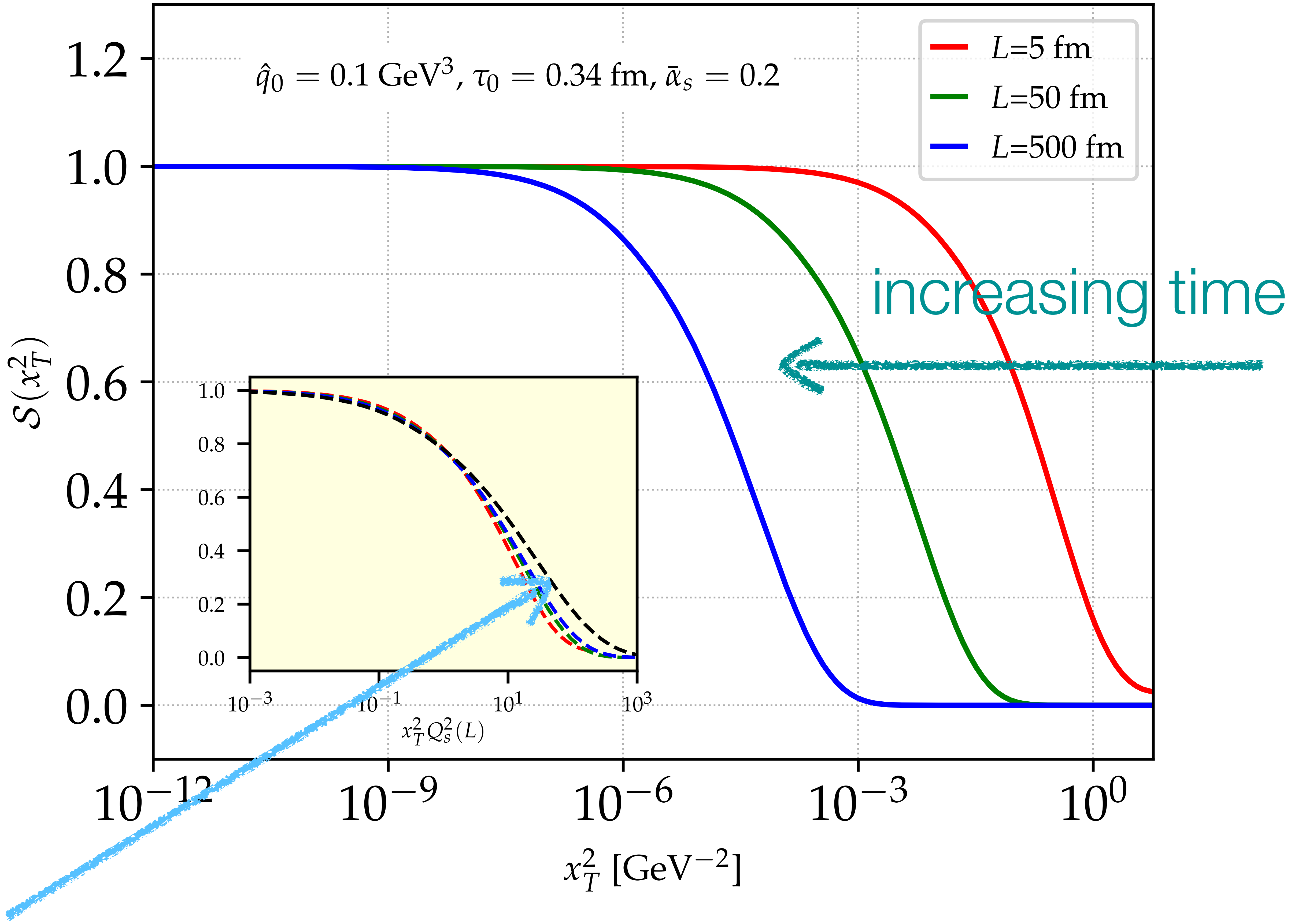
Traveling waves solutions

- Dipole S-matrix admits TW solutions

$$S(x_\perp) = \mathrm{e}^{-\frac{1}{4}x_\perp^2 L\,\hat{q}(1/x_\perp, L)}$$

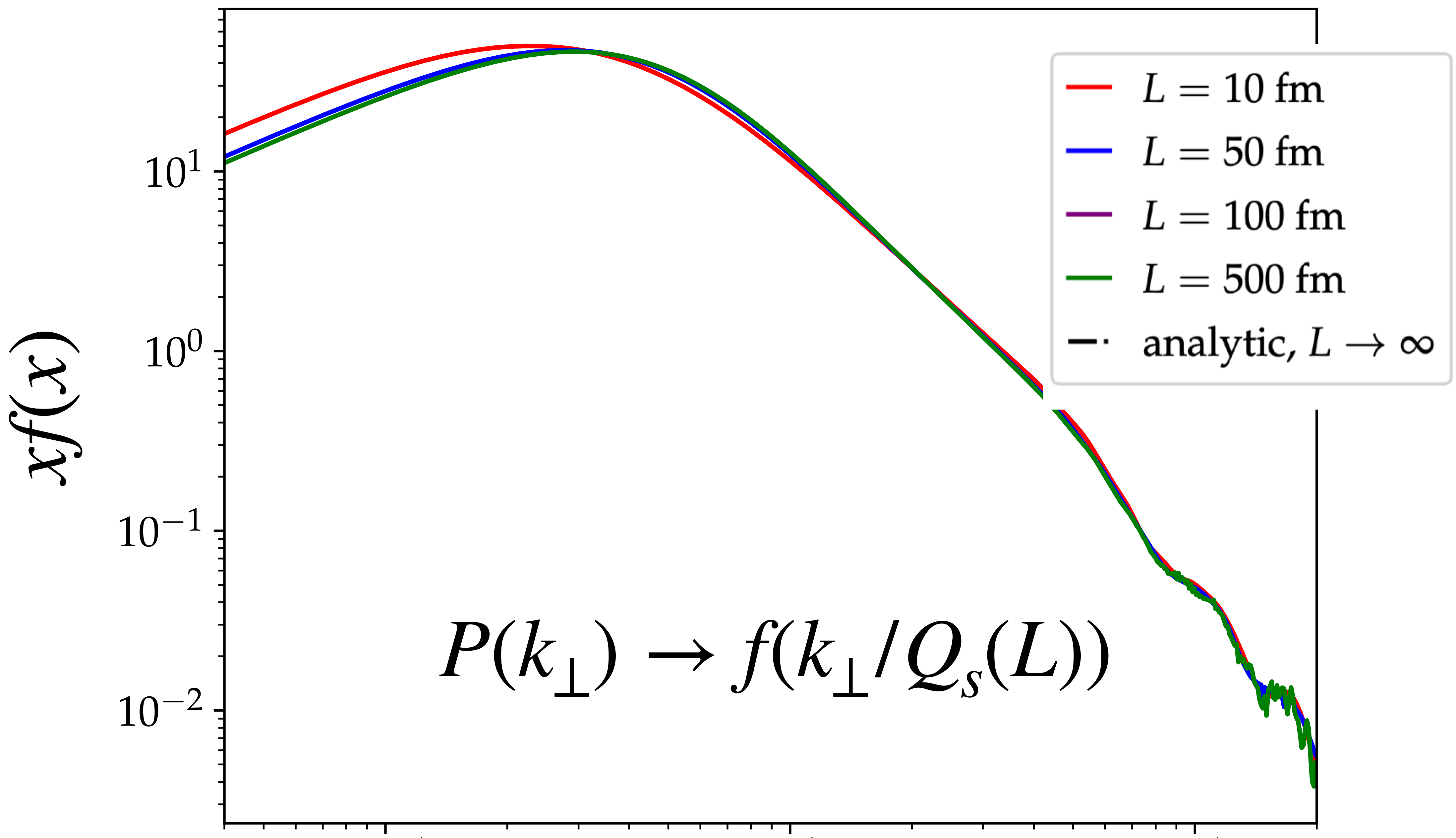
- Related to TMB by a Fourier transform

$$P(k_\perp) \equiv \int \mathrm{d}^2 x_\perp\; S(x_\perp)\; \mathrm{e}^{-i x_\perp \cdot k_\perp}$$



- NB: exact scaling solution (dashed black curve): are the deviations to the asymptotic limit universal?

Geometric scaling in TMB



$x \equiv k_\perp/Q_s$

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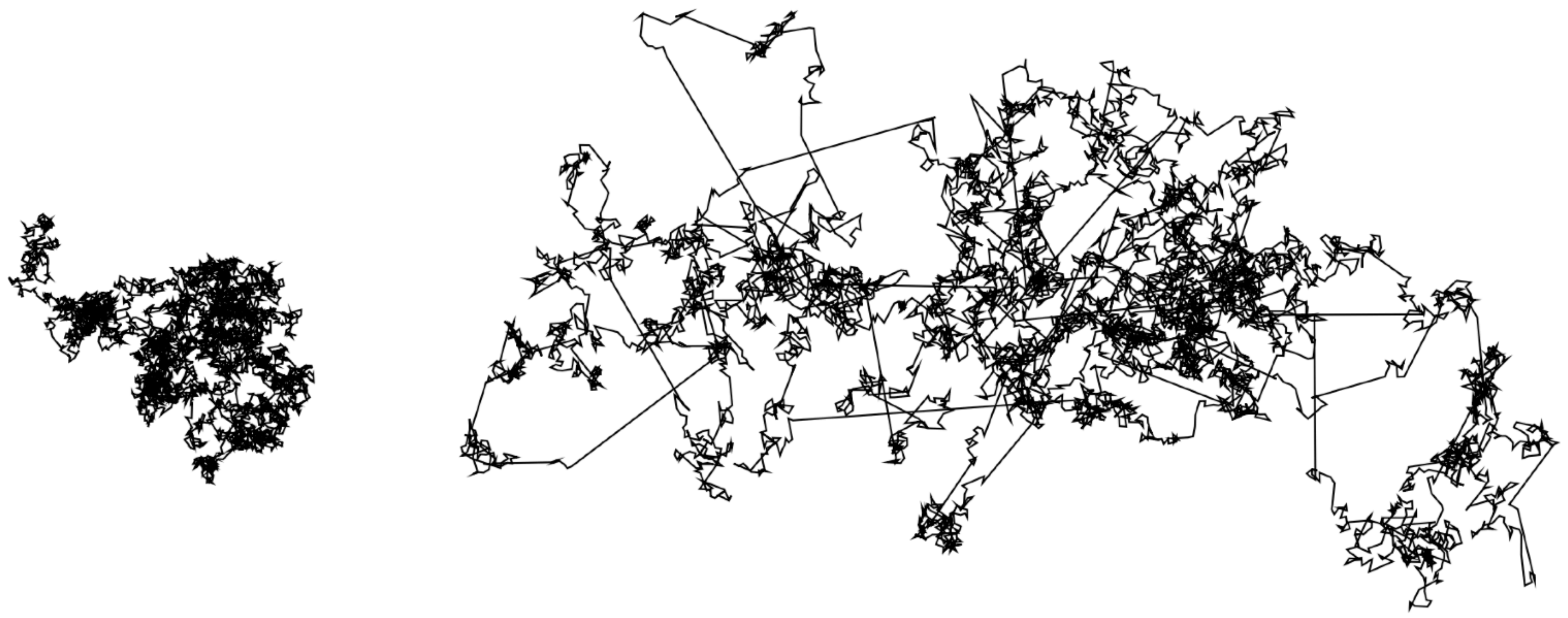


Brownian motion

Lévy flight

- Geometric scaling: TMB distribution depends on a single scaling variable
- Transverse momentum is a super diffusive process exhibiting scale invariance akin to Lévy flights

studied in the context of saturation physics in DIS: Mueller, Triantafyllopoulos, Iancu, McLerran, Itakura, Munier, Peschanski (2002-2004)

Sub-asymptotic solutions

- Using Brunet and Derrida's (1988) approach to FKPP Munier and Peschanski (2003) to BK equations
- Velocity and shape of the wave front of the form

$$\log Q_s^2(Y) = cY + b\log Y + \text{const}\,.$$

$$Y \equiv \log \frac{L}{\tau_0}$$

$$\frac{\hat{q}(Y,k_\perp)\,L}{Q_s^2} = \mathrm{e}^{\beta x}\, Y^\alpha\, G\left(\frac{x}{Y^\alpha}\right)$$

$$x \equiv \log \frac{k_\perp^2}{Q_s^2}$$

- The function G accounts for the diffusion of the wave front
- Recall that Q_s^2 is the typical transverse momentum scale

Universal pre-asymptotic solutions for $\hat{q}$

$$\frac{\hat{q}(Y,k_\perp)\,L}{Q_s^2} = \exp\left(\beta x - \frac{\beta x^2}{4cY}\right)\left[1 + \beta x + \frac{bx}{c^2 Y}\left(1 + \frac{\beta(c+4)x}{6}\right) + \mathcal{O}(Y^{-2})\right]$$

Blue: asymptotic limit. Orange: pre-asymptotic $\mathcal{O}(1/Y)$

$$x \equiv \log \frac{k_\perp^2}{Q_s^2}$$

$$c = 1 + 2\sqrt{\bar{\alpha} + \bar{\alpha}^2} + 2\bar{\alpha} \simeq 1 + 2\sqrt{\bar{\alpha}}$$

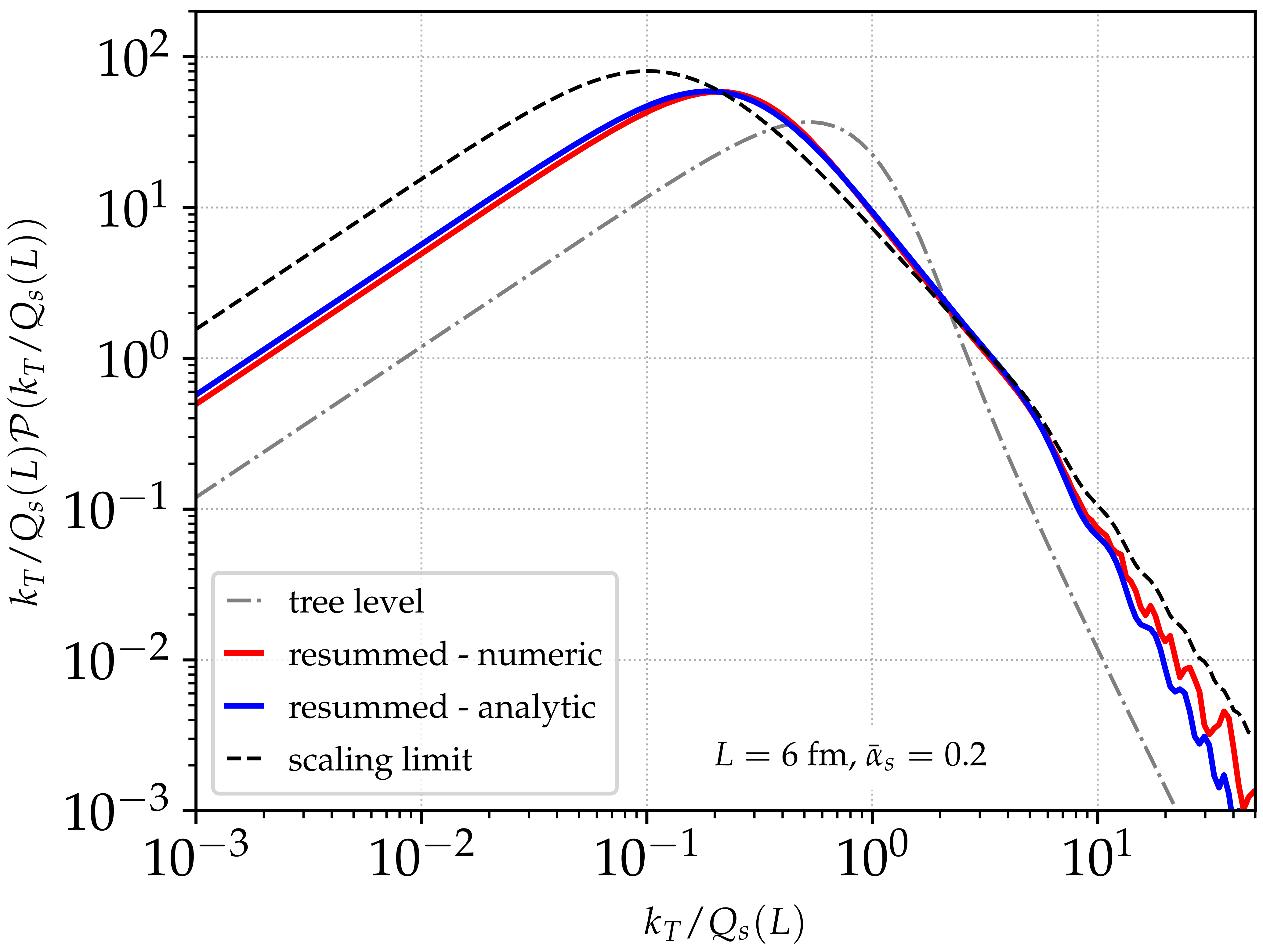
$$b = -\frac{2}{3(1-\beta)}$$

Slope of the wave front

$$\beta = \frac{c-1}{2c}$$

Analytic vs numerics

$$\frac{\hat{q}(Y,k_\perp)\,L}{Q_s^2} = \exp\left(\beta x - \frac{\beta x^2}{4cY}\right)\left[1 + \beta x + \frac{bx}{c^2 Y}\left(1 + \frac{\beta(c+4)x}{6}\right) + \mathcal{O}(Y^{-2})\right]$$



⤳ Universal pre-asymptotic solution provides a good description of numerical simulations for $L = 6\,\mathrm{fm}$ and $\bar{\alpha} = 0.2$

⤳ wider distribution due to heavy Lévy tail

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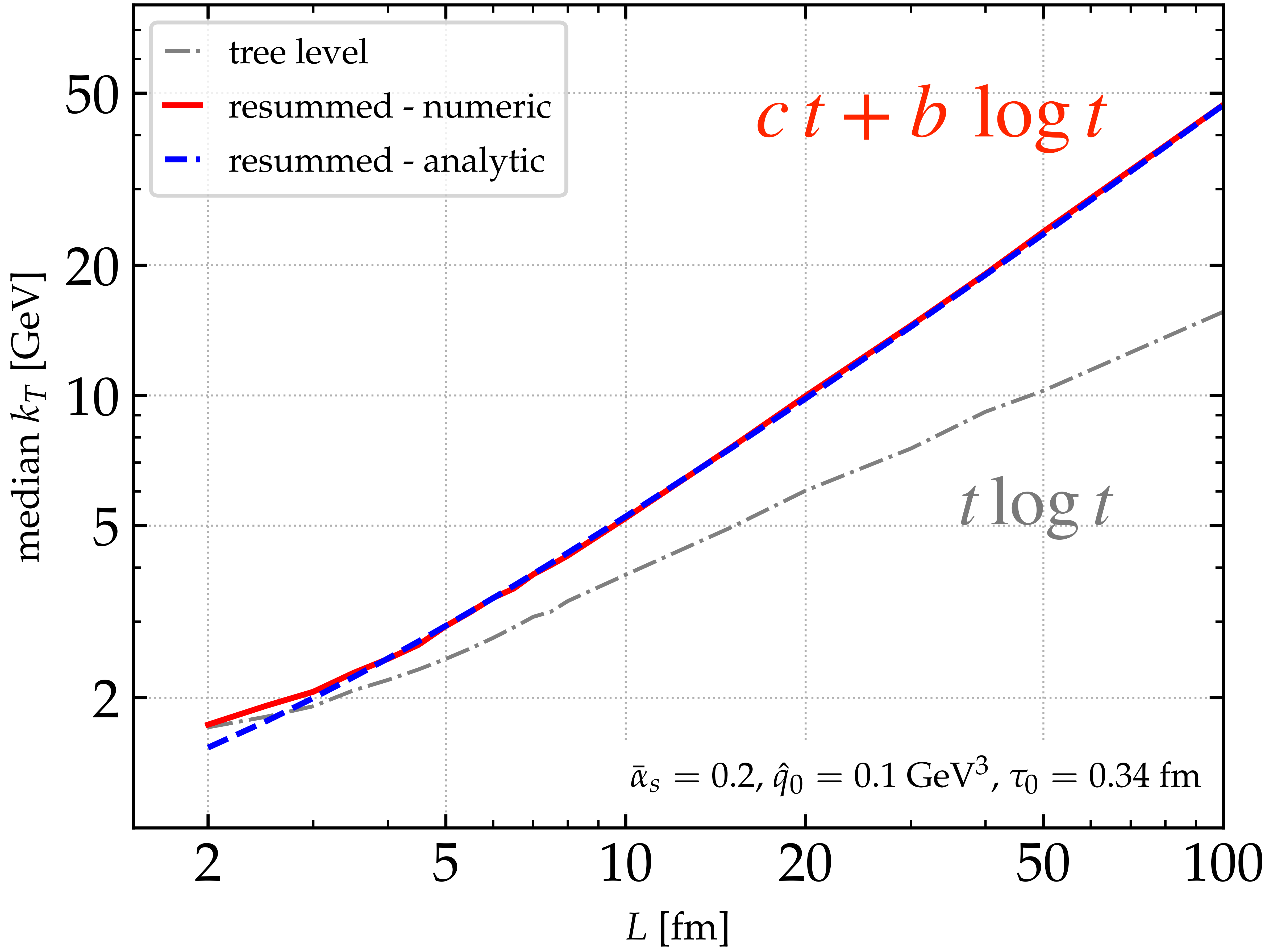
Time dependence of the typical transverse momentum

$$L_s(Y) = \log Q_s^2(Y) = cY + b \log Y + \text{const}\,.$$



Nonlocal quantum corrections:
anomalous system size dependence (super diffusion)

$$Q_s^2 \simeq \langle k_\perp^2 \rangle_{\text{median}} \propto L^{1+2\beta}$$

$$\beta \simeq \sqrt{\bar{\alpha}}$$

Universal behavior observed down to $L \simeq 3$ fm

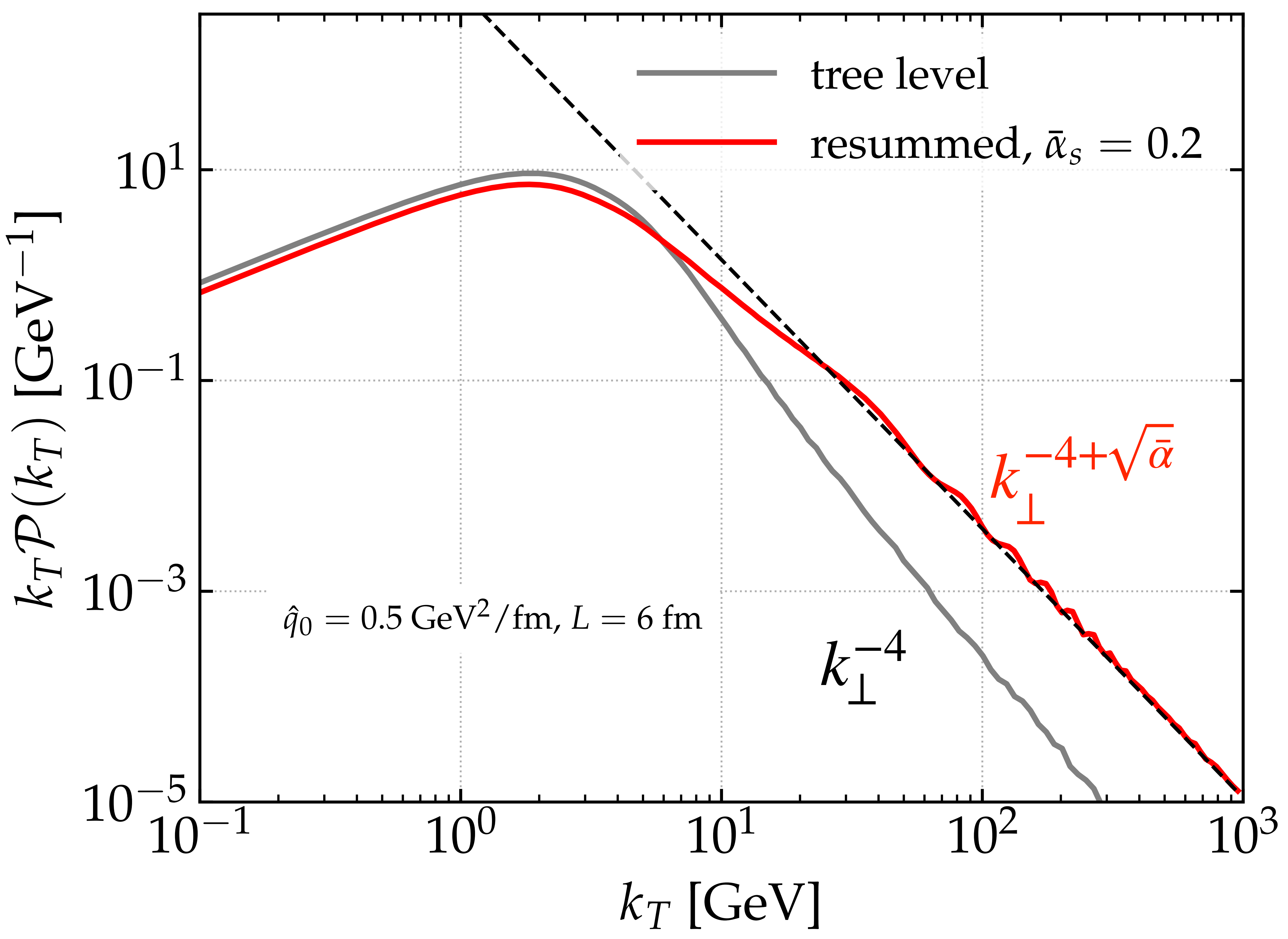
Heavy tail - Lévy flight

- Extended geometric scaling window:

$$Q_s(L) \; < \; k_\perp \; \ll \; \frac{Q_s^2}{\mu}$$

- Asymptotic behavior after resummation (heavy tail)

$$P(k_\perp) \to f(k_\perp/Q_s(L)) \simeq \left(\frac{Q_s^2(L)}{k_\perp^2} \right)^{4-\sqrt{\bar{\alpha}}}$$



- For $k_\perp \gg Q_s^2/\mu \;\; \Rightarrow \;\;$ expect Coulomb scaling $k_\perp^{-4}$

Applications

- TMD's, jet quenching, etc. New transverse momentum dependent distribution beyond gaussian approximation
- Initial condition for small x evolution
- Search for Rutherford scattering (point-like scatterers) in HIC [D'Eramo, Lekaveckas , Liu, Rajagopal (2012)]
- System size dependence of momentum broadening: probe the anomalous diffusion in the QGP

Summary

- TMB is a super-diffusive process due to logarithmically enhanced quantum corrections that
- TMB exhibits geometric scaling and heavy tails akin to Lévy random walks
- Exploiting an analogy with saturation physics and reaction-diffusion processes we find exact universal asymptotic and pre-asymptotic solutions for the transverse momentum distribution
- Outlook: investigate anomalous diffusion in QCD matter in HIC and in particular the modification of the Rutherford scattering, running coupling

Back up

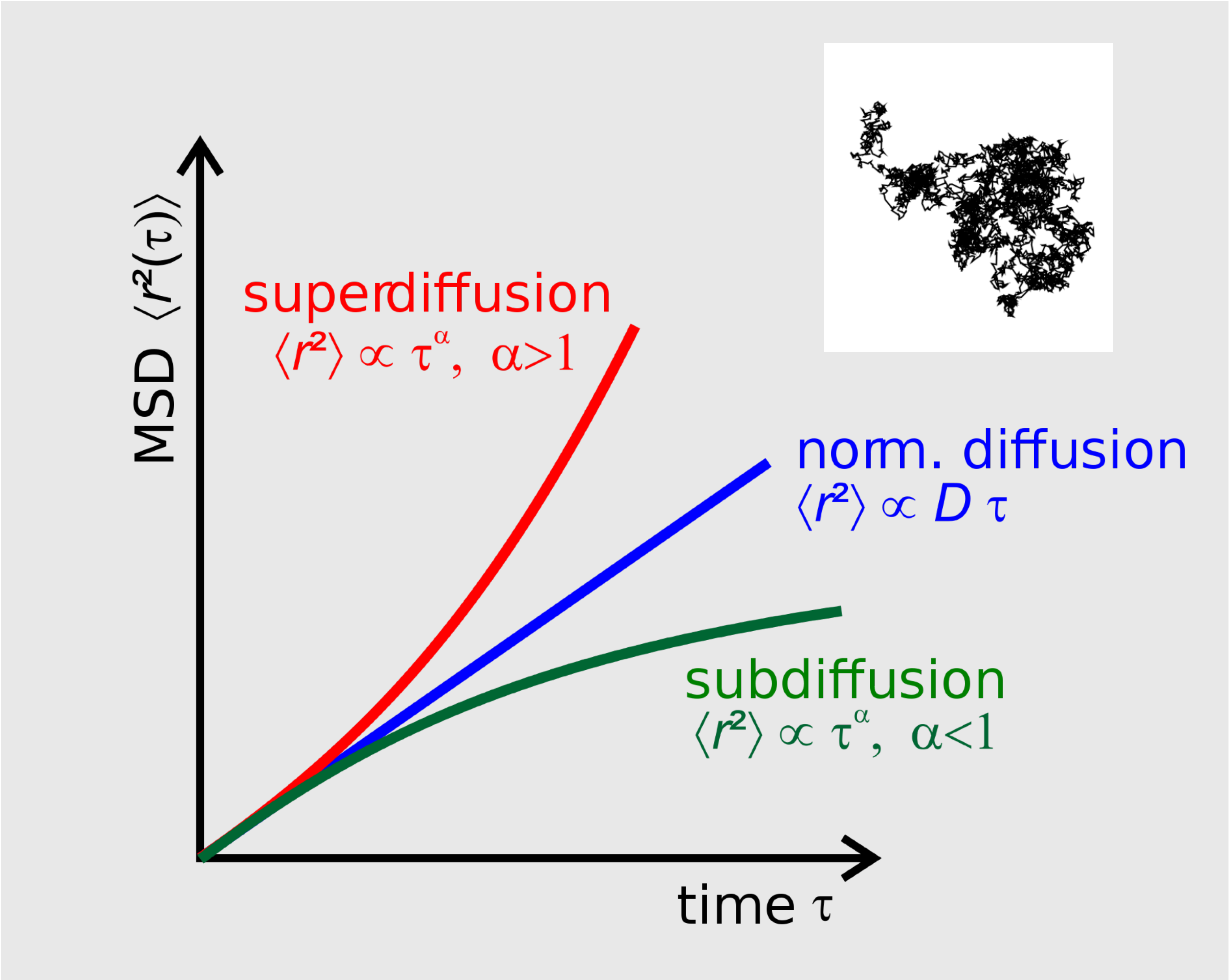
Anomalous diffusion (generalities)

- Mean squared displacement (MSD) in brownian motion

$$\langle r^2 \rangle = Dt$$

- Anomalous diffusion $\alpha \neq 1$:

$$\langle r^2 \rangle \propto t^\alpha$$



Compare to exact solution of linear evolution

Analytic solution for $Q_s(\tau) \sim \hat{q}_0 \tau$

Iancu, Triantafyllopoulos (2015)

$$Q_s^2(L) \equiv \hat{q}(Y,Y)\tau = \hat{q}_0 \frac{I_0\left(2\sqrt{\bar{\alpha}}Y\right)}{\sqrt{\alpha Y}} \simeq L^{2\sqrt{\bar{\alpha}}}$$

$$Y \equiv \log \frac{L}{\tau_0}$$

$$\Rightarrow \qquad b_{\rm lin} = \frac{3}{2} \neq b_{\rm non-lin} = \frac{3}{2(1-\beta)}$$

↝ Also, only non-linear evolution yields exact geometric scaling

- TMB in QCD matter
- Reaction-diffusion (FKPP)
- Gluon saturation (BK)
- Lévy flights

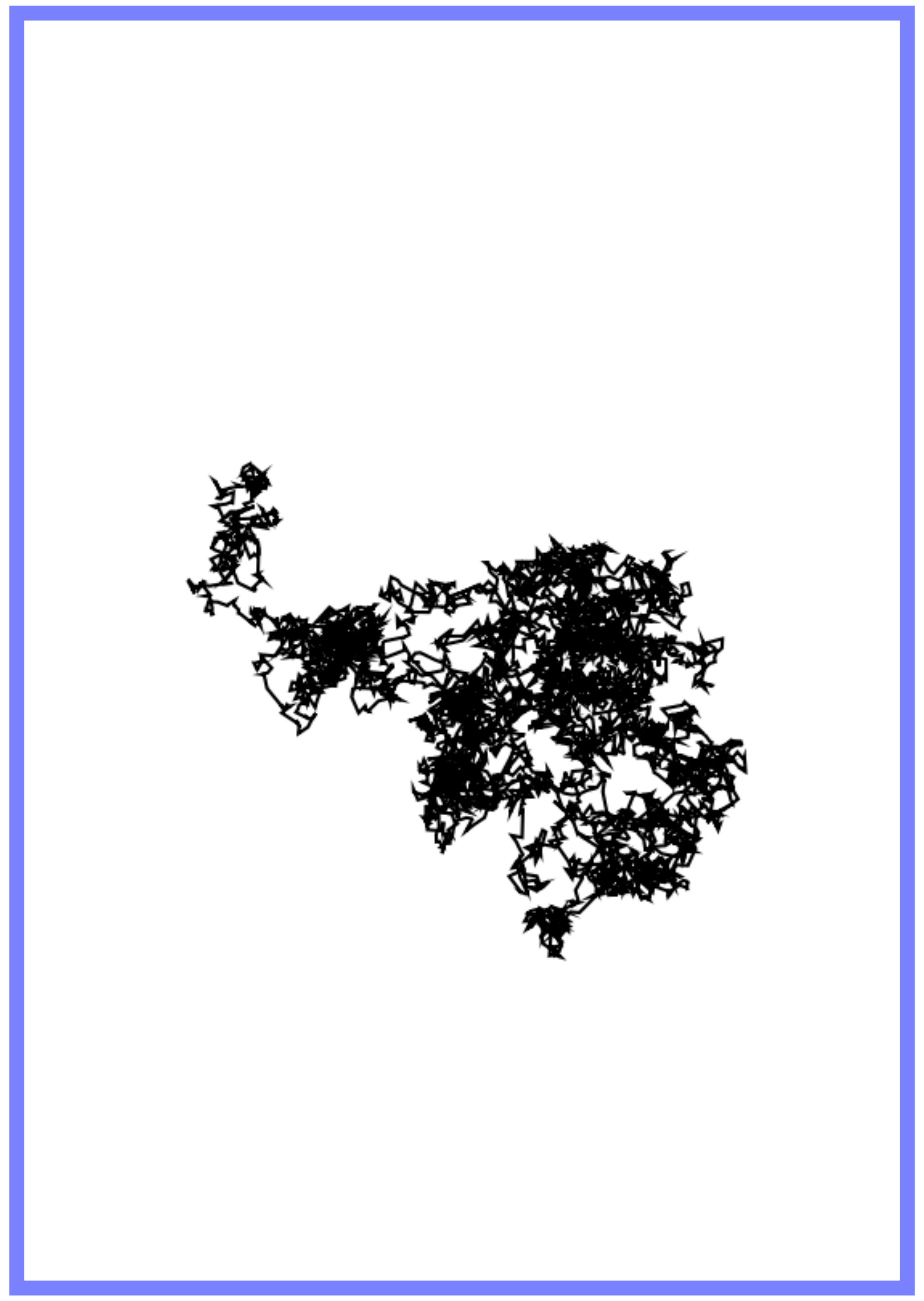
$\Rightarrow$

stochasticity

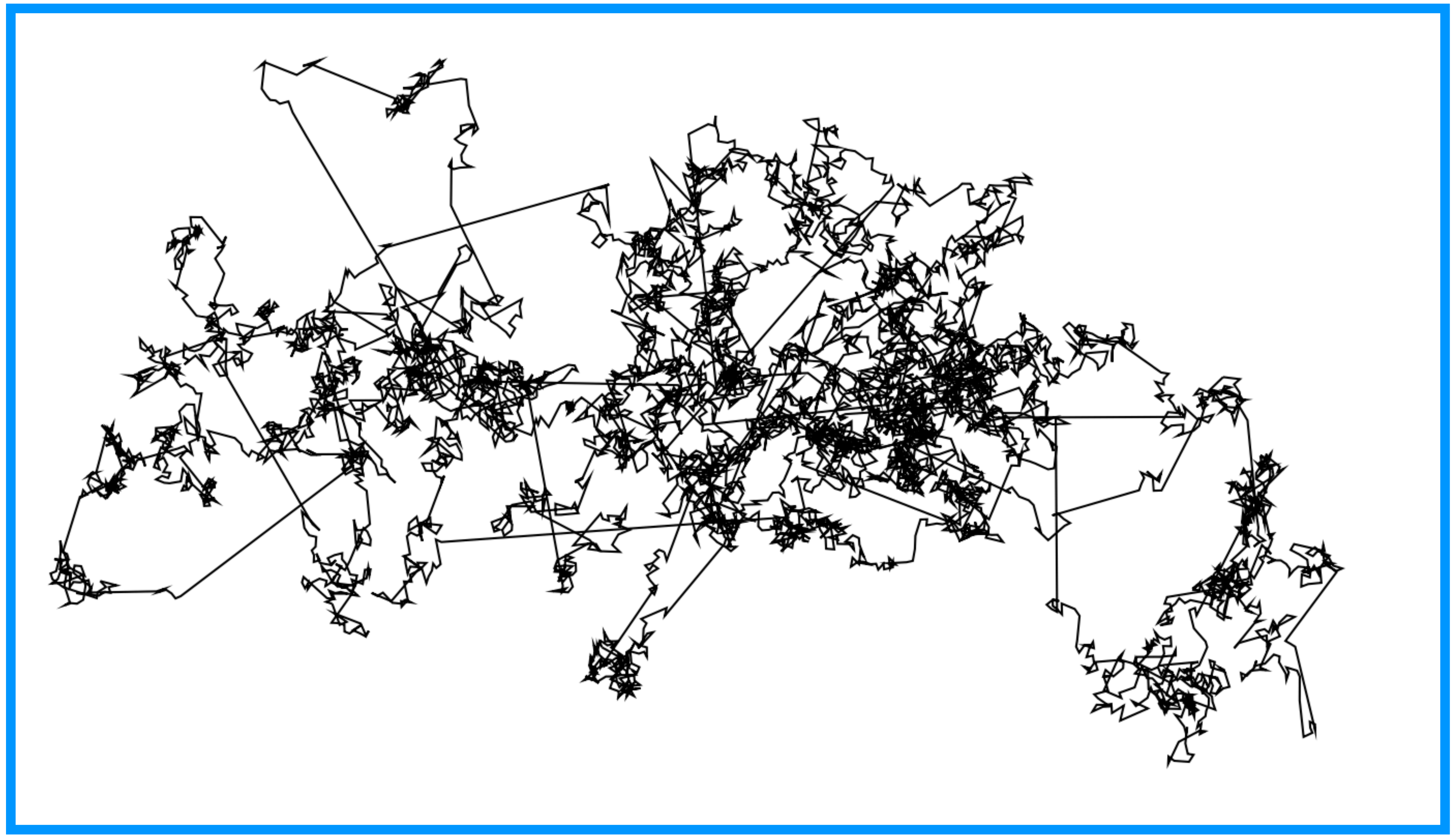
non-linearity

self-similarity

Normal diffusion
(Brownian motion)



Super-diffusion
(Lévy walks)

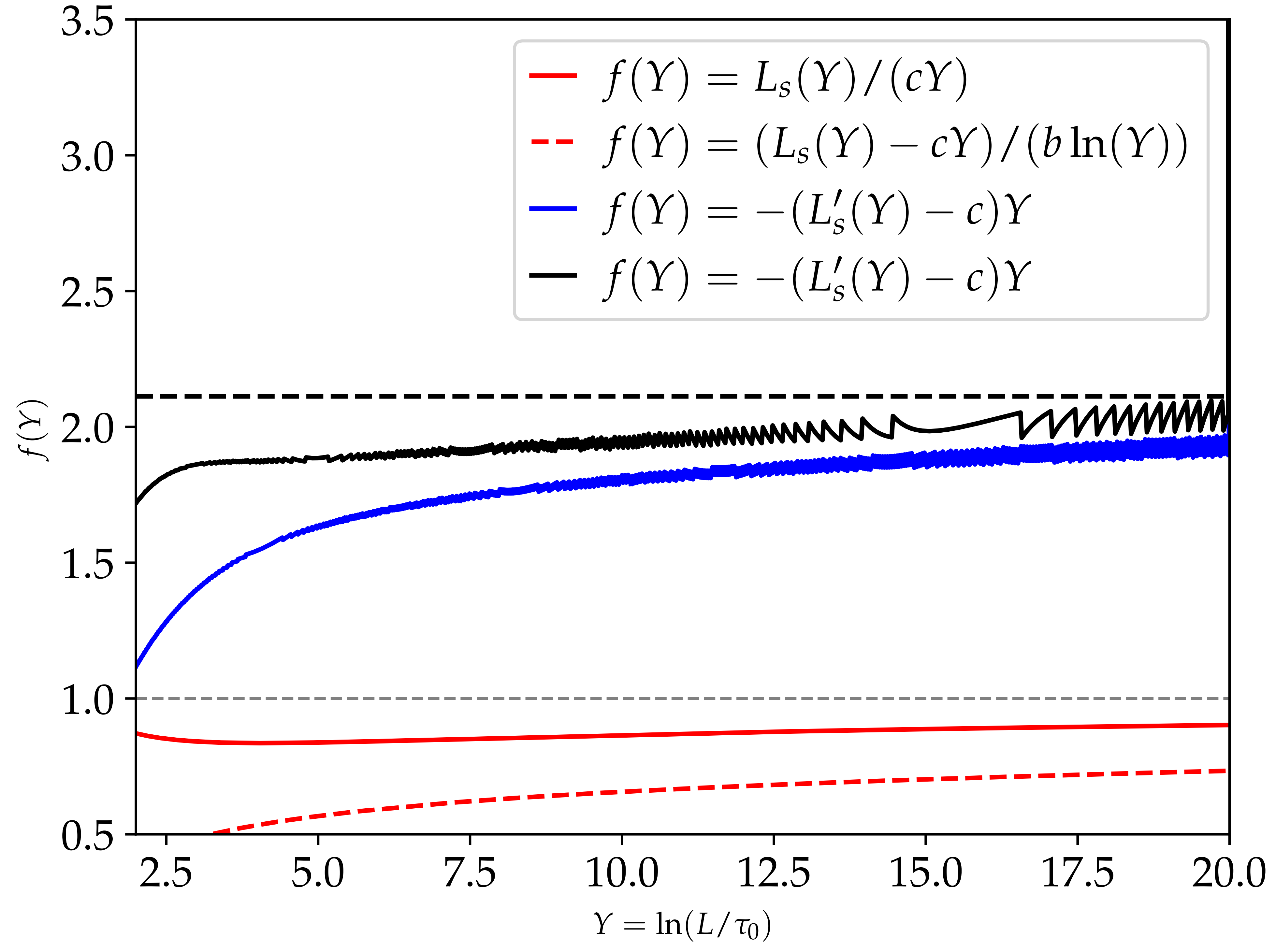


Large step length

Numerical cross-check for the sub-leading term

$$L_s(Y) = \log Q_s^2(Y) = cY + b \log Y + \text{const}\,.$$

$$b = -\frac{2}{3(1-\beta)}$$

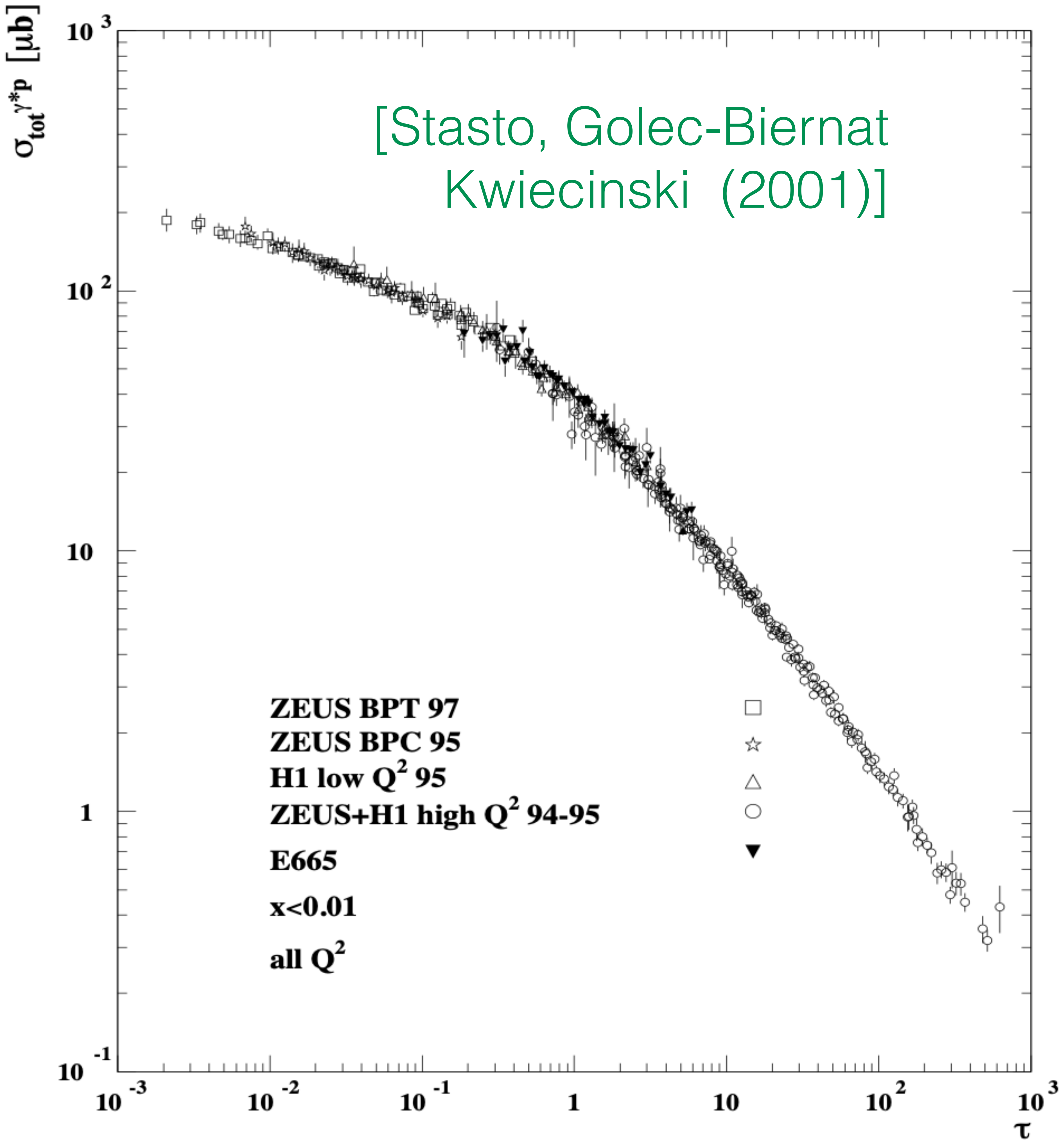


Geometric scaling in DIS at small x

- cross-section depends on τ

$$\tau = \frac{Q^2}{Q_s^2(x)}$$

$$\sigma_{\gamma^* p}(x, Q^2) \rightarrow \sigma_{\gamma^* p}(Q^2/Q_s^2(x))$$



[Stasto, Golec-Biernat Kwiecinski (2001)]

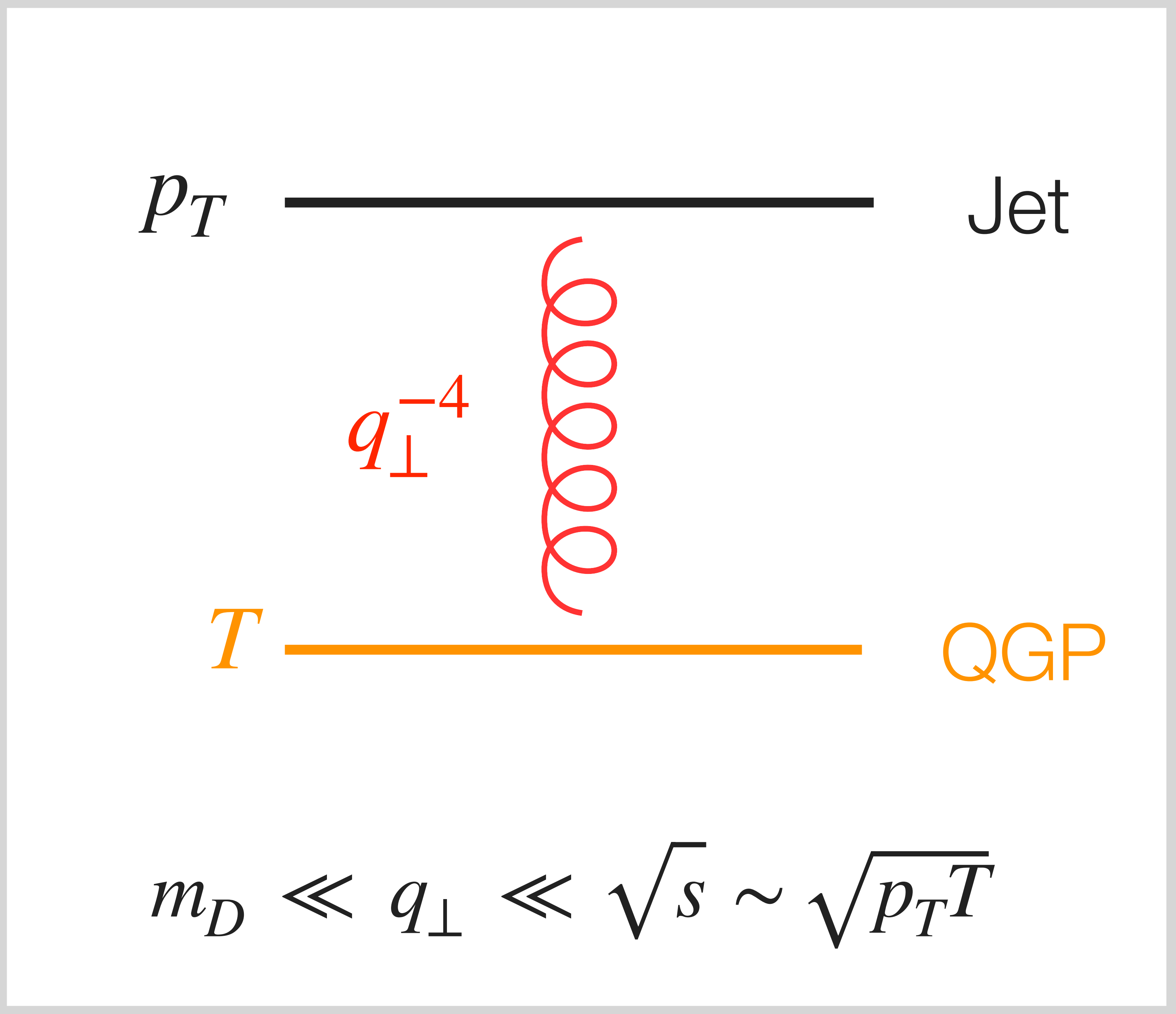
Leading order picture

- Independent multiple scattering approximation, $m_D^{-1} \ll \ell_{\rm mfp} \ll L$, TMB described by the rate equation

$$\frac{\partial}{\partial t} P(k_\perp) = \int_{q_\perp} \gamma_{\rm el}(q_\perp) \left[P(q_\perp) - P(k_\perp) \right]$$

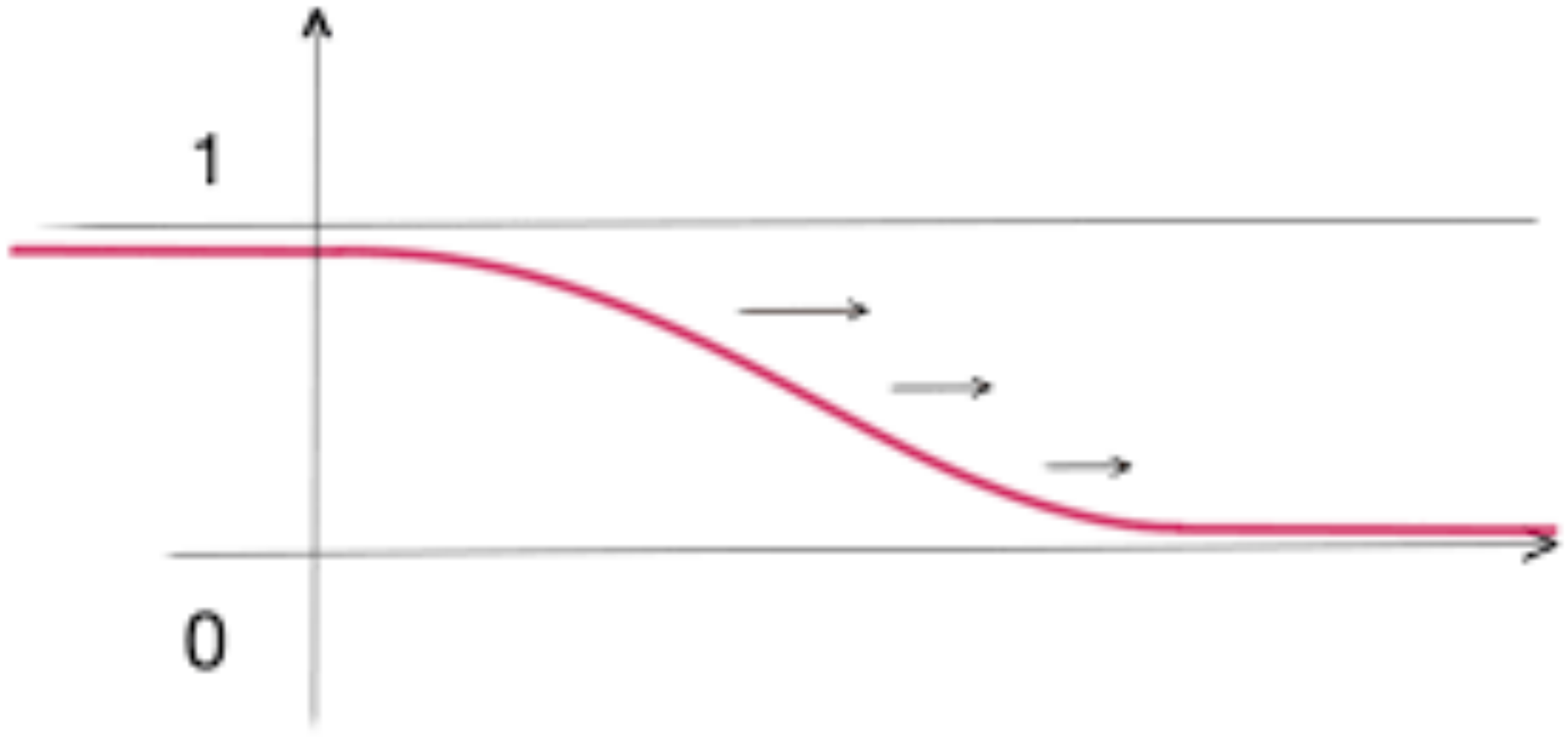
- Collision rate related to the LO elastic cross-sec

$$\gamma_{\rm el}(q_\perp) = C_R \frac{{\rm d}^2\sigma}{{\rm d}^2 q_\perp} \simeq (4\pi)^2 \frac{\alpha_s^2 C_R\, n}{q_\perp^4}$$



Traveling waves solutions

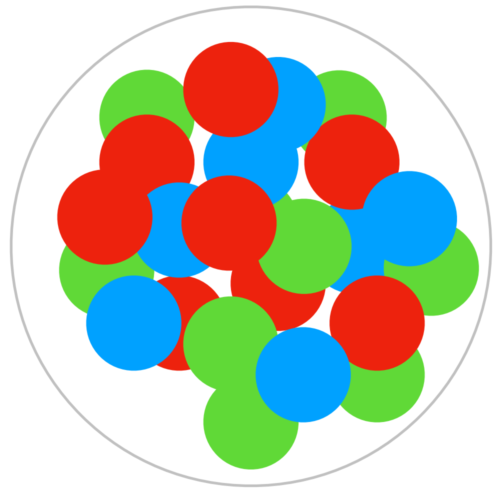
- To derive sub-asymptotic behavior we follow Brunet and Derrida's (1988) approach to FKPP equation (Fisher-Kolmogorov-Petrovsky, Piskunov) - (population growth, wave propagation, etc)



$$\frac{\partial}{\partial t}u = D\frac{\partial^2}{\partial^2 x}u + u(1-u)$$

- Also used by Munier and Peschanski (2003) to find traveling wave solutions to the Balitsky-Kovchegov (1997) equation describing gluon saturation

$$\frac{\partial}{\partial Y}N = \bar{\alpha}\chi(-\partial_L)N - \bar{\alpha}N^2 \qquad L \equiv \log k_\perp^2$$



Exact scaling solution for large media $L \to \infty$

$$\hat{q}(k_\perp, L)\, L \equiv Q_s^2(L)\; g\left(x = \frac{k_\perp^2}{Q_s^2(L)} \right)$$

$$g(x) = x^{-\beta}\left(1 + \beta \log x\right) \qquad x > 1$$

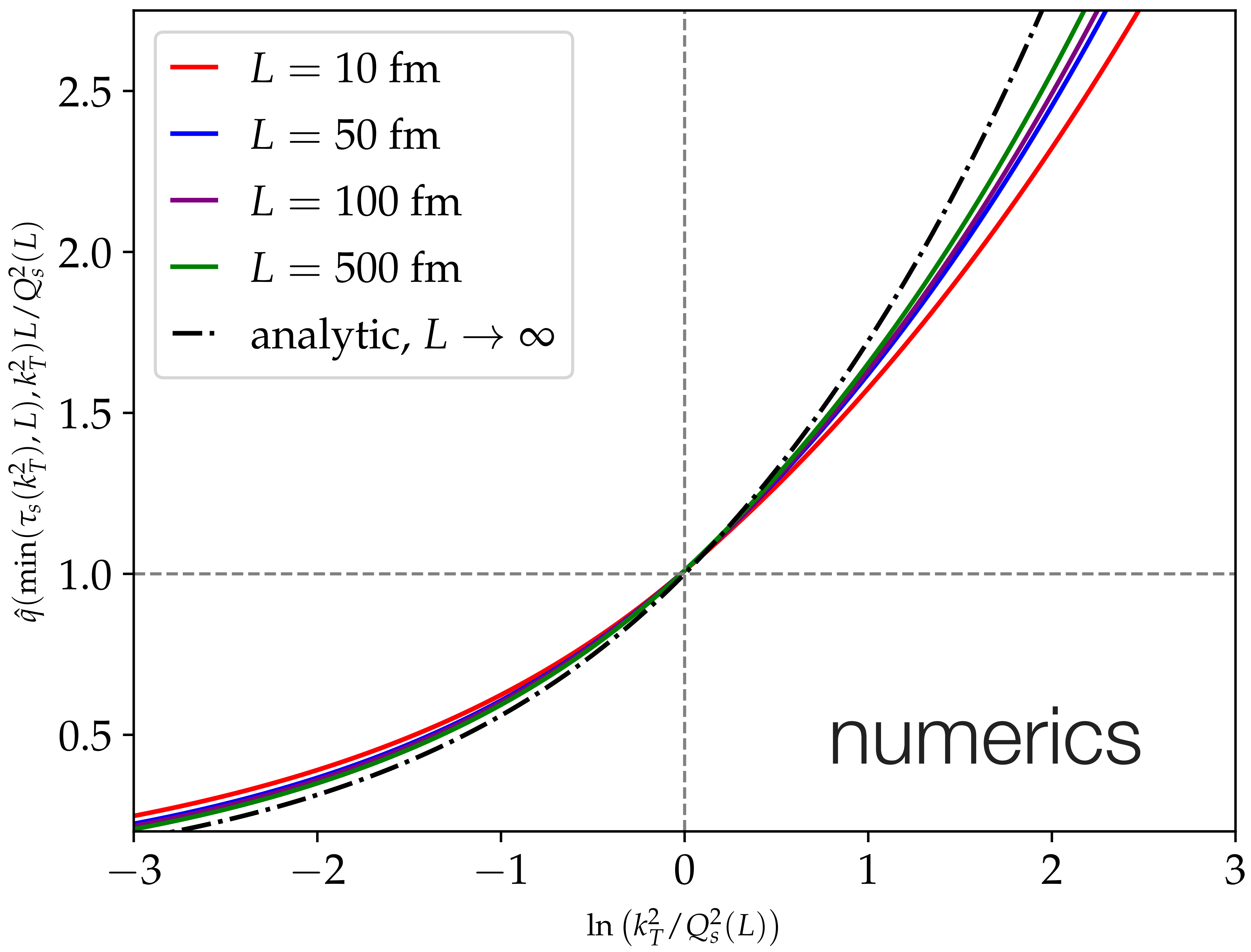
$$g(x) = x^{-2\beta} \qquad x < 1$$

- Anomalous scaling: super diffusive process

$$Q_s^2(L) \propto L^{1+2\sqrt{\bar{\alpha}}}$$

⇝ nonlocal quantum corrections

$\beta \simeq \sqrt{\bar{\alpha}}$



$\hat{q}$ as function of the scaling variable x for medium length L = 10 - 500 fm