RECENT RESULTS ON EXTRACTIONS OF QUARK TMDS

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MARIE CURIE ACTIONS

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RESULTS OBTAINED WITH CONTRIBUTIONS FROM

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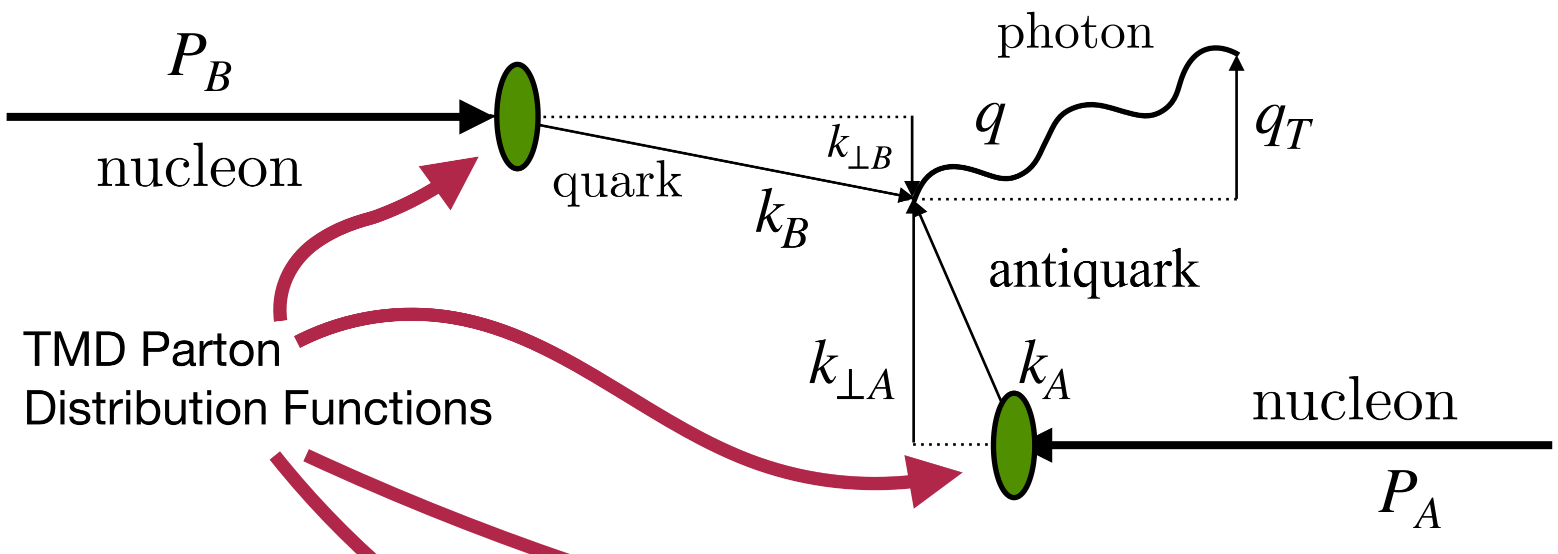
Marco Radici

Andrea Signori

Matteo Cerutti

UNPOLARISED QUARK TMDS

TMDS IN DRELL-YAN PROCESSES



W term

$$F_{UU}^{1}(x_A, x_B, \boldsymbol{q}_T^2, Q^2)$$

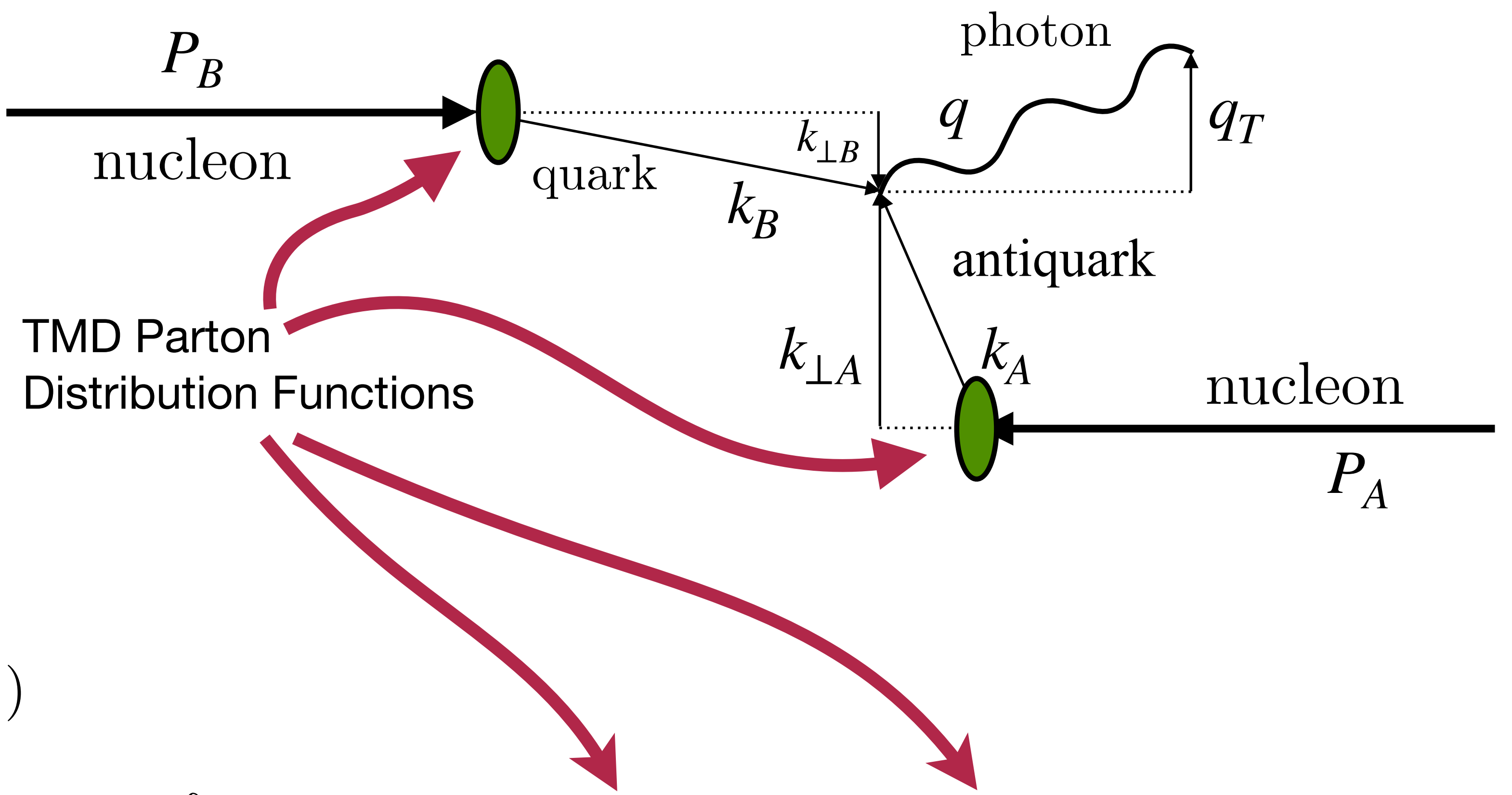
$$= \sum_a \mathcal{H}_{UU}^{1a}(Q^2, \mu^2) \int d^2\boldsymbol{k}_{\perp A}\, d^2\boldsymbol{k}_{\perp B}\, f_1^a(x_A, \boldsymbol{k}_{\perp A}^2; \mu^2)\, f_1^{\bar{a}}(x_B, \boldsymbol{k}_{\perp B}^2; \mu^2)\, \delta^{(2)}(\boldsymbol{k}_{\perp A} - \boldsymbol{q}_T + \boldsymbol{k}_{\perp B})$$

$$+ Y_{UU}^{1}(Q^2, \boldsymbol{q}_T^2) + \mathcal{O}(M^2/Q^2)$$

The W term, dominates at low transverse momentum ($q_T \ll Q$)

So far, the Y term has been excluded in the Pavia analyses

TMDS IN DRELL-YAN PROCESSES



$$F^1_{UU}(x_A, x_B, \boldsymbol{q}_T^2, Q^2)$$

$$\approx \sum_q \mathcal{H}^{1q}_{UU}(Q^2,\mu^2) \int d^2\boldsymbol{k}_{\perp A}\, d^2\boldsymbol{k}_{\perp B}\, f_1^q\big(x_A, \boldsymbol{k}^2_{\perp A};\mu^2\big)\, f_1^{\bar{q}}\big(x_B, \boldsymbol{k}^2_{\perp B};\mu^2\big)\, \delta^{(2)}\big(\boldsymbol{k}_{\perp A} - \boldsymbol{q}_T + \boldsymbol{k}_{\perp B}\big)$$

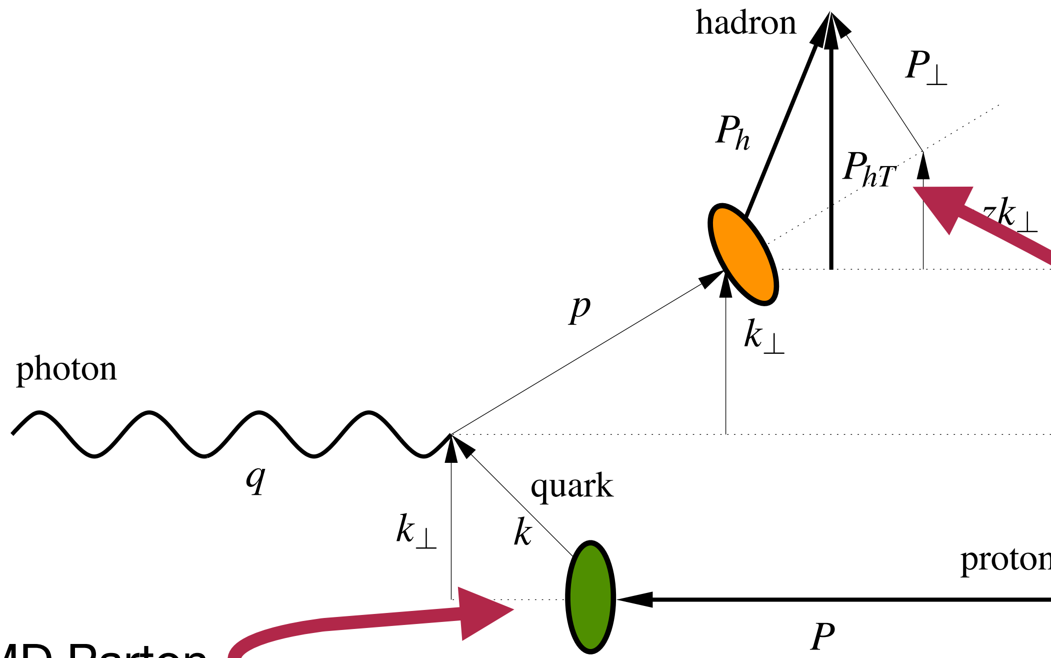
$$= \sum_q \mathcal{H}^{1q}_{UU}(Q^2,\mu^2) \int db_T b_T J_0(b_T|\boldsymbol{q}_T|) \hat{f}_1^q\big(x_A, b_T^2;\mu^2\big)\, \hat{f}_1^{\bar{q}}\big(x_B, b_T^2;\mu^2\big)$$

At small transverse momentum, the dominant part is given by TMDs.

The analysis is usually done in Fourier-transformed space

TMDs formally depend on two scales, but we set them equal.

TMDS IN SEMI-INCLUSIVE DIS



TMD Parton
Distribution Functions

TMD Parton
Fragmentation Functions

$$F_{UU,T}(x, z, \boldsymbol{P}_{hT}^2, Q^2)$$

$$= x \sum_q \mathcal{H}_{UU,T}^q(Q^2, \mu^2) \int d^2\boldsymbol{k}_\perp \, d^2\boldsymbol{P}_\perp \, f_1^a(x, \boldsymbol{k}_\perp^2; \mu^2) \, D_1^{a\to h}(z, \boldsymbol{P}_\perp^2; \mu^2) \, \delta(z\boldsymbol{k}_\perp - \boldsymbol{P}_{hT} + \boldsymbol{P}_\perp)$$

$$= x \sum_a \mathcal{H}_{UU,T}^q(Q^2, \mu^2) \int db_T b_T J_0(b_T|\boldsymbol{P}_{h\perp}|) \hat{f}_1^q(x, z^2 b_\perp^2; \mu^2) \, \hat{D}_1^{a\to h}(z, b_\perp^2; \mu^2)$$

hard factor

TMD STRUCTURE

$$\hat{f}_1^q(x, b_T; \mu^2) = \int d^2\boldsymbol{k}_\perp e^{i\boldsymbol{b}_T\cdot\boldsymbol{k}_\perp} f_1^q(x, \boldsymbol{k}_\perp^2; \mu^2)$$

perturbative Sudakov form factor

$$\hat{f}_1^q(x, b_T; \mu^2) = \sum_i \left(C_{qi} \otimes f_1^i\right)(x, b_*; \mu_b) e^{\tilde{S}(b_*;\mu_b,\mu)} e^{g_K(b_T)\ln\frac{\mu}{\mu_0}} \hat{f}_{\mathrm{NP}}^q(x, b_T)$$

$$\mu_b = \frac{2e^{-\gamma_E}}{b_*}$$

collinear PDF

matching coefficients (perturbative)

nonperturbative part of evolution

nonperturbative part of TMD

see, e.g., Collins, "Foundations of Perturbative QCD" (11)

PAVIA 2019 b_* PRESCRIPTION

$$\hat{f}_1^q(x, b_T; \mu^2) = \sum_i \left(C_{qi} \otimes f_1^i\right)(x, b_*; \mu_b) e^{\tilde{S}(b_*;\mu_b,\mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}_{\rm NP}^q(x, b_T)$$

$$\mu_0 = 1\,\text{GeV}$$

$$b_* \equiv \frac{b_T}{\sqrt{1 + b_T^2/b_{\max}^2}}$$

Collins, Soper, Sterman, NPB250 (85)

$$\mu_b = 2e^{-\gamma_E}/b_*$$

$$\bar{b}_* \equiv b_{\max} \left(\frac{1 - e^{-b_T^4/b_{\max}^4}}{1 - e^{-b_T^4/b_{\min}^4}} \right)^{1/4}$$

$$b_{\max} = 2e^{-\gamma_E}$$

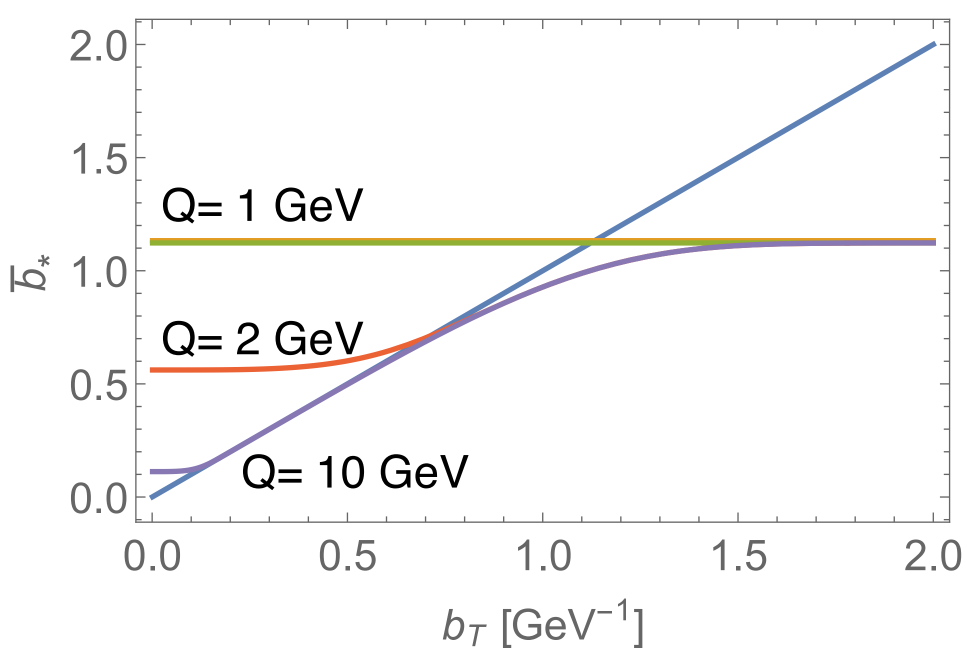
$$b_{\min} = \frac{2e^{-\gamma_E}}{Q}$$

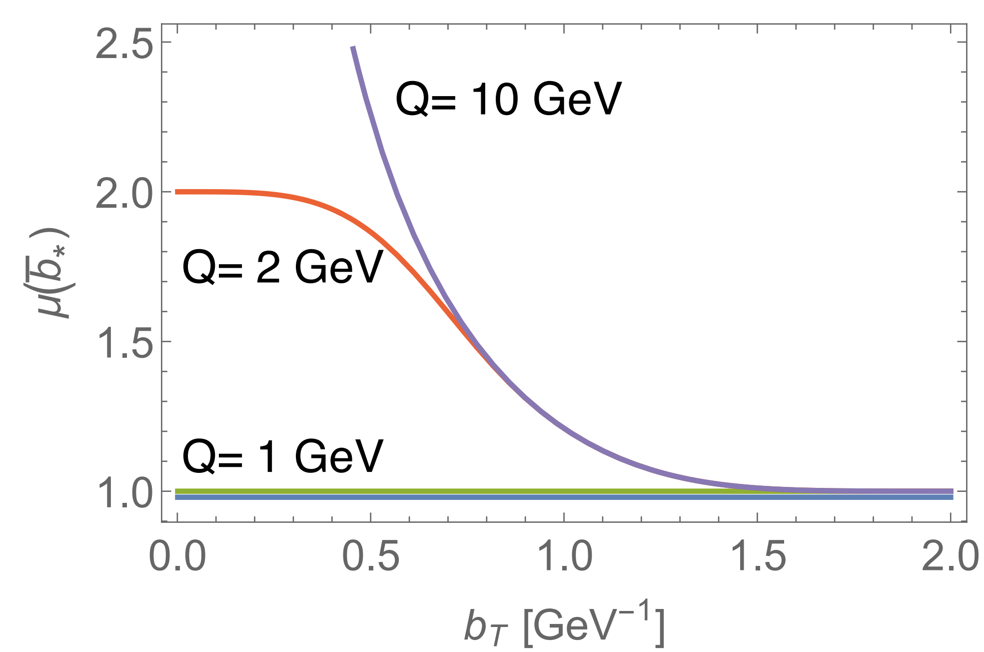
These are all choices that should be at some point checked/challenged

EFFECTS OF b_* PRESCRIPTION

$$\mu_b = 2e^{-\gamma_E}/b_* \qquad \bar{b}_* \equiv b_{\max}\left(\frac{1-e^{-b_T^4/b_{\max}^4}}{1-e^{-b_T^4/b_{\min}^4}}\right)^{1/4} \qquad b_{\max} = 2e^{-\gamma_E}$$

$$b_{\min} = \frac{2e^{-\gamma_E}}{Q}$$





No significant effect at high Q, but large effect at low Q (inhibits perturbative contribution)

EFFECTS OF b_* PRESCRIPTION

- At $b_T > b_{max}$, the perturbative contributions are inhibited and replaced by nonperturbative contributions (roughly speaking, at small transverse momentum)
- At $b_T < b_{min}$ the TMD is modified, but it should influence the region outside the applicability of TMD factorization (roughly speaking, at high transverse momentum)
- At Q=1, only, the integral of the TMD is by construction equal to the collinear PDF (see also talk by Z. Sun)

LOW-b_T MODIFICATIONS

$$\log\left(Q^2 b_T^2\right) \to \log\left(Q^2 b_T^2 + 1\right)$$

see, e.g., Bozzi, Catani, De Florian, Grazzini
hep-ph/0302104

$$b_*(b_c(b_{\rm T})) = \sqrt{\frac{b_{\rm T}^2 + b_0^2/(C_5^2 Q^2)}{1 + b_{\rm T}^2/b_{\rm max}^2 + b_0^2/(C_5^2 Q^2 b_{\rm max}^2)}}$$

$$b_{\rm min} \equiv b_*(b_c(0)) = \frac{b_0}{C_5 Q}\sqrt{\frac{1}{1 + b_0^2/(C_5^2 Q^2 b_{\rm max}^2)}}$$

Collins et al.
arXiv:1605.00671

- The justification is to recover the integrated result ("unitarity constraint")
- Modification at low b_T is allowed because resummed calculation is anyway unreliable there

RECENT TMD FITS OF UNPOLARIZED DATA

	Framework	HERMES	COMPASS	DY	Z production	N of points	χ^2/N_{points}
Pavia 2017 arXiv:1703.10157	NLL	✔	✔	✔	✔	8059	1.55
SV 2017 arXiv:1706.01473	NNLL′	✘	✘	✔	✔	309	1.23
BSV 2019 arXiv:1902.08474	NNLL′	✘	✘	✔	✔	457	1.17
SV 2019 arXiv:1912.06532	NNLL′	✔	✔	✔	✔	1039	1.06
Pavia 2019 arXiv:1912.07550	N^3LL	✘	✘	✔	✔	353	1.02

What have we been doing in the last two years ?

PV17 → MAP21 ← PV19

MAP21

work still in progress…

NANGA PARBAT: A PUBLIC PLATFORM FOR TMD STUDIES

https://github.com/MapCollaboration/NangaParbat

Nanga Parbat: a TMD fitting framework

Nanga Parbat is a fitting framework aimed at the determination of the non-perturbative component of TMD distributions.

Download

You can obtain NangaParbat directly from the github repository:

https://github.com/vbertone/NangaParbat/releases

For the last development branch you can clone the master code:

```
git clone git@github.com:vbertone/NangaParbat.git
```

PERTURBATIVE ORDER OF EACH INGREDIENT

Order in powers of α_S

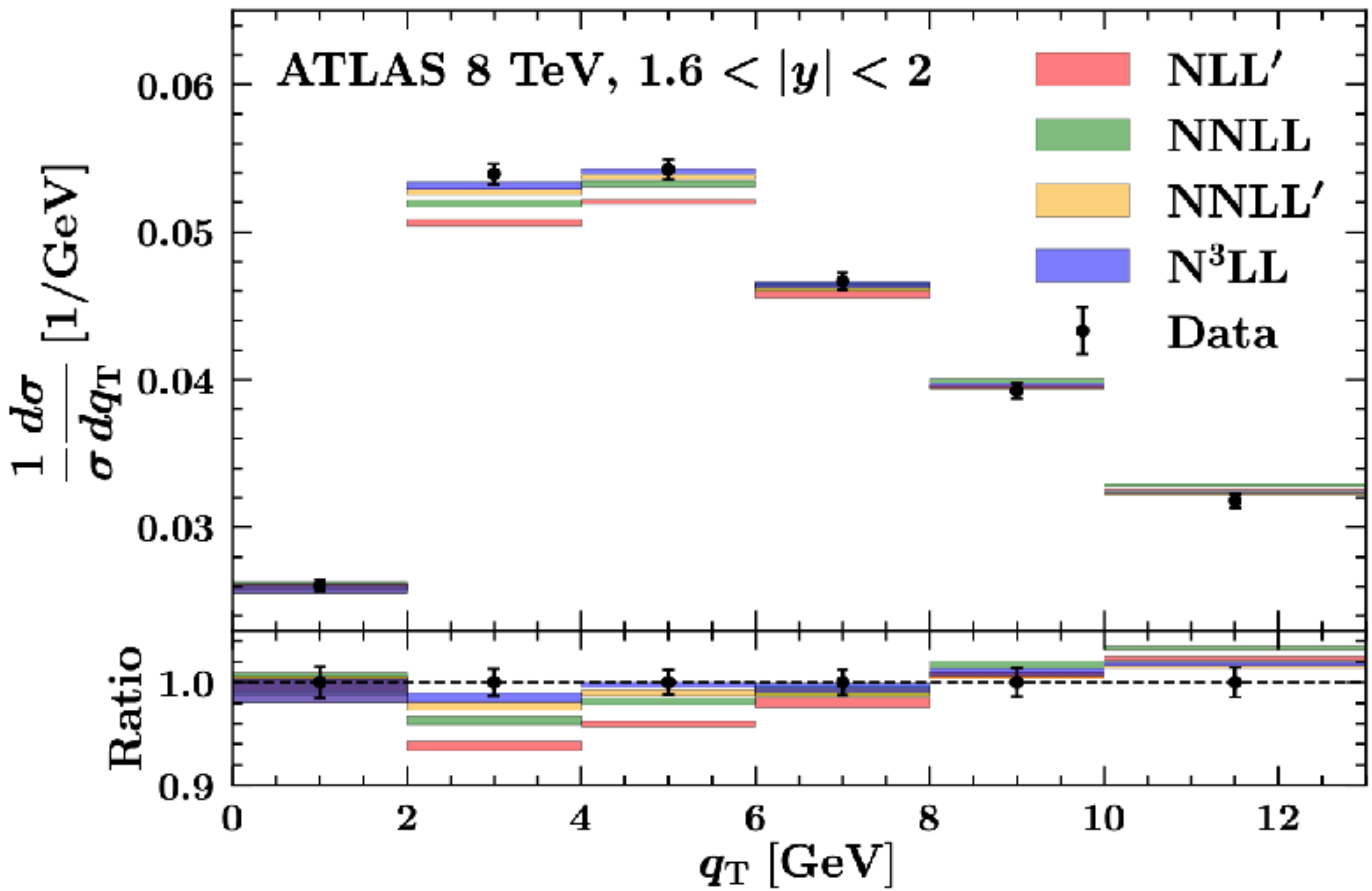
hard factor and matching coefficients

ingredients in perturbative Sudakov form factor

Accuracy	H and C	K and γ_F	γ_K	PDF and α_S evol.
LL	0	-	1	-
NLL	0	1	2	LO
NLL'	1	1	2	NLO
NNLL	1	2	3	NLO
NNLL'	2	2	3	NNLO
N^3LL^-	2	3	4	NLO
N^3LL	2	3	4	NNLO
N^3LL'	3	3	4	N^3LO

Collinear fragmentation functions not available beyond NLO

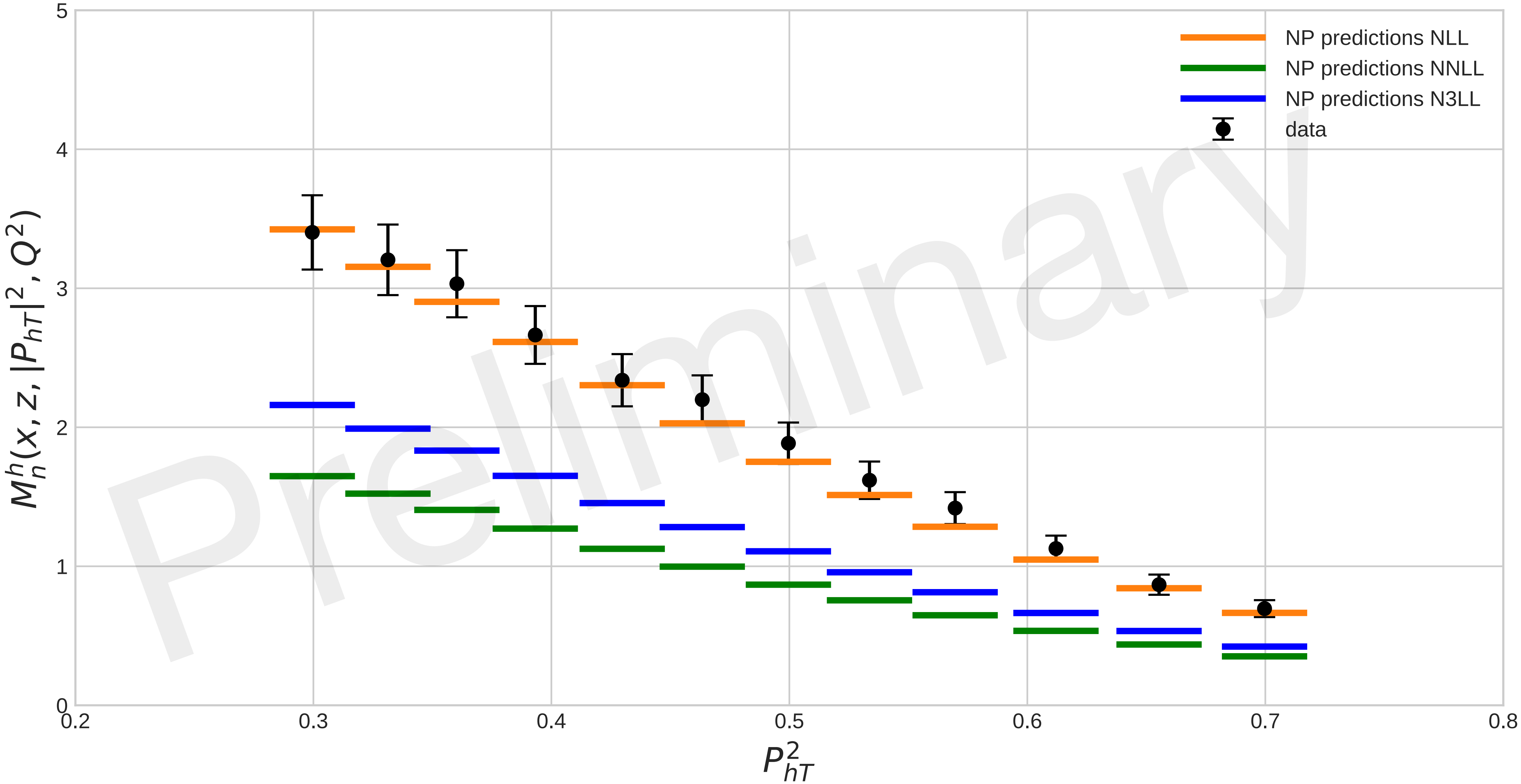
COMPARISON OF DIFFERENT ORDERS IN DY



Bacchetta, Bertone, Bissolotti, Bozzi, Delcarro, Piacenza, Radici, arXiv:1912.07550

COMPARISON OF DIFFERENT ORDER SIDIS

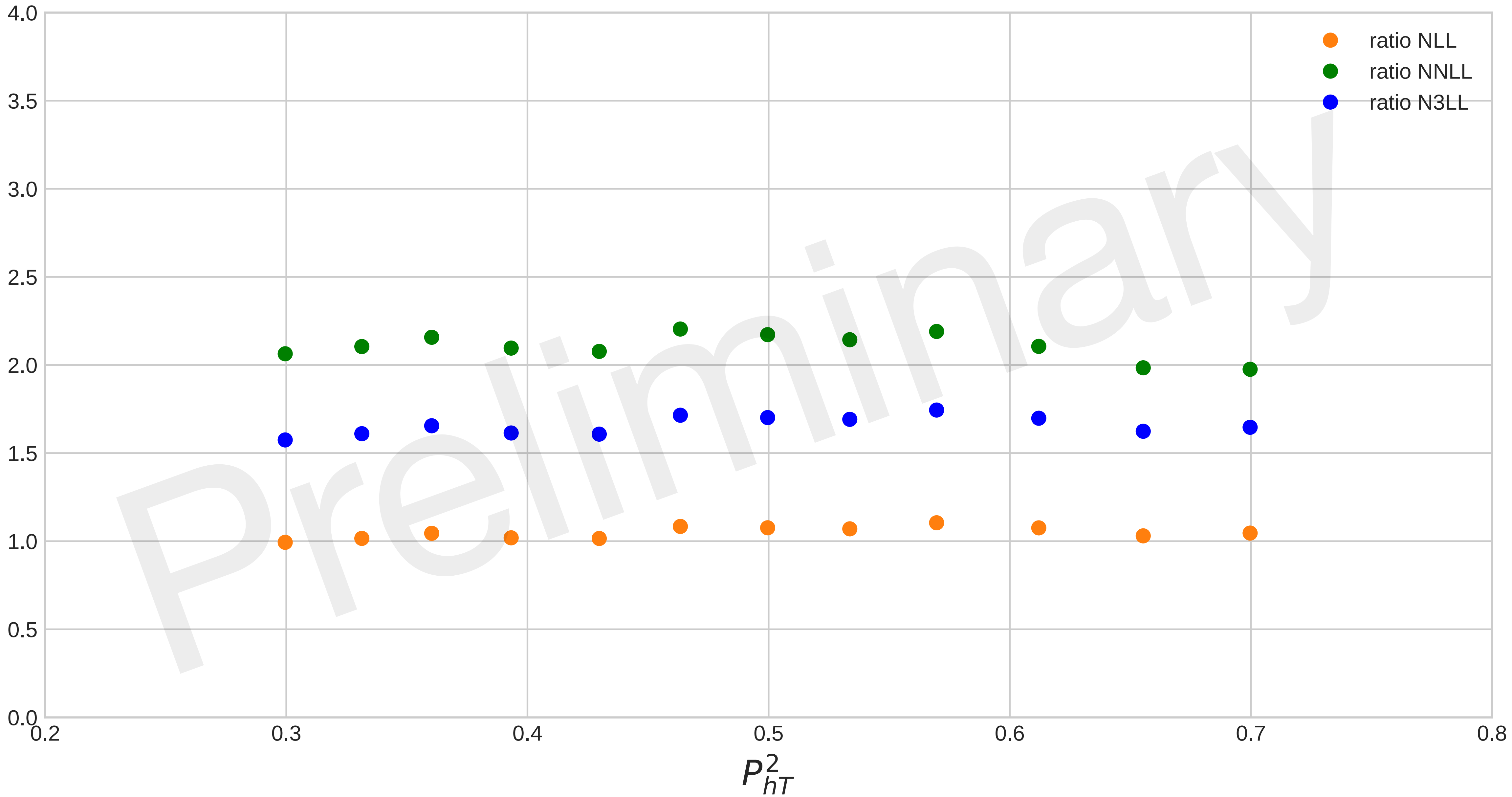
COMPASS multiplicities (one of many bins)



The description considerably worsens at higher orders

RATIO DATA/PREDICTIONS

COMPASS multiplicities (one of many bins)



The discrepancy amounts to a constant factor

OUR TENTATIVE SOLUTION

PREFACTOR $= \dfrac{\left.\dfrac{d\sigma}{dxdzdQ^2}\right|_{\text{nonmix.}}}{\int W d^2 q_T}$

$$\left.\frac{d\sigma^h}{dxdQ^2dz}\right|_{O(\alpha_S)} = \sigma_0 \sum_{f,f'} \frac{e_f^2}{z^2}(\delta_{f'f} + \delta_{f'g})\frac{\alpha_S}{\pi}\left.\left\{\left[D_1^{h/f'} \otimes C_1^{f'f} \otimes f_1^{f/N}\right](x,z,Q)\right\}\right|_{\text{nonmix.}}$$

$$\int W \Big|_{O(\alpha_S)} = \sigma_0 \sum_{f,f'} \frac{e_f^2}{z^2}(\delta_{f'f} + \delta_{f'g})\frac{\alpha_S}{\pi}\left[D_1^{h/f'} \otimes C_{\text{TMD}}^{f'f} \otimes f_1^{f/N}\right](x,z,Q)$$

The prefactor is independent of the fitting parameters

NONMIXED TERMS IN COLLINEAR SIDIS CROSS SECTION

$$\left.\frac{\mathrm{d}\sigma^h}{\mathrm{d}x\mathrm{d}Q^2\mathrm{d}z}\right|_{\mathcal{O}(\alpha_s^1)} = \sigma_0 \sum_{ff'} \frac{e_f^2}{z^2} \left(\delta_{f'f} + \delta_{f'g}\right) \frac{\alpha_s}{\pi} \Bigg\{ \left[D_1^{h/f'} \otimes C_1^{f'f} \otimes f_1^{f/N}\right](x,z,Q)$$

$$+ \frac{1-y}{1+(1-y)^2} \left[D_1^{h/f'} \otimes C_L^{f'f} \otimes f_1^{f/N}\right](x,z,Q)\Bigg\},$$

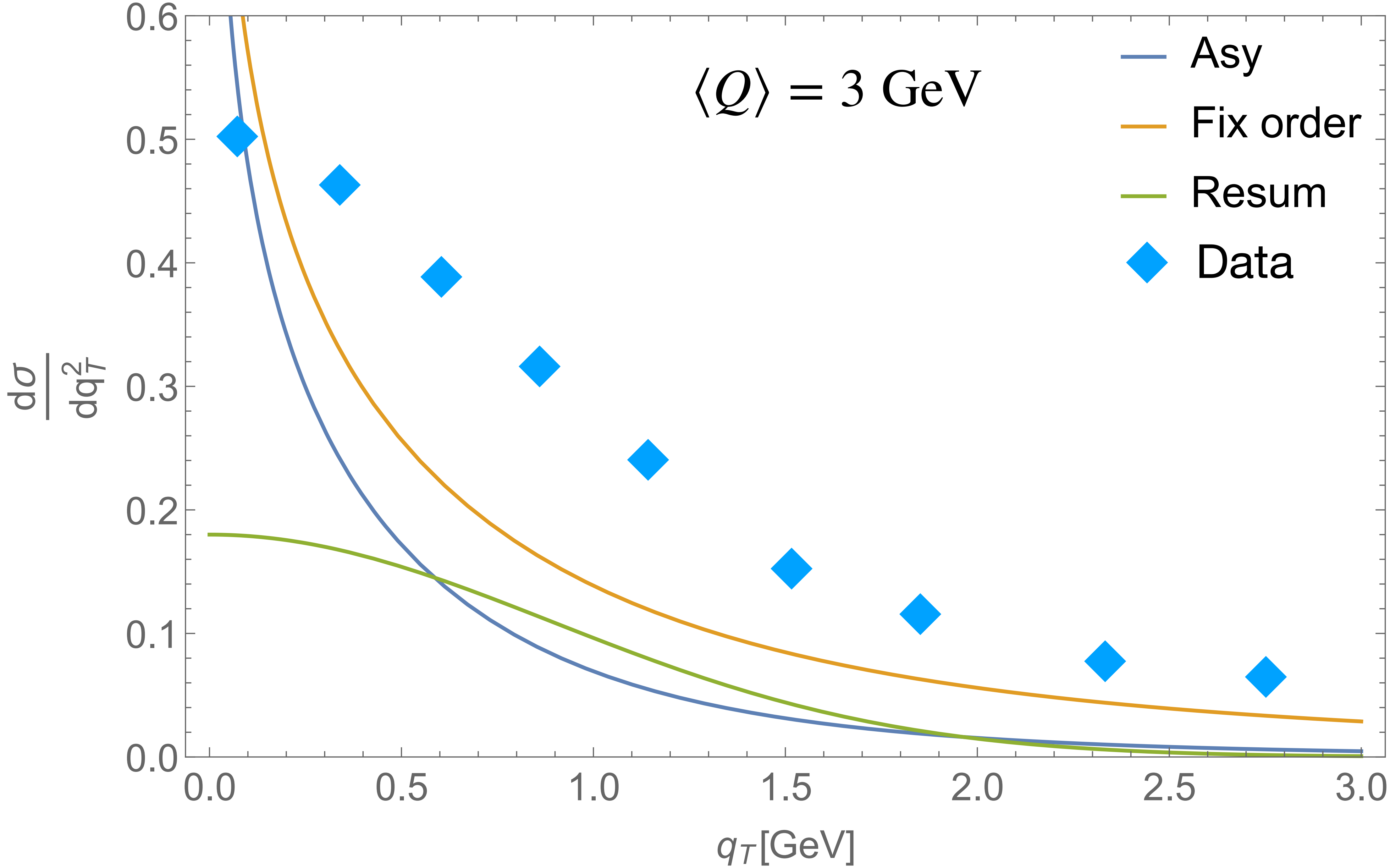
$$C_1^{qq} = \frac{C_F}{2}\Bigg\{ -8\delta(1-x)\delta(1-z)$$

$$+ \delta(1-x)\left[P_{qq}(z)\ln\frac{Q^2}{\mu_F^2} + L_1(z) + L_2(z) + (1-z)\right]$$

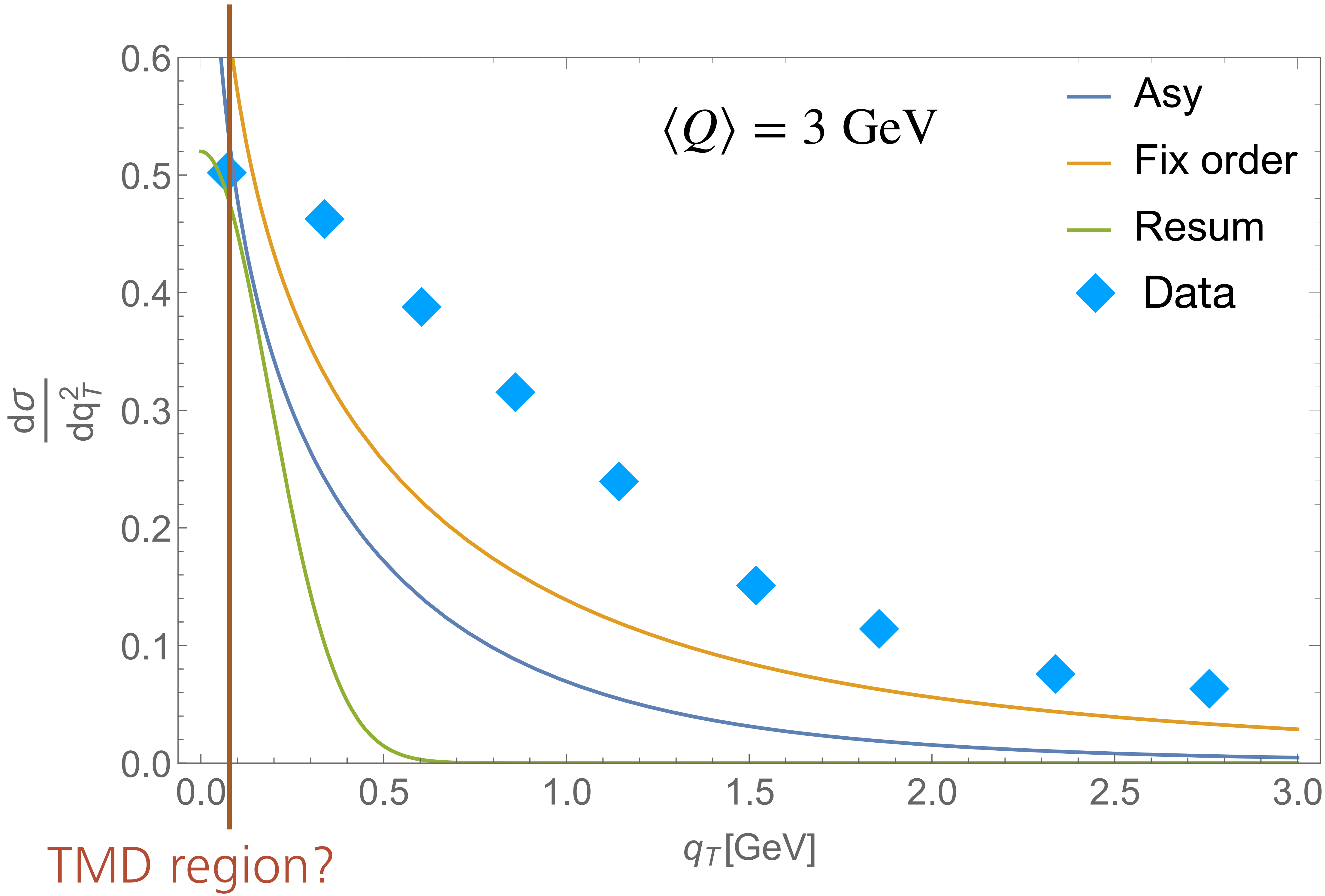
$$+ \delta(1-z)\left[P_{qq}(x)\ln\frac{Q^2}{\mu^2} + L_1(x) - L_2(x) + (1-x)\right]$$

$$+ 2\frac{1}{(1-x)_+}\frac{1}{(1-z)_+} - \frac{1+z}{(1-x)_+} - \frac{1+x}{(1-z)_+} + 2(1+xz)\Bigg\},$$

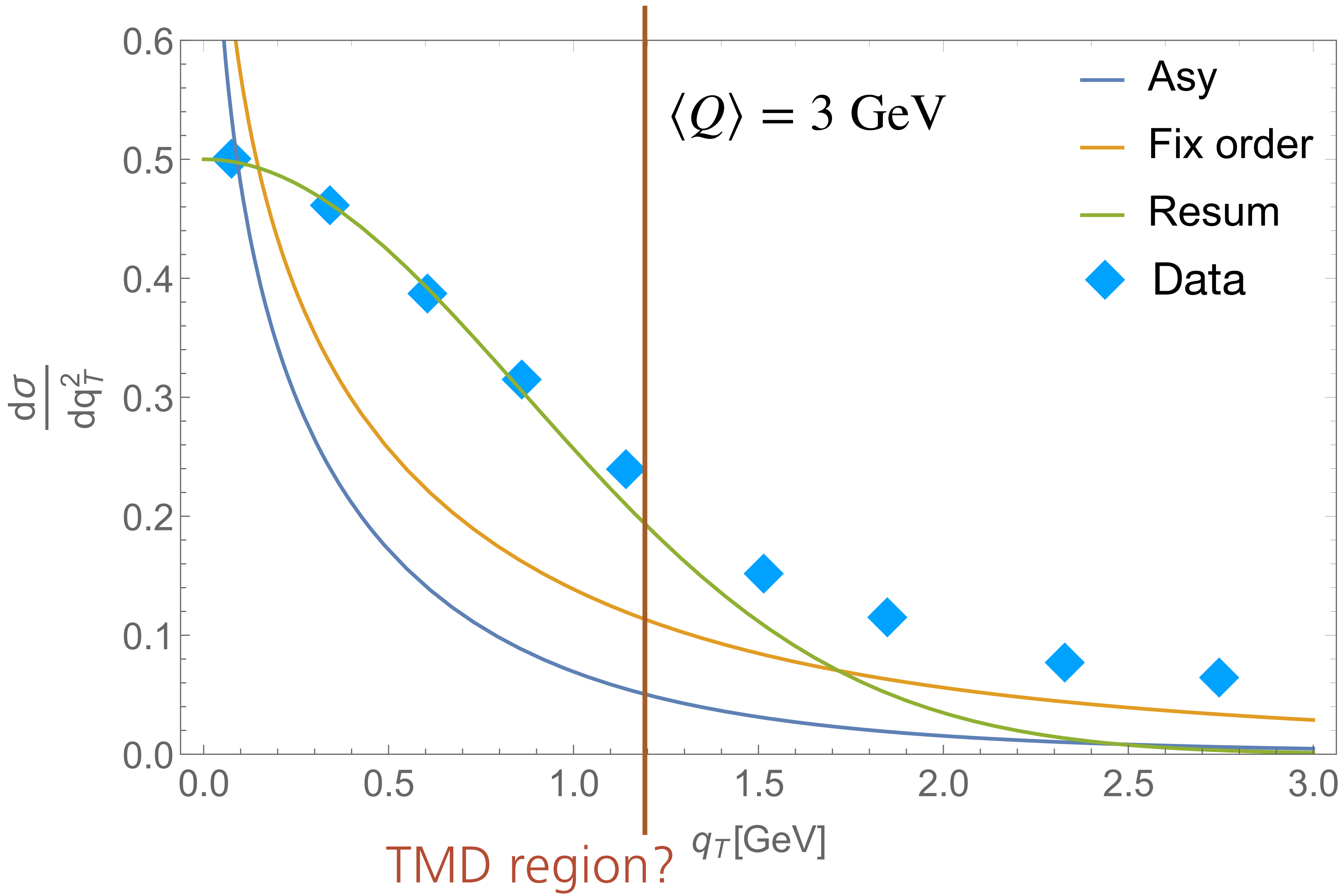
SOME JUSTIFICATION: INITIAL SITUATION



SOLUTION 1: RESTRICT TMD REGION

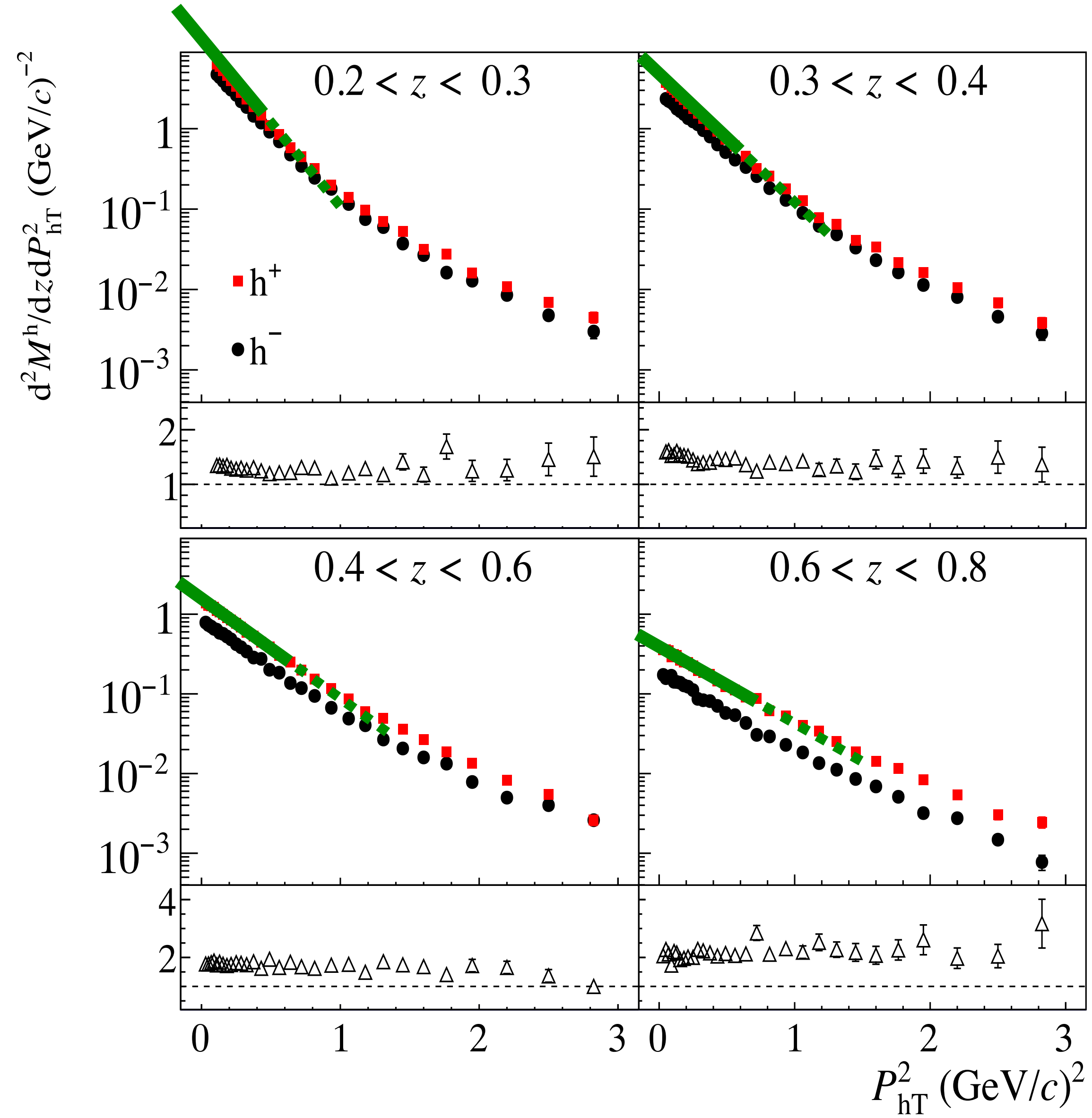


SOLUTION 2: ENHANCE TMD CONTRIBUTIONS



TMD REGIONS AND DATA

COMPASS Collab., arXiv:1709.07374



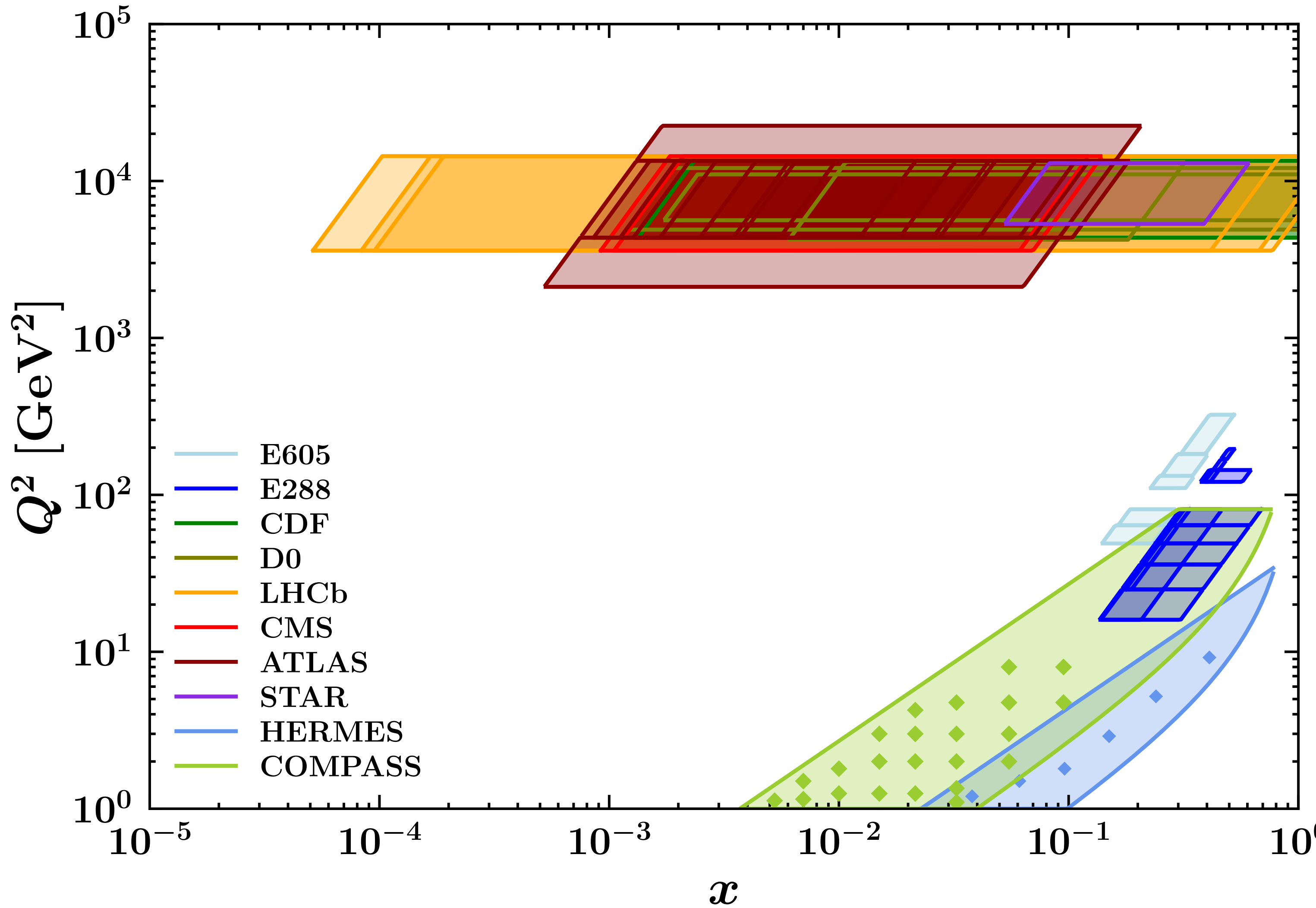
$\langle Q \rangle = 3.1$ GeV

It seems that the same "physics" is dominating at least for

$0 \leq P_{hT} \leq 0.7$ GeV,

which means $0 \leq q_T \leq 2.8$ GeV in the lowest-z bin

MAP21 TMD FIT CHOICES (PRELIMINARY)



$Q^2 > 1.4\ \text{GeV}^2$

$0.2 < z < 0.6$

$q_T < 0.2\,Q \quad \text{(DY)}$

$P_{hT} < \min\left[\min\left[0.2\,Q, 0.5zQ\right] + 0.3\,\text{GeV}, zQ\right] \quad \text{(SIDIS)}$

Number of points: 1520

FUNCTIONAL FORM

$$f_{1\mathrm{NP}}(x, b_T^2) \propto \text{F.T. of } \left(e^{-\frac{k_\perp^2}{g_{1A}}} + \lambda_B k_\perp^2 e^{-\frac{k_\perp^2}{g_{1B}}} + \lambda_C e^{-\frac{k_\perp^2}{g_{1C}}} \right)$$

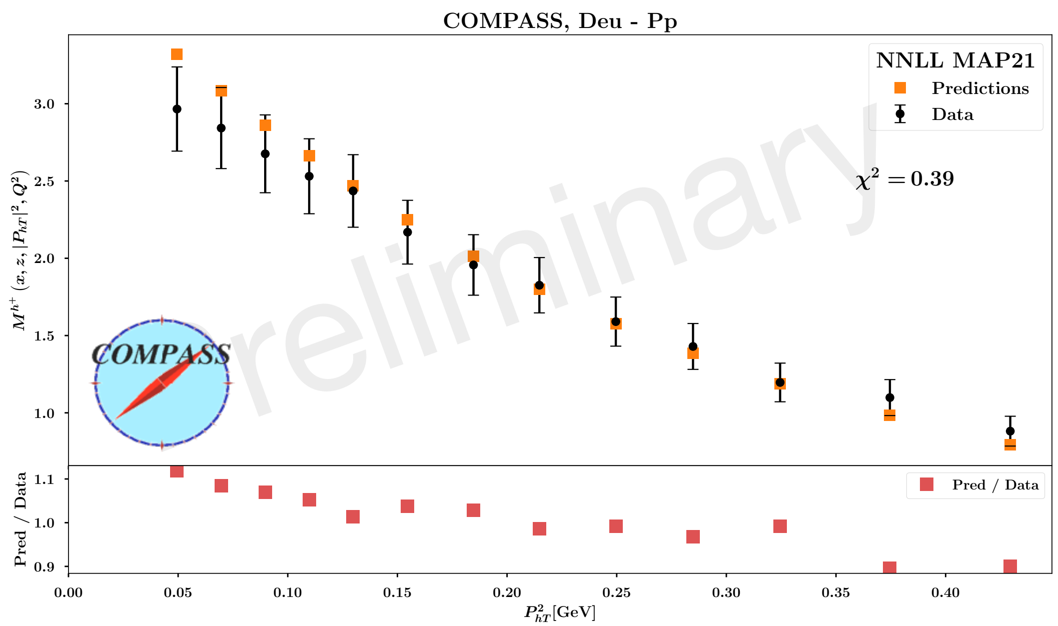
Still working on the final form

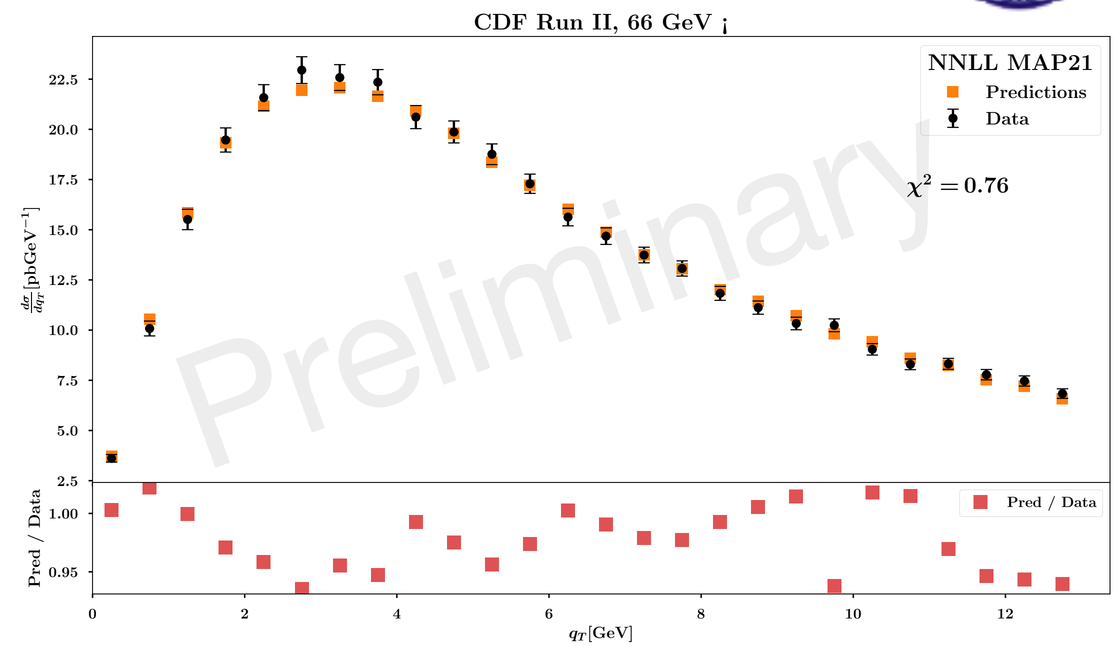
$$g_1(x) = N_1 \frac{(1-x)^\alpha \; x^\sigma}{(1-\hat{x})^\alpha \; \hat{x}^\sigma}$$

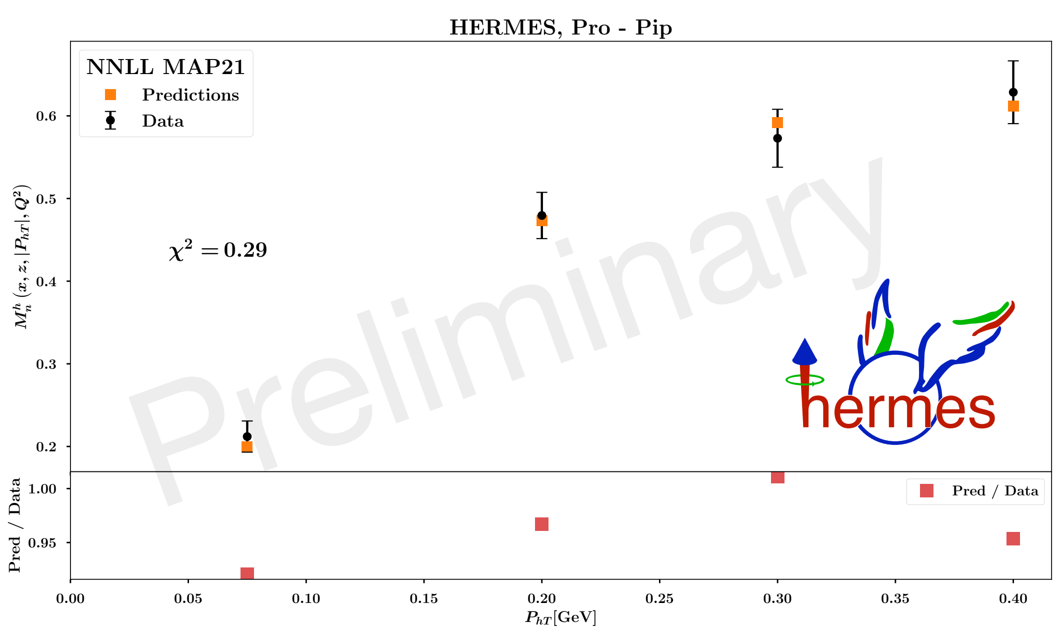
$$g_K(b_T^2) = -\frac{g_{2A}}{2} b_T^2 - \frac{g_{2B}}{2} b_T^4$$

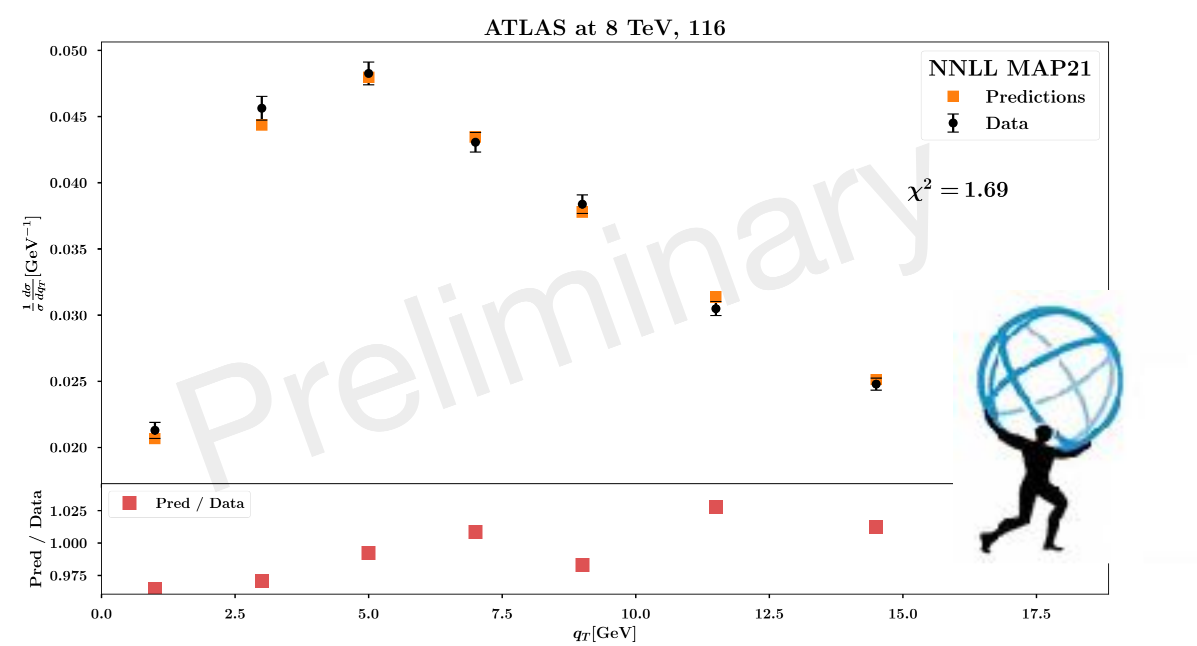
11 parameters for TMD PDF
+ 2 for NP evolution +14 for FF
= 27 free parameters

N²LL: EXAMPLE OF GOOD BINS

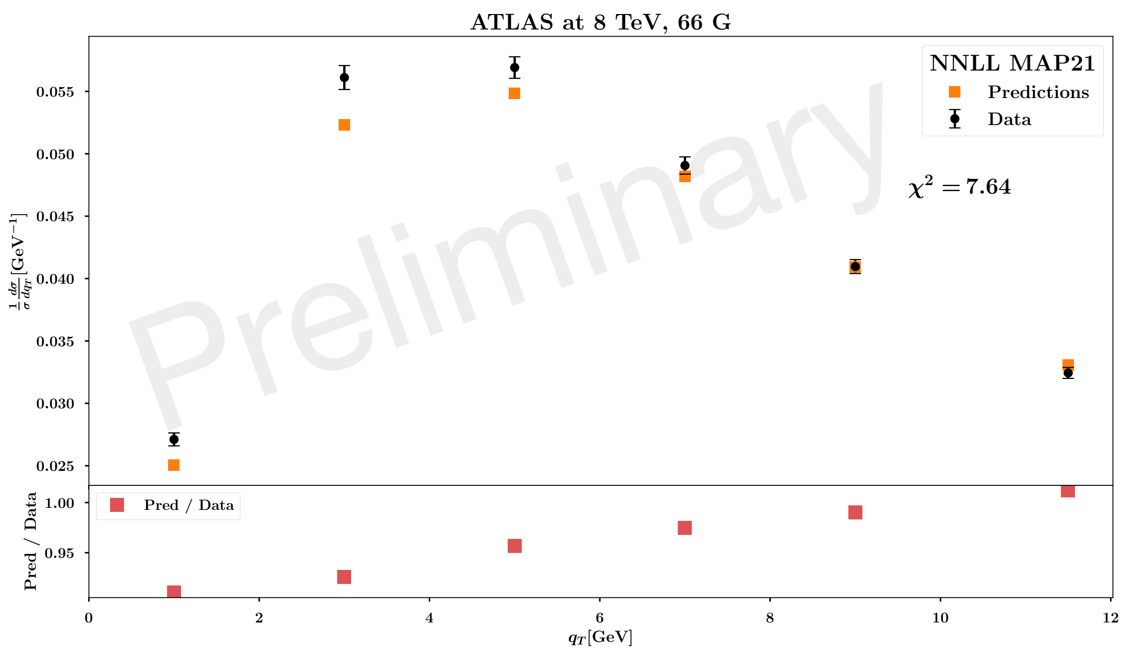


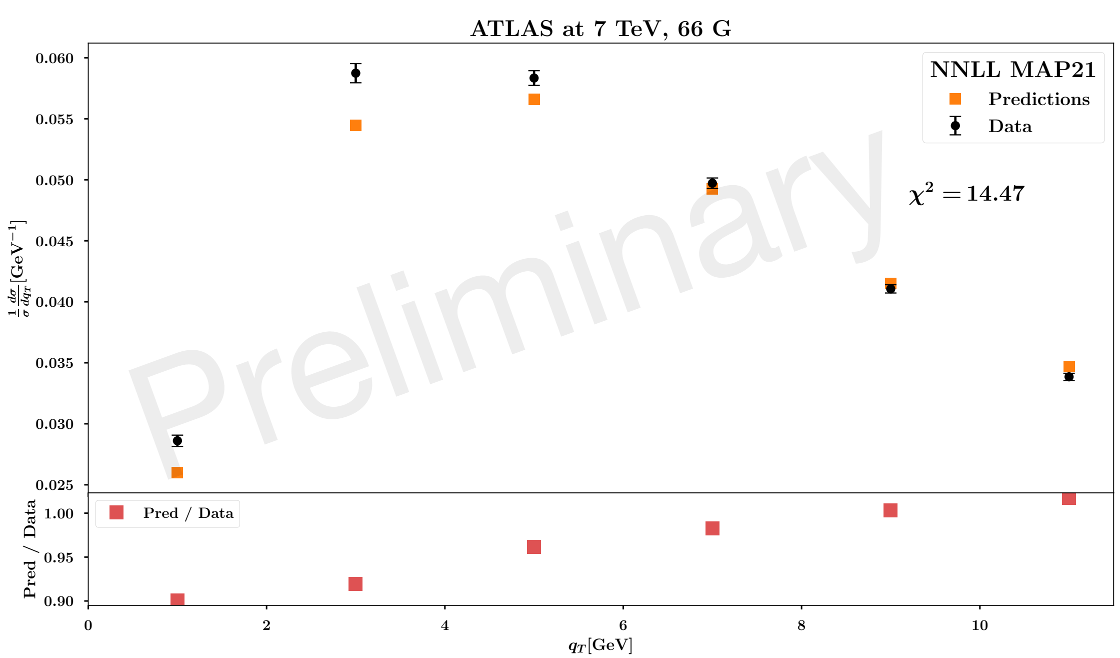


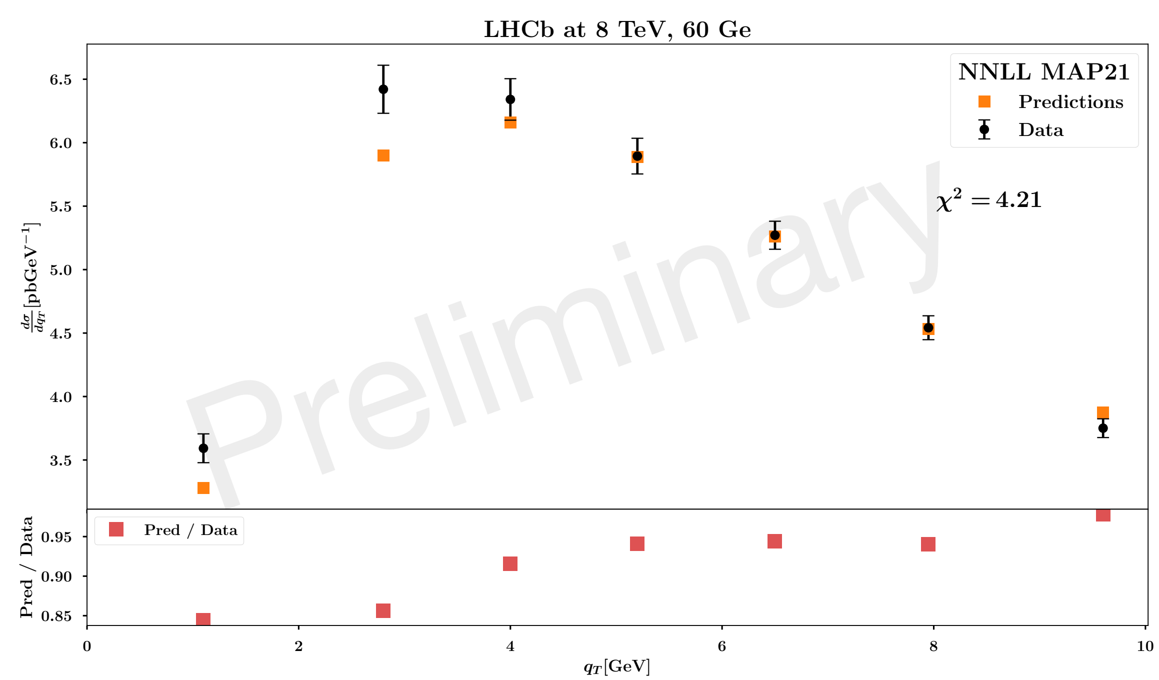




N2LL: EXAMPLE OF BAD BINS





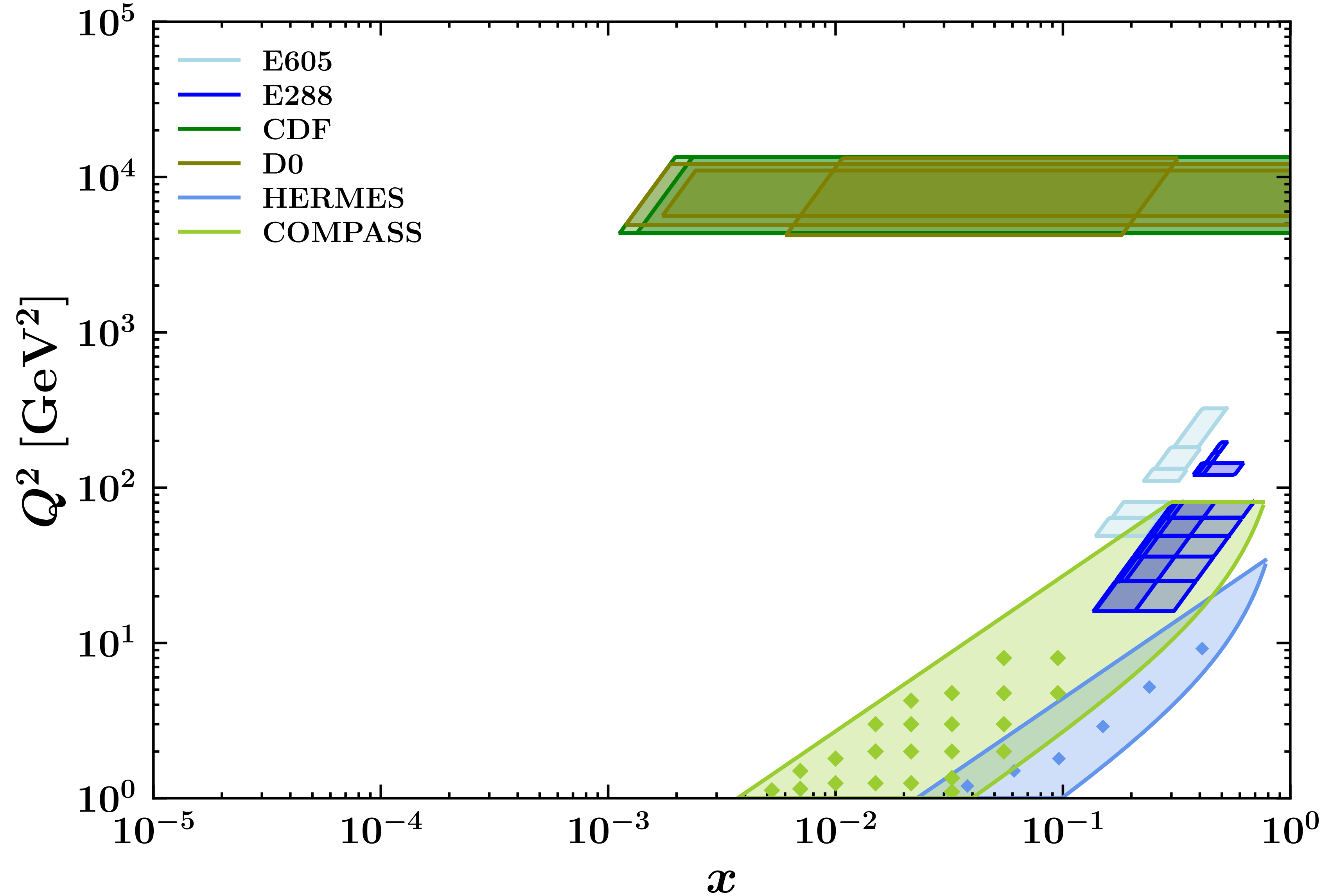


GENERAL CONSIDERATIONS ABOUT UNPOLARIZED TMDS

- SIDIS data can be described very well at NLL, but require normalisation factors at NLL′ or higher
- The identification of the region of applicability of the TMD formalism is still an open issue
- Good global χ^2 can be reached at N^2LL, but some LHC data remain hard to describe

BACKUP SLIDES

x–Q² COVERAGE PV17



Bacchetta, Delcarro, Pisano, Radici, Signori, arXiv:1703.10157

DATA SELECTION IN PAVIA 2017

$$Q^2 > 1.4 \text{ GeV}^2$$

$$0.2 < z < 0.7$$

$$P_{hT}, q_T < \text{Min}[0.2\ Q, 0.7\ Qz] + 0.5 \text{ GeV}$$

Total number of data points: 8059
Total χ^2/dof = 1.55

DATA SELECTION IN PAVIA 2017

$$Q^2 > 1.4 \ \mathrm{GeV}^2$$

$$0.2 < z < 0.7$$

$$P_{hT}, q_T < \mathrm{Min}[0.2 \ Q, 0.7 \ Qz] + 0.5 \ \mathrm{GeV}$$

Total number of data points: 8059
Total χ^2/dof = 1.55

We checked also

$$P_{hT} < \mathrm{Min}[0.2\, Q, 0.5\, Qz] + 0.3\, \mathrm{GeV}$$

Total number of data points: 3380
Total χ^2/dof = 0.96

$$P_{hT} < 0.2\, Qz$$

Total number of data points: 477
Total χ^2/dof = 1.02

PV17 – RESULTING TMDS

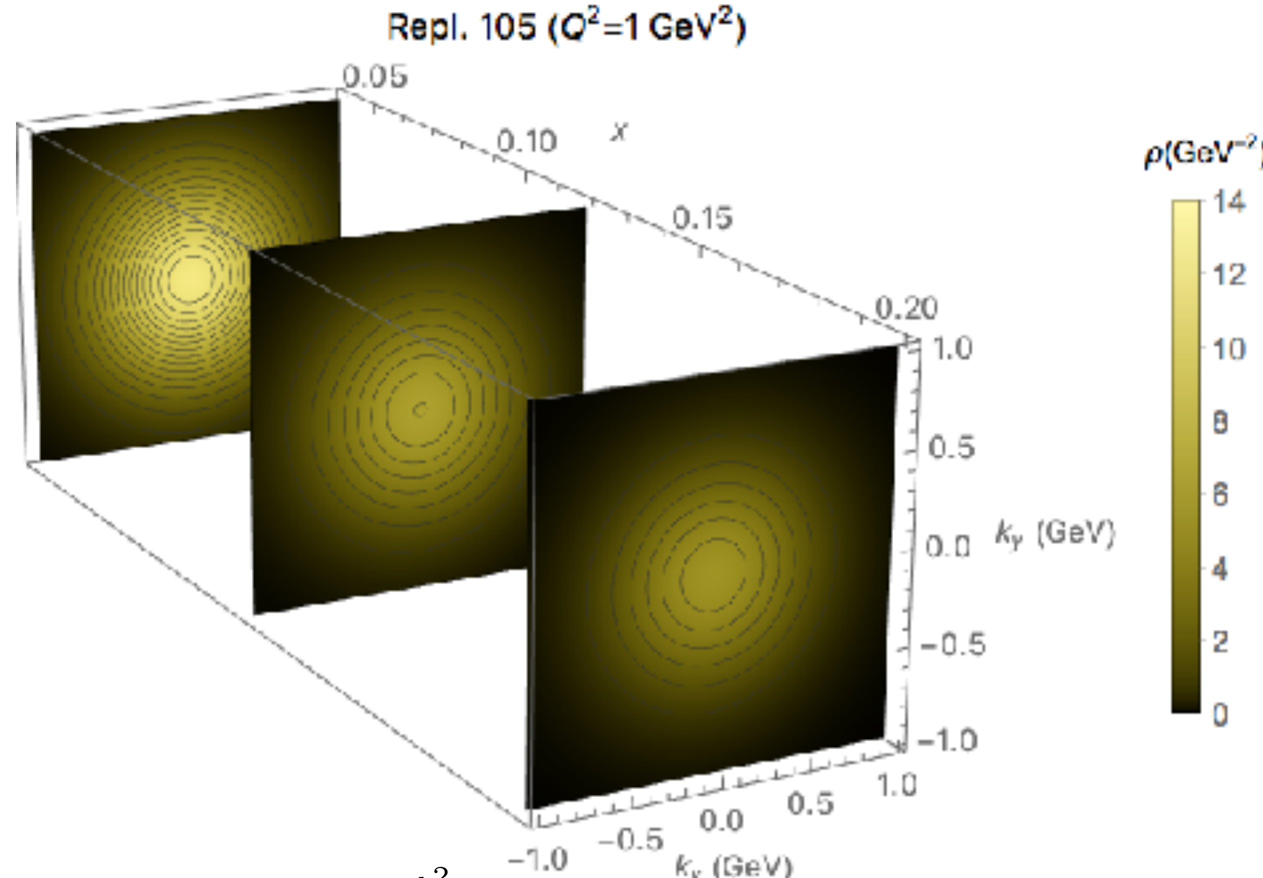
Bacchetta, Delcarro, Pisano, Radici, Signori, arXiv:1703.10157

expression in b_T space

$$\hat{f}_{\rm NP}(x, b_T) = e^{-g_1(x)\frac{b_T^2}{4}}\left(1 - \frac{\lambda g_1^2(x)}{1+\lambda g_1(x)}\frac{b_T^2}{4}\right)$$

- Guassian + weighted Gaussian
- nontrivial x dependence
- no flavor dependence

plot in $k_\perp$ space



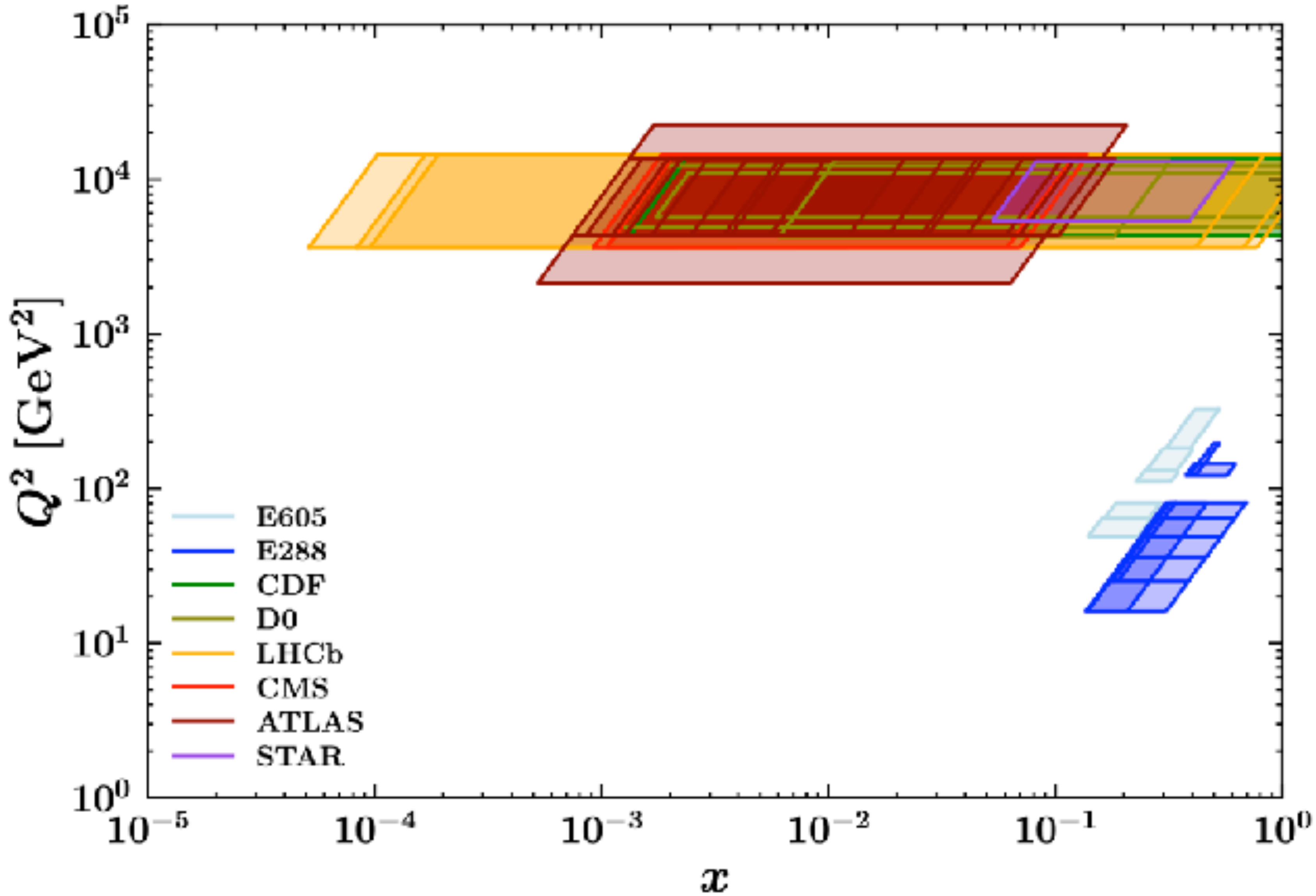
$$g_K(b_T) = -\frac{g_2}{2} b_T^2$$ Gaussian

$$\hat{D}_{\rm NP}(z, b_T) = \frac{g_3(z)\; e^{-g_3(z)\frac{b_T^2}{4z^2}} + (\lambda_F/z^2) g_4^2(z)\left(1 - g_4(z)\frac{b_T^2}{4z^2}\right)\; e^{-g_4^2(z)\frac{b_T^2}{4z^2}}}{z^2\left(g_3(z) + (\lambda_F/z^2) g_4^2(z)\right)}$$

TMD Frag. Func.

11 free parameters

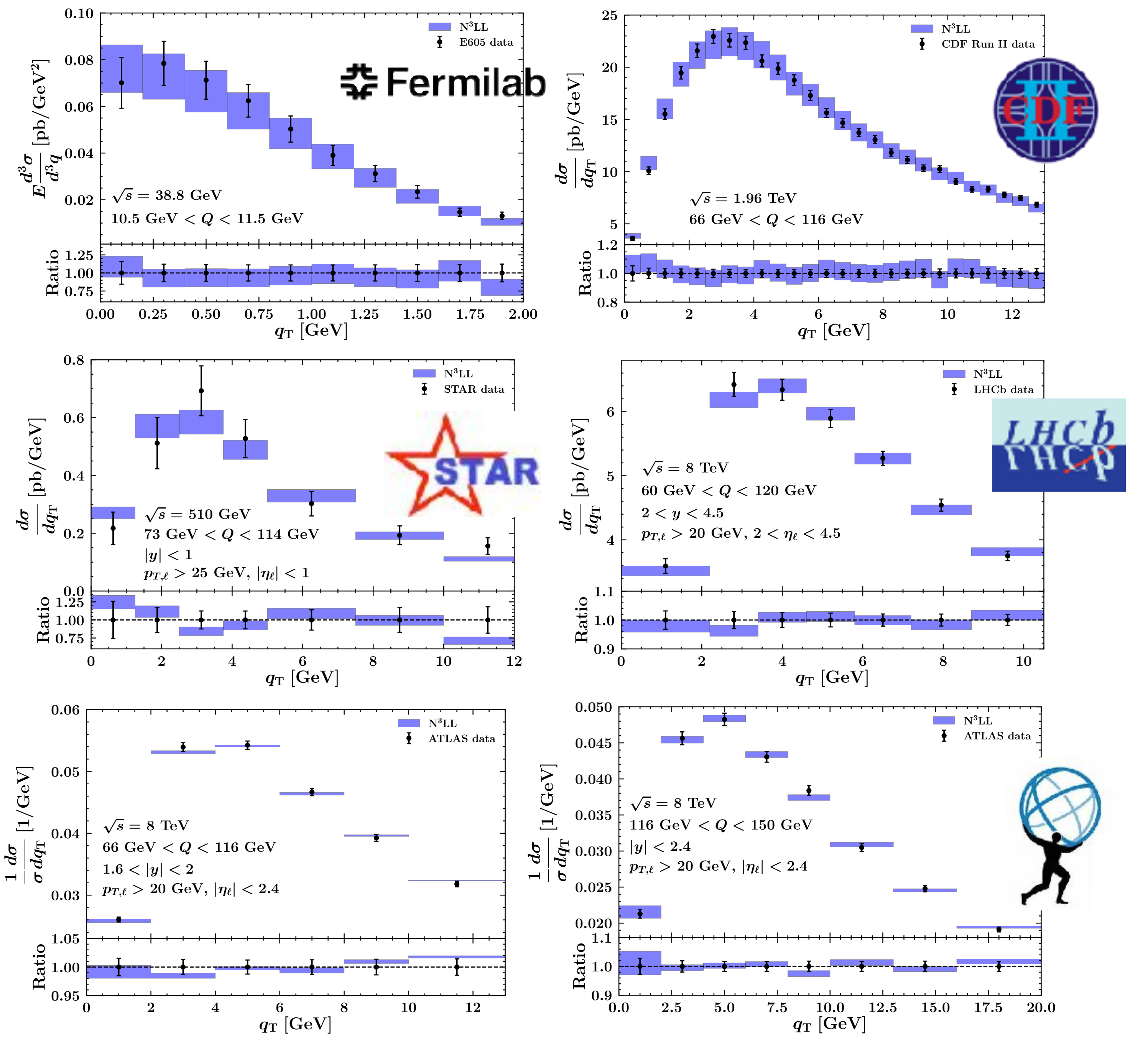
x–Q^2 COVERAGE PV19



Bacchetta, Bertone, Bissolotti, Bozzi, Delcarro, Piacenza, Radici, arXiv:1912.07550

THE PAVIA19 EXTRACTION

Bacchetta, Bertone, Bissolotti, Bozzi, Delcarro, Piacenza, Radici, arXiv:1912.07550

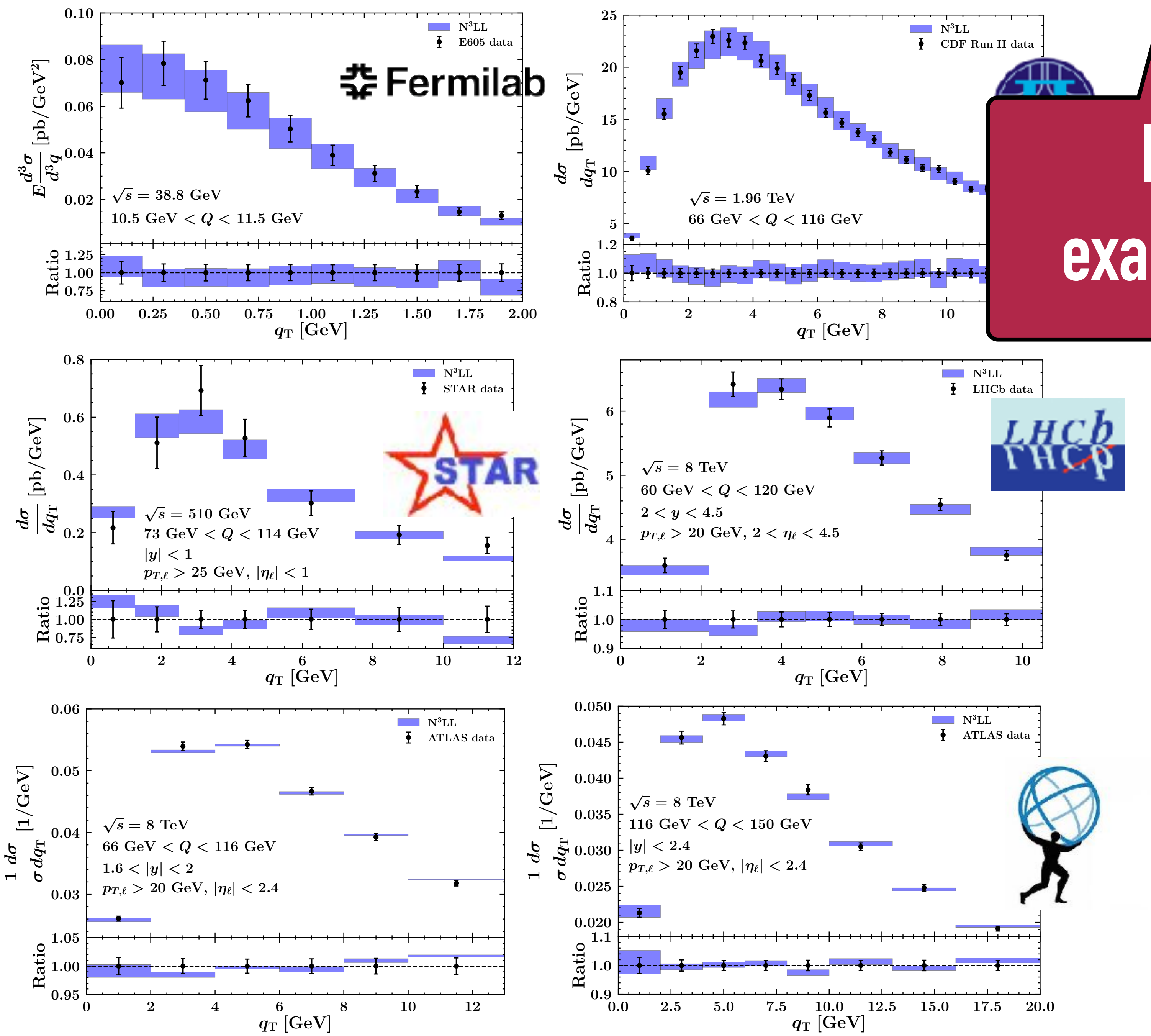


Data selection: $q_T/Q < 0.2$

Number of data points: 353

PV19 – DATA COMPARISION

Bacchetta, Bertone, Bissolotti, Bozzi, Delcarro, Piacenza, Radici, arXiv:1912.07550



Pavia19: first DY fit at N³LL, exactly reproduces normalization

Data selection: $q_T/Q < 0.2$

Number of data points: 353

Global χ^2/dof = 1.02

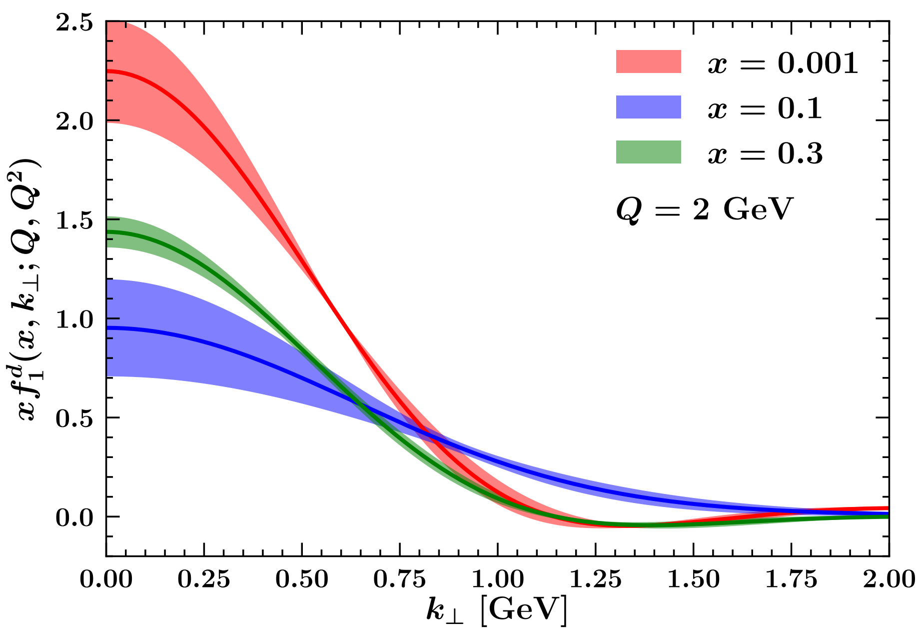
PV19 – RESULTING TMDS

expression in b_T space

$$f_{\rm NP}(x, b_T, \zeta) = \left[\frac{1-\lambda}{1+g_1(x)\frac{b_T^2}{4}} + \lambda \exp\left(-g_{1B}(x)\frac{b_T^2}{4}\right)\right]$$

$$\times \exp\left[-\left(g_2 + g_{2B} b_T^2\right) \ln\left(\frac{\zeta}{Q_0^2}\right) \frac{b_T^2}{4}\right] ,$$

plot in $k_\perp$ space



- q-Guassian + Gaussian
- nontrivial x dependence
- no flavor dependence
- non-Gaussian nonperturbative TMD evolution

9 free parameters

X DEPENDENCE IN TMDS

PV17

$$g_1(x) = N_1 \frac{(1-x)^{\alpha}\ x^{\sigma}}{(1-\hat{x})^{\alpha}\ \hat{x}^{\sigma}}$$

PV19

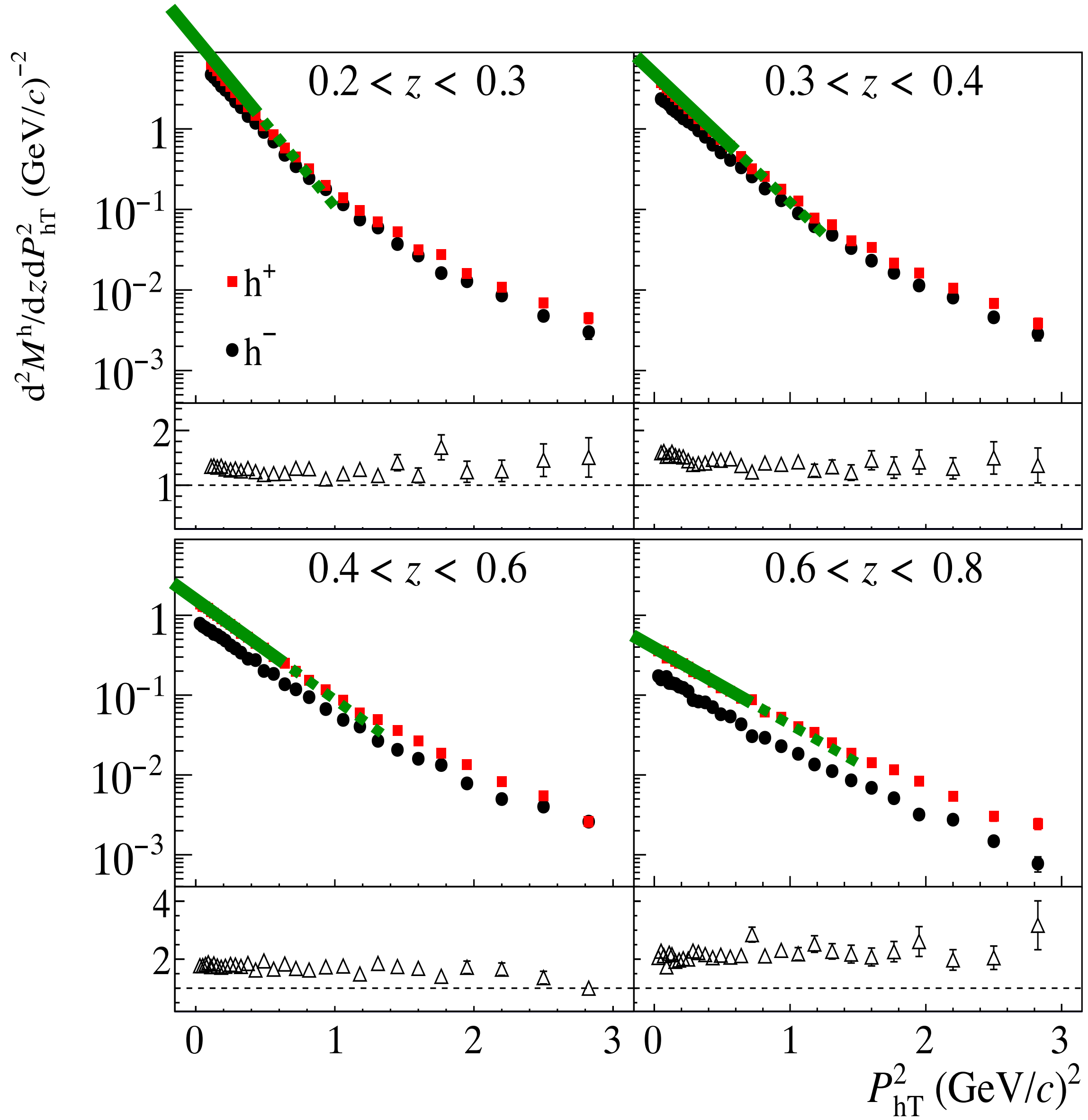
$$g_1(x)\ = \frac{N_1}{x\sigma} \exp\left[-\frac{1}{2\sigma^2} \ln^2\left(\frac{x}{\alpha}\right)\right] \ ,$$

$$g_{1B}(x)\ = \frac{N_{1B}}{x\sigma_B} \exp\left[-\frac{1}{2\sigma_B^2} \ln^2\left(\frac{x}{\alpha_B}\right)\right] \ .$$

TMD REGIONS AND DATA

COMPASS Collab., arXiv:1709.07374

$\langle Q \rangle = 3.1$ GeV



It seems that the same "physics" is dominating at least for $0 \leq P_{hT} \leq 0.7$ GeV,

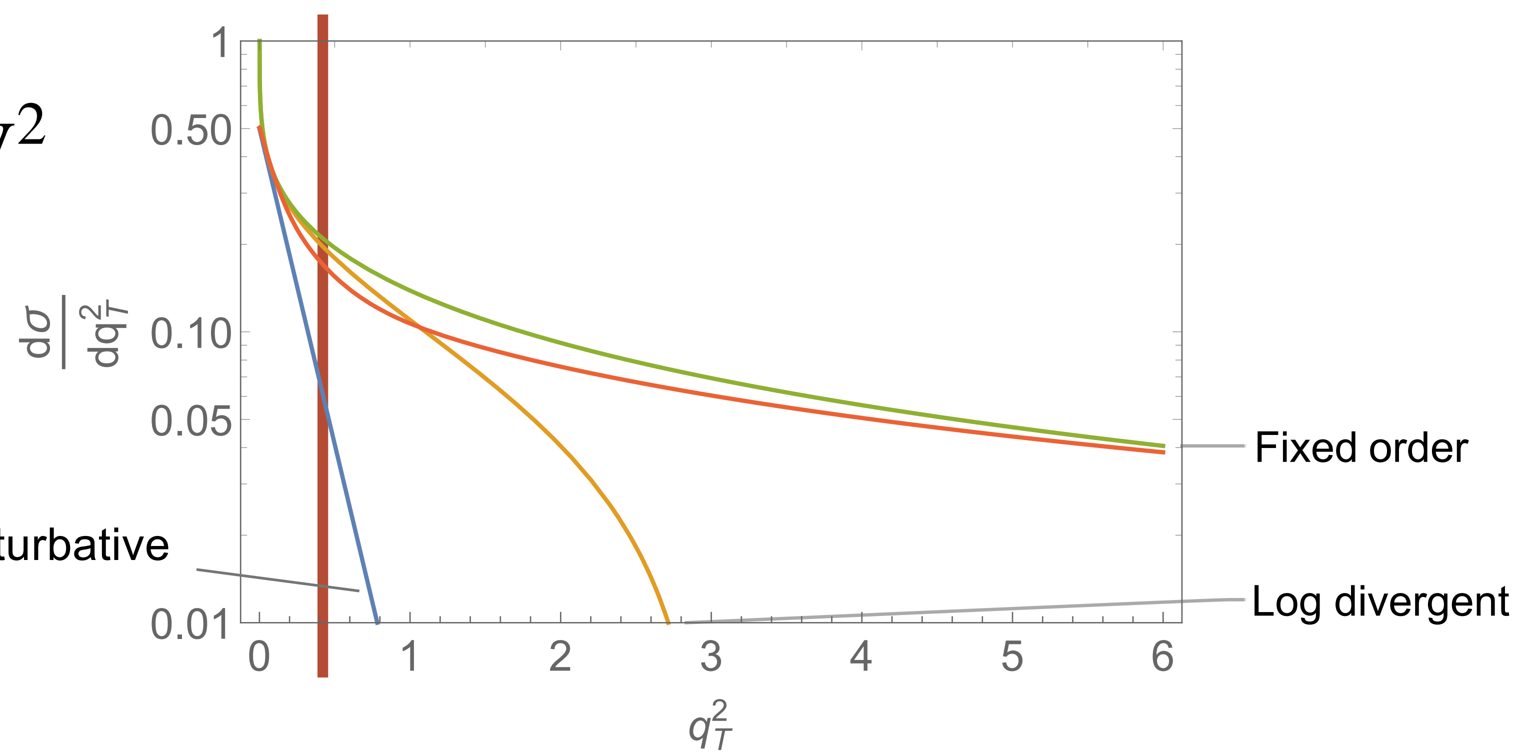
which means $0 \leq q_T \leq 2.8$ GeV in the lowest-z bin

TMD REGIONS: PERTURBATIVE VS. NONPERTURBATIVE

Perturbative approach:
TMD region = where the log divergence of the fixed-order calculation dominates (resummation is required)

Nonperturbative approach:
TMD region = where either the log divergence OR the nonperturbative contributions dominate

$$\langle Q^2 \rangle = 4\ \mathrm{GeV}^2$$



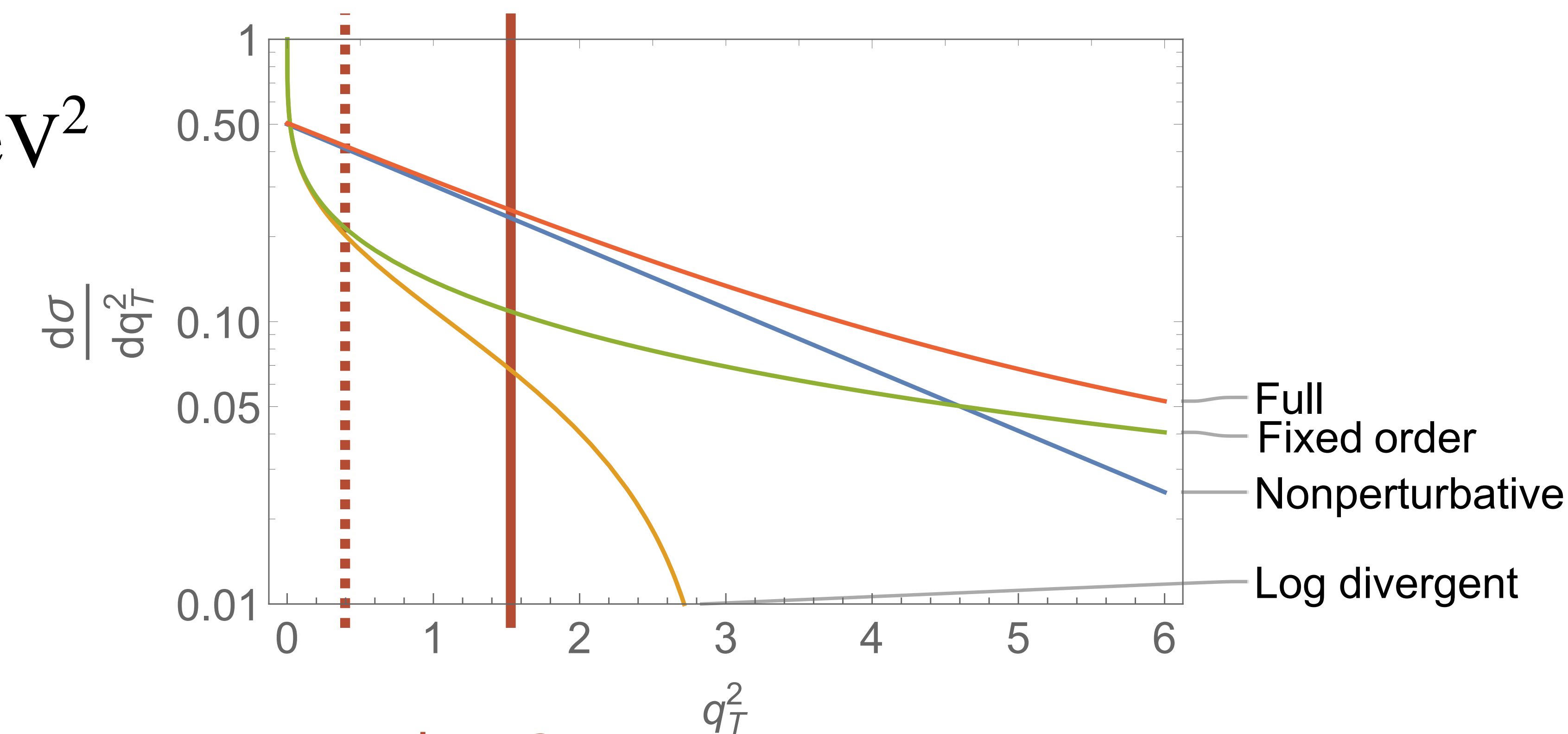
TMD region (ideal situation)

TMD REGIONS: PERTURBATIVE VS. NONPERTURBATIVE

Perturbative approach:
TMD region = where the log divergence of the fixed-order calculation dominates (resummation is required)

Nonperturbative approach:
TMD region = where either the log divergence OR the nonperturbative contributions dominate

$$\langle Q^2 \rangle = 4 \text{ GeV}^2$$



TMD region?