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SFB 676 – Project B2

SUSY Parameter and Mass Determination at the LHC

C. Sander, *Hamburg University*

Cambridge Phenomenology Seminar – 25th February 10

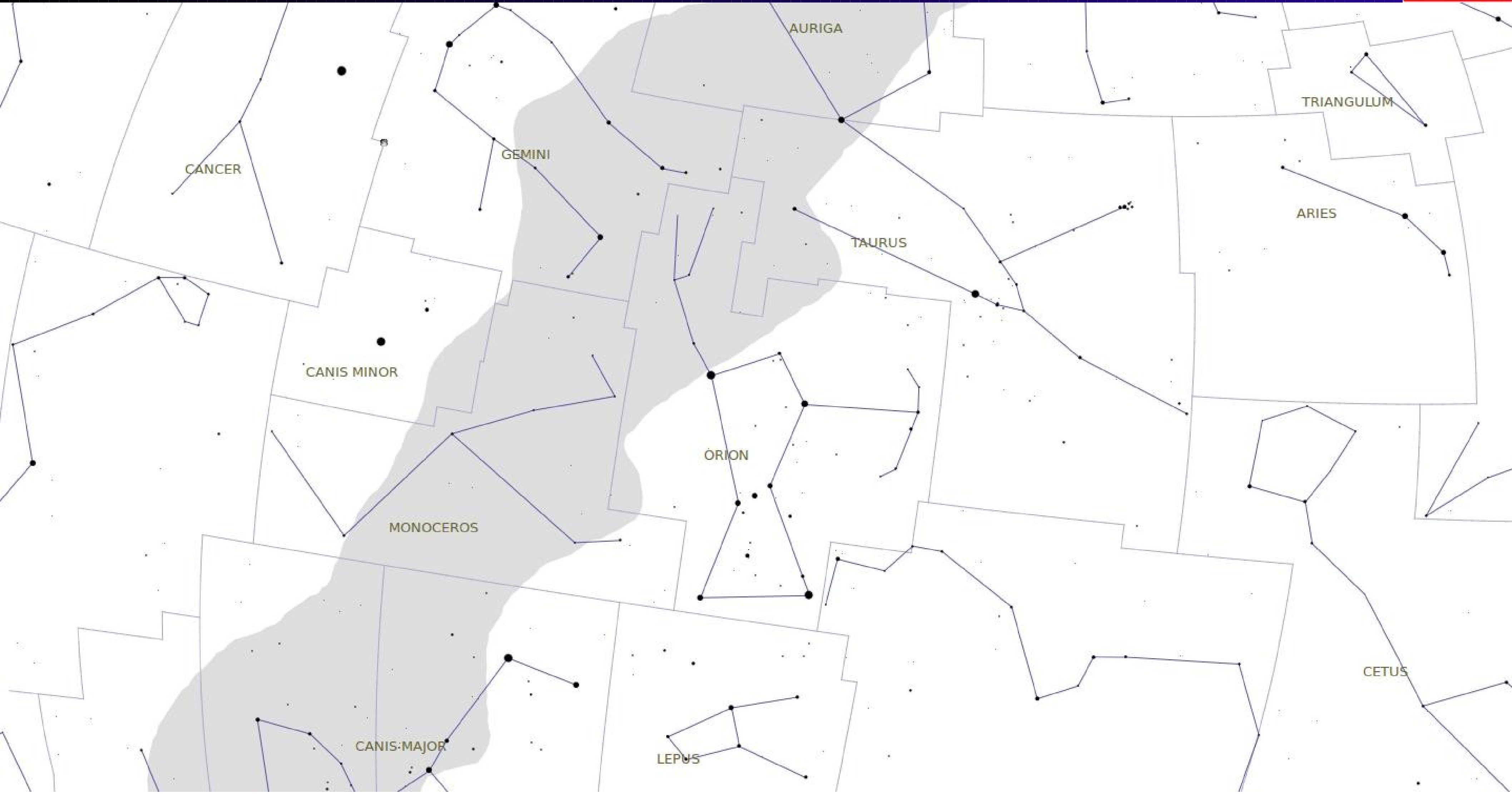
kstars – Cambridge 25th Feb 4pm

Example: Search for Underlying Theory

Obvious questions in prehistoric times:

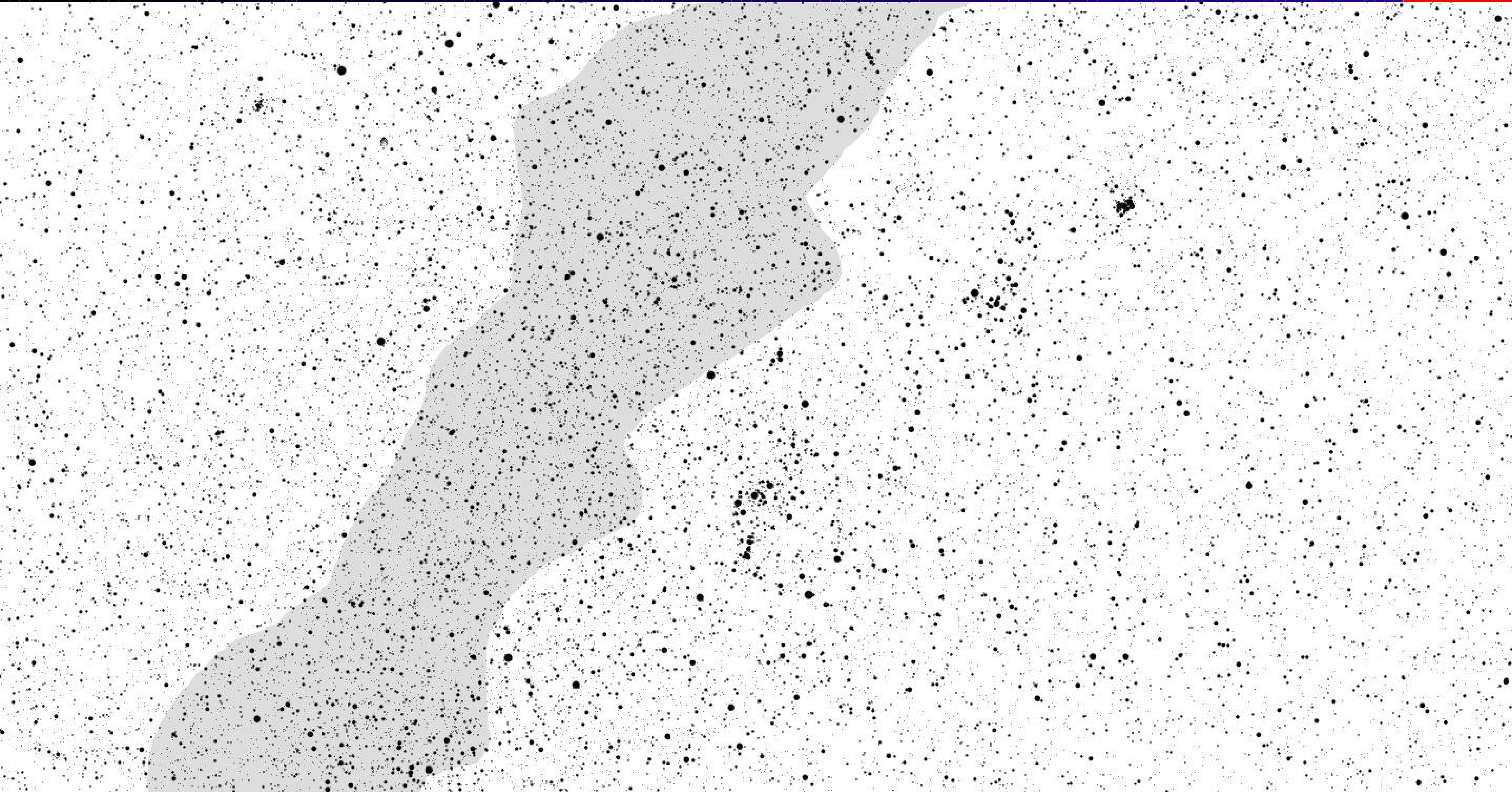
Do stars in particular sky regions belong to a constellation?

Example: Search for Underlying Theory



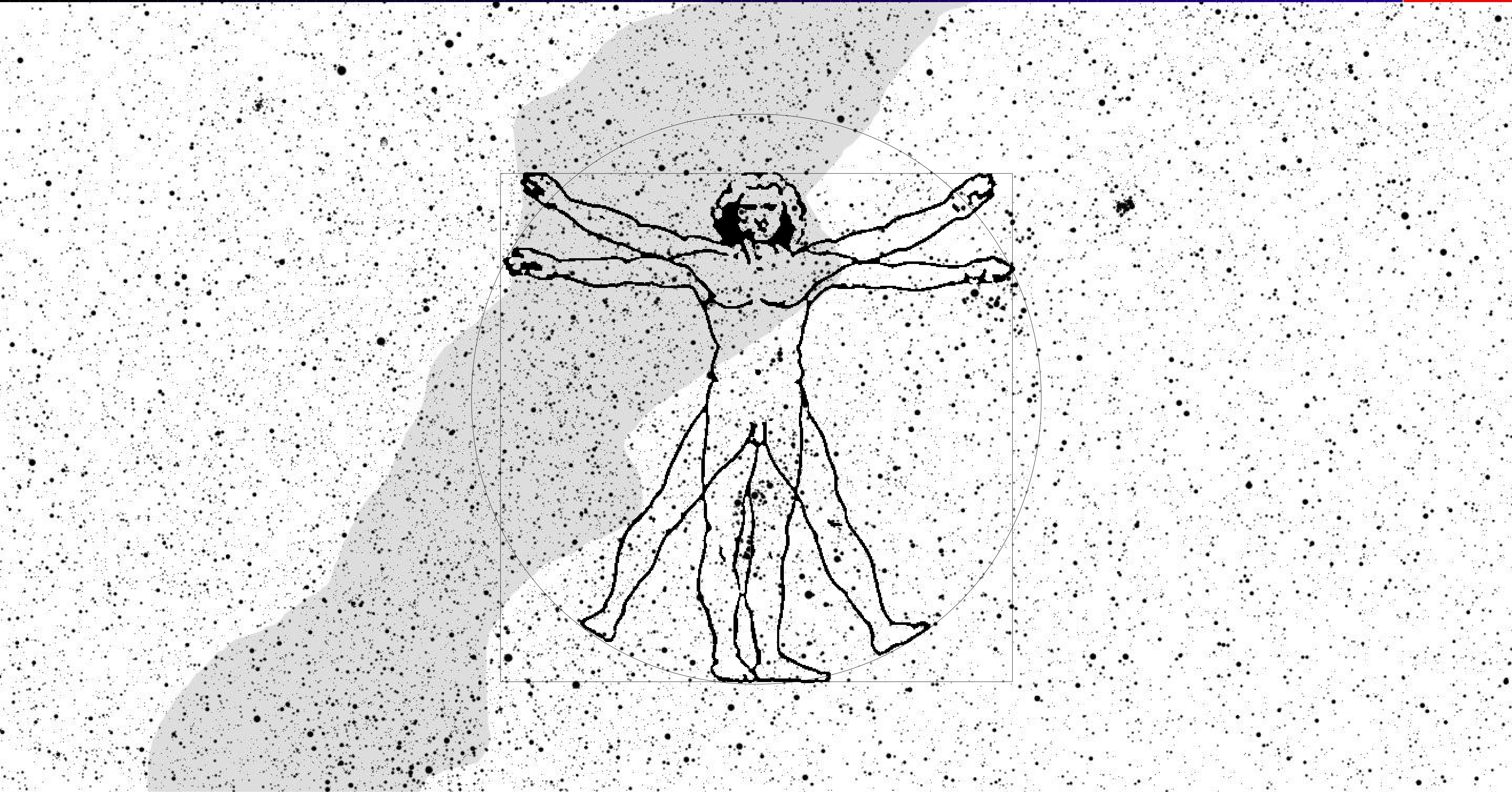
This is what astrologists -of our culture- agreed on!

Example: Search for Underlying Theory



Including *noise*, *pile-up*, *underlying event* …

Example: Search for Underlying Theory



Including *noise*, *pile-up*, *underlying event* … **it is possible to find everything in the sky!**

Introduction

Part I: Review of sensitive observables at the LHC

- Kinematic end-points, mass edges, thresholds ...
- m_T, m_{T2} , Kinks ...
- Weak boson and top production rates
- Invariant multi jet masses

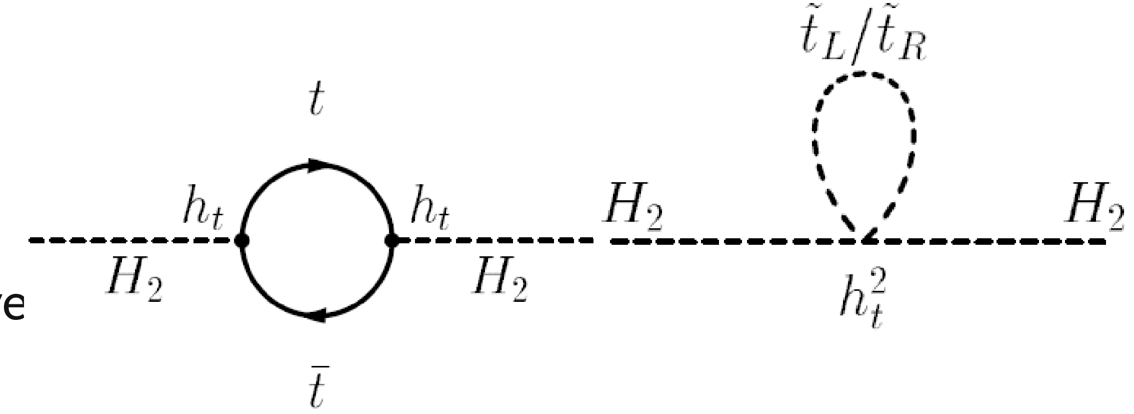
Part II: SUSY mass determination using kinematic fits

- Global fits
- Event reconstruction methods:
  - Hybrid method
  - Multi event method
  - Single event method
  - Kinematic method

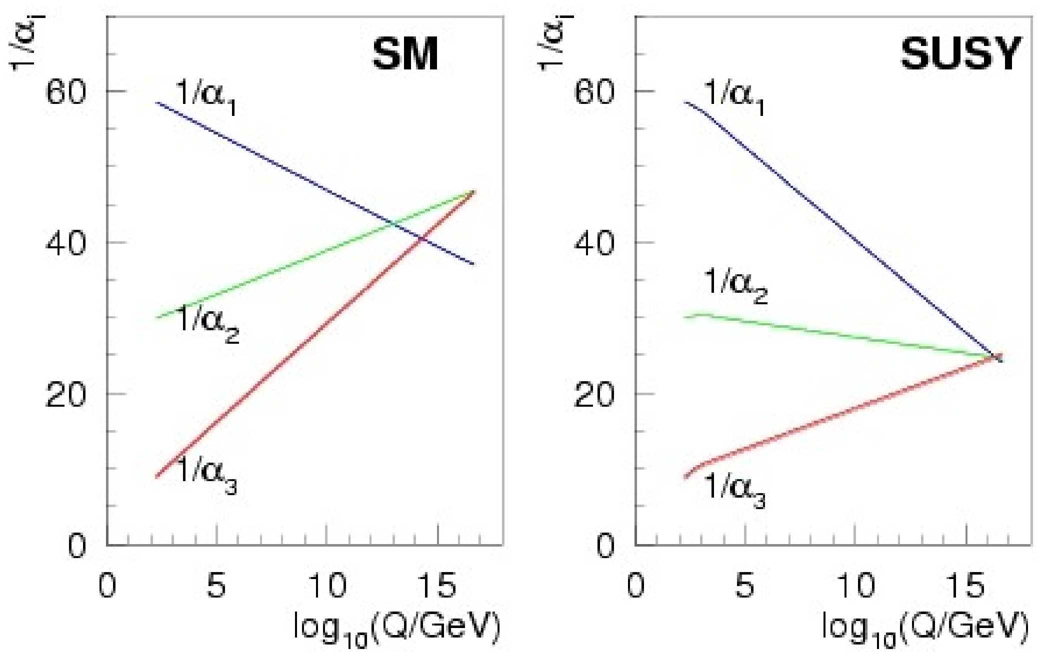
Summary

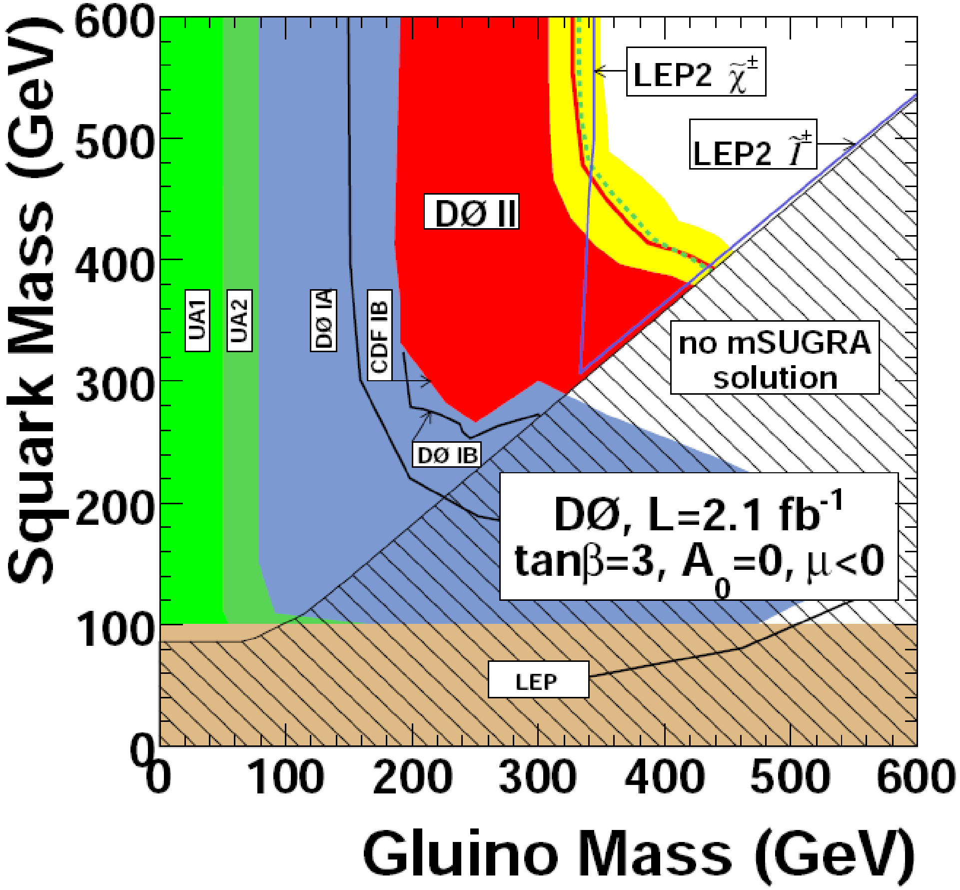
- **Last possible symmetry:** between fermions and bosons
- Each SM particle gets a SUSY partner equal in all quantum numbers except for spin (±½)
  - → Opposite sign of loop corrections solve fine tuning problem
  - → New particles change slope of running couplings → gauge unification
  - → Graviton ($s = 2$) ↔ $g/W/Z/\gamma$ ($s = 1$)
  - → Provides perfect DM candidate
  - → "Natural" EWSBreaking
- No candidates for supersymmetric partners discovered so far
  - → SUSY has to be broken, but sparticles should have masses of ~1 TeV to keep advantages of SUSY

cancellation of loop corrections:



unification of gauge couplings:



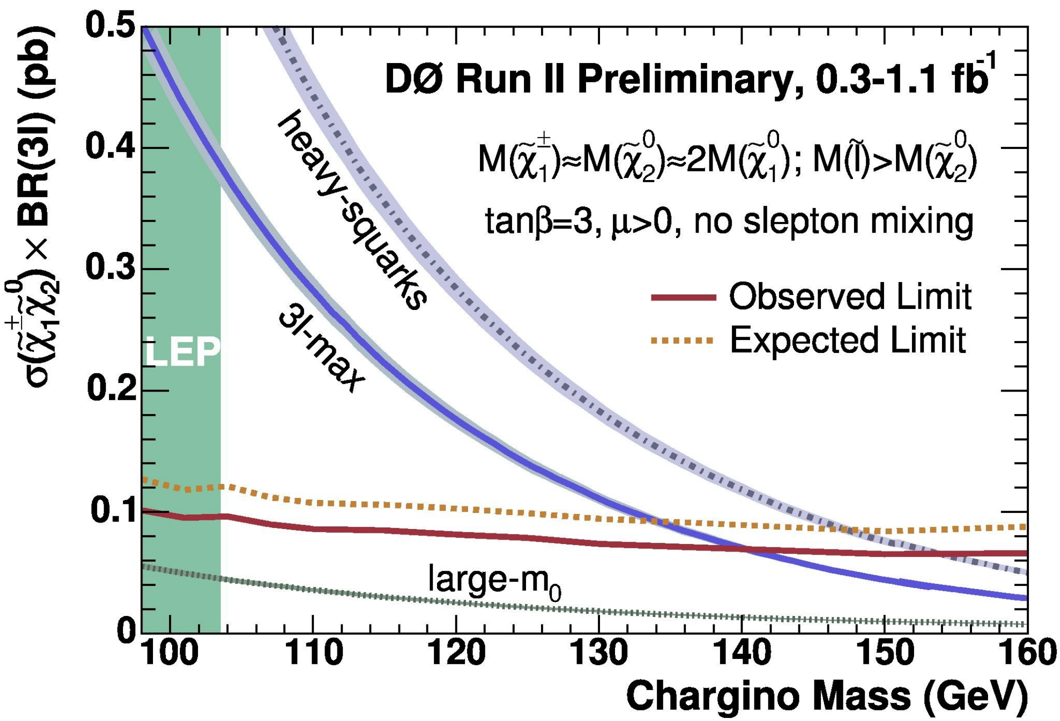


Hadronic searches:

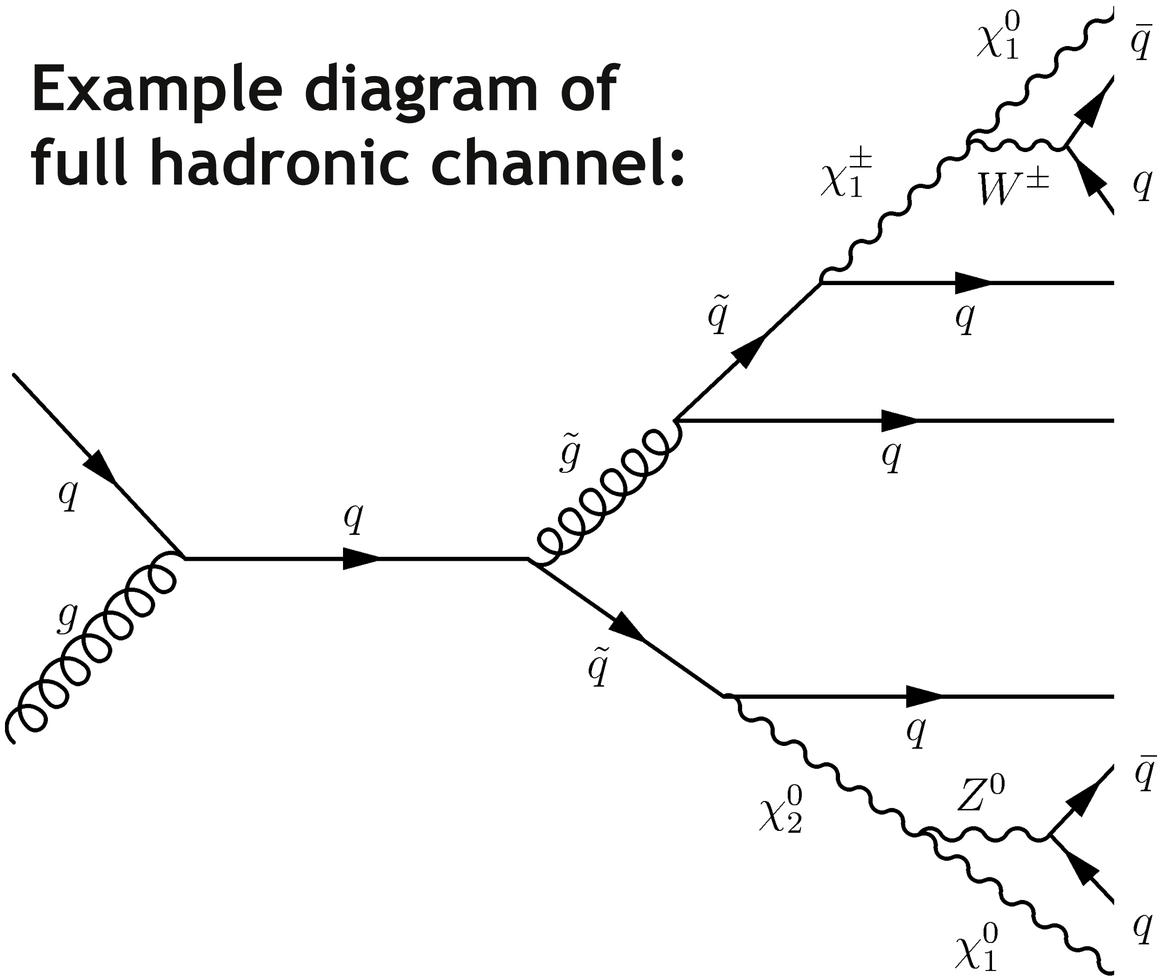
- Best limits from CDF and DØ
- Within mSUGRA: $m_{\tilde{g}} \gtrsim 300$ GeV
  $m_{\tilde{q}} \gtrsim 400$ GeV

Leptonic searches:

- Best limits on SUSY masses from Tevatron and LEP experiments
- "Golden channel at Tevatron"
  $q\bar{q} \to \chi_1^{\pm}\chi_2^0 \to 3l + \not{E}_T$



Example diagram of full hadronic channel:



- R-parity conserved:
- SUSY particles are produced in pairs
- Cascade decay down to stable LSP
  - $\not{E}_T$
  - large number of jets/leptons
  - jet/lepton pairs compatible with weak gauge boson masses
  - ...
- Fully hadronic decay mode has large branching ratio

Two goals:

(1) Discover SUSY at the LHC

(2) Determine model parameters of underlying theory

Part I

Review of sensitive observables at the LHC

Observables

Pre-LHC era:

- Electroweak precision data (*LEP/SLAC*)
- Anomalous magnetic moment of muon (*BNL*)
- Rare decays (*B-factories*, *Tevatron*)
- Relic density constraints from astrophysical experiments (*WMAP* + *SNIa* + ...)
- Higgs, SUSY, and ... mass limits (*Tevatron*, *HERA*, *LEP*)

LHC (+ILC) era:

- Kinematic end-points, kinks, ...

LHC inverse problem: Inverse map of observables to parameter space shows many degeneracies. Roughly spoken: **Too less constraints for the large number of unknowns!**

Arkani-Hamed, Kane, Thaler & Wang 05

Needed: New observables providing additional information

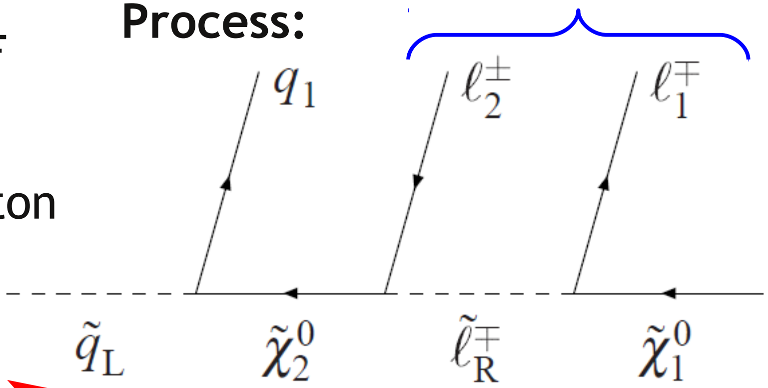
- Branching ratios
- Invariant multi-jet masses, ...

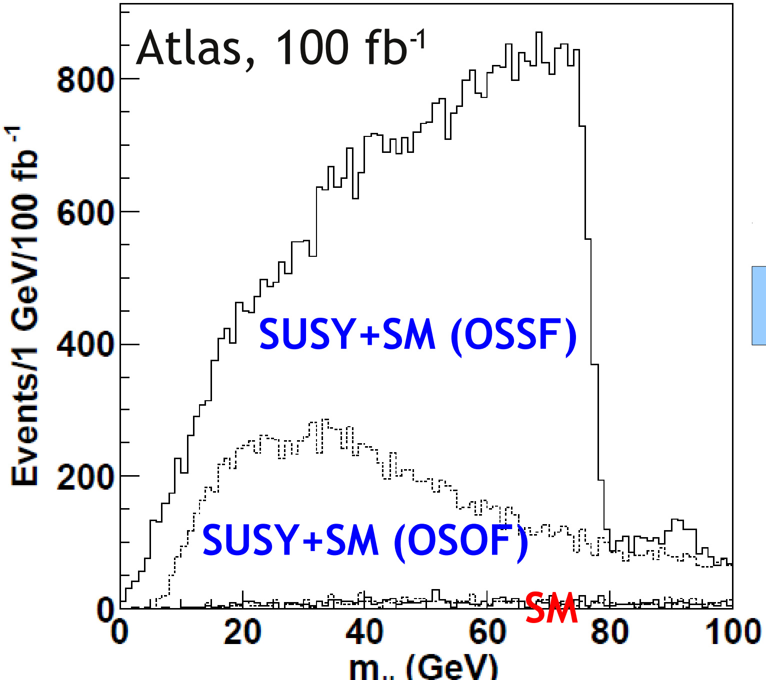
- Unmeasured LSP → no peak in invariant mass distributions but endpoints, thresholds ...
- Background (SUSY and SM) can be suppressed by OSOF (opposite sign, opposite flavor) subtraction
- Accurate reproduction of theoretical expected di-lepton edge:

$$\left(m^2_{ll}\right)^{\mathrm{edge}} = \frac{\left(m^2_{\tilde{\chi}^0_2} - m^2_{\tilde{l}_R}\right)\left(m^2_{\tilde{l}_R} - m^2_{\tilde{\chi}^0_1}\right)}{m^2_{\tilde{l}_R}} = 77\,\mathrm{GeV}$$

Opposite Sign, same flavour (OSSF)lepton pair

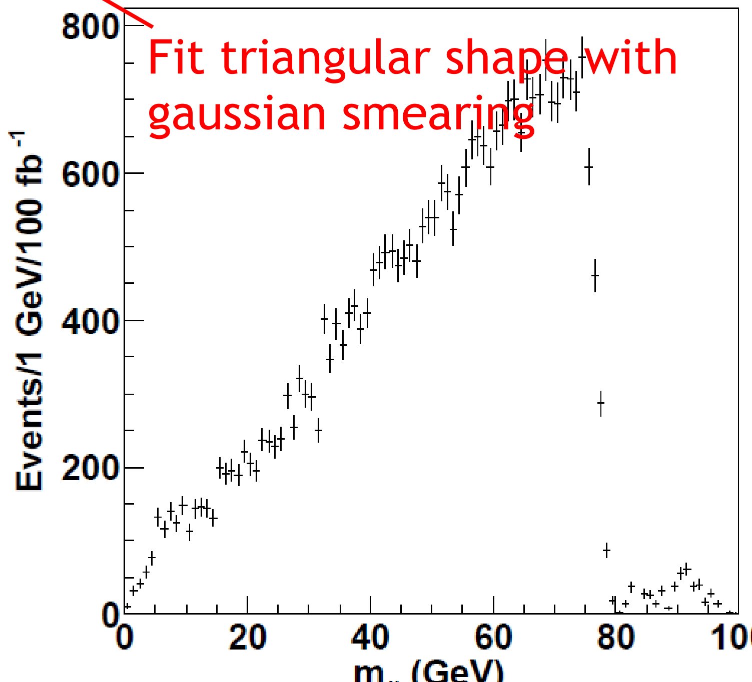
Process:



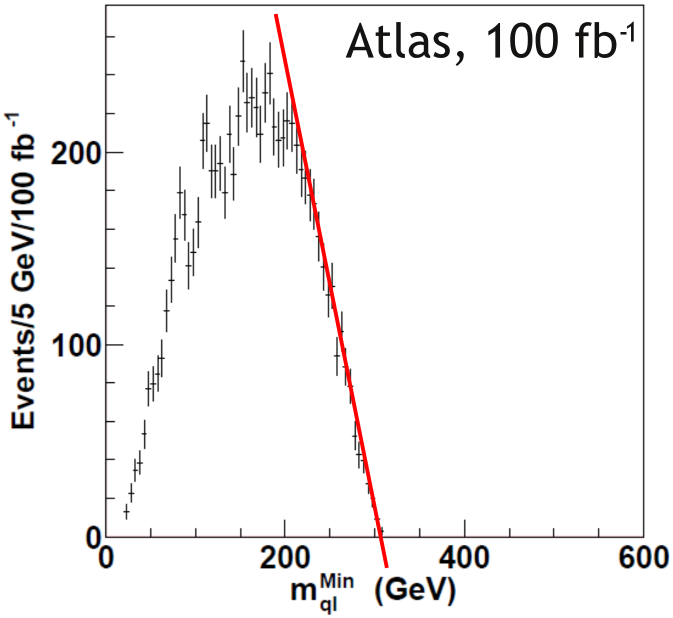


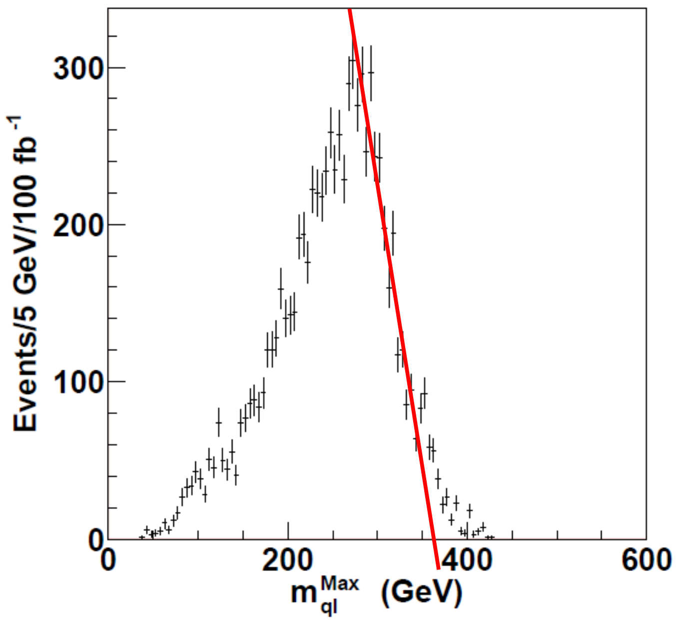
Model: SPS1a

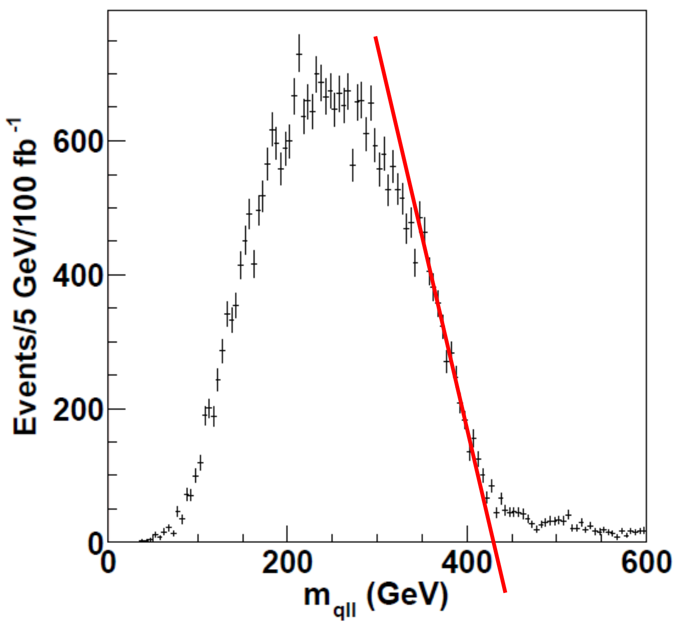
OSOS subtraction →



Gjelsten, Hisano, Kawagoe, Lytken, Miller, Nojiri, Osland & Polesello (in LHC/IC study group) 04

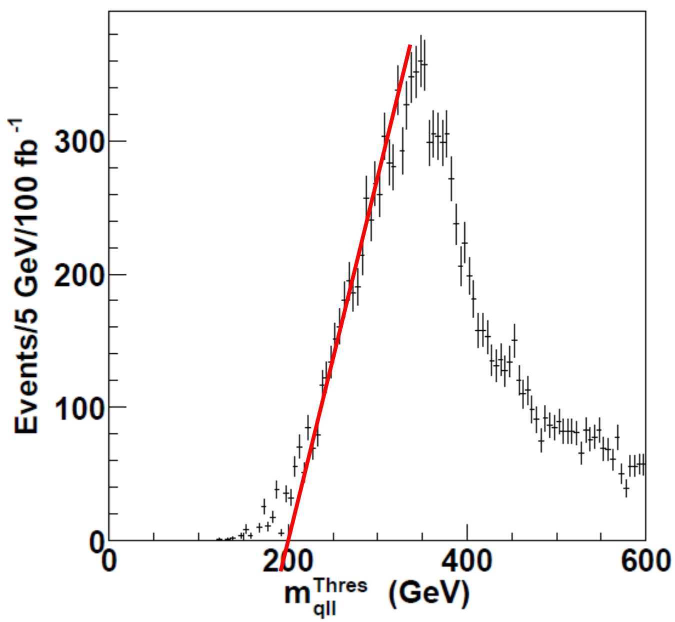






- "Min"/"Max" w.r.t. choice of lepton
- $m(qll)^{\text{thres}}$ refers to threshold of subset, where angle of two leptons in slepton rest frame exceeds $\pi/2$

Edge	Nominal Value	Fit Value	Syst. Error Energy Scale	Statistical Error
$m(ll)^{edge}$	77.077	77.024	0.08	0.05
$m(qll)^{edge}$	431.1	431.3	4.3	2.4
$m(ql)^{edge}_{min}$	302.1	300.8	3.0	1.5
$m(ql)^{edge}_{max}$	380.3	379.4	3.8	1.8
$m(qll)^{thres}$	203.0	204.6	2.0	2.8
$m(bll)^{thres}$	183.1	181.1	1.8	6.3



Accurate reproduction of "leptonic" mass edges

All plots: OSOF subtracted

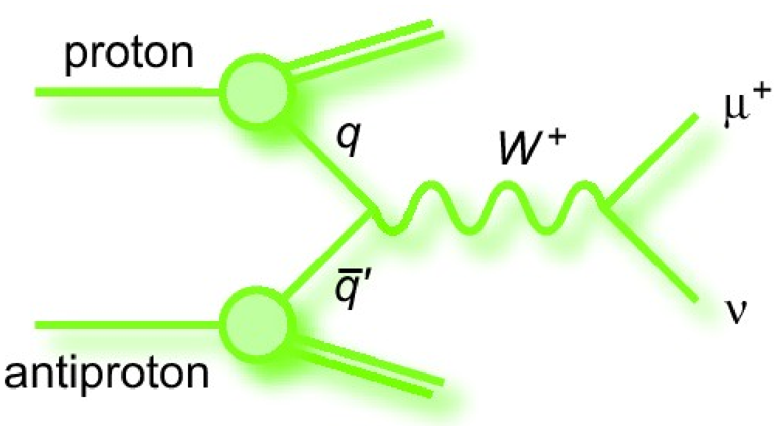
Reminder: W-mass measurement at hadron collider (unknown longitudinal ν momentum component)

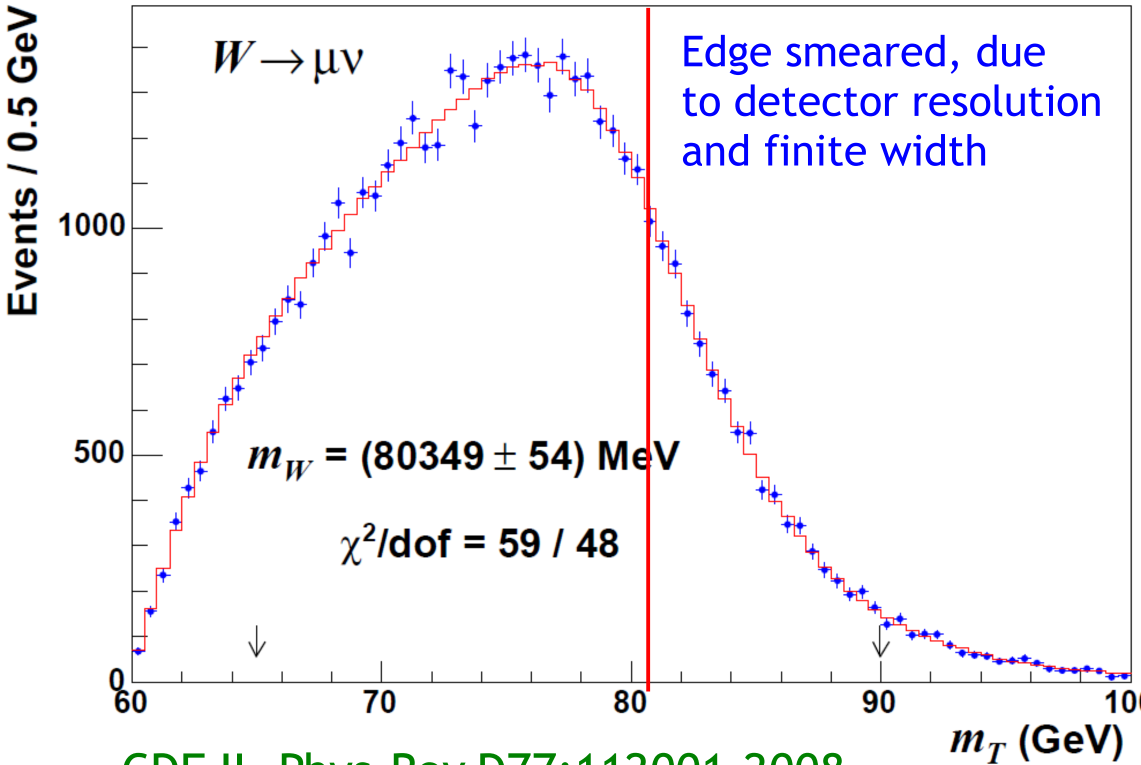
$$(p_\mu + p_\nu)^2 = m_W^2$$

$$p_\mu^2 + p_\nu^2 + 2p_\mu p_\nu = m_W^2$$

$$2\left(E_T^\mu E_T^\nu \cosh \Delta y - \mathbf{p}_T^\mu \cdot \mathbf{p}_T^\nu\right) = m_W^2 \quad \text{with} \quad m_\nu = 0 \approx m_\mu$$

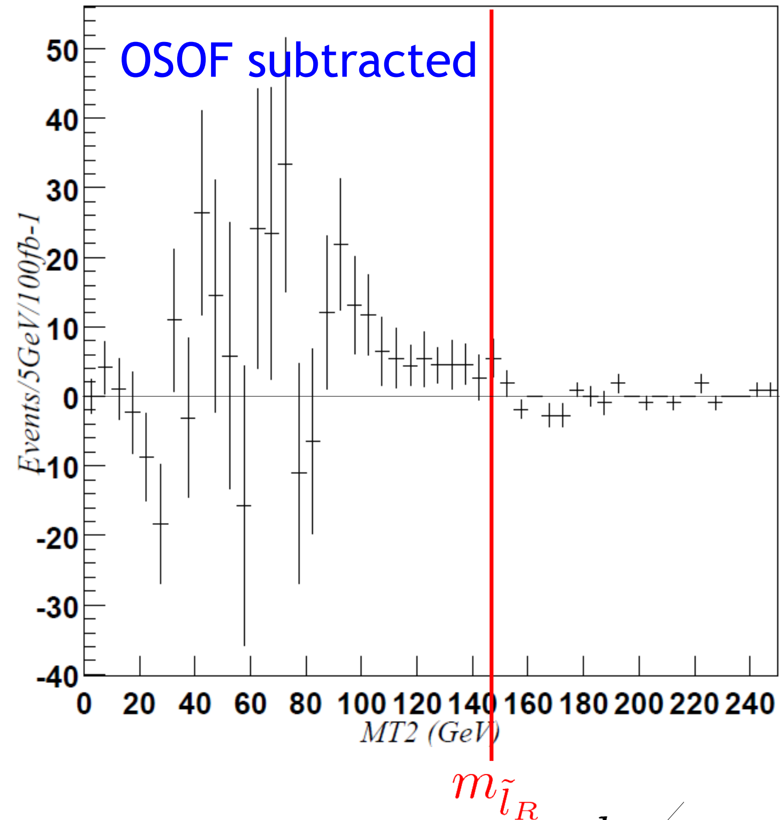
$$M_T^2 = 2\left(E_T^\mu \not{E}_T - \mathbf{p}_T^\mu \cdot \not{\mathbf{p}}_T\right) \leq m_W^2 \quad \text{with} \quad \not{E}_T = \not{\mathbf{p}}_\mathbf{T}$$





CDF II, Phys.Rev.D77:112001,2008

Question: **Similar definition for more than one massive escaping particles, e.g. LSP?**



For arbitrary momenta:

$$m_{\tilde{l}}^2 = m_l^2 + m_{\tilde{\chi}}^2 + 2(E_T^l E_T^{\tilde{\chi}} \cosh\Delta\eta - \mathbf{p}_T^l \cdot \mathbf{p}_T^{\tilde{\chi}})$$

since $\cosh\Delta\eta \leq 1$

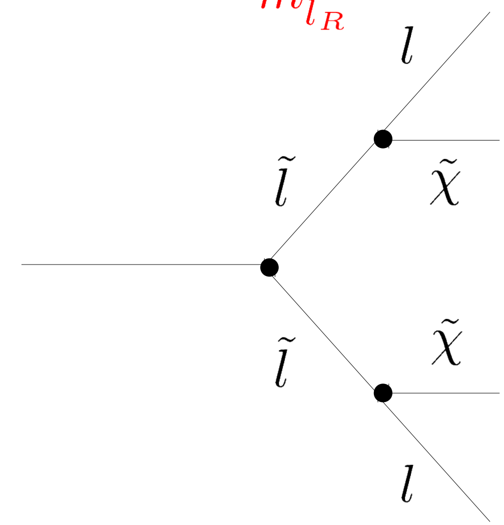
$$m_{\tilde{l}}^2 \geq m_T^2(\mathbf{p}_T^l, \mathbf{p}_T^{\tilde{\chi}}) = m_l^2 + m_{\tilde{\chi}}^2 + 2(E_T^l E_T^{\tilde{\chi}} - \mathbf{p}_T^l \cdot \mathbf{p}_T^{\tilde{\chi}})$$

Not usable, since both neutralinos contribute to $\not{E}_T$

$$\not{E}_T = \mathbf{p}_T^{\tilde{\chi}_a} + \mathbf{p}_T^{\tilde{\chi}_b}$$

If both LSP momenta are known

$$m_{\tilde{l}}^2 \geq \max\{m_T^2(\mathbf{p}_T^{l^-}, \mathbf{p}_T^{\tilde{\chi}_a}), m_T^2(\mathbf{p}_T^{l^+}, \mathbf{p}_T^{\tilde{\chi}_b})\}$$



Since splitting not known, the best one can do is:

$$m_{\tilde{l}}^2 \geq M_{T2}^2 = \min_{\not{\mathbf{p}}^a + \not{\mathbf{p}}^b = \not{\mathbf{p}}_T} \{\max\{m_T^2(\mathbf{p}_T^{l^-}, \not{\mathbf{p}}_T^a), m_T^2(\mathbf{p}_T^{l^+}, \not{\mathbf{p}}_T^b)\}\}$$

Useful, if enough events populate region near end-point

Lester & Summers 99

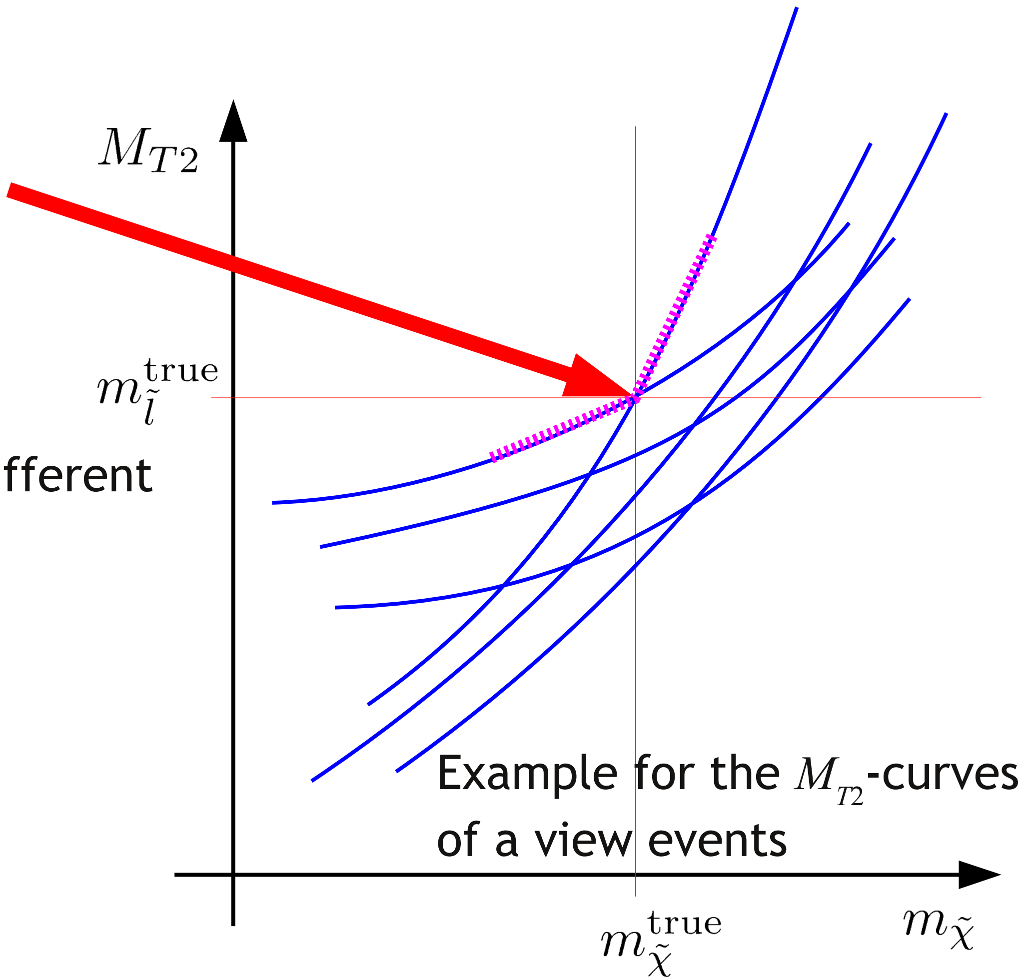
- LSP mass $m_{\tilde{\chi}}$ needs to be input for M_{T2}; **can it be determined from data ?**

$$m_{\tilde{l}}^2 \geq M_{T2}(m_{\tilde{\chi}})^2 = \min_{\not{\mathbf{p}}^a + \not{\mathbf{p}}^b = \not{\mathbf{p}}_T} \{\max\{m_T^2(\mathbf{p}_T^{l^-}, \not{\mathbf{p}}_T^a; m_{\tilde{\chi}}), m_T^2(\mathbf{p}_T^{l^+}, \not{\mathbf{p}}_T^b; m_{\tilde{\chi}})\}\}$$

- Per definition M_{T2} is montonically increasing as function of $m_{\tilde{\chi}}$
- If event set has enough variety:

  → kink of $M_{T2}^{max}(m_{\tilde{\chi}})$ at true masses

- **Simple explanation of kink:**
  - M_{T2}-bound $M_{T2}(m_{\tilde{\chi}}^{true}) \leq m_{\tilde{l}}$
  - But M_{T2}-curve have different slopes for different kinematic configurations



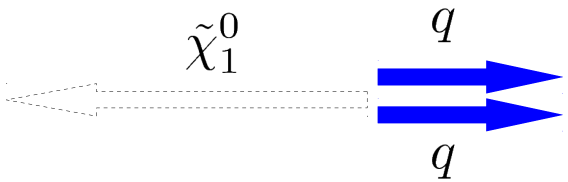
Cho, Choi, Kim & Park 07,08,09

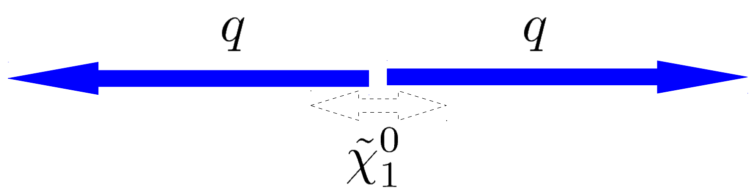
Two origins kinks:

Details in: Cho, Choi, Kim & Park arXiv:0709.0288 & arXiv.0909.4853

(A) N(>2)-body decays, e.g. $\tilde{g} \rightarrow qq\tilde{\chi}_1^0$

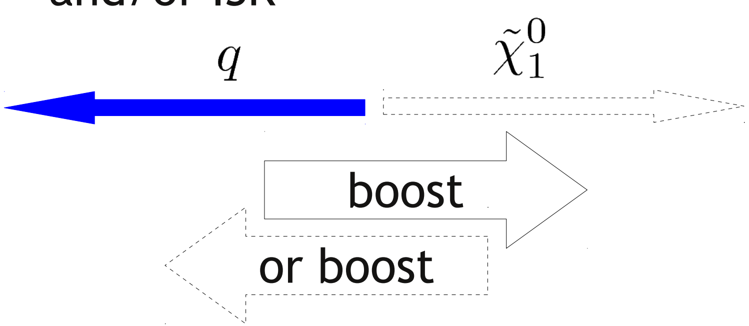
- Extreme momentum configurations → different slopes of $M_{T2}(m_{\tilde{\chi}}^{\text{true}})$
- No such kink for $\tilde{l} \rightarrow l\tilde{\chi}_1^0$

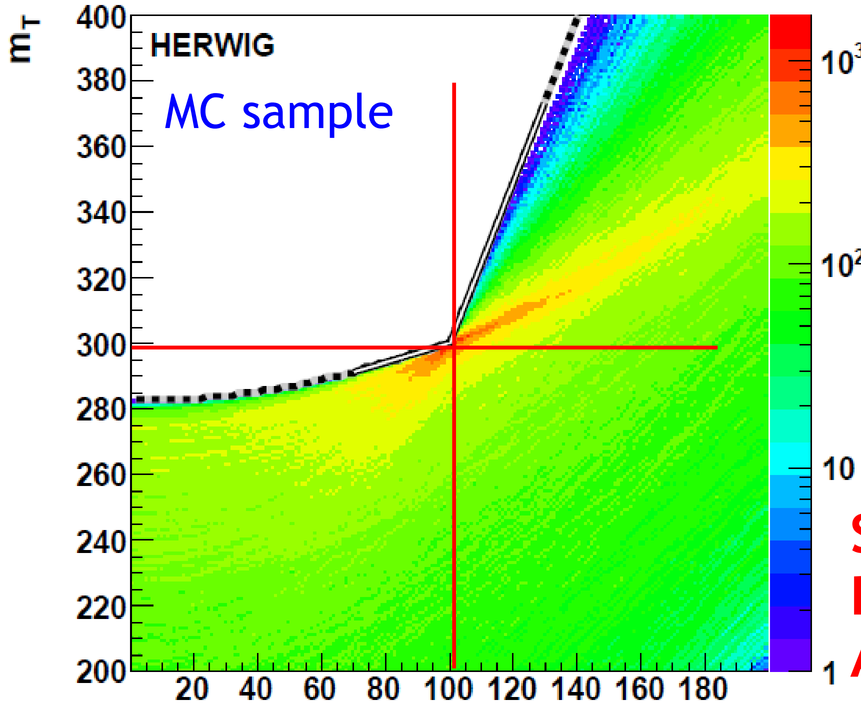




(B) Boost dependence, e.g. $\tilde{l} \rightarrow l\tilde{\chi}_1^0$

- boosted parents
- and/or ISR





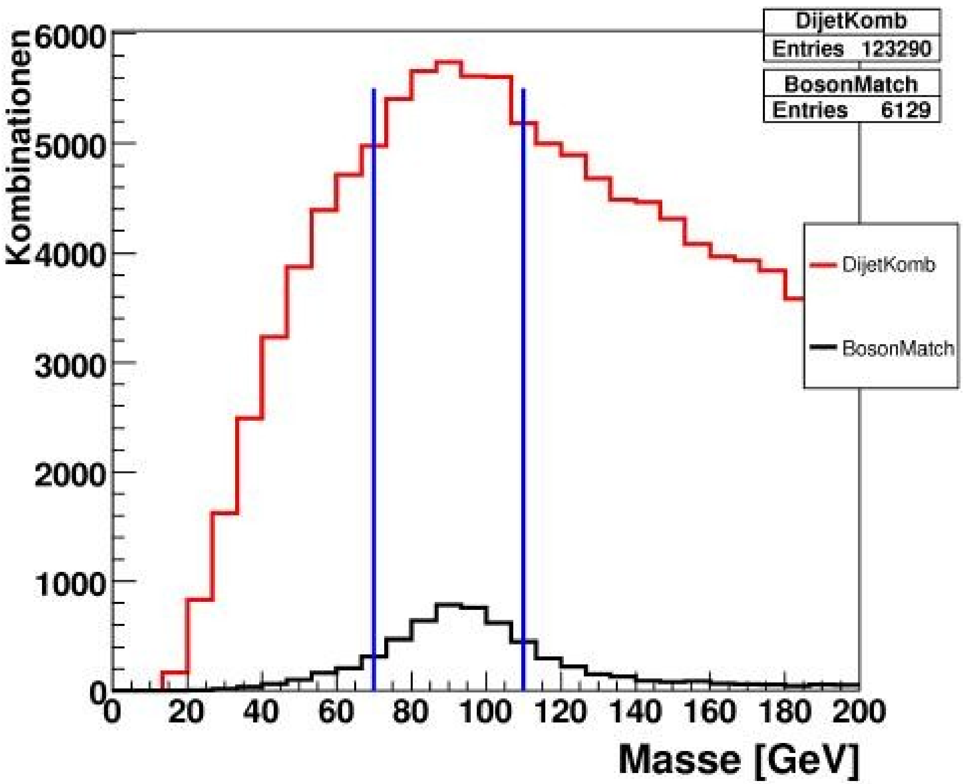
Barr, Gripaios & Lester 07

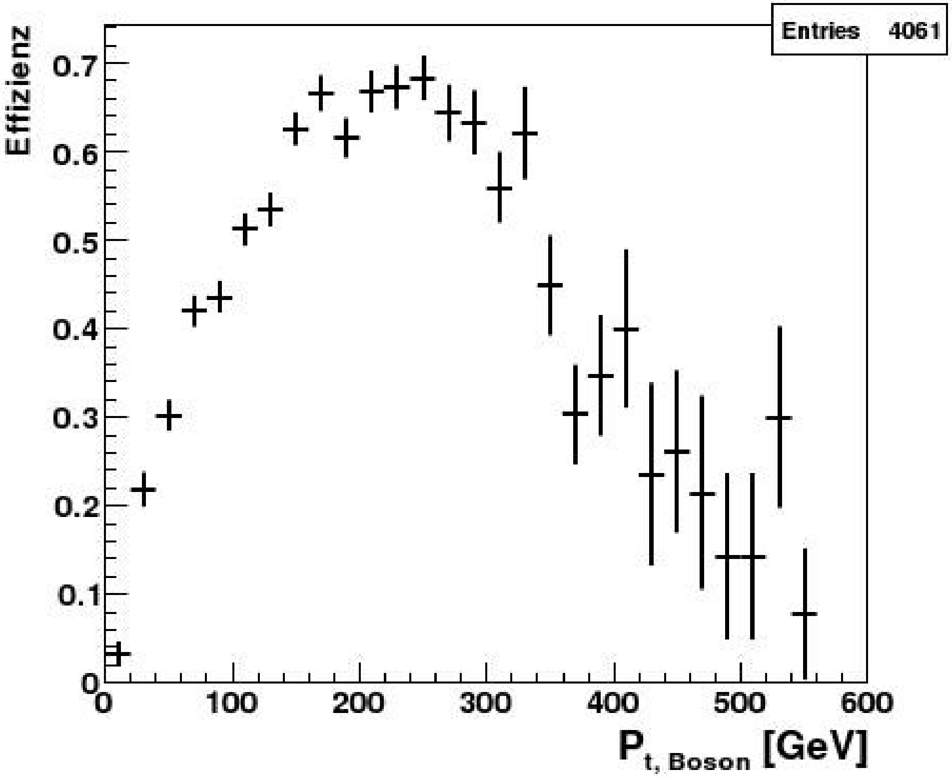
Statistics? Resolution? Acceptance?

- Jet algorithm: iterative cone 0.5
- Jet cuts $p_T > 20$ GeV and $|\eta| < 2.5$
- Candidates: dijets with

$$70 \text{ GeV} < M_{\text{inv}} < 110 \text{ GeV}$$

- Large combinatorial background



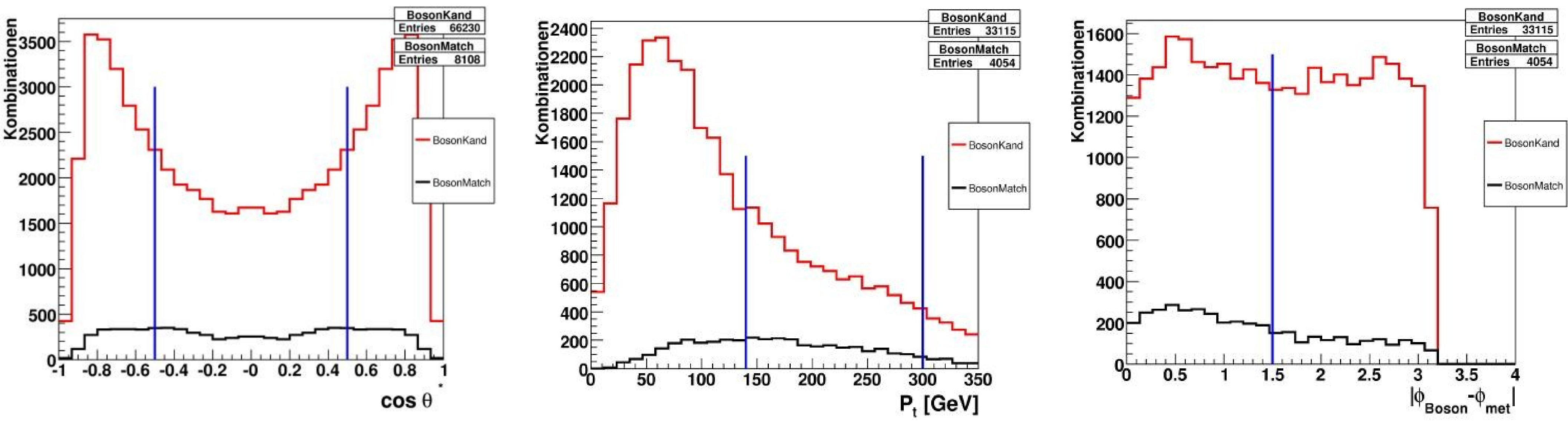


Reconstruction efficiency:

- Low efficiency at small boson p_T due to small jet reconstruction efficiency
- Low efficiency at large boson p_T due to jet merging

Friederike Nowak

Supression of Combinatorial Bg



Discriminating variables:

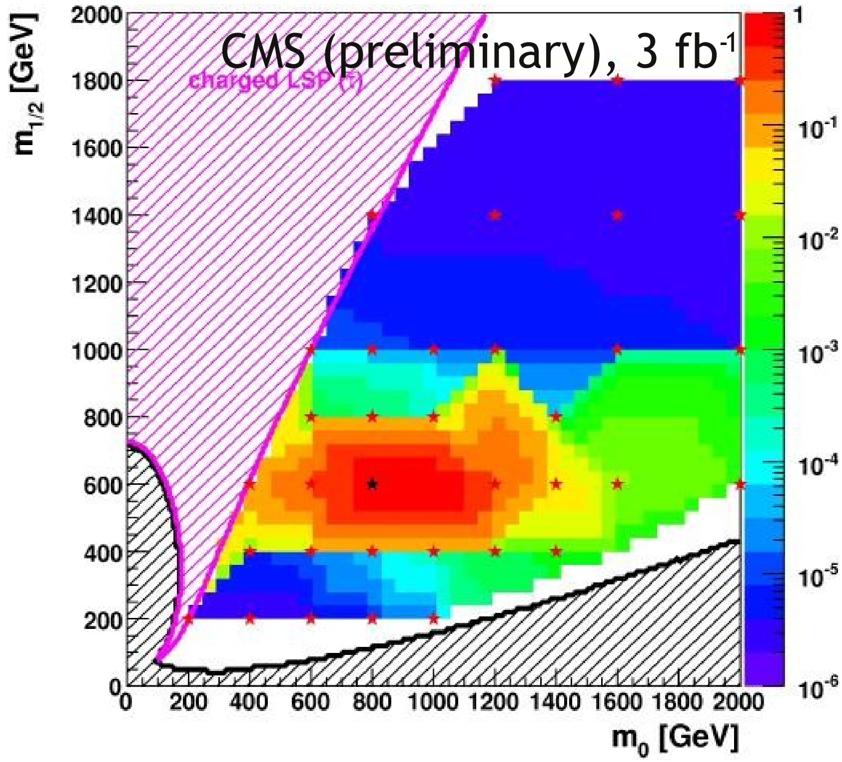
- θ^*: angle (in the W rest frame) between a W jet and the flight direction
- p_T of W candidate
- Angle between $\not\!\!E_T$ and W candidate

→ Reduction of combinatorial background by factor up to ~3

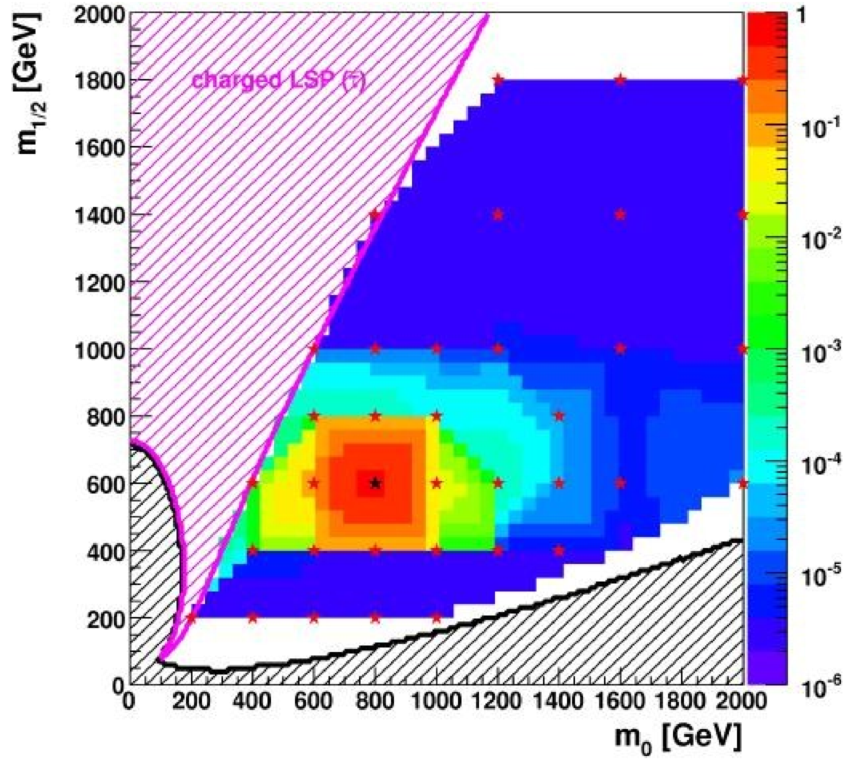
- If W candidate can be combined with third jet to m_{top} → top candidate

Constrain Parameter Space

Signal event rate only



+ information about W/Z and top candidates



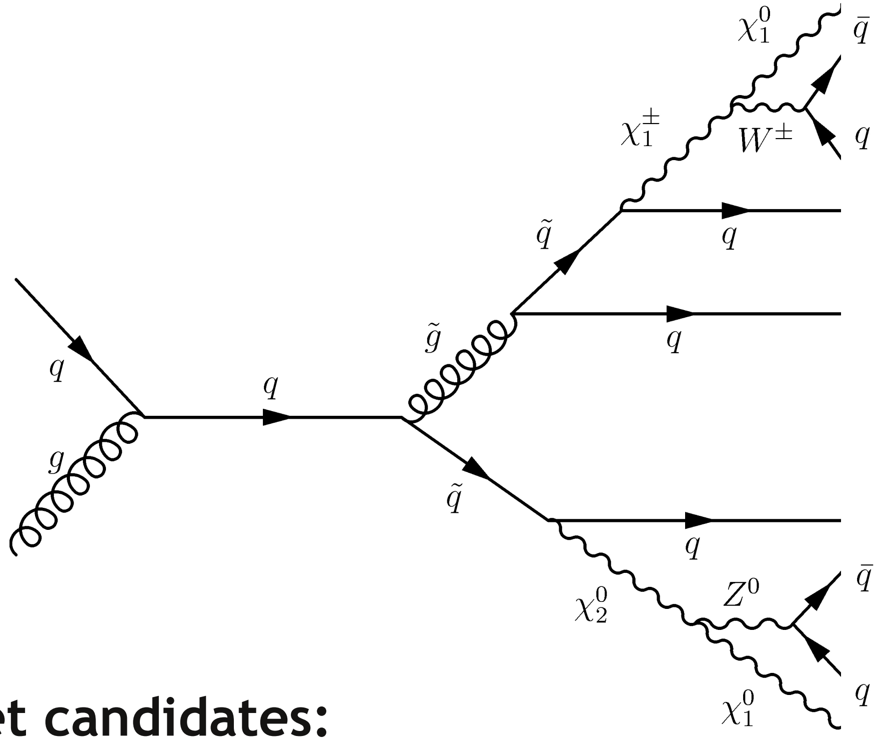
- Scan hypothesis and compare (χ^2 test) with pseudo data (here: $m_0 = 800$ GeV and $m_{1/2} = 600$ GeV)
- Boson candidate rate contains information in addition to absolute event rate → larger parts of the parameter space can be excluded

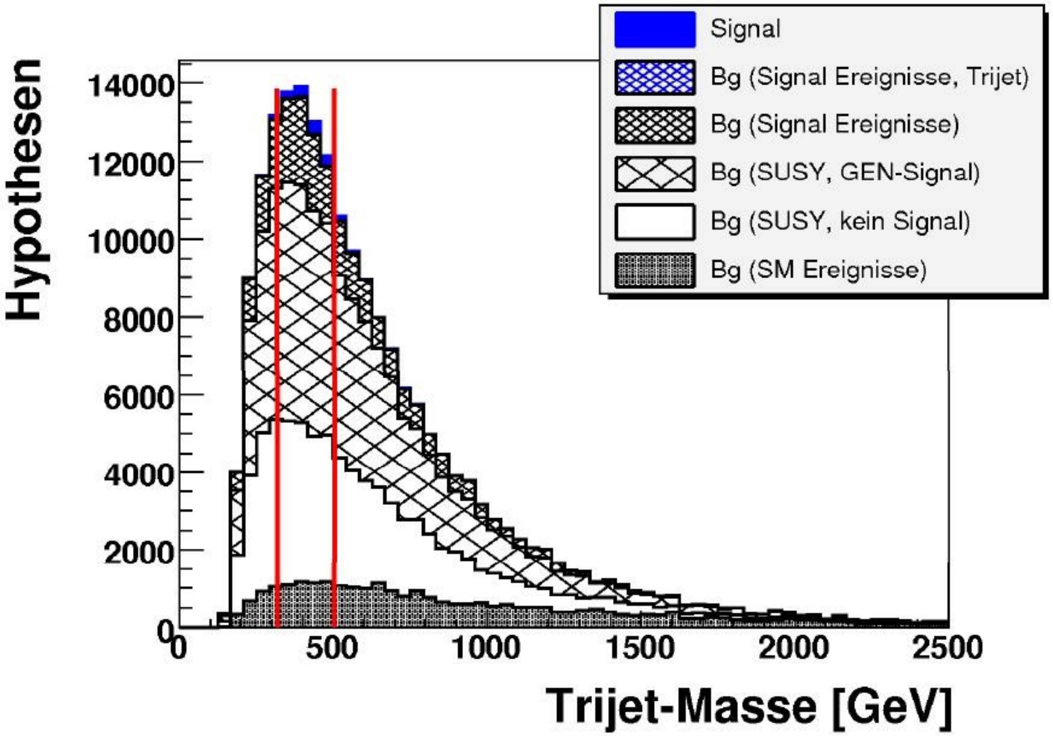
Reconstruction of Mass Edges

Search for other discriminating variables:

Ulla Gebbert

- Hadronic decay of squarks: invariant trijet mass distribution
- No sharp peak but upper and lower mass edge (due to unmeasured LSPs)
- Non-degenerated squark mass spectra: define only 1. and 2. generation as signal

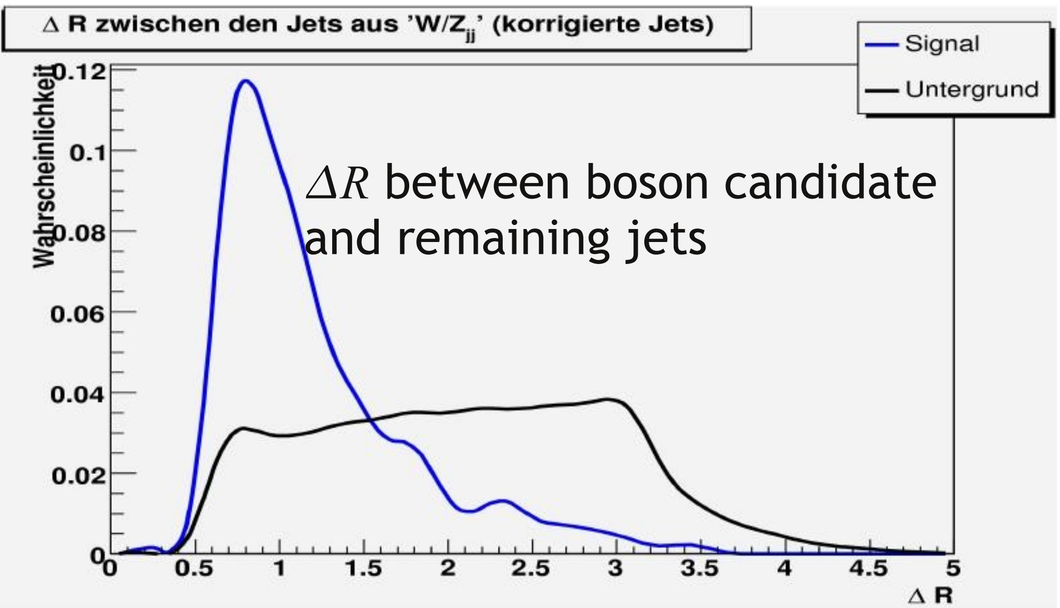


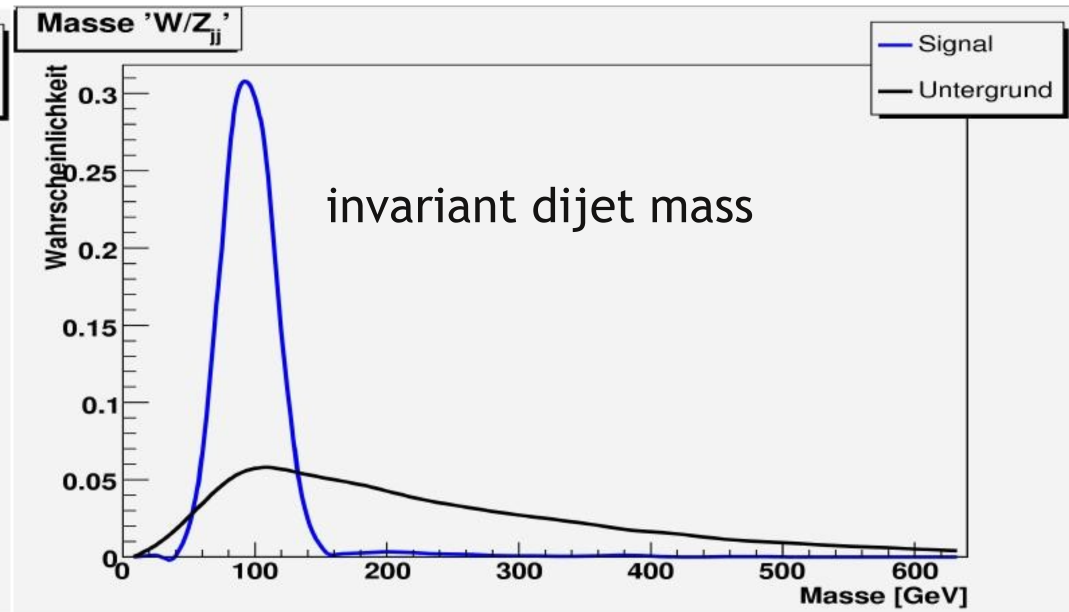


Trijet candidates:

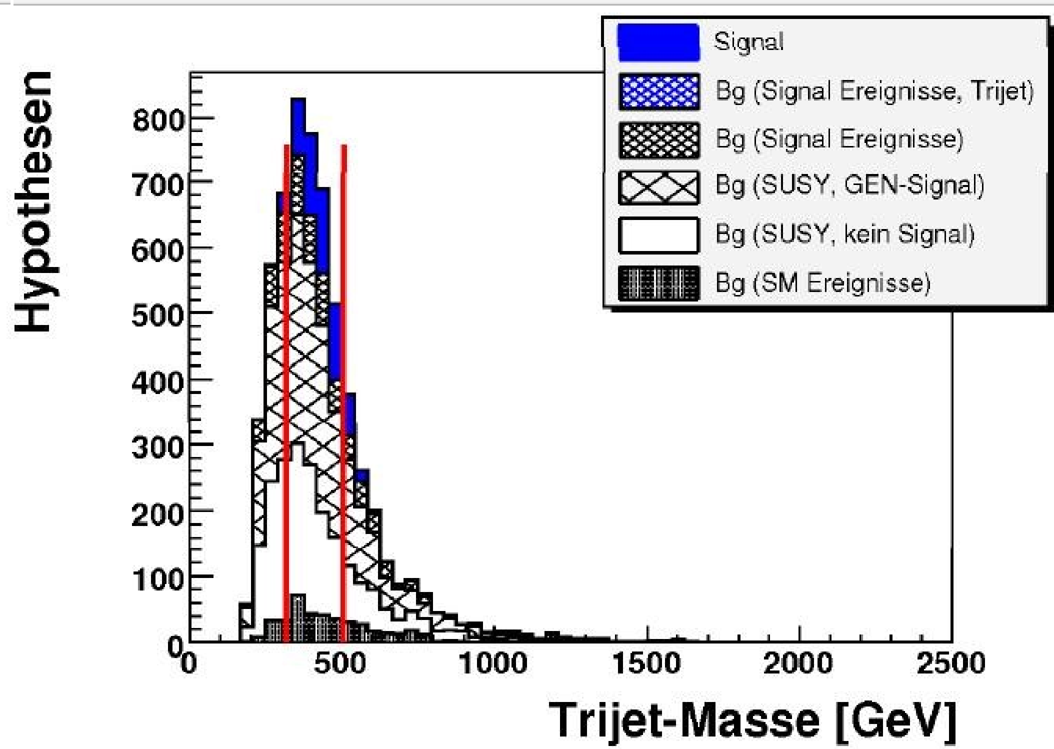
- W/Z candidate combined with one of two p_T hardest jets (large mass gap between $\tilde{q}$ and $\tilde{\chi}^\pm$)
- Up to 20 combinations per event
- Start with small S/B of ~1/100

Reconstruction of Mass Edges

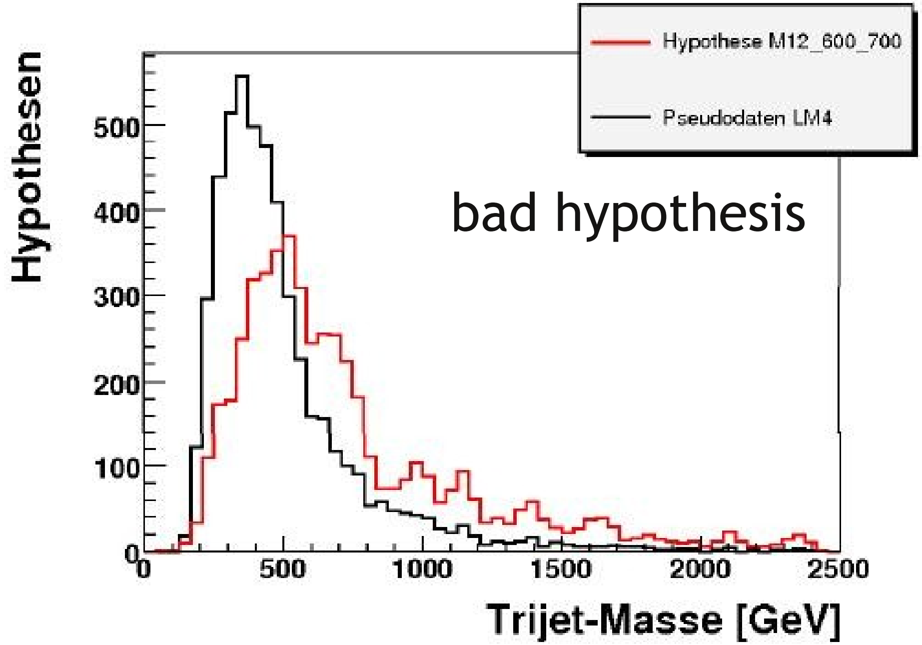


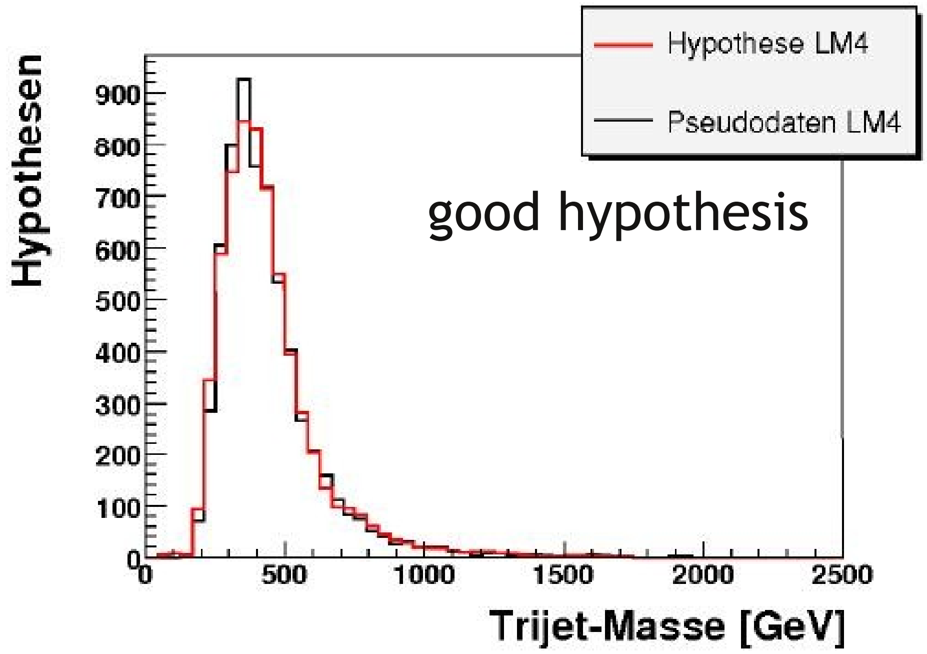


- Select out of 17 kinematic variables up to 5 best separating and least correlated variables
- Use likelihood ratio method to separate signal from background
- Improve S/B from ~1/100 to ~1/10
- Background might be "signal like"

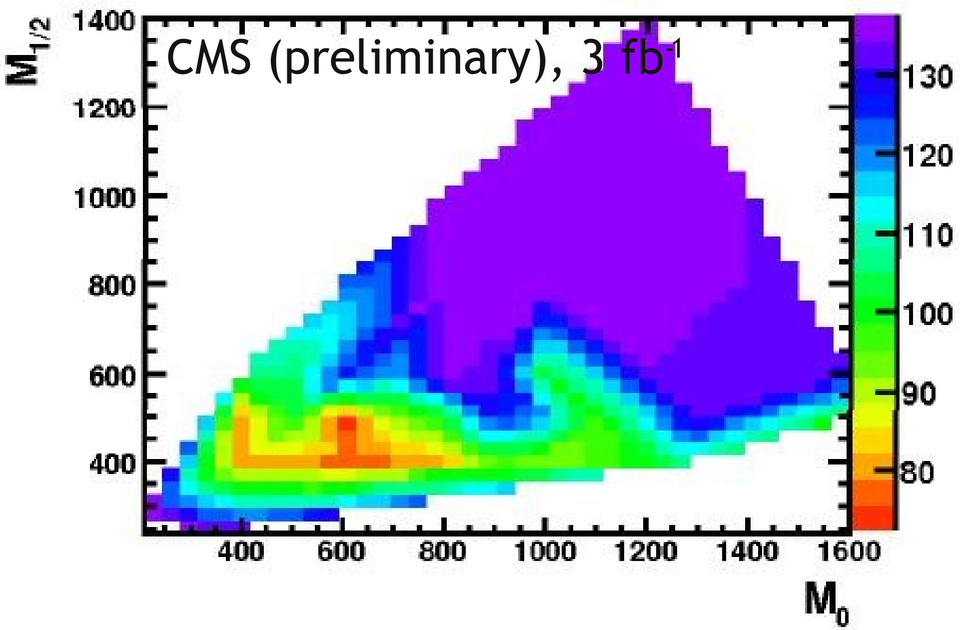


Constrain Parameter Space





- Scan over hypotheses
- Compare with pseudo data (here: $m_0 = 600$ GeV and $m_{1/2} = 400$ GeV) via binned maximum Likelihood (hypotheses normalized to data)
- Shape of trijet mass distribution provides enough information to constrain the parameter space



Part II

Susy Mass/Parameter Determination

Observables:
Electroweak Precision Data
LHC (+ Tevatron/LEP/ILC)
Cosmology
Rare Decays ...

"easy" ←

→ challenging

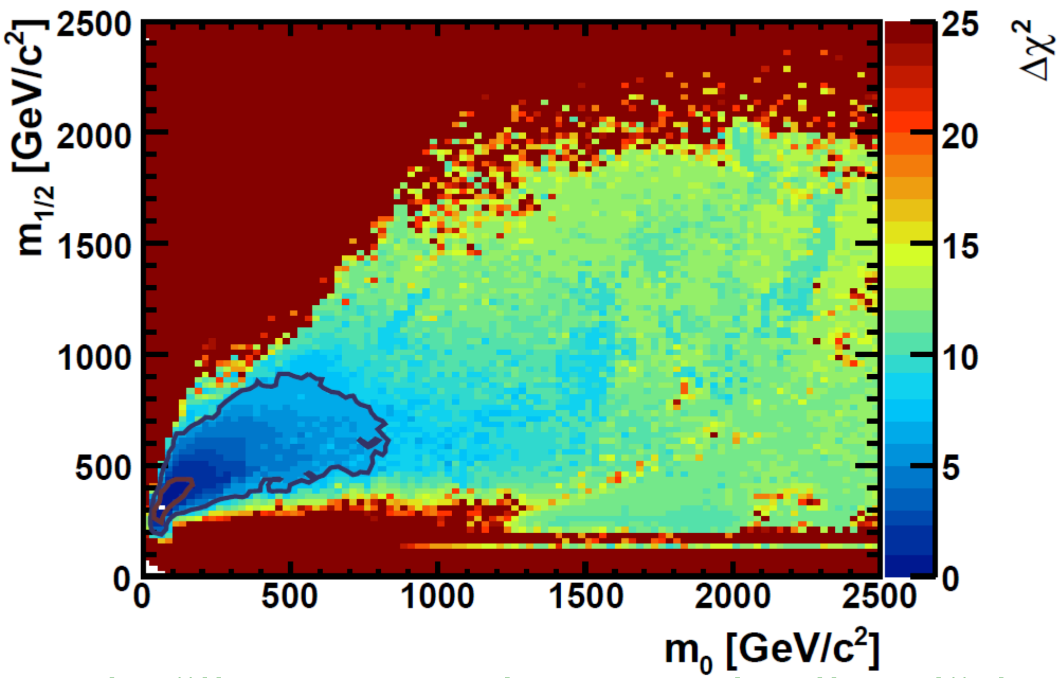
Model Parameters:
SM Parameters
mSUGRA / GMSB / MSSM
...

In particular, if realized model is not known

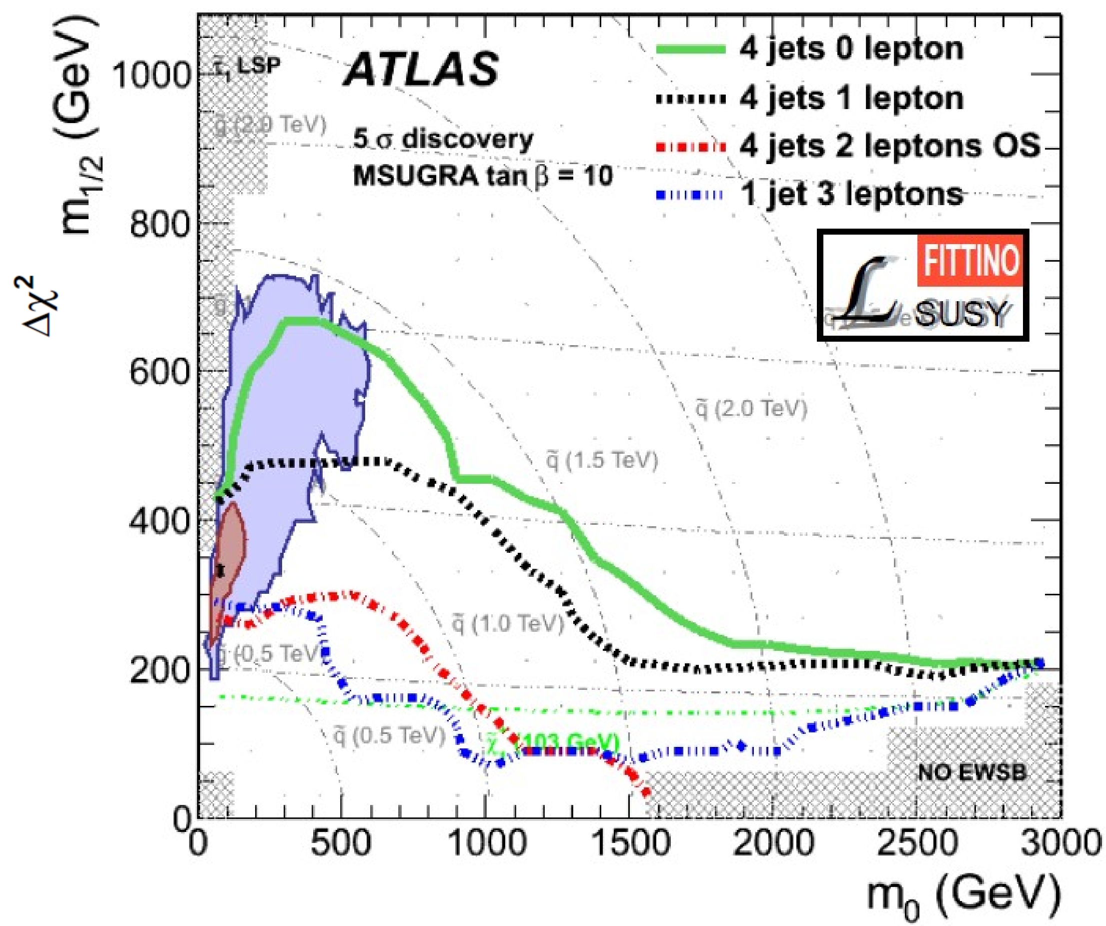
- Number of independent observables limited → not possible to fit most general models, e.g. MSSM (>100 parameter)
- "Coverage" of multidimensional parameter space → high computational cost
- Discrimination between models
- Systematic uncertainties (common or individual)
- In case of end-points: chain ambiguities
- ...

Global Fits - Present Picture

- **Very active research field:** Fittino, SFitter, GFitter, MasterCode, Baltz *et al.*, Allanach *et, al.*, Baer *et al.*, de Boer *et al.* … and many more
- **As examples:** Mastercode and Fittino (pre-LHC observables only)
  - Results compatible
  - **Present best fit within reach of LHC**
  - **But:** Upper limits driven by Δa_μ



Buchmüller, Cavanaugh, De Roeck, Ellis, Flächer, Heinemeyer, Isidori, Olive, Ronga & Weiglein 09



Bechtle, Desch, Uhlenbrock & Wienemann 09

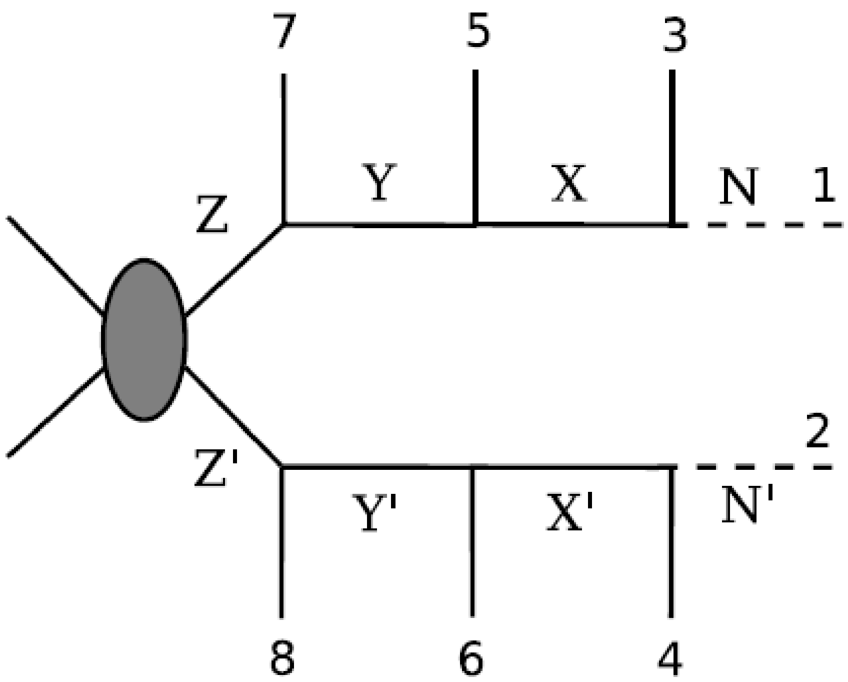
Event Reconstruction

Reconstruction of full kinematics of SUSY events ➔ access to masses

- **For one event:** In general **more unknowns** (LSP momenta, SUSY masses) **than constraints** (p_T balance, invariant masses)
- **For a set of events:** **some unknowns** (SUSY masses) **are common** ➔ problem can be over-constrained
- **Possible approaches:**
  - **Hybrid method:** combine kinematic end-point measurements and event-wise reconstruction quality
  - **Multi-event method:** Choose number of events such, that number of unknowns equals number of constraints ➔ look at parameter space covered by "real solutions"
  - **Single-event method:** Scan over common unknowns and reconstruct each event; use some measure (like unused constraints) for goodness of fit
  - **… + kinematic fit (our approach):** Constrained least square fit of many events taking into account uncertainties of measurements

Event topology:

e.g. $\tilde{q} \to q\tilde{\chi}_2^0 \to ql\tilde{l} \to ql\bar{l}\tilde{\chi}_1^0$



- Two identical cascade decays:
  - Z = Z', Y = Y', X = X': Intermediate heavy particles
  - 1 = N = N' = 2: Stable WIMP
  - 7, 5, 3, 8, 6, 4: SM particles/final states (jets/leptons)
- **General idea:**
  - Number of measured uncorrelated kinematic endpoints c_{end} and number of intermediate masses n_{mass} → degrees of freedom for experiment-wise end-point fit $d_{end} = n_{mass} - c_{end}$
    →**Use event wise information to improve mass resolution**
  - **Requirement:** number of kinematic constraints c_{ext} larger than number of unknown momentum components per event n_{mom}

Nojiri, Polesello & Tovey 08

Hybrid Method

- **8 mass shell conditions:**

$$(p_1+p_3+p_5+p_7)^2 = M_Z^2 = (p_2+p_4+p_6+p_8)^2$$
$$(p_1+p_3+p_5)^2 = M_Y^2 = (p_2+p_4+p_6)^2$$
$$(p_1+p_3)^2 = M_X^2 = (p_2+p_4)^2$$
$$(p_1)^2 = M_N^2 = (p_2)^2$$

- 2 transverse momentum constraints:

$$p_1^x + p_2^x = E_{\text{miss}}^x$$
$$p_1^y + p_2^y = E_{\text{miss}}^y$$

- **Overall:** 10 constraints and 12 unknowns (2×4 WIMP momenta + 4 masses)
- For each M, unknown p_1 and p_2 are determined by solving **system of equation from mass shell conditions**, giving 2 solutions for each leg (4 combinations)
- Each event is fitted minimizing $\chi^2(\mathrm{M})$ with free parameters $\mathrm{M} = (M_Z, M_Y, M_X, M_N)$

Endpoints(M) — Measured endpoints — Reconstructed momenta(M)

$$\chi^2(M) = \sum_{i=1}^{c_{\text{end}}} \left(\frac{m_{\text{evt}}^{\text{end},i} - m_{\text{exp}}^{\text{end},i}}{\sigma_{m_{\text{exp}}^{\text{end},i}}} \right)^2 + \left(\frac{p_1^x + p_2^x - E_{\text{miss}}^x}{\sigma_{E_{\text{miss}}^x}} \right)^2 + \left(\frac{p_1^y + p_2^y - E_{\text{miss}}^y}{\sigma_{E_{\text{miss}}^y}} \right)^2$$

Experimental resolutions

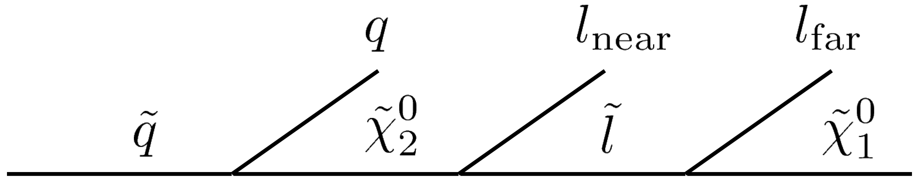
Hybrid Method: Example

- **Process:** $\tilde{q} \to q\tilde{\chi}_2^0 \to ql\tilde{l} \to ql\bar{l}\tilde{\chi}_1^0$ **Model:** SPS1a
- Five kinematic end-points (100 fb^{-1}):

Gjelsten, Hisano, Kawagoe, Lytken, Miller, Nojiri, Osland & Polesello (in LHC/IC study group) 04

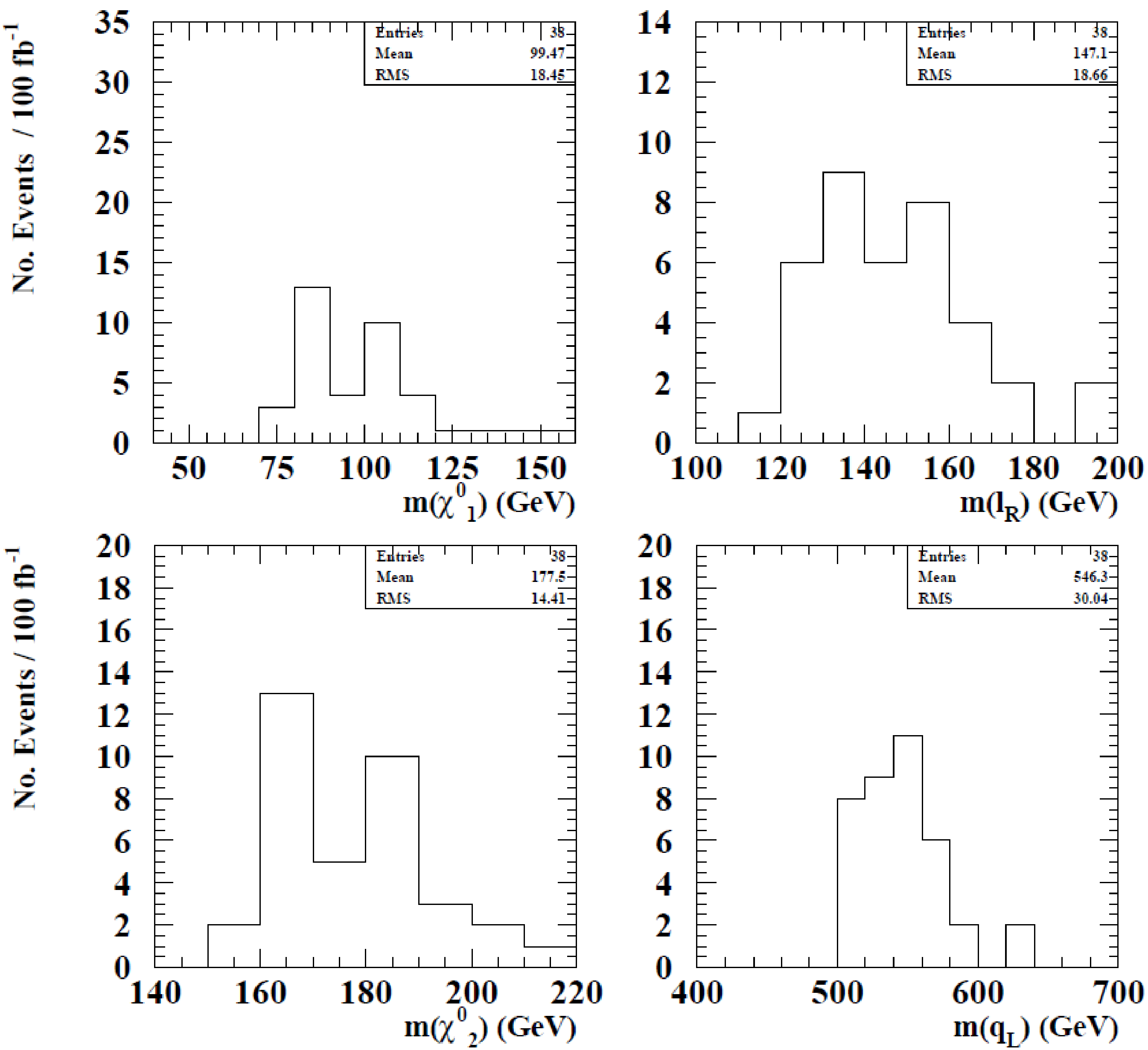
Edge	Nominal Value	Fit Value	Syst. Error Energy Scale	Statistical Error
$m(ll)^{\text{edge}}$	77.077	77.024	0.08	0.05
$m(qll)^{\text{edge}}$	431.1	431.3	4.3	2.4
$m(ql)^{\text{edge}}_{\text{min}}$	302.1	300.8	3.0	1.5
$m(ql)^{\text{edge}}_{\text{max}}$	380.3	379.4	3.8	1.8
$m(qll)^{\text{thres}}$	203.0	204.6	2.0	2.8
$m(bll)^{\text{thres}}$	183.1	181.1	1.8	6.3

- **Selection:**
  - “standard” cuts: 2 Jets, 4 leptons, M_{eff} and missing E_T
  - 2 opposite sign same flavor (OSSF) lepton pairs; if same flavor: only one of two possible pairings must give two $m(ll) < m(ll)^{\text{edge}}$ → **allocate leptons to each leg of event**
  - Only one possible pairings of two leading jets with two OSSF lepton pairs must give two $m(llq) < m(llq)^{\text{edge}}$ → **allocate jets to each leg of event**
  - For each leg: maximum(minimum) of two $m(lq) < m(lq)^{\text{edge}}_{\text{min(max)}}$ → **allocate leptons to the near and far position in each leg of event**



Hybrid Method: Results

- **Results:** each entry corresponds to $\chi^2(M)$ minimization



No inclusion of momentum balance in goodness-of-fit function (no additional information):

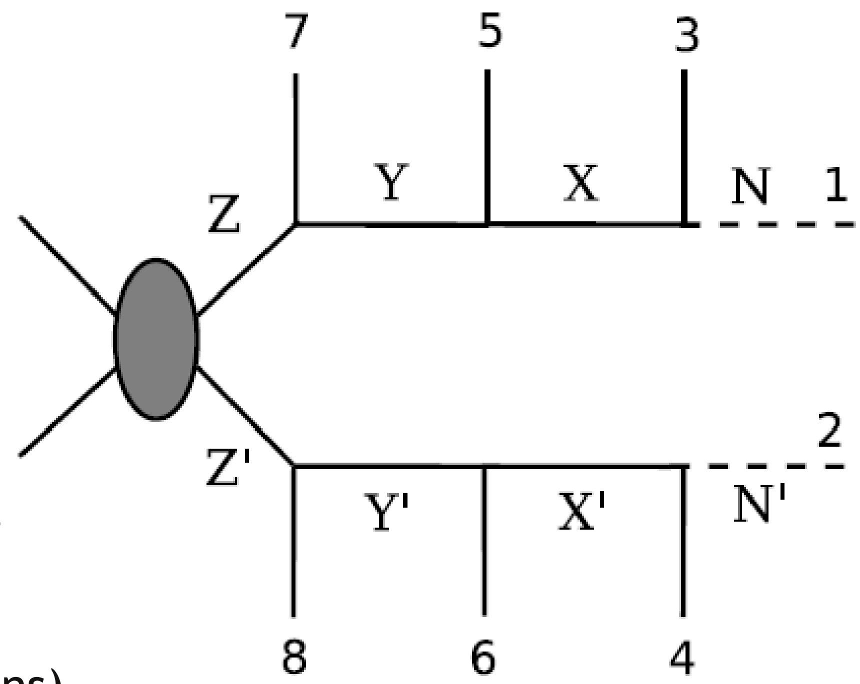
Narrow mass distributions, but results equivalent with end-point fit (not shown)

Inclusion of momentum balance (additional information):

Wider distributions (see plots) but mean values more accurate (~30%)

Event topology:

e.g. $\tilde{q} \rightarrow q\tilde{\chi}_2^0 \rightarrow ql\tilde{l} \rightarrow ql\bar{l}\tilde{\chi}_1^0$



- Two identical cascade decays:
  - $Z = Z'$, $Y = Y'$, $X = X'$: Intermediate heavy particles
  - $1 = N = N' = 2$: Stable WIMP
  - 7, 5, 3, 8, 6, 4: SM particles/final states (jets/leptons)
- **General idea:**
  - Formulate constraints as linear equations of unknown momenta
  - Choose number of events such, that number of unknowns equals equations
  - Solve system of equation → in general more than one complex solution
  - Reconstruct invariant masses from measured and reconstructed particles

Cheng, Engelhardt, Gunion, Han & McElrath 08
Cheng, Gunion, Han & McElrath 09

Multi-Event Method

- **One event:** 8 unknowns (2×4 WIMP momentum components) and 6 equations
  - 4 mass constraints:

$$\begin{aligned}
(M_Z^2 =)\quad (p_1+p_3+p_5+p_7)^2 &= (p_2+p_4+p_6+p_8)^2,\\
(M_Y^2 =)\quad (p_1+p_3+p_5)^2 &= (p_2+p_4+p_6)^2,\\
(M_X^2 =)\quad (p_1+p_3)^2 &= (p_2+p_4)^2,\\
(M_N^2 =)\quad p_1^2 &= p_2^2.
\end{aligned}$$

  - 2 transverse momentum constraints:

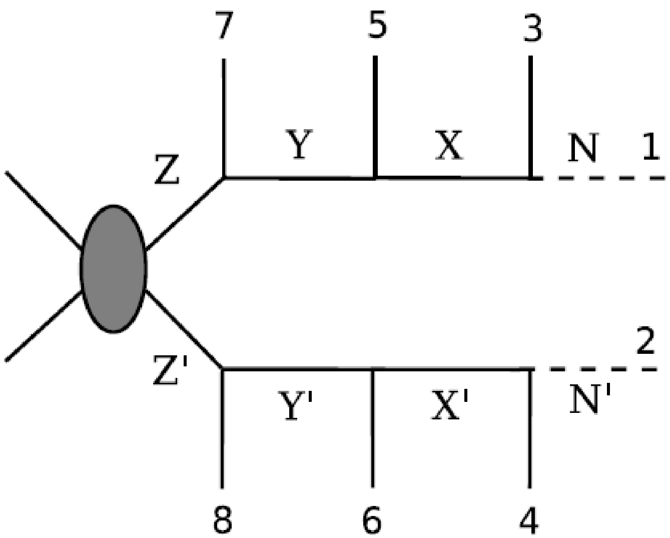
$$p_1^x + p_2^x = p_{miss}^x, \quad p_1^y + p_2^y = p_{miss}^y.$$

- **Two events:** +8 unknowns +10 equations = **16 equations for 16 unknowns**
  - 8 mass constraints:

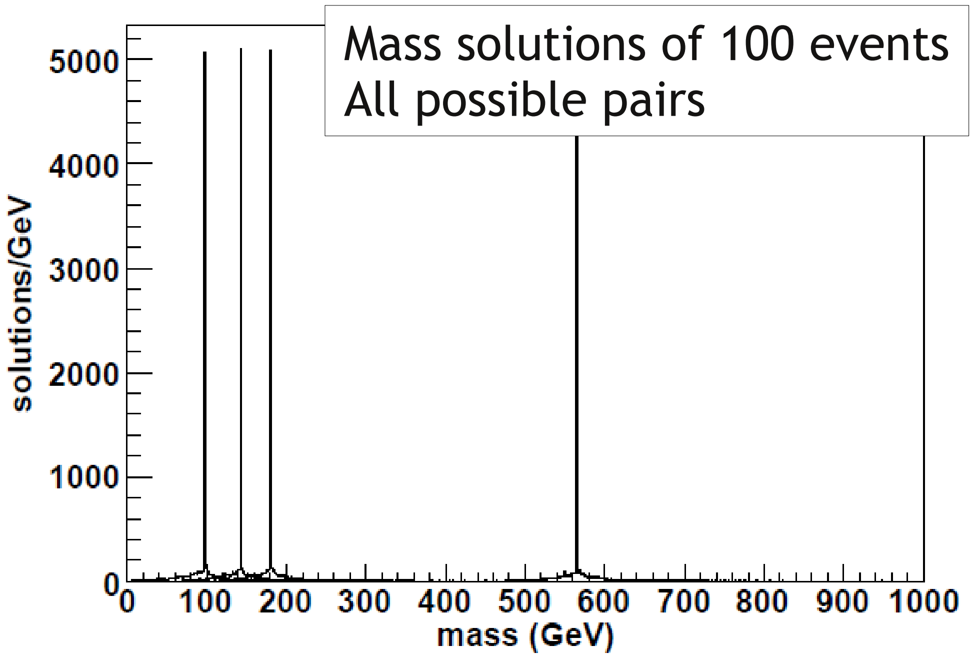
$$\begin{aligned}
q_1^2 &= q_2^2 = p_2^2,\\
(q_1+q_3)^2 &= (q_2+q_4)^2 = (p_2+p_4)^2,\\
(q_1+q_3+q_5)^2 &= (q_2+q_4+q_6)^2 = (p_2+p_4+p_6)^2,\\
(q_1+q_3+q_5+q_7)^2 &= (q_2+q_4+q_6+q_8)^2\\
&= (p_2+p_4+p_6+p_8)^2,
\end{aligned}$$

  - 2 transverse momentum constraints:

$$q_1^x + q_2^x = q_{miss}^x, \quad q_1^y + q_2^y = q_{miss}^y.$$



Multi-Event Method - Results



Solution of system of equations:

→ up to 8 complex solutions

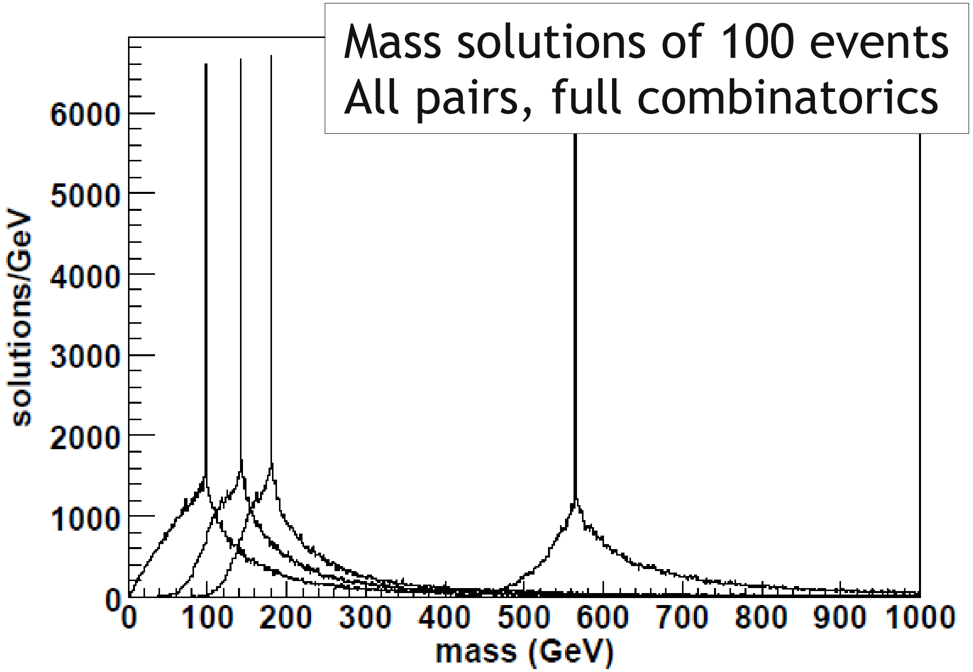
→ consider only real solutions

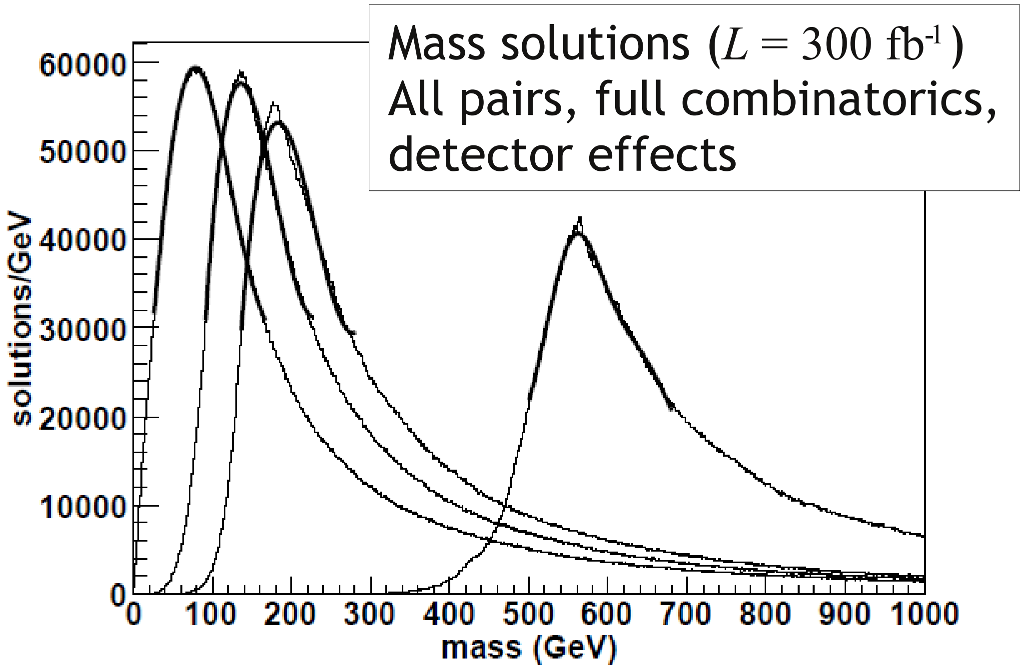
Results for process: $\tilde{q} \rightarrow q\tilde{\chi}_2^0 \rightarrow ql\tilde{l} \rightarrow ql\bar{l}\tilde{\chi}_1^0$

Model: SPS1a

High luminosity, signature with leptons

→ **Good prospects for mass determination**

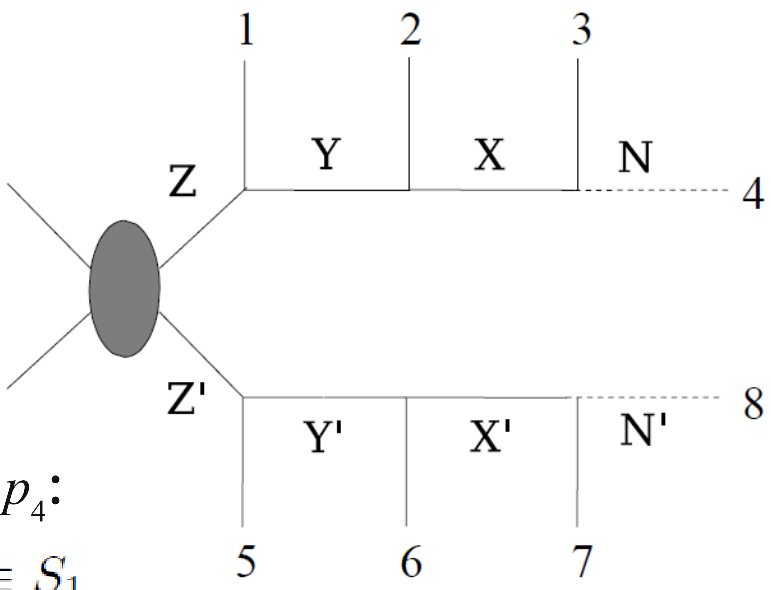




Single-Event Method

- Same event topology as before

$$(p_1+p_2+p_3+p_4)^2 = M_Z^2$$
$$(p_2+p_3+p_4)^2 = M_Y^2$$
$$(p_3+p_4)^2 = M_X^2$$
$$p_4^2 = M_N^2$$



- Leaving aside last equation 3 linear equations in p_4:

$$-2p_1\cdot p_4 = M_Y^2 - M_Z^2 + 2p_1\cdot p_2 + 2p_1\cdot p_3 + m_1^2 \equiv S_1$$
$$-2p_2\cdot p_4 = M_X^2 - M_Y^2 + 2p_2\cdot p_3 + m_2^2 \equiv S_2$$
$$-2p_3\cdot p_4 = M_N^2 - M_X^2 + m_3^2 \equiv S_3$$

- Same for second cascade:

$$-2p_5\cdot p_8 = M_{Y'}^2 - M_{Z'}^2 + 2p_5\cdot p_6 + 2p_5\cdot p_7 + m_5^2 \equiv S_5$$
$$-2p_6\cdot p_8 = M_{X'}^2 - M_{Y'}^2 + 2p_6\cdot p_7 + m_6^2 \equiv S_6$$
$$-2p_7\cdot p_8 = M_{N'}^2 - M_{X'}^2 + m_7^2 \equiv S_7$$

- Transverse momentum balance:

$$p_4^x + p_8^x = p_{\text{miss}}^x \equiv S_4$$
$$p_4^y + p_8^y = p_{\text{miss}}^y \equiv S_8$$

Webber 09

General idea:

- Scan over mass hypothesis (M_Z, M_Y, M_X, M_N); for each mass hypothesis solve system of 8 equations for 8 unkowns $\mathbf{P}$

$$\mathbf{P} = (p_4^x, p_4^y, p_4^z, E_4, p_8^x, p_8^y, p_8^z, E_8)$$

$$\mathbf{AP} = \mathbf{S}$$

$$\mathbf{P} = \mathbf{A}^{-1}\mathbf{S}$$

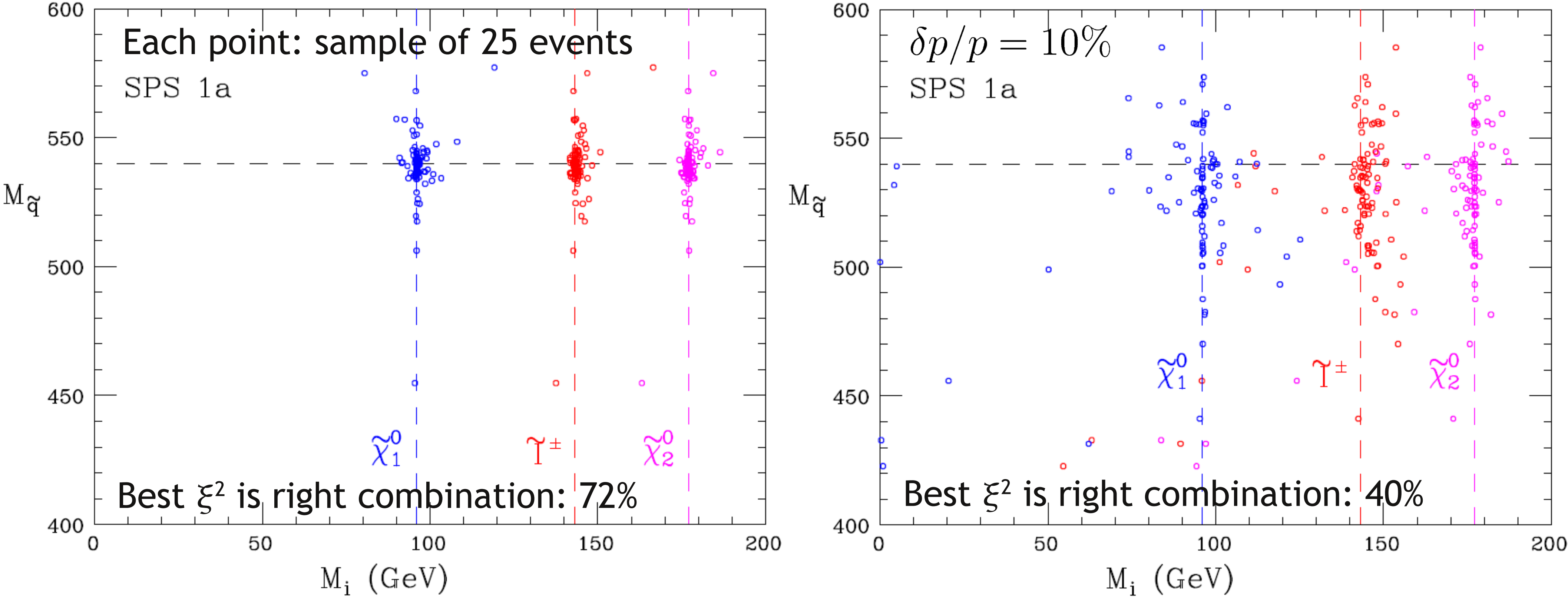
- Measure of goodness of fit: M_N constraint (remember: not used so far)

$$\xi^2(\mathbf{M}) = \sum_n \left[(p_4^2)_n - M_N^2\right]^2 + \sum_n \left[(p_8^2)_n - M_{N'}^2\right]^2$$

sum over all events

- **Find the mass hypothesis $\mathbf{M}$ with smallest ξ^2 for all combinations per event**

Single-Event Method - Results



- **Process:** $\tilde{q} \to q\tilde{\chi}_2^0 \to ql\tilde{l} \to ql\bar{l}\tilde{\chi}_1^0$ **Model:** SPS1a
- **Advantage:** each event contributes independently and additive to goodness-of-fit function
- **Challenge:** minimization non-trivial
- **Bias:** shift of determined masses (~5 GeV / ~25 GeV (squarks)) might be corrected for by MC

Kinematic Fit Method

Our approach: complete reconstruction of SUSY events using kinematic fits in combination with mass scan — Autermann, Mura, CS, Schettler & Schleper 09

Advantage: **Kinematic fit takes into account uncertainties of measurements**

Potential problems:
- Many jets → huge combinatorial bg (7 jets: 1260 combinations)
- Effect of SM and SUSY backgrounds
- Detector resolution and acceptance
- Initial and final state radiation
- No perfect mass degeneration
- Width of virtual particles

For N events:

$N\times$**21 local measurements** (7 jets)

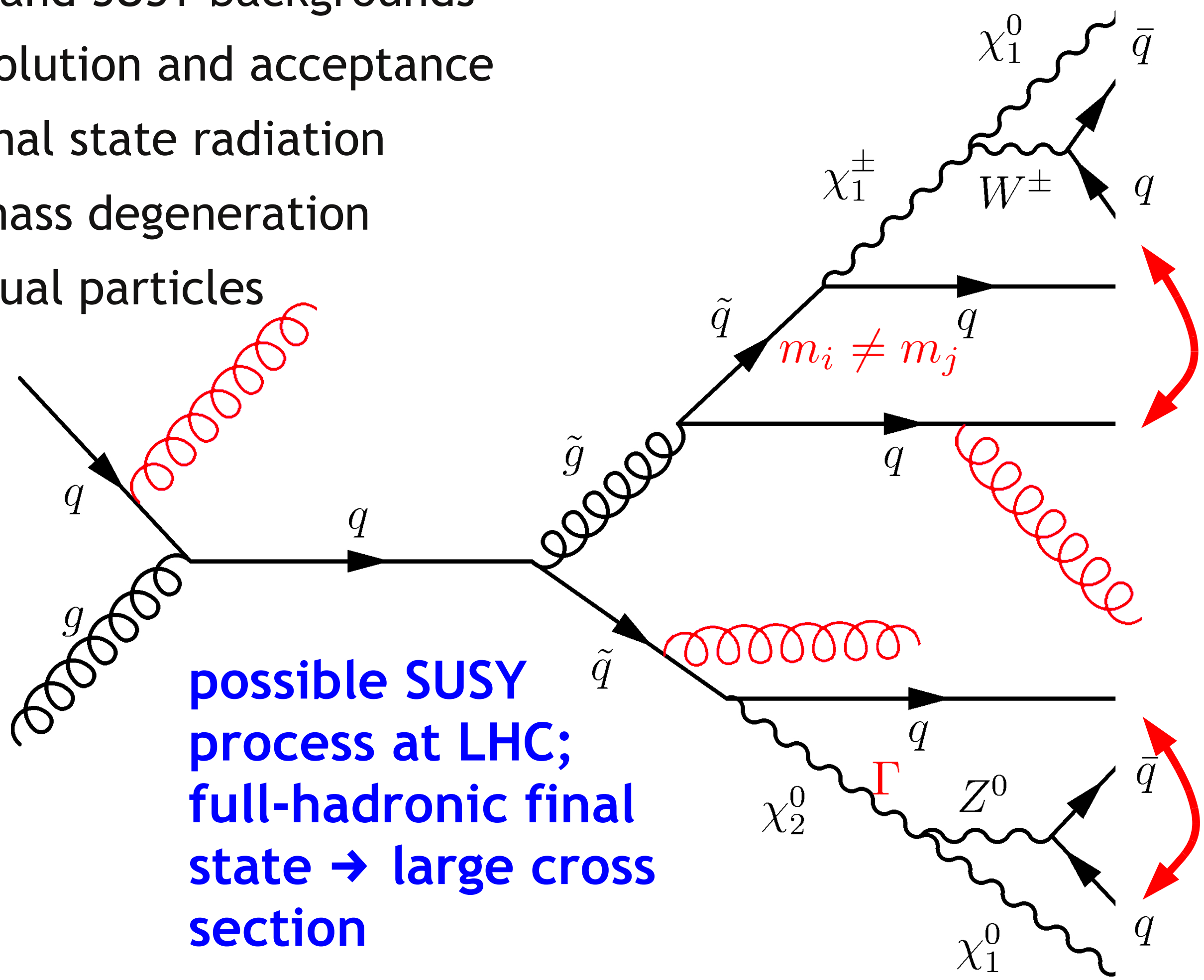
4 global unknowns (SUSY masses)

$N\times$**6 local unknowns** (2 LSPs)

$N\times$**7 local constraints:**

- p_x, p_y
- $1\times M_{gluino}, 2\times M_{squark}, 2\times M_{chargino/neutralino}$

Over constrained for $N > 4$



possible SUSY process at LHC; full-hadronic final state → large cross section

Non-linear Constrained Fits

Method for constrained fits: **Method of Lagrangian Multiplier**

But: invariant mass constraints in general not linear

→ Linearization via Taylor expansion

→ Iterative solution

Problems:

- Linearization of constraints only good approximation “near” solution → if “away” from solution iterative procedure might results in **too large** or **too small steps**, or even **wrong direction**
- Definition of convergence criterion

Used fitting code: `KinFitter`

- `C++` implementation ... (V. Klose and J. Sundermann)
- ... of **`ABCFIT`** from ALEPH collaboration (O. Buchmüller and J. B. Hansen)
- Additional modifications (step scaling and scaling of constraints)

Alt. Fitting Technique: Genetic Algorithm

- Formulation of constraints as additional χ^2 term → "cost function"
- Interpret cost function as χ^2 → **carefully chosen errors**

$$\chi^2_{M^2} = \left(\frac{M^2_{\text{inv}}(j_1, j_2, j_3) - M^2}{\sigma_{M^2}} \right)^2 \quad \text{and} \quad \chi^2_{p_{x/y}} = \left(\frac{\sum_{\text{all particles}} p_{x/y}}{\sigma_p} \right)^2$$

Minimize cost function: gradient, simplex, LBFGS, simulated annealing and **genetic algorithm (GA)**

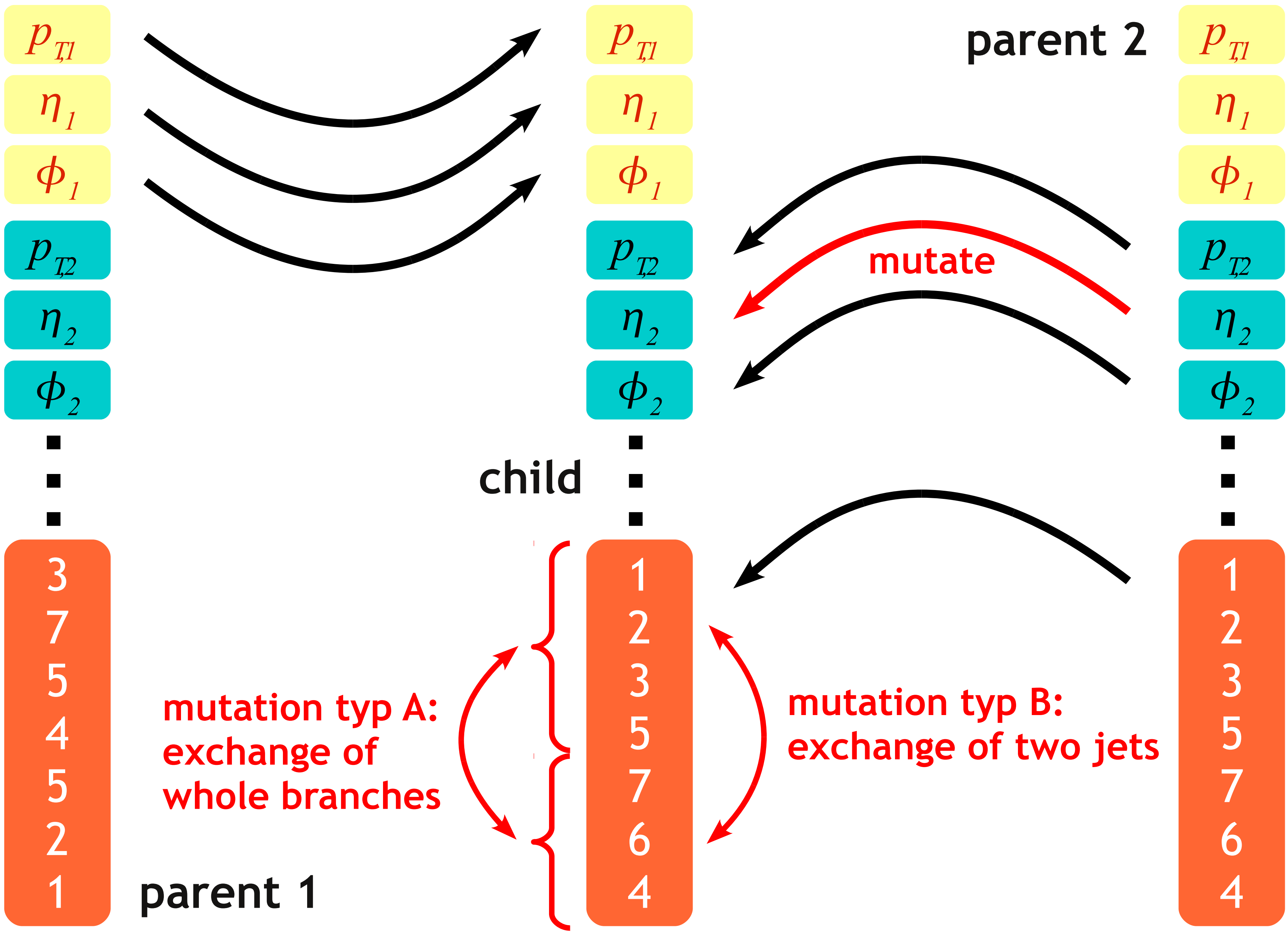
GA: Final state 4-momenta are genome of individual; jet combination is one additional gene. Fitness function (here χ^2) defines **which individual is fittest**

Algorithm:

1) Create first generation of individuals (starting population)
2) Select best fitting individuals
3) Create new individuals by selecting randomly two parents and inherit randomly genes from either one or other parent
4) For each child mutate each genome with small probability
5) Back to step 2) until convergence

Advantage: no linearization needed ⇔ **Disadvantage:** high computational cost

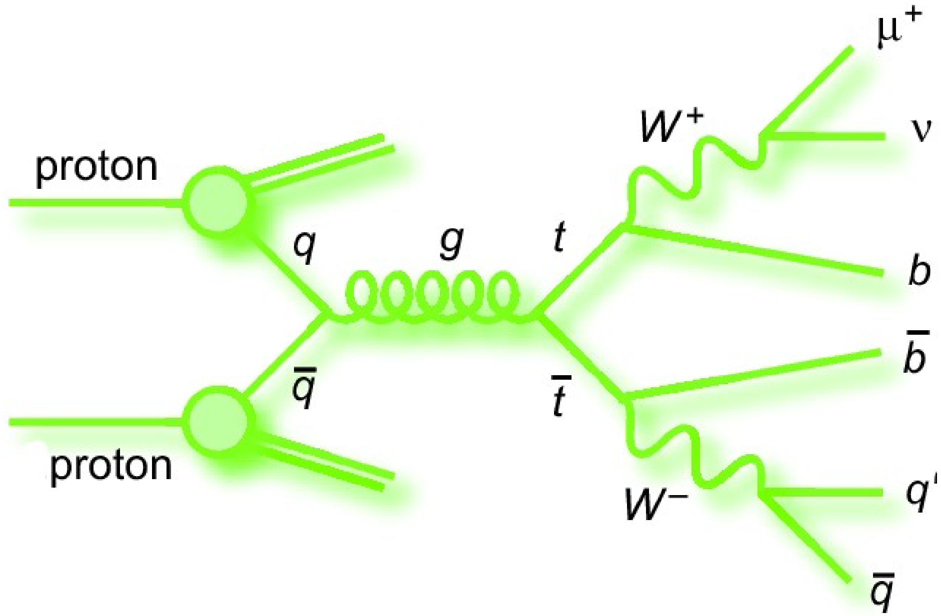
Genetic Algorithm - Schematic Picture



Proof of Principle: Semi-Leptonic $t\bar{t}$

Counting unknowns and constraints:

- 4 jets + 1 lepton = 15 measured parameters
- 1 neutrino = **3 unmeasured parameters**
- **6 constraints** (p_x, p_y, $2\times M_W$ and $2\times M_{top}$)



Combinatorics:

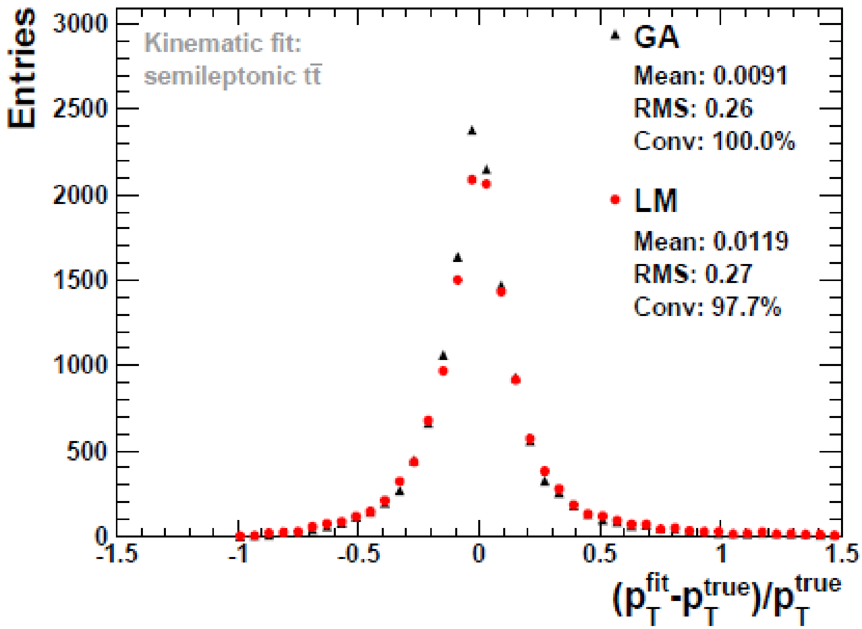
- No b-tagging used

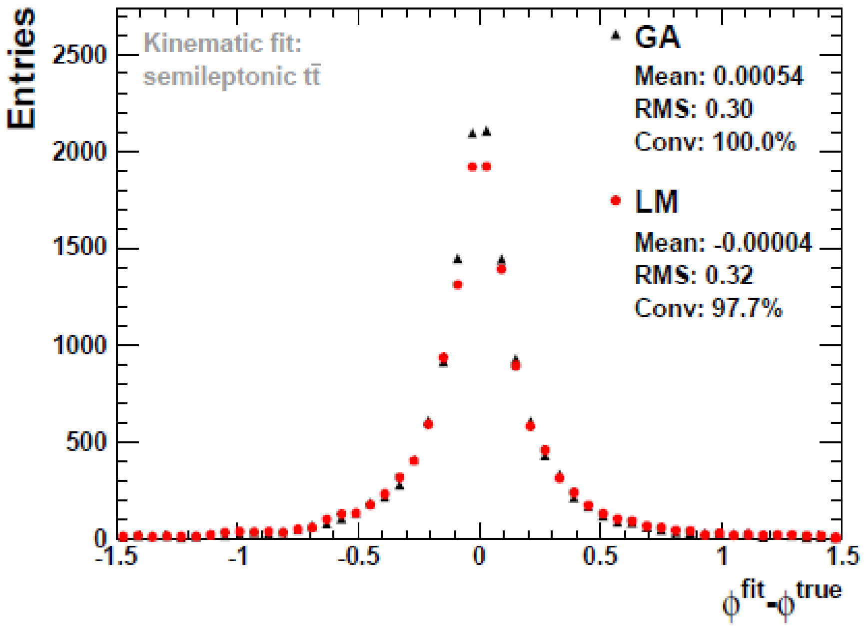
  → 12 possible jet configurations

Event generation and detector simulation:

- **`Pythia6`** generated events including ISR and FSR
- Each final state jet smeared according to typical jet momentum and angular resolutions at ATLAS/CMS
- Jet/lepton selection cuts: Four jets and one lepton with
  - $p_T > 20$ GeV
  - $|\eta| < 3.0$

Resolution of fitted neutrinos:





Scenario:
- No bg from other processes
- Full combinatorial bg
- ISR and FSR
- Detector resolution and acceptance

Genetic algorithm:

Right jet combinations has smallest χ^2 for **75.6%** events

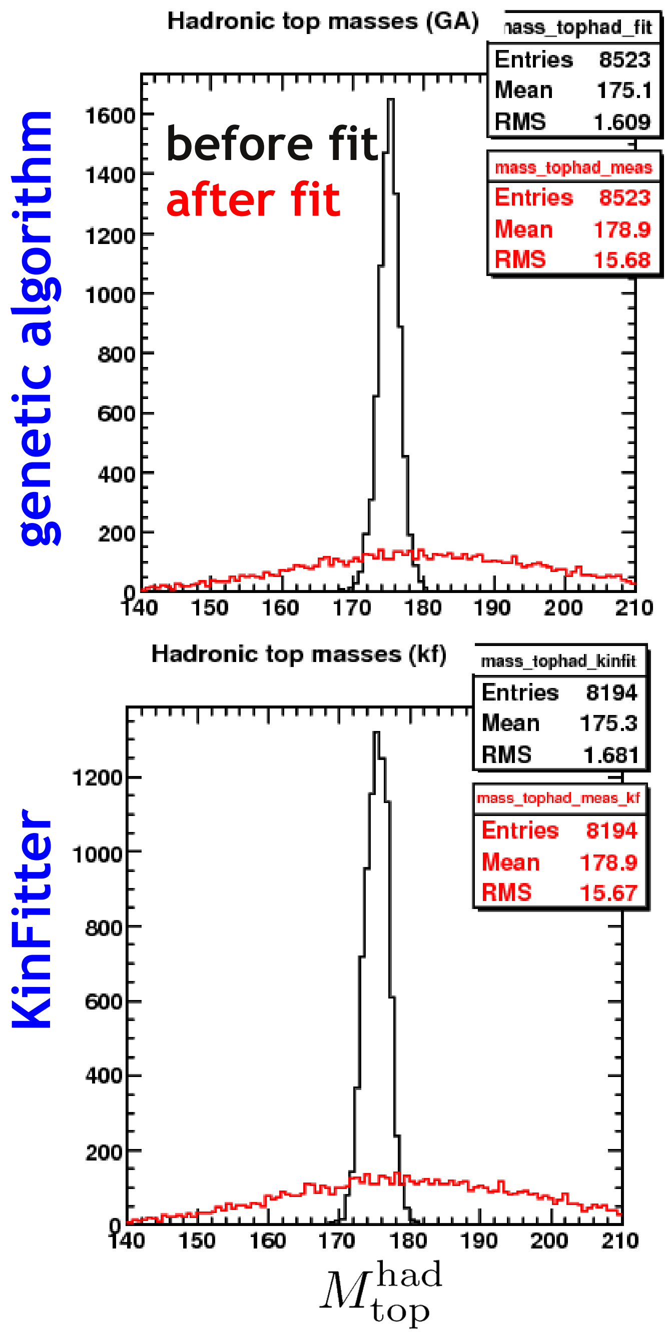
KinFitter:

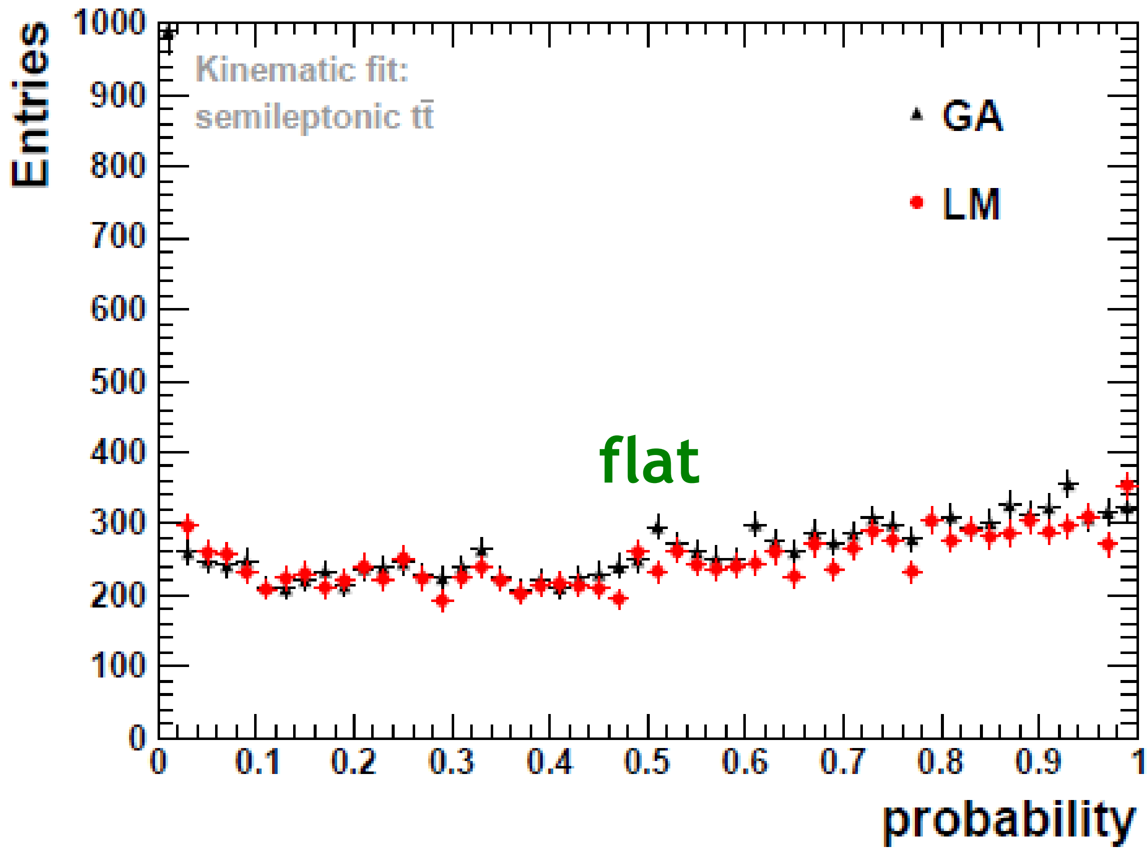
Converged for **98.0%** events

Right jet combinations has smallest χ^2 for **72.9%** events

→ Similar performance of both methods for neutrino resolution

Proof of Principle: Semi-Leptonic $t\bar{t}$

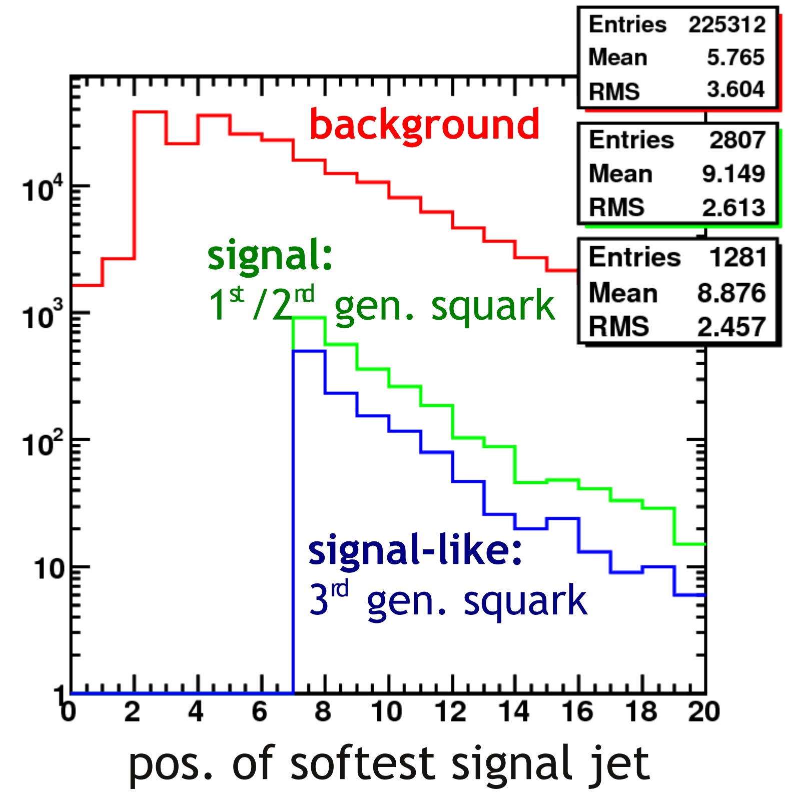




- **Comparable** and **reasonable** results for both algorithms
- Increase at lowest fit probabilities due to acceptance cuts and non-Gaussian (Breit-Wigner) tail of invariant mass distributions

SUSY Sample

- mSUGRA test point:
  - Parameters: $m_0 = 230\ \text{GeV}, m_{1/2} = 360\ \text{GeV}$
    $A_0 = 0, \tan\beta = 10, \text{sign}\ \mu = +$
  - Masses: $m_{\tilde{q}} \approx 810\ \text{GeV}, m_{\tilde{g}} \approx 860\ \text{GeV}$
    $m_{\chi_1^\pm} \approx 273\ \text{GeV}, m_{\chi_1^0} \approx 147\ \text{GeV}$
  - Cross section at LHC: $\sigma_{\text{tot}} = 7.8\ \text{pb(LO)}$
  - Branching ratios: $Br(\chi_2^0 \to h^0 \chi_1^0) \approx 85\%$
    $Br(\chi_1^\pm \to W^\pm \chi_1^0) \approx 97\%$



- **Pythia6** generated events including ISR and FSR
- Each final state jet smeared according to typical jet momentum and angular resolutions at ATLAS/CMS
- Jet selection cuts: 7 jets with
  - $p_T > 30\ \text{GeV}$
  - $|\eta| < 3.0$

→ **Dominant background of other SUSY processes (S/B ~ 1/40)**

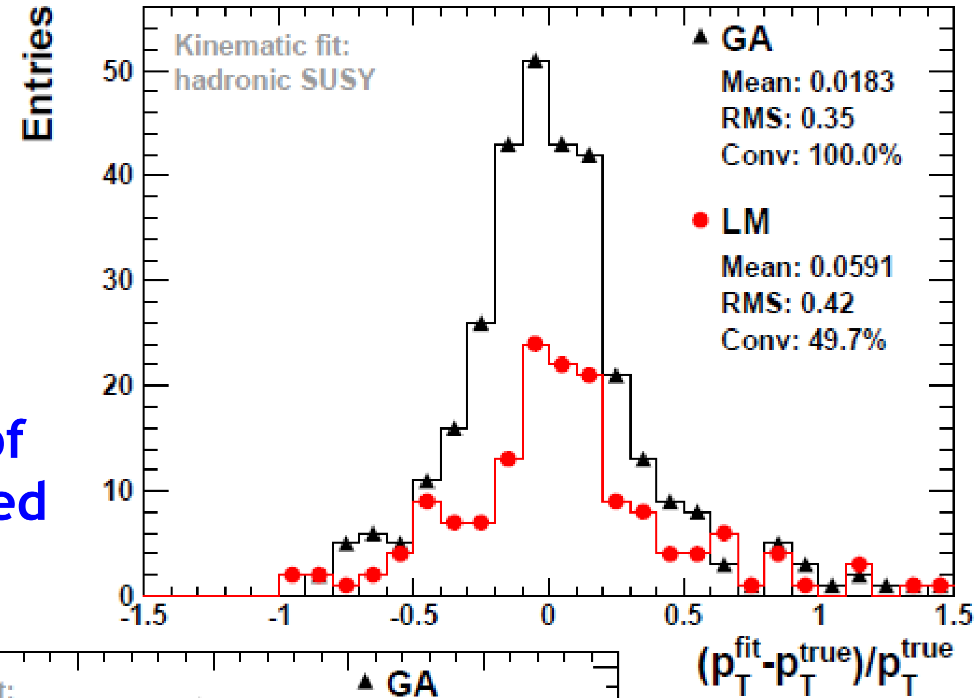
Fit of SUSY Events with Genetic Algorithm

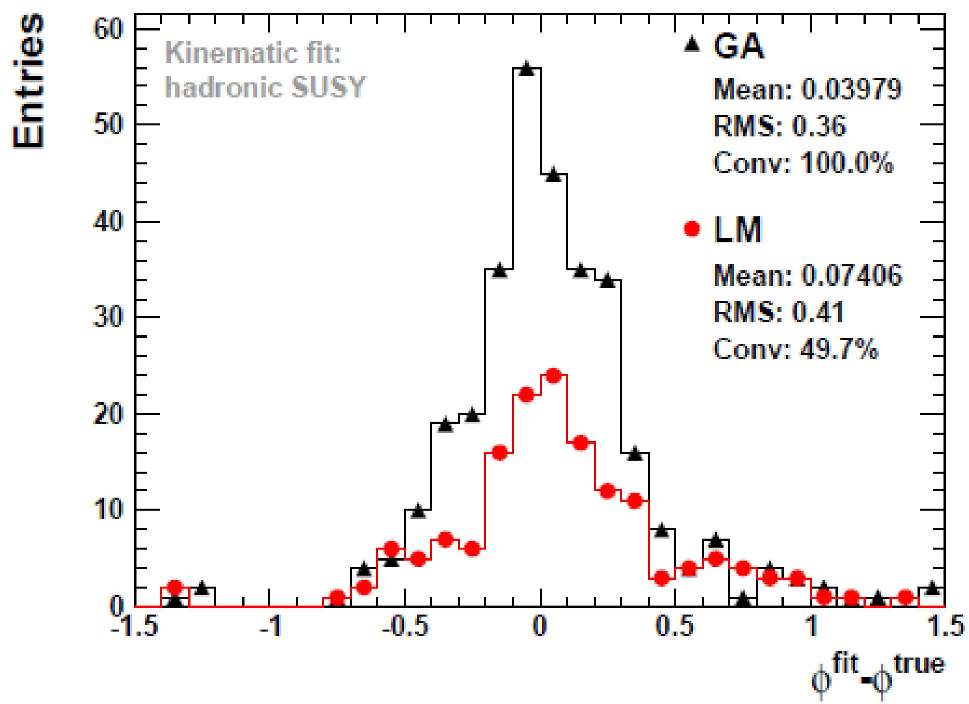
- No SM bg
- No SUSY bg
- Full combinatorial bg
- ISR and FSR
- Detector resolution and acceptance
- Mass hypothesis = true masses

First step: Reconstruct SUSY events with true mass hypothesis

- Reasonable resolution of unmeasured particles
- Neutralino Starting momenta: set to fulfill chargino mass constraint

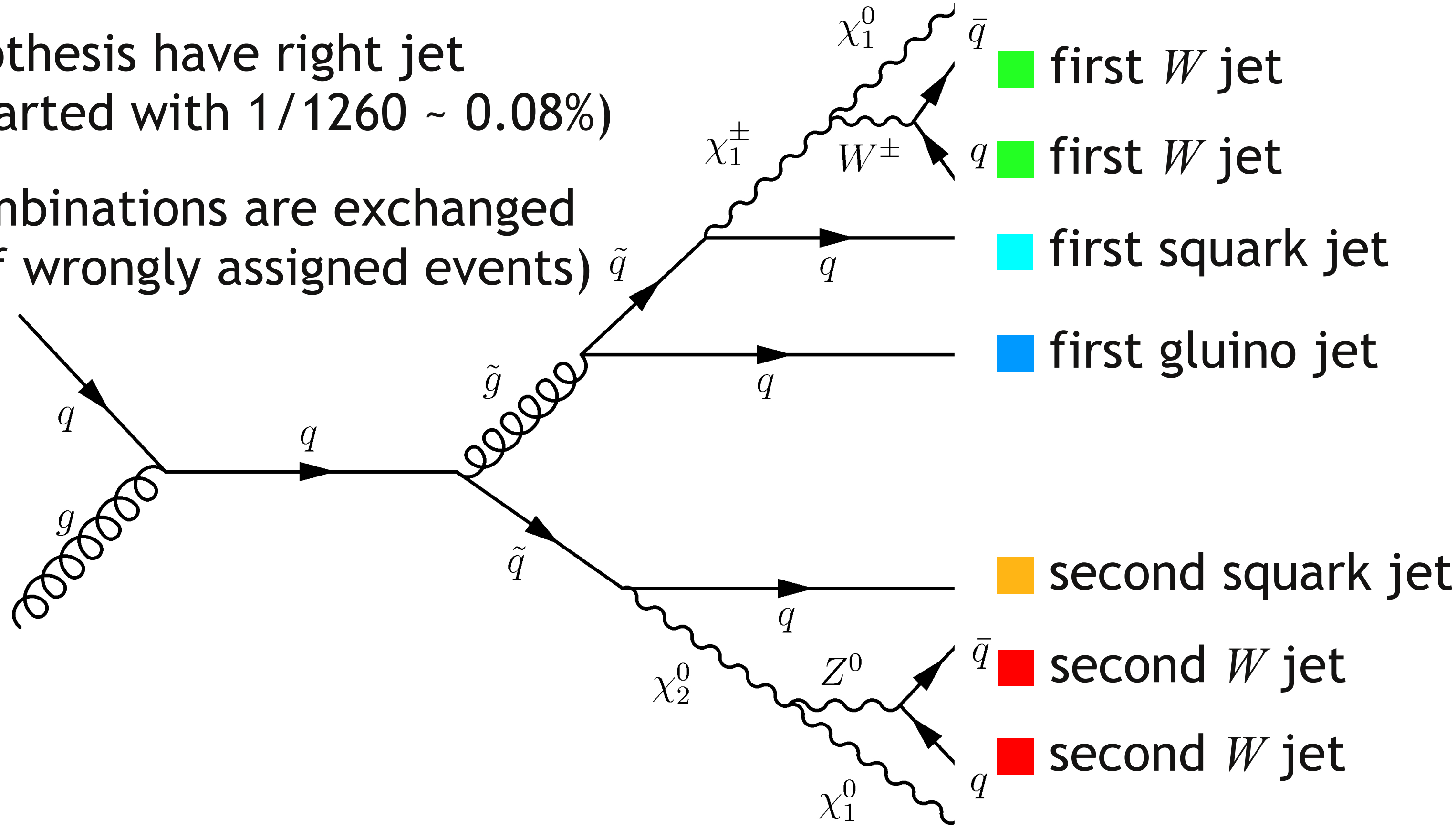
Resolution of reconstructed neutralinos

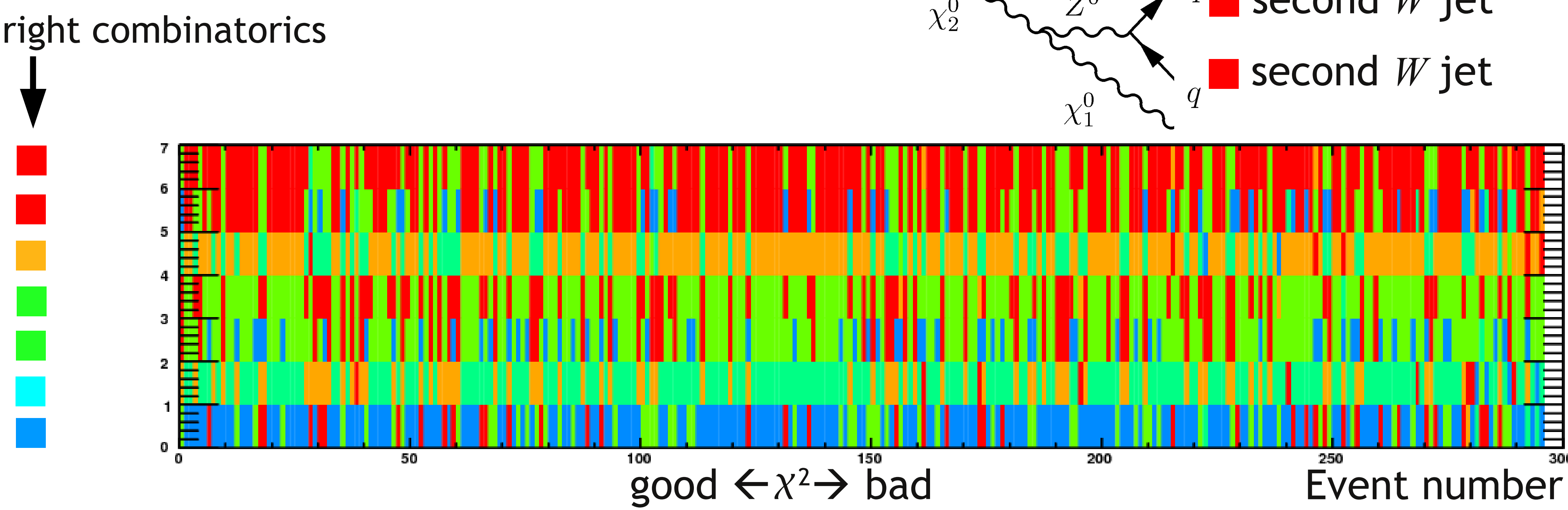




Reduction Combinatorial Background

- ~45% of best hypothesis have right jet combinatorics (started with 1/1260 ~ 0.08%)
- Typical wrong combinations are exchanged branches (~10% of wrongly assigned events)





Probability Distribution

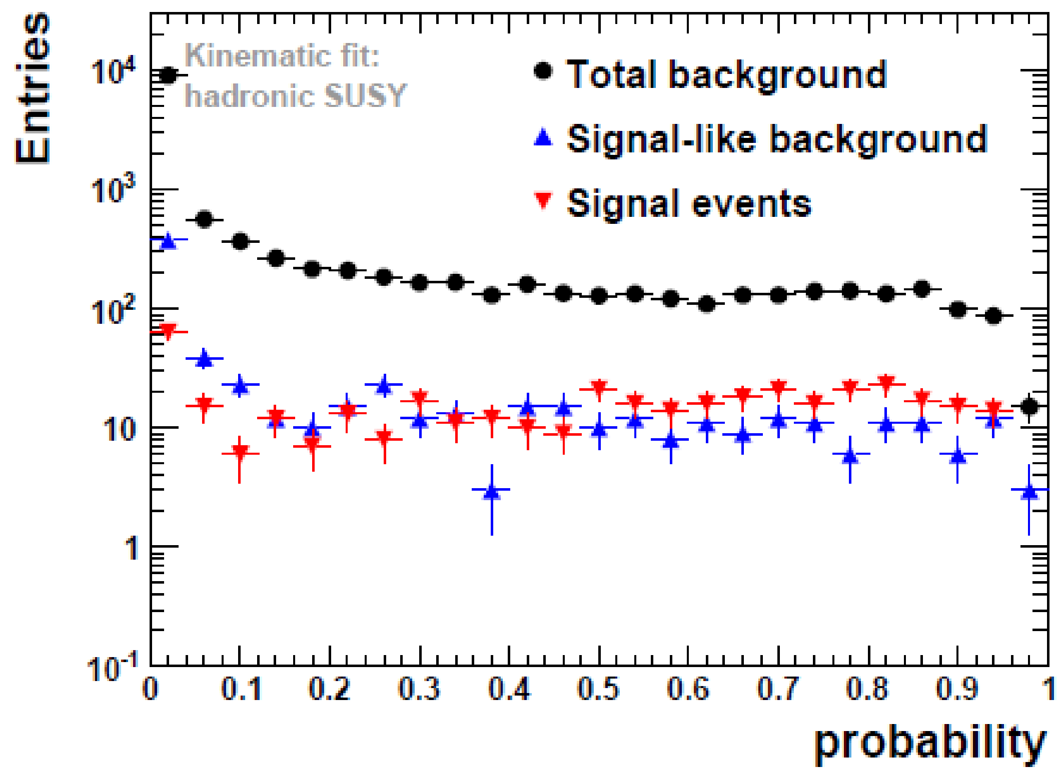
Similar probability distribution of SUSY background:

- "Signal like" cascade topologies, e.g. decays via heavier charginos or neutralinos
- Signal cascades but different squark mass (3rd generation)
- Signal cascades but one soft jet replaced by ISR jet
- Huge jet combinatorics

Fit probability distribution flat for signal (slight systematic shift toward higher probabilities due to combinatorics)

Background peaks at lower values: cut on 0.1(0.3) improves S/B from ~1/33 to ~1/11 (~1/8)

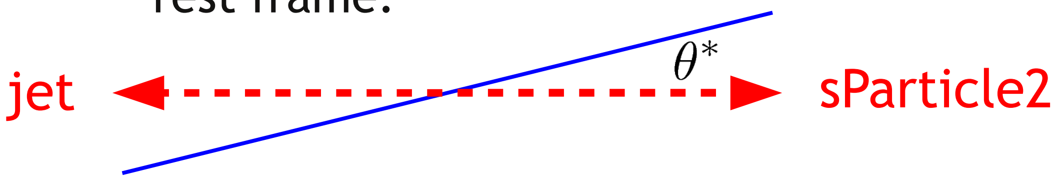
- No SM bg
- Full SUSY bg
- Full combinatorial bg
- ISR and FSR
- Detector resolution and acceptance
- Mass hypothesis = true masses



Angular Distributions

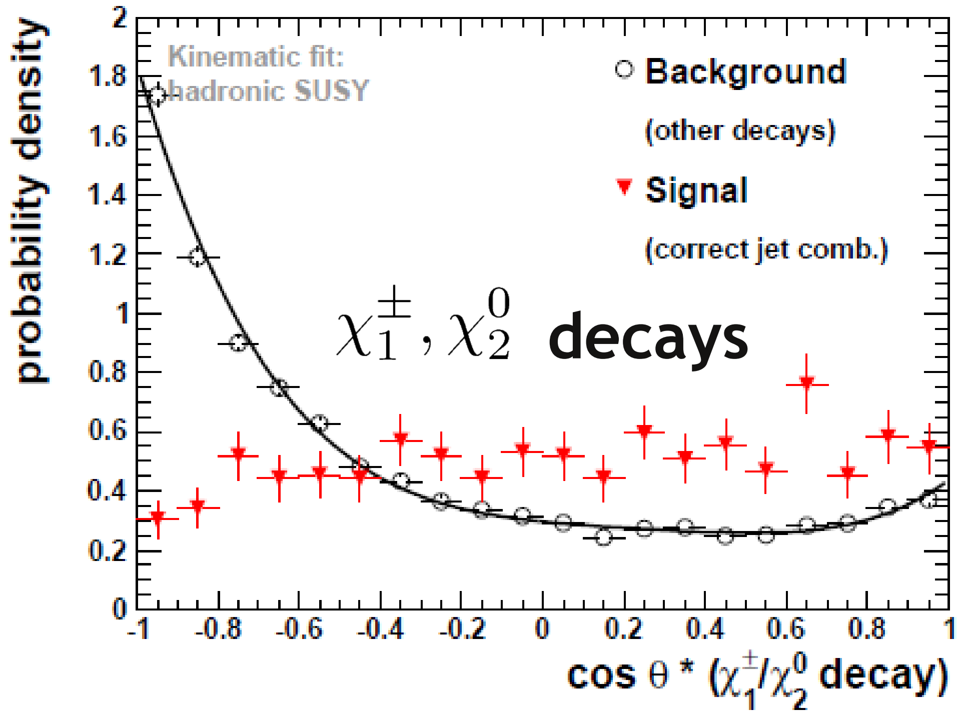
- Huge combinatorial background → Large invariant mass combinations, e.g.
- In rest frame of SUSY particles: angular distribution $\cos\theta^*$ of decay products with respect to flight direction of decaying particle should be ~isotropic (for spin 0)
- $\cos\theta^*$ for typical background 4-vector configurations are not uniformly distributed (smaller angles preferred)

rest frame:



→ **Define new likelihood including angular information:**

$$\mathcal{L} = p \cdot \prod_{k=1}^{N_{\mathrm{decays}}} LR_k = p \cdot \prod_{k=1}^{N_{\mathrm{decays}}} \frac{1}{C_{\mathrm{norm}}} \frac{f_k^{\mathrm{signal}}}{f_k^{\mathrm{signal}} + f_k^{\mathrm{bg}}}$$



Many decay angles in SUSY cascades

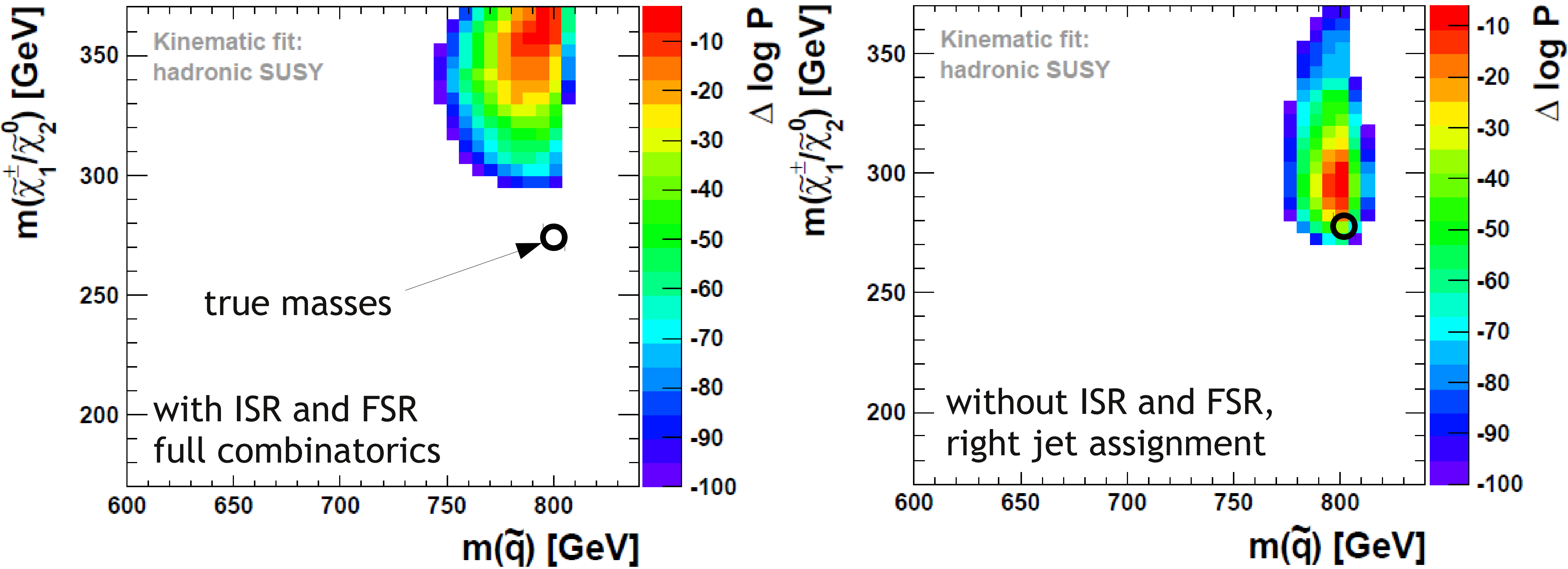
→ Use event kinematics to reduce combinatorial bg reduction

Mass Scan

- Fix gluino and neutralino mass to true values
- Vary two remaining masses (squark and chargino)

$$\log \mathcal{P} = \sum_{i=1}^{N_{\mathrm{tot}}} \log \max(p_i, p_{\mathrm{cut}})$$

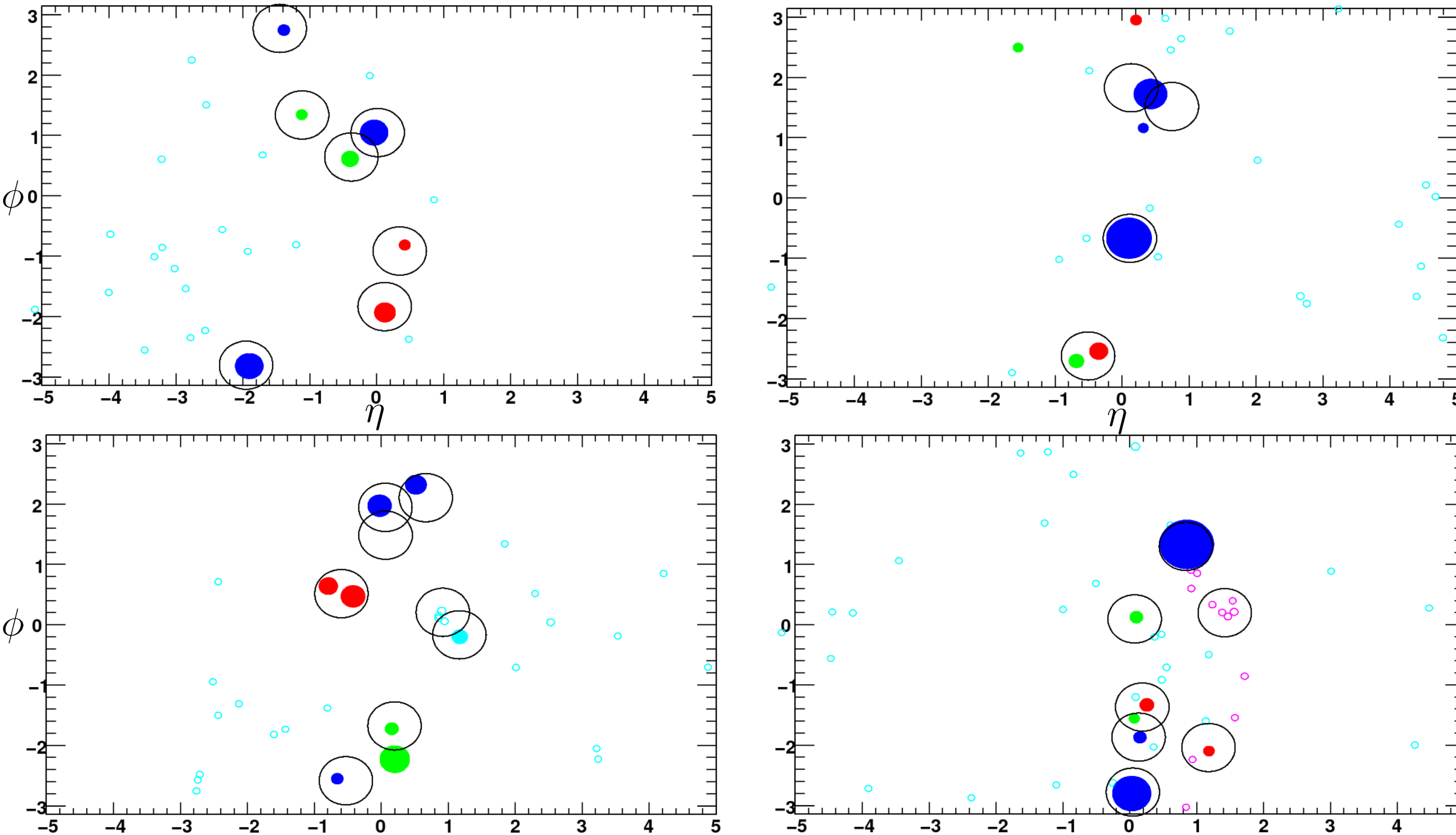
- No SM bg
- No SUSY bg
- Full combinatorial bg
- ISR and FSR
- Detector resolution and acceptance
- **Scan mass hypothesis**



- Concordance between maximum likelihood and true values (bias due to non perfect momentum balance)

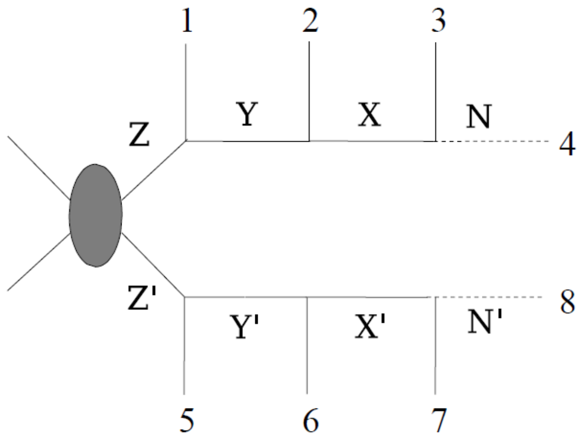
- Partons → PartonJets (here: AntiKt4): "one-to-one" matching for ~1/4 of events

gluino/squark parton W/Z/h parton W/Z/h parton W/Z/h parton FSR parton Jets



Leptonic Cascade Fit

Event topology: $\tilde{q} \to q\tilde{\chi}_2^0 \to ql\tilde{l} \to ql\bar{l}\tilde{\chi}_1^0$ **Model:** SPS1a



- **Toy MC:**
  - 140 fb^{-1} for LHC @14 TeV
  - Final stated smeared according to typical resolutions
- **Event Selection:**

N	pT [GeV]	|η|
Jets		
4	> 30.	< 3.5
Leptons		
2x2 OSSF	> 10.	< 2.5

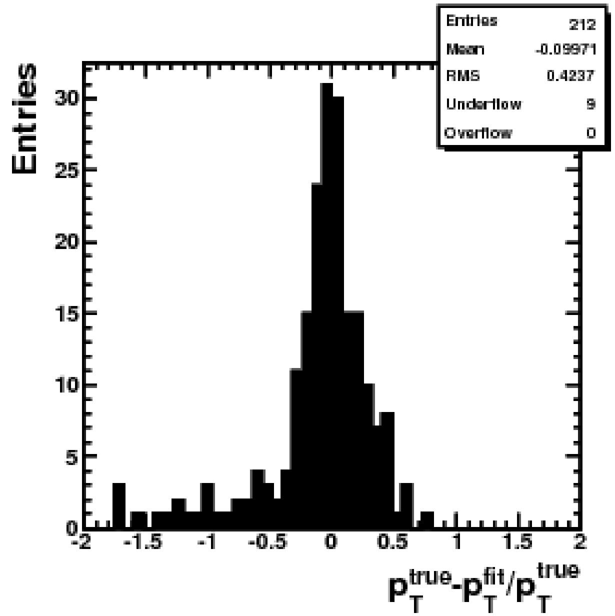
  - Efficiency: 53% (217 events, exactly this cascade)
  - Background fraction: 57%

- **Kinematic fit with Lagrange Multiplier**
  - 6 mass & 2 momentum-balance constraints
  - Uncertainties for masses & momentum sum from MC
- Choose best result from a set of random starting values
  - 400 tries/mass hypothesis
  - Exclude events with low convergence rates (<0.2)
- Efficiently reduces fluctuations in the final likelihood when repeating the fit

Benedikt Mura

Leptonic Cascde Fit: Preliminary Results

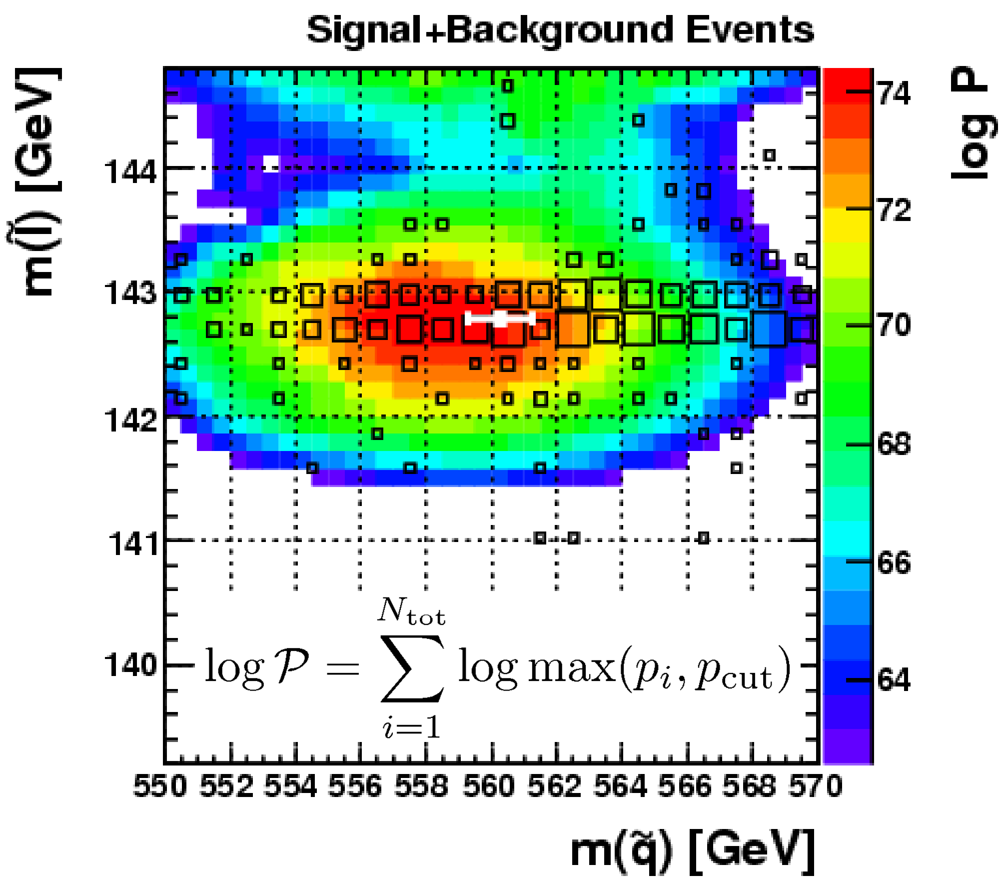
- Reconstruction of LSP momenta in signal (using correct masses in the fit, full combinatorics)



- Performance in combinatorial problem (16/32 possibilities):

Correct assignment	42%
Lepton exchanged on same branch	42%
Particle assigned to the wrong branch	6%

- Visualization in 2D mass plane:
  - $m(\tilde{\chi}_2^0)$ and $m(\tilde{\chi}_1^0)$ fixed to central values
  - Vary squark & slepton masses
  - **Nice agreement of maximum and true masses** (boxes) **and their mean value** (white marker)



Summary

- The LHC will provide new observables for determination or constraining of parameters of BSM-models: kinematic end-points, rates, invariant multi-jet distributions ...
- Various methods available to determine the masses: Global fits (rather model dependent), hybrid methods, multi- or single-event method (less model dependent)
- New approach: kinematic fit for event reconstruction in combination with mass scan
  - Genetic algorithm yields comparable results to Lagrangian Multipliers and is well suited for highly non linear problems
  - Kinematic fits provide a powerful tool to reconstruct SUSY cascades
  - Invariant mass constraints reduce combinatorial background of signal cascades (0.08% → ~45%)
  - Combinatorial SUSY background dominant for studied mSUGRA scenario → further discriminating variables needed, e.g. $\cos\theta^*$
  - Fully hadronic channel challenging, promising first results for leptonic channels


SUSY

Backup

SPS1a

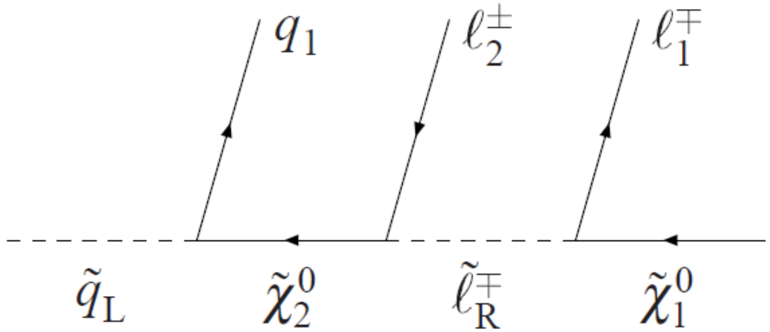
- MSUGRA model:

$$m_0 = 100 \text{ GeV}, m_{1/2} = 250 \text{ GeV}$$
$$A_0 = 0, \tan\beta = 10, \text{sign } \mu = +$$

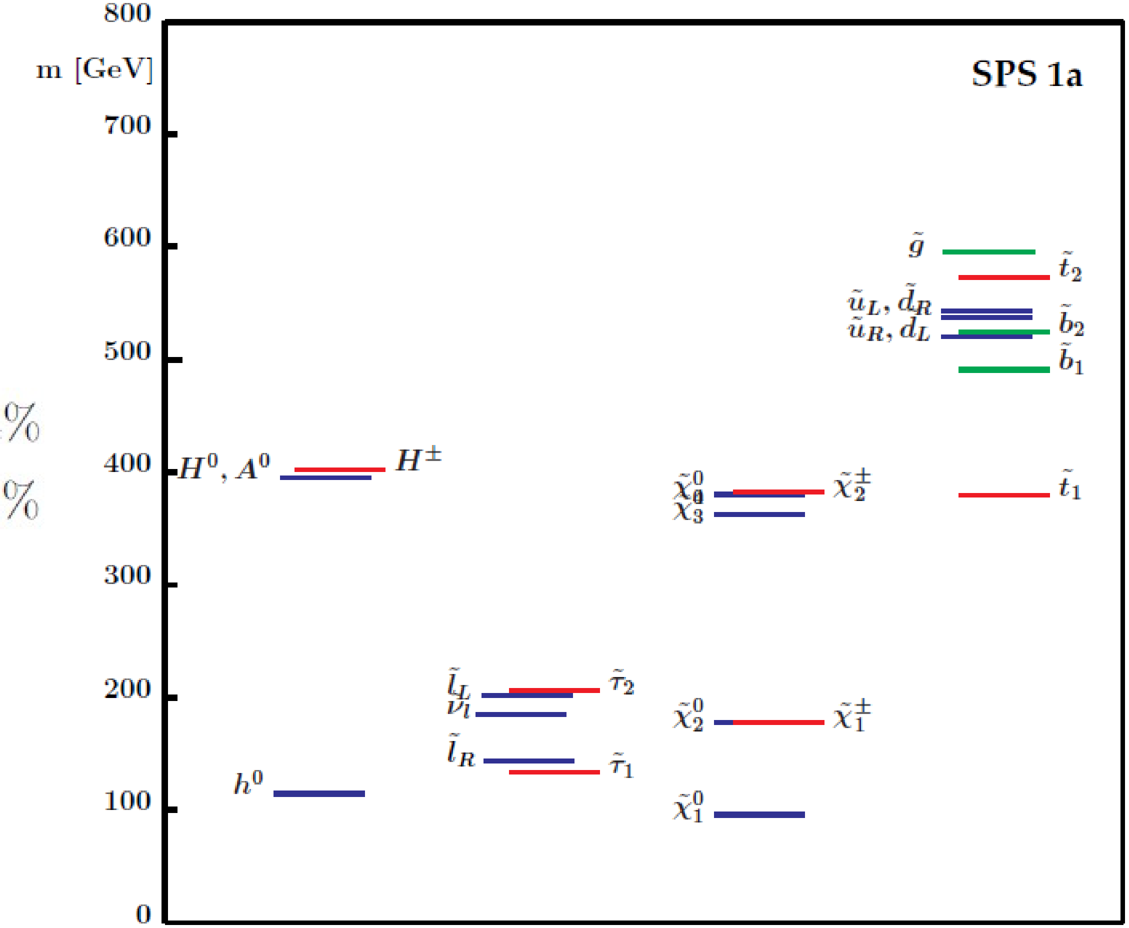
- Cross section:

$$\sigma(\tilde{q}_L) = 33 \text{ pb}, \quad BR(\tilde{q}_L \to q\tilde{\chi}_2^0) = 31.4\%$$
$$\sigma(\tilde{b}_1) = 7.6 \text{ pb}, \quad BR(\tilde{b}_1 \to b\tilde{\chi}_2^0) = 35.5\%$$

- Process:

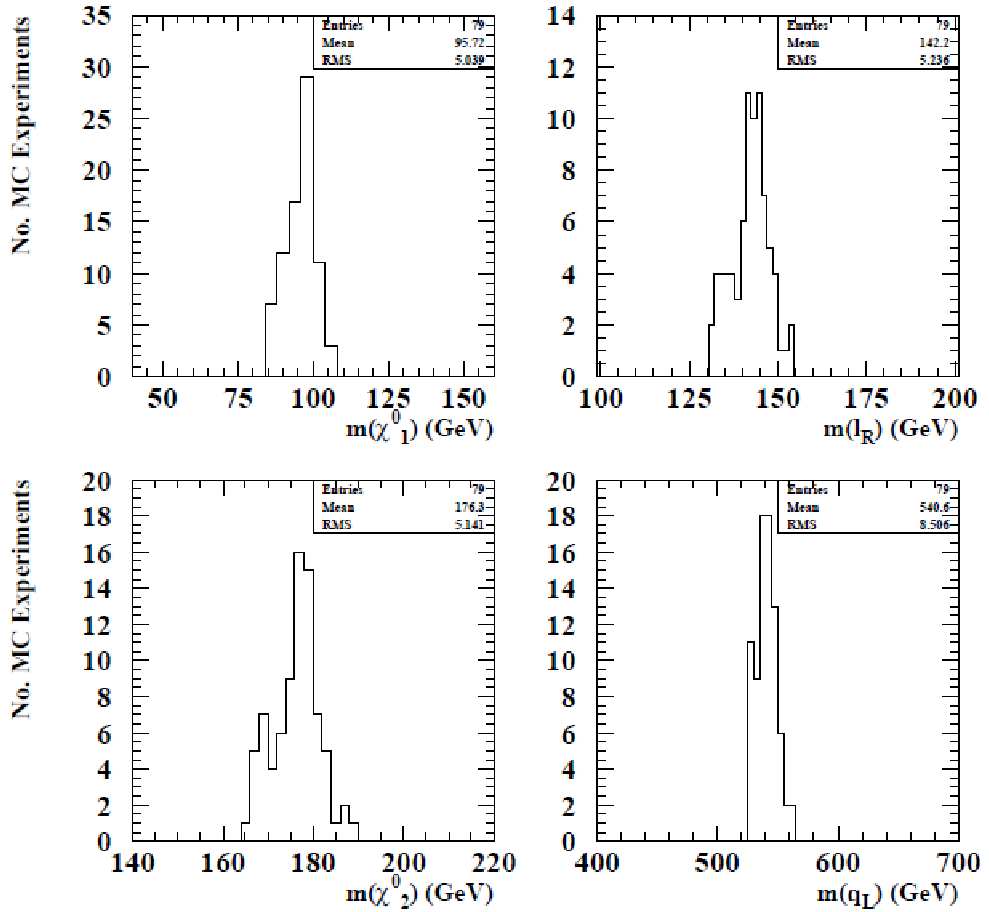


$$\tilde{q}_L \xrightarrow[\sim 32\%]{} q\tilde{\chi}_2^0 \xrightarrow[12.1\%]{} q l_2^\pm \tilde{l}_R^\mp \xrightarrow[100\%]{} q l_2^\pm l_1^\mp \tilde{\chi}_1^0$$



Hybrid Method: Results

Distribution of mean mass values for many MC experiments (each corresponding to 100 fb^{-1}):



Comparison of results with or without transverse momentum balance:

State	Input	End-Point Fit		Hybrid Method, E_T^{miss}		Hybrid Method, no E_T^{miss}	
		Mean	Error	Mean	Error	Mean	Error
$\tilde{\chi}_1^0$	96.05	96.5	8.0	95.8(92.2)	5.3(5.5)	97.7(96.9)	7.6(8.0)
$\tilde{l}_R$	142.97	143.3	7.9	142.2(138.7)	5.4(5.6)	144.5(143.8)	7.8(8.1)
$\tilde{\chi}_2^0$	176.81	177.2	7.7	176.4(172.8)	5.3(5.4)	178.4(177.6)	7.6(7.9)
$\tilde{q}_L$	537.2–543.0	540.4	12.6	540.7(534.8)	8.5(8.7)	542.9(541.4)	12.2(12.7)

Single-Events Solution - Results

Dependence of determined masses on momentum resolution and goodness-of-fit quality cut $\xi^2_{\max}$:

$\delta p/p$	$\xi^2_{\max}$	f_ξ	f_{cor}	$M_{\tilde{q}}$ (540)	$M_{\tilde{\chi}^0_2}$ (177)	$M_{\tilde{\ell}}$ (143)	$M_{\tilde{\chi}^0_1}$ (96)
0	∞	100%	72%	538 ± 20	176 ± 12	143 ± 7	95 ± 10
0	100	80%	76%	539 ± 7	177 ± 1	144 ± 1	96 ± 2
5%	∞	100%	52%	534 ± 28	176 ± 11	143 ± 10	95 ± 13
5%	100	57%	55%	539 ± 9	178 ± 3	144 ± 2	96 ± 4
10%	∞	100%	40%	522 ± 37	171 ± 18	140 ± 17	88 ± 26
10%	200	42%	43%	530 ± 22	173 ± 12	140 ± 12	89 ± 20