Measurement of Temporal Distribution using Accelerating Radio Frequency Cavity in Low Emittance Injector

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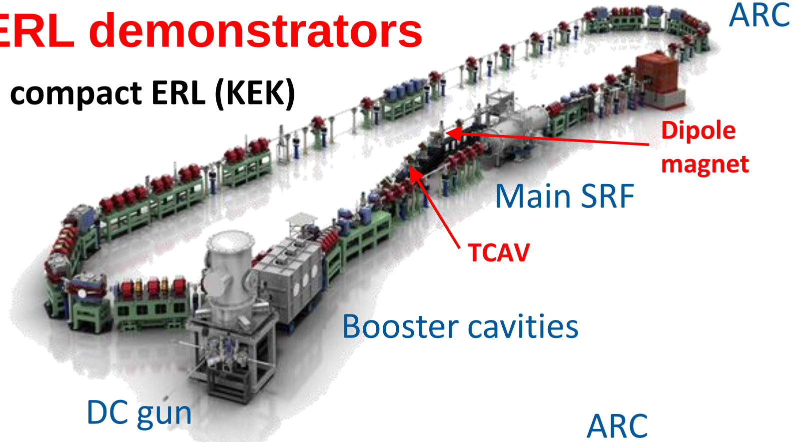
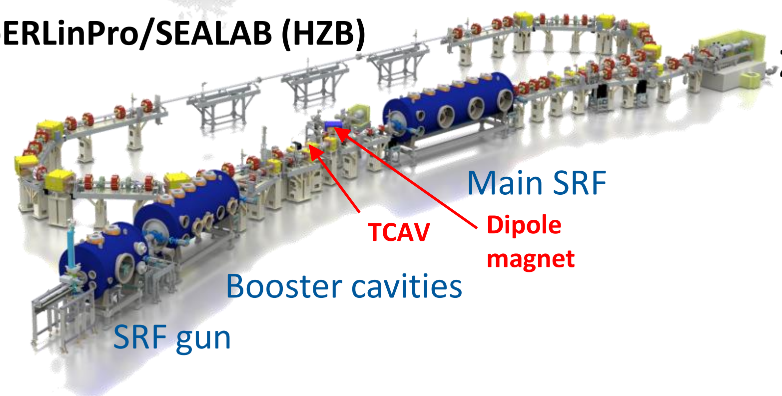
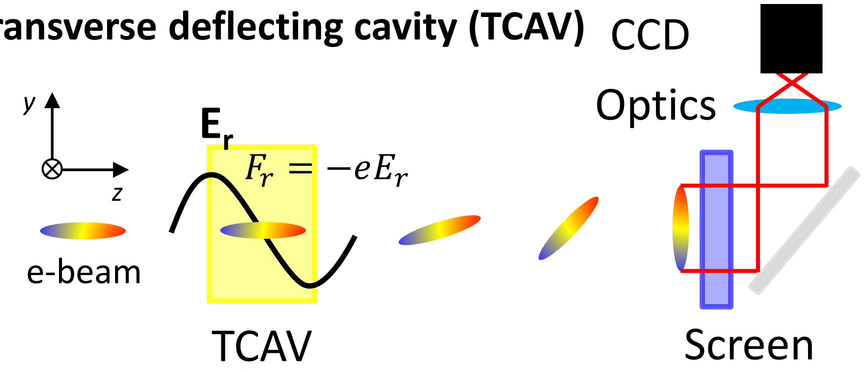
ERL demonstrators





Transverse deflecting cavity (TCAV)



Pros: **Space-charge free**

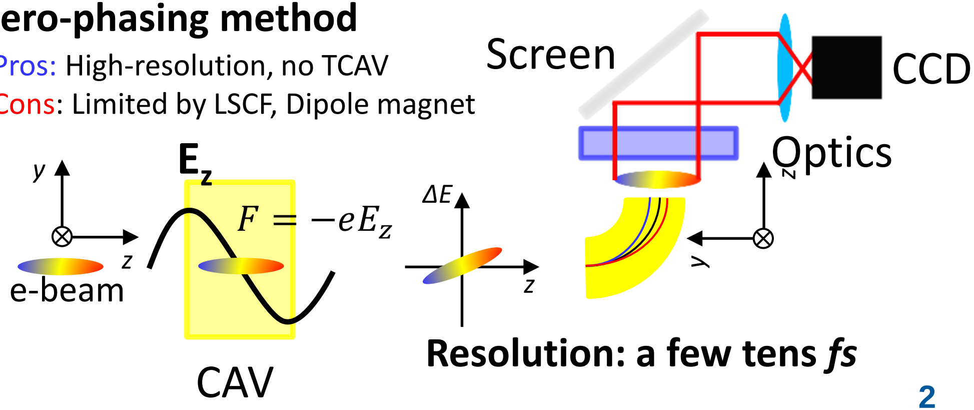
Cons: Difficult to install TCAV **near e-gun**

Resolution: a few hundred *fs*

Zero-phasing method

Pros: High-resolution, no TCAV

Cons: Limited by LSCF, Dipole magnet

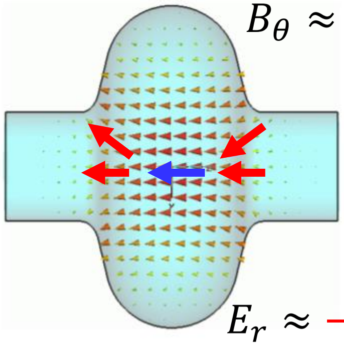


Resolution: a few tens *fs*

2-cell booster cavity in compact-ERL

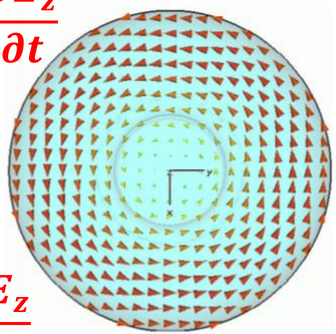
EM field map of TM01 mode

E-field

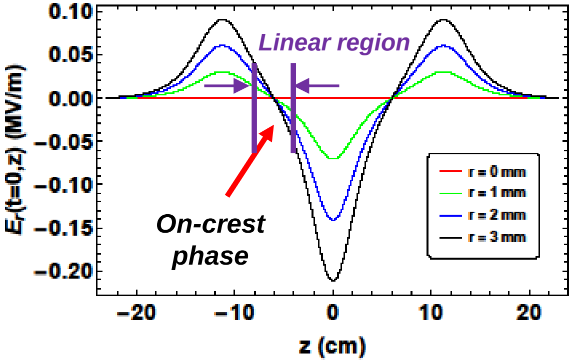


$$B_\theta \approx \frac{r}{2c^2}\frac{\partial E_z}{\partial t}$$

H-field



$$E_r \approx -\frac{r}{2}\frac{\partial E_z}{\partial z}$$



An **accelerating RF cavity** can provide a **phase-dependent transverse kick** to the electrons, resulting in the linear coupling of the trajectory angle with the longitudinal position.

The equation of motion with the maximum acceleration phase (on-crest phase)

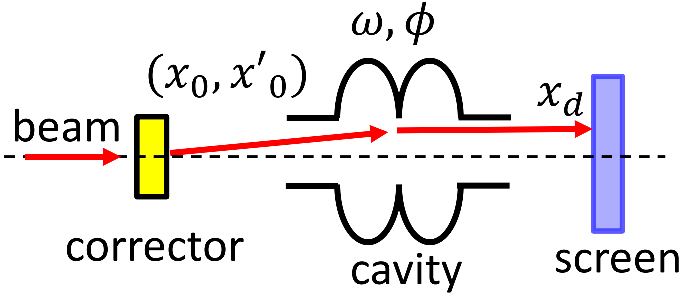
$$\frac{dp_r}{dt} = e(E_r - \beta c B_\theta) = -\frac{er}{2}\left[\frac{\partial E_z}{\partial z} - \frac{\beta}{c}\frac{\partial E_z}{\partial t}\right]$$

For high energy beams ($\beta \simeq 1$) , the transfer matrix can be calculated analytically (in Rosenzweig's paper) which is well known as a **SRF focusing effect**. J. Rosenzweig and L. Serafini, Phys. Rev. E **49**, 1599 (1994)

Bunch length measurement using TM01 cavity

For low energy beams, the transverse motion is complicated by the significant changes in the velocity of an electron inside the cavity owing to its low initial kinetic energy.

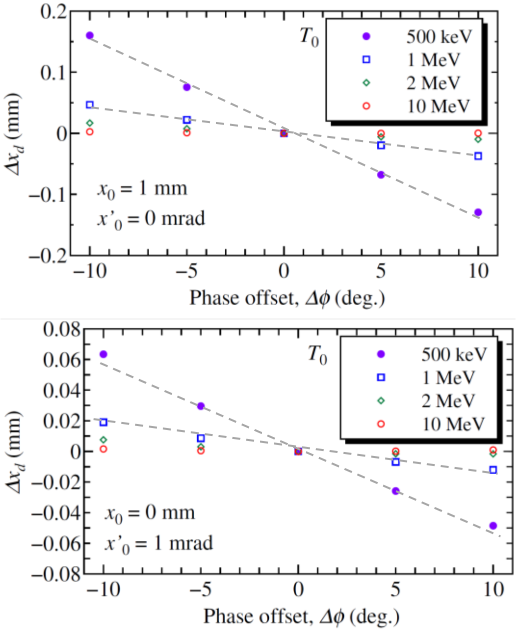
→ Numerical simulation using **General Particle Tracer**



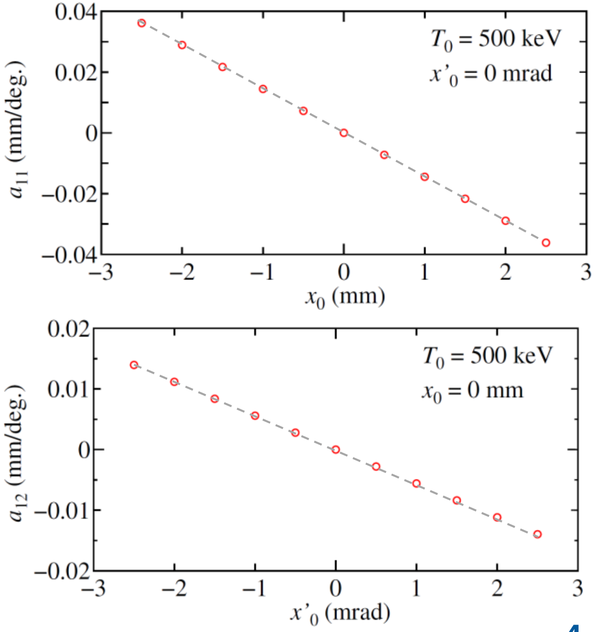
With a paraxial approximation, we can describe the transverse motion using the transfer-matrix as

$$\boldsymbol{x_d} = a_{11}x_0 + a_{12}x'_0$$
$$= x_c + (d_{11}\phi\, x_0 + d_{12}\phi\, x'_0)$$
$$= \boldsymbol{x_c} + (\boldsymbol{d_{11}x_0} + \boldsymbol{d_{12}x'_0})\ \boldsymbol{\phi}$$

Is it linear on the phase ϕ?



Is it linear on the x_0 and x'_0?



Since the phase can be expressed as $\boldsymbol{\phi = \omega t}$, the horizontal distribution convoluted by the initial temporal distribution is given by

$$G(x) = \int f(\tau) g(x - (d_{11}x_0 + d_{12}x_0')\omega\tau)\, d\tau$$

, where $f(x)$ and $g(x)$ are a temporal and horizontal distribution, respectively.

If the beam has a Gaussian distribution and is transversally small, the transverse beam size is determined by the bunch length, initial offset and angle, and accelerating RF cavity parameters as

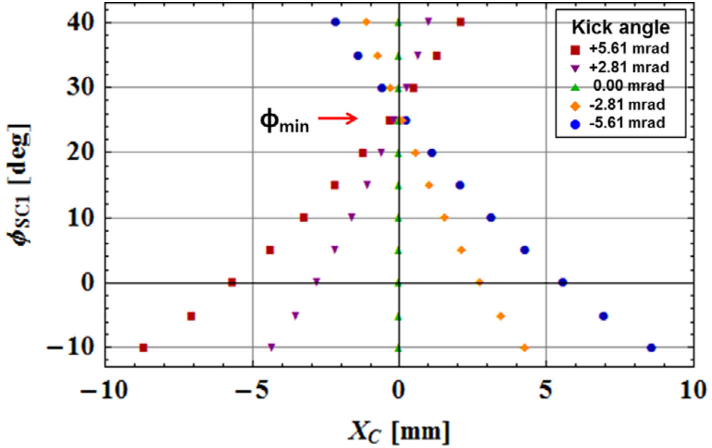
$$\sigma_x = \sqrt{\boldsymbol{\sigma_{x0}^2} + \left((d_{11}x_0 + d_{12}x_0')\omega\sigma_t\right)^2}$$

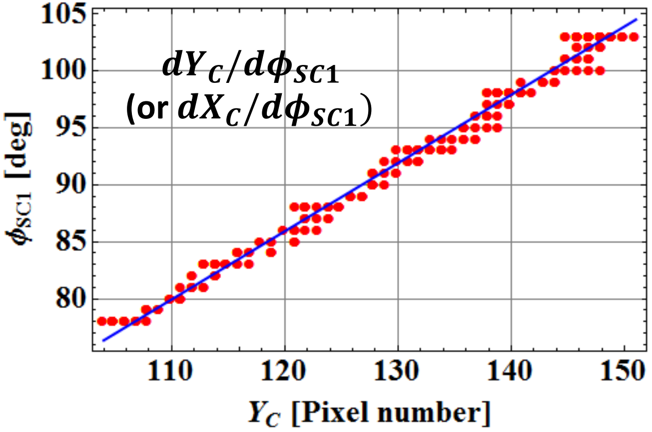
, where σ_{x0} is the initial beam size at the screen without offset and σ_t is the bunch length.

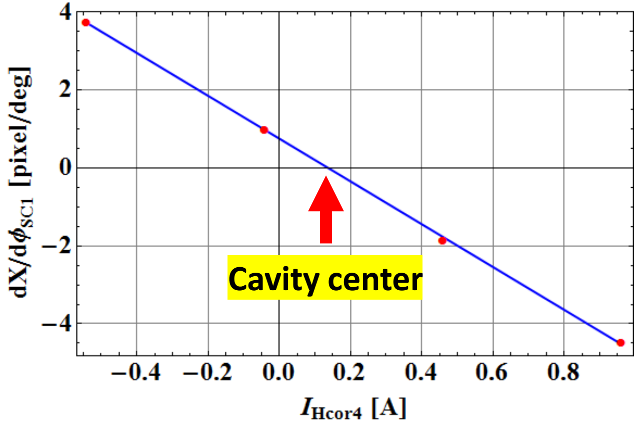
Remark:

1. Small σ_{x0} (focusing on the screen downstream) improves the resolution of the method.
2. Since the σ_{x0} is determined by the integration of complex physics processes including optics, energy variation as well as low-energy beam dynamics, **it is not necessary to deconvolute the emittance dilution by space charge effects**.

How to find the center of RF cavity?







The focusing strength due to the RF kick is linearly proportional to the phase of cavity only around the on-crest phase.

$$\Delta y = k_{SC1} L \quad \text{where} \;\; k_{SC1} \propto \phi_{SC1} r$$

The center of the cavity can be estimated by this equation.

$$r \propto \Delta y/\phi_{SC1}$$

How to find the center of RF cavity?

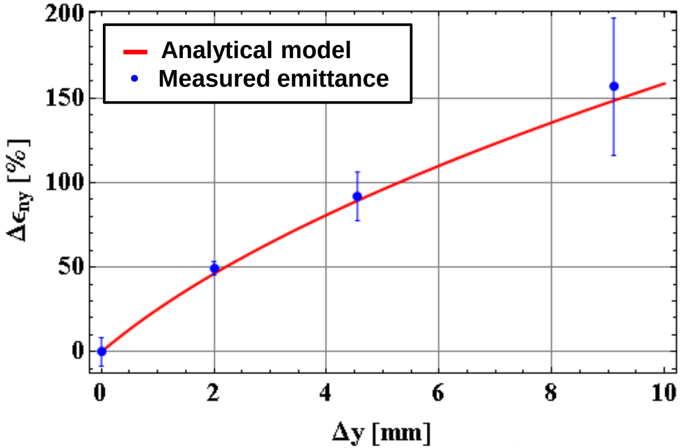
Transverse emittance growth due to all the bunch slices that align along the orientation of the phase ellipse in the injector cavity can be expressed as

$$\varepsilon_{nr}{}^{2} = (\varepsilon_{nr0}\beta + D^2)(\varepsilon_{nr0}\gamma + D'^2) - (\varepsilon_{nr0}\alpha - DD')^2$$

where β, α and γ are the Twiss parameters at the observation point, and D and D' represent the rms displacement of the bunch slice on the transverse phase space, respectively. The D' of the particles laid on the rms bunch length, φ_{rms}, with a beam offset of r can be given by

$$D'(r, \phi_{rms}) = \frac{\Delta p_r}{p_z} = \frac{\sqrt{(W_r(r, \phi_{rms})/mc^2 + 1)^2 - 1}}{\sqrt{\gamma_z{}^2 - 1}}$$

J.-G. Hwang et.al., Nucl. Instrum. Methods Phys. Res. Sect. A, 753, 97 (2014)

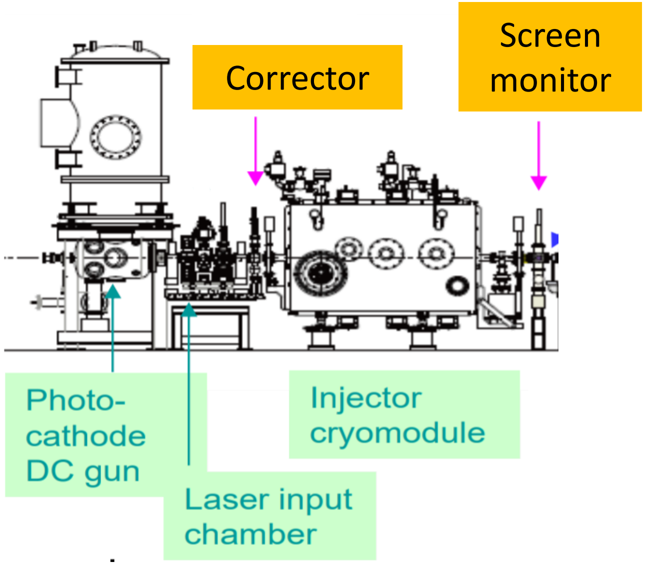


Measured transverse emittance with various beam offsets.

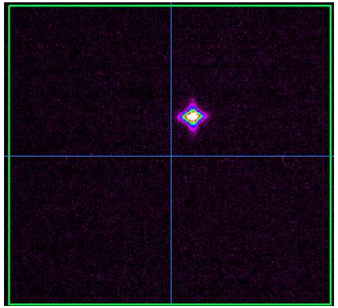
Δx (mm)	Δy (mm)	ε_{nx} (mm-mrad)	ε_{ny} (mm-mrad)
-12.4	0	1.070±0.053	0.970±0.033
0	0	0.212±0.0064	0.259±0.023
0	-4.55	0.243±0.0057	0.497±0.037
0	-9.10	0.240±0.015	0.665±0.11

Δx (mm)	Δy (mm)	ε_{nx} (mm-mrad)	ε_{ny} (mm-mrad)
0	2.00	-	0.399±0.011
0	0	-	0.267±0.017
0	-2.00	-	0.439±0.010

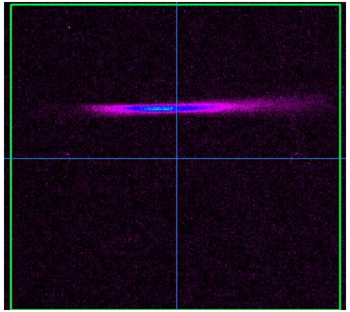
Proof-of-principle test at compact-ERL



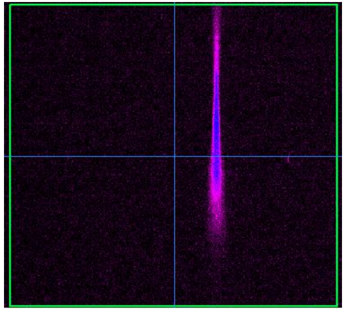
First proof-of-principle test with a long pulse generation by a laser stacking of 8 pulses with 2 ps separation



Beam on cavity center



TM01 cavity with Δx' = 23 mrad



TM01 cavity with Δy' = 23 mrad

Condition for the PoP studies

Parameters	Values
K.E.	0.39 keV (@ ent. cavity) ~ 2 MeV (@ after cavity)
Bunch charge	10 fC
$\varepsilon_{nx,ny}$	~ 0.1 mm-mrad
E_{acc}	7.95 MV/m @ 1st booster cavity

Bunch length measurement using TM01 cavity

For determining the absolute bunch length, we must know: $\boldsymbol{\omega}, \boldsymbol{d_{11}}, \boldsymbol{d_{12}}, \boldsymbol{x_0}, \boldsymbol{x_0'}$.

To make the problem simple, we performed our experiments with $x_0 = 0$.

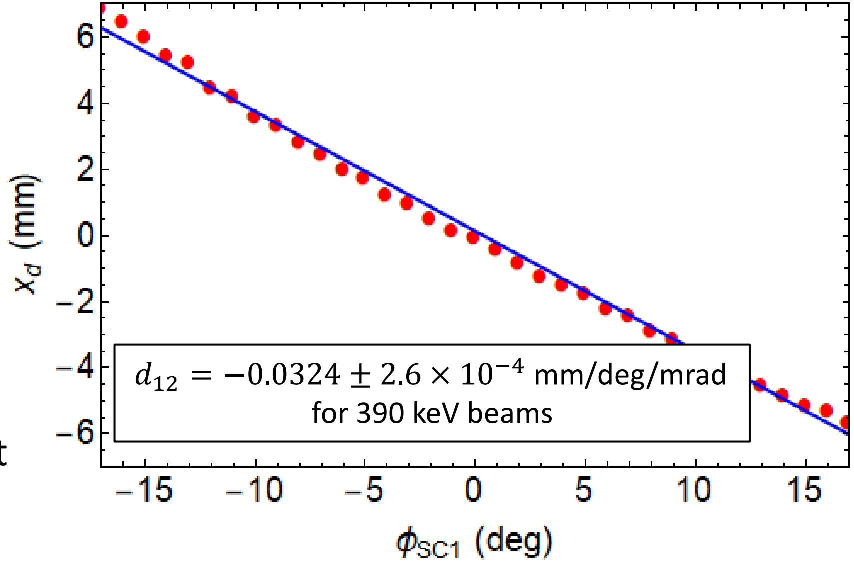
$$\sigma_x = \sqrt{\sigma_{x0}^2 + (\boldsymbol{d_{12} x_0' \omega} \sigma_t)^2}$$

The $\boldsymbol{\omega}, \boldsymbol{x_0'}$ can be estimated by pre-calibrations, and then, unknown parameter is only $\boldsymbol{d_{12}}$.

Can we measure $\mathbf{d_{12}}$ in the experiment?

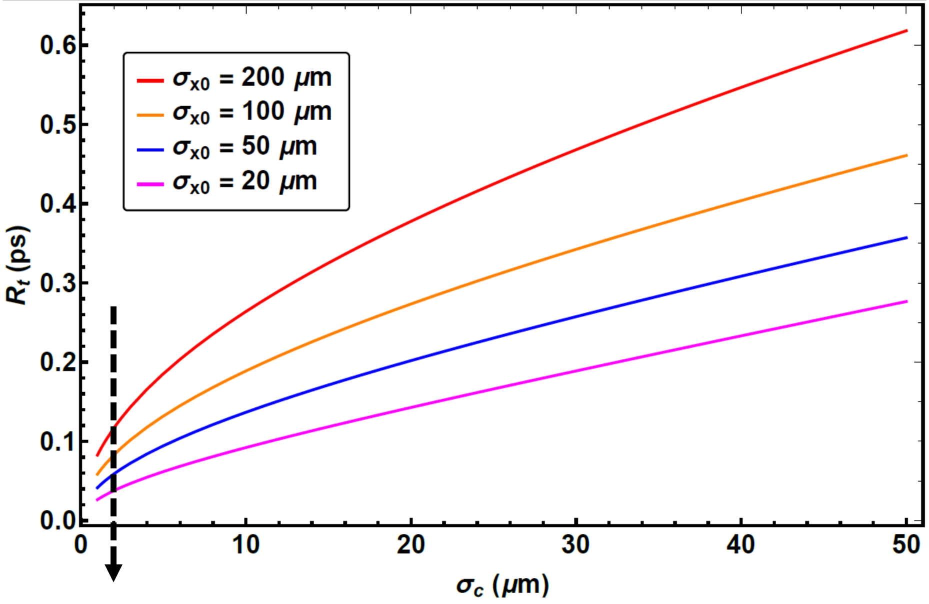
Yes! From the definition, $\Delta x = d_{12} x_0' \phi$, the $d_{12} = \Delta x / (x_0' \phi)$.

Then it is necessary to measure the position variation as a function of the phase with a fixed initial angle.



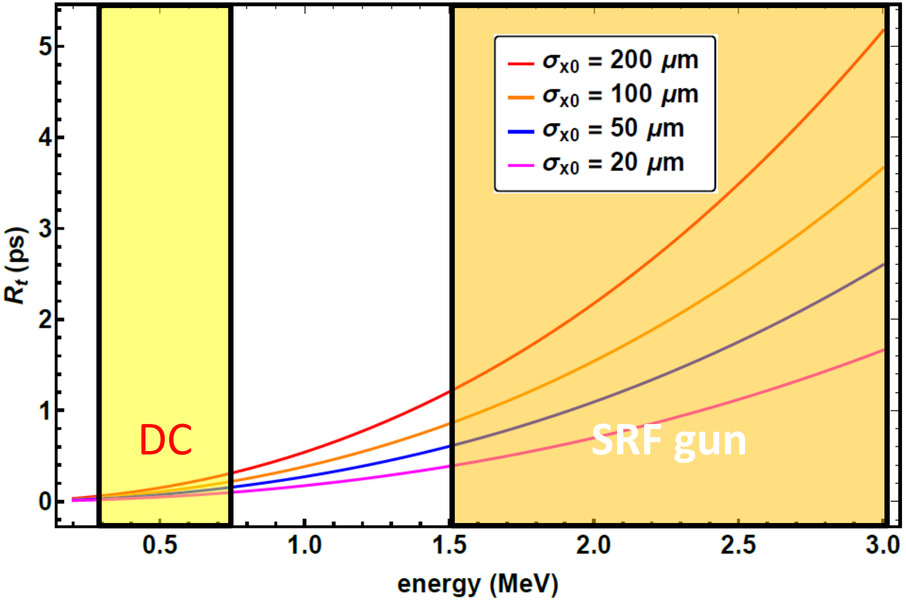
Resolution: about **0.98 ps** for 100 um resolution (It can be improved by increasing the field gradient of the cavity or improving the spatial resolution or lowering the initial beam energy.)

Bunch length measurement using TM01 cavity



State-of-the-art screen monitor (1.44 µm)

G. Kube *et al.*, "Transverse Beam Profile Imaging of Few-Micrometer Beam Sizes Based on a Scintillator Screen", in *Proc. Of IBIC'15*, pp. 330–334

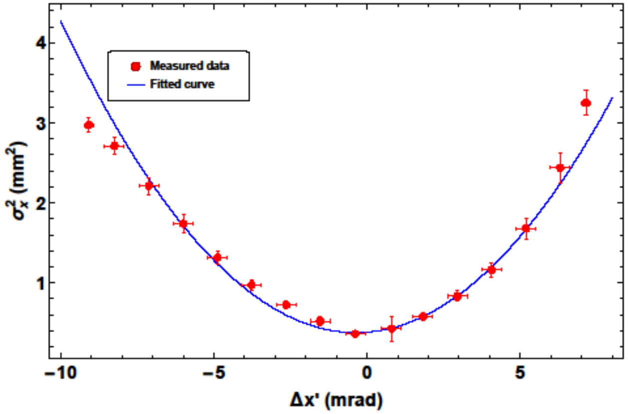


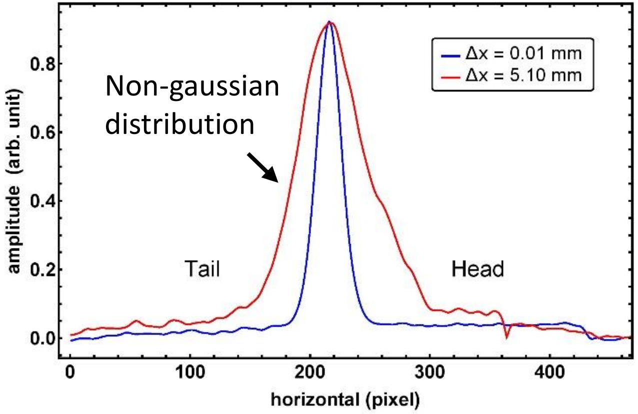
With an initial beam size of 50 μm and a spatial resolution of 1.44 um, the temporal resolution is

- 76 fs for 0.5 MeV (Photocathode DC Gun)
- 272 fs for 1.0 MeV
- 1 ps for 2.0 MeV (Photocathode SRF Gun)

Long pulse
: laser stacking of 8 pulses with 2 ps separation

13.51 +/- 0.03 ps
for E_{acc} = 7.21 MV/m

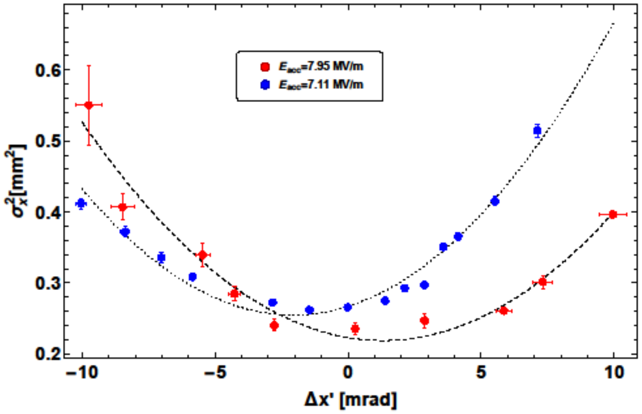


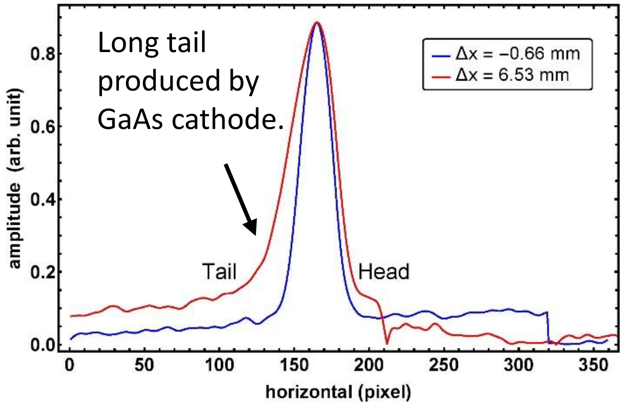


Short pulse
: a single laser with a duration of 3.3 ps rms

3.39 +/- 0.12 ps (rms)
for E_{acc} = 7.11 MV/m

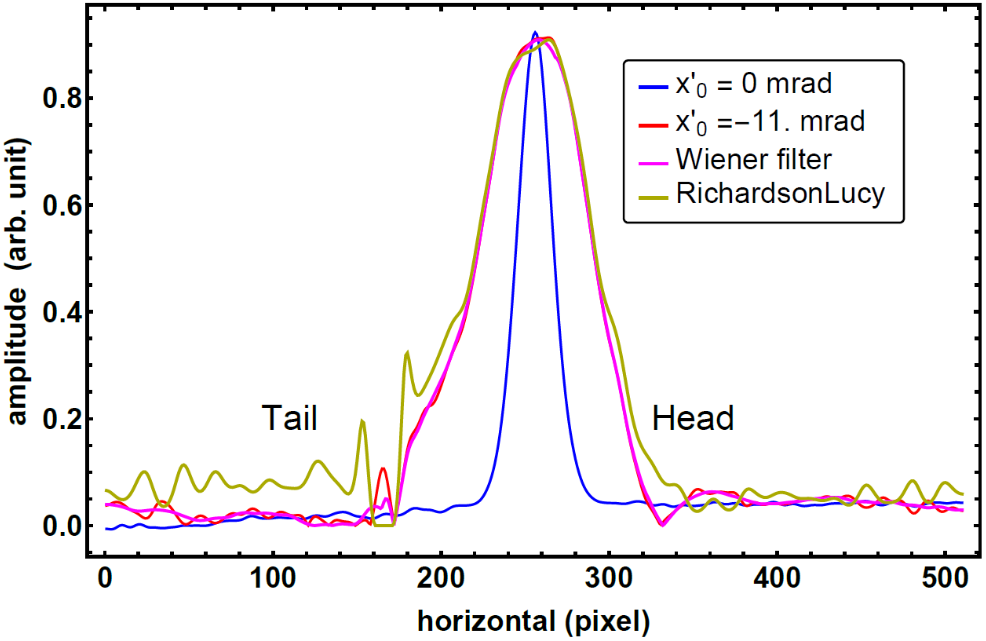
2.94 +/- 0.15 ps (rms)
for E_{acc} = 7.95 MV/m





J.-G. Hwang et.al., Sci. Rep., 10, 18905 (2020)

Temporal distribution deconvolution



J.-G. Hwang et.al., "Absolute bunch length measurements of low energy beams using accelerating RF cavity", in *Proc. Of IBIC'21*, WEOA07.

Wiener filter

- Original spectrum is estimated by taking the product of FFT of measured signal *I*(*x*) with the Wiener filter *W*(*u*):

$$\hat{S}(u) = W(u)\, I(u)$$

where

$$W(u) = \frac{\hat{H}(u)^* P_s(u)}{\left|\hat{H}(u)\right|^2 P_s(u) + P_n(u)}$$

, and $\hat{H}(f)$ is FFT of the point-spread function $H(x)$.

Richardson-Lucy algorithm

- Deconvolution by the Richardson-Lucy algorithm is achieved by minimizing the convex loss function given by

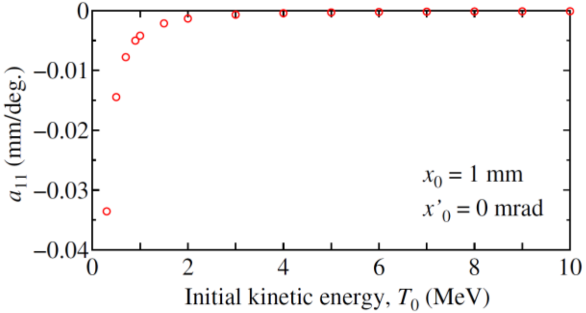
$$J(O) = \sum O * H(x) - I(x) \cdot \ln(O * H(x))$$

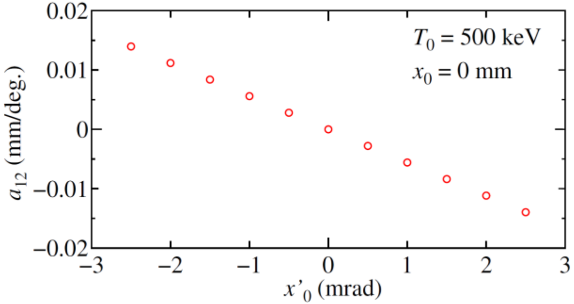
, where O is the output image resulting from deconvolution.

Summary

Radial electric-field always **exists** in a TM01-mode accelerating cavity.
It was considered **harmful and interventive** (dilutes emittance) for high-quality low-energy beams.

Here, we propose to use it for measuring a **temporal profile/bunch length** of the high-quality beam with only existing instruments in the beamline. (below 3 MeV relying on the field gradient)





The center of the cavity should be defined to use this method.

We have proven that this can be done without special instruments

1. Measure $dY_C/d\phi_{SC1}$ with various offset (at least three point)
2. Estimate the electromagnetic center by $dY_C/d\phi_{SC1} = 0$
3. Confirm it by varying beam offset inside a cavity

THANK YOU FOR YOUR ATTENTION

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Special thanks to compact-ERL collaborators for their efforts in constructing and performing the machine commissioning.