Gopakumar-Vafa Hierarchies in Winding Inflation and Uplifts

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Gopakumar-Vafa (GV) invariants

- a question:

  compactify type IIA string theory on a
  Calabi-Yau (CY) manifold X

  $h^{1,1}$ 2-cycles and Kahler moduli (2-cycle volumes)

  $h^{2,1}$ 3-cycles and complex structure (c.s.) moduli (3-cycle sizes)

  X contains 2-dim submanifold, so-called holomorphic
  2-cycles — smooth 2-dim manifolds are either a 2-sphere
  or a Riemann surface (“Pretzel”) of genus g
  (g = number of holes in the “Pretzel”)

  2-cycles come in equivalence classes defined by the homology
  group $H_2(X,\mathbb{Z})$

  wrap D2-branes on the 2-cycles = supersymmetric 4D particles
  = BPS states

Gopakumar-Vafa (GV) invariants

- Question: how many such BPS states are there for each 2-cycle from wrapped D2-branes?
  (how many different ways to wrap
  a D2-brane on a 2-cycle?

  Answer: the GV invariants $n^g_{(d_1,\ldots,d_{h^{1,1}})}$ count # of BPS states we get from D2-branes wrapped on a 2-cycle of genus g in homology class;

  GV invariants hence are integer numbers

  beautiful mathematical objects describing part of the topological data of CY manifold

  — What do they have to do with inflation or the cosmological constant?

Gopakumar-Vafa (GV) invariants

- method to compute the g = 0 GV invariants:
  - by using mirror symmetry of the space of CY manifolds

  for every $X(h^{1,1}, h^{2,1})$ a mirror $\tilde{X}(\tilde{h}^{1,1}, \tilde{h}^{2,1})$ with $\tilde{h}^{1,1} = h^{2,1}$, $\tilde{h}^{2,1} = h^{1,1}$

  - and the Picard-Fuchs system of differential equation governing the 3rd cohomology group of X which determines the 3-cycles and c.s. moduli

  can compute the GV invariants of $\tilde{X}$
- use existing computer programm "instanton" by Klemm & Kreutzer (2001) to this for all CYs given by complete-intersection polynomials in projective spaces (CICYs)

  about 8000 CICYs in total, after removing redundancies

  we computed all g = 0 GV invariants for all CICYs with $h^{1,1} \leq 9$

Gopakumar-Vafa (GV) invariants

$$h^{1,1} = 2$$

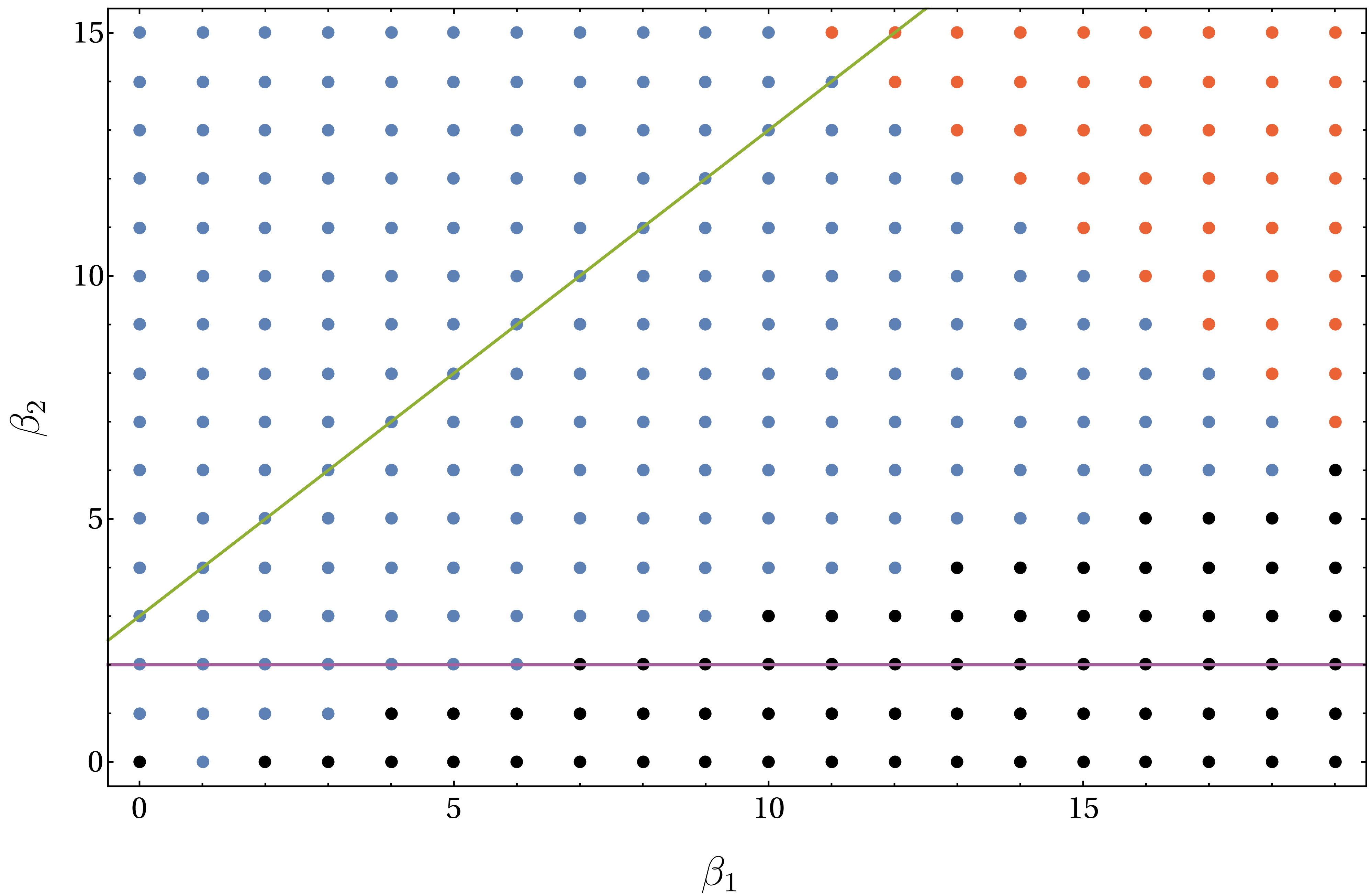


Figure 5: Occupation sites for the CICY 7858.

[Carta, Mininno, Righi & AW '21]

GV invariants & flux compactificaton of string theory

- look at a CICY with 2 Kahler moduli, which is the mirror of a CY X with 2 c.s. moduli:

n_{β_1,β_2} genus-0 GV invariants of CICY $\tilde{X}$ with $h^{1,1}=2$ – gives K and periods of c.s. moduli of mirror CY X

- Kahler potential:

$$
\begin{aligned}
K = -\ln\Bigg(&-\frac{4}{3}\kappa_{ijk}\mathrm{Im}\,(z^i)\mathrm{Im}\,(z^j)\mathrm{Im}\,(z^k)+ic+\\
&-2\sum_{\beta_1,\beta_2}^{\infty} n_{\beta_1,\beta_2}\left(\mathrm{Li}_3\left(e^{i\beta_i z^i}\right)+\mathrm{Li}_3\left(e^{-i\beta_i \overline{z}^i}\right)\right)+\\
&-2\sum_{\beta_1,\beta_2}^{\infty} n_{\beta_1,\beta_2}\beta_i\mathrm{Im}\,(z^i)\left(\mathrm{Li}_2\left(e^{i\beta_i z^i}\right)+\mathrm{Li}_2\left(e^{-i\beta_i \overline{z}^i}\right)\right)\Bigg)
\end{aligned}
$$

GV invariants & flux compactificaton of string theory

- look at a CICY with 2 Kahler moduli and its mirror:

n_{β_1,β_2} genus-0 GV invariants of CICY $\tilde{X}$ with $h^{1,1}=2$ – gives K and periods of c.s. moduli of mirror CY X

- flux superpotential from 3-form fluxes in type IIB string theory:

$$W=(N_F-\tau N_H)^T\cdot\Sigma\cdot\Pi$$

$$\Pi=\begin{pmatrix} 1 \\ z^i \\ \frac{1}{2}\kappa_{ijk}z^jz^k+\frac{1}{2}a_{ij}z^j+b_i-\sum_{\beta_1,\beta_2}^{\infty}n_{\beta_1,\beta_2}\beta_i\mathrm{Li}_2\left(e^{i\beta_iz^i}\right) \\ -\frac{1}{3!}\kappa_{ijk}z^iz^jz^k+b_iz^i+\frac{c}{2}+2i\sum_{\beta_1,\beta_2}^{\infty}n_{\beta_1,\beta_2}\mathrm{Li}_3\left(e^{i\beta_iz^i}\right)-\sum_{\beta_1,\beta_2}^{\infty}n_{\beta_1,\beta_2}\beta_iz^i\mathrm{Li}_2\left(e^{i\beta_iz^i}\right)\end{pmatrix}$$

GV invariants & flux compactificaton of string theory

- compute scalar potential from flux-induced F-terms

  can choose fluxes such, that all 3 of the 4 real scalars of the 2 c.s. moduli of X are stabilized with SUSY unbroken:

$$V = e^K K^{I\bar{J}} D_I W D_{\bar{J}} \overline{W}$$
$$\sim e^{K_0} (K^0)^{I\bar{J}} \Delta D_I W|_{GV} \Delta D_{\bar{J}} \bar{W}|_{GV}$$

$$D_I W = D_I|_0 W_0 + K_{0,I} \Delta W_{GV} + \Delta K_{GV,I} W_0 \equiv D_I W|_0 + \Delta D_I W|_{GV}$$

$$V_{\rm inf}(\varphi) \sim e^{K_0} \kappa\, \varepsilon^2 \left[\sin\left(\frac{M}{N}\varphi + \theta\right)\right]^2 \sim e^{K_0} \kappa\, \varepsilon^2 \left[1 - \cos\left(2\frac{M}{N}\varphi + 2\theta\right)\right]$$

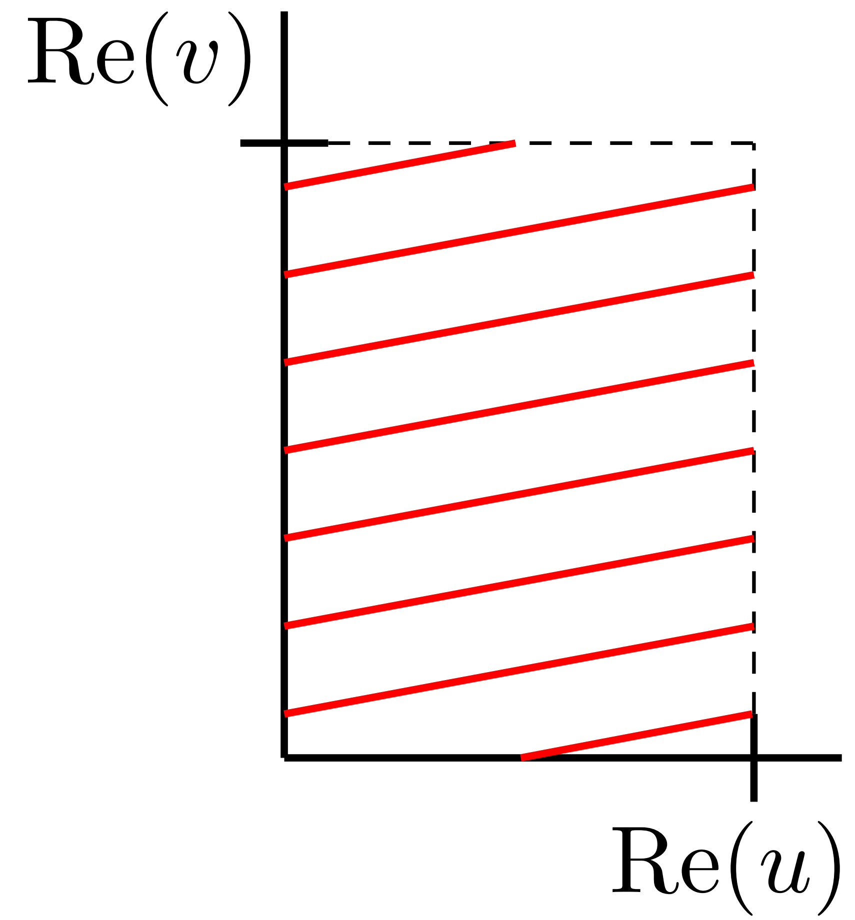
[Hebecker, Mangat, Rompineve & Witkowski '15]

GV invariants & flux compactificaton of string theory

- this is the leading scalar potential if:

$$z_1 = u \text{ and } z_2 = v$$

$$n_{1,0} e^{-\mathrm{Im}\,(u)} \ll n_{0,1} e^{-\mathrm{Im}\,(v)} \ll 1$$



- and hence small parameter:

$$\varepsilon = n_{0,1} e^{-\mathrm{Im}\,(v)}$$

- can do this either by stabilizing VEVs u, v differently, or by hierarchies of GV values:

[Hebecker, Mangat, Rompineve & Witkowski '15]

GV invariants & flux compactificaton of string theory

- can do this either by stabilizing VEVs u, v differently, or by hierarchies of GV values:

$$n_{0,1} \gg n_{1,0}$$

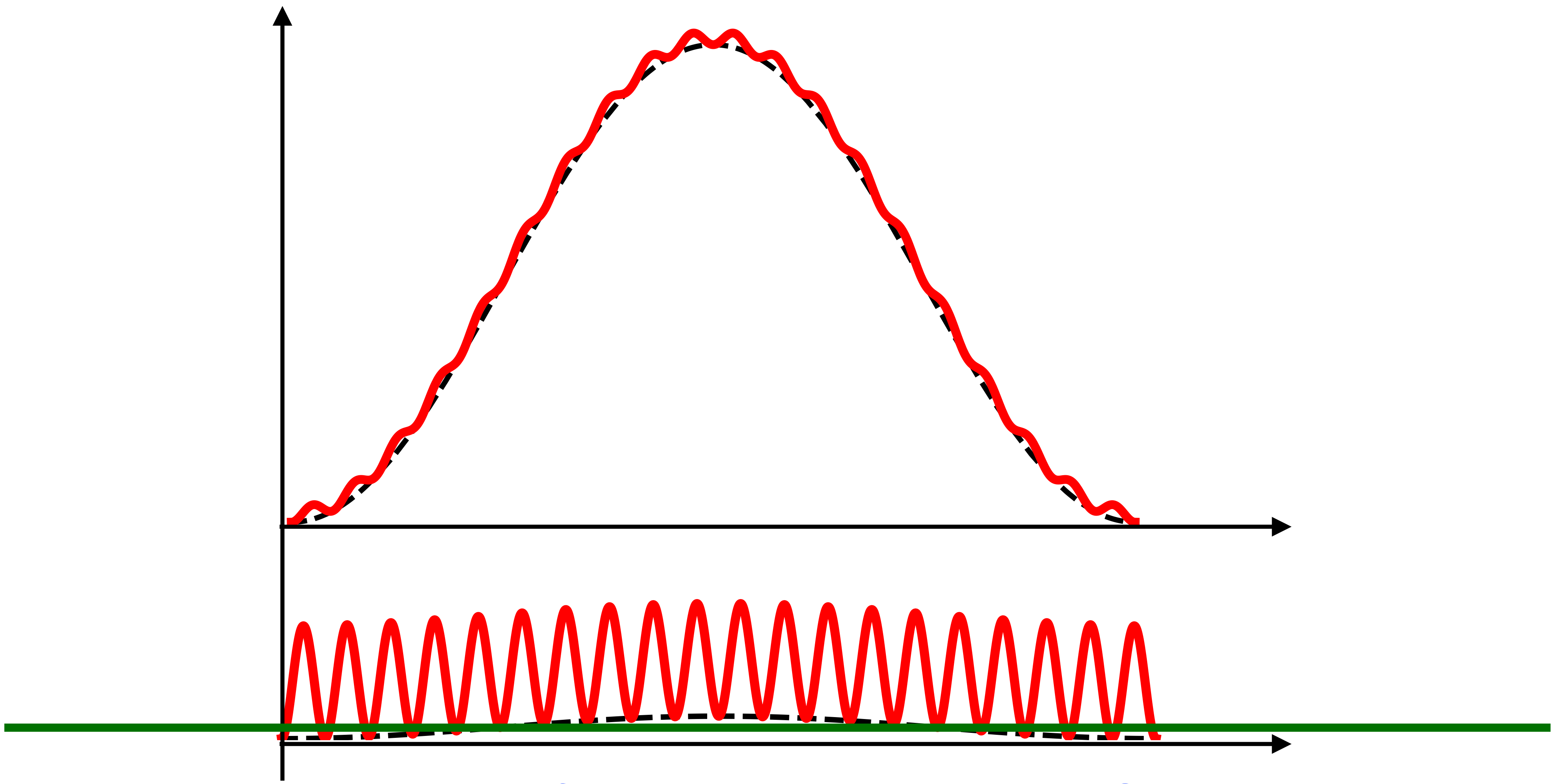
$$\mathrm{Im}\,(u) \sim \mathrm{Im}\,(v) \gg \ln \frac{n_{0,2}}{n_{0,1}}$$

$$\mathrm{Im}\,(u) \gg \ln \frac{n_{1,1}}{n_{0,1}} \quad \text{and} \quad \mathrm{Im}\,(v) \gg \ln \frac{n_{1,1}}{n_{1,0}}$$

- we have searched GV database of CICYs for such hierarchies - and find examples — can help build winding inflation models

[Carta, Mininno, Righi & AW '21]

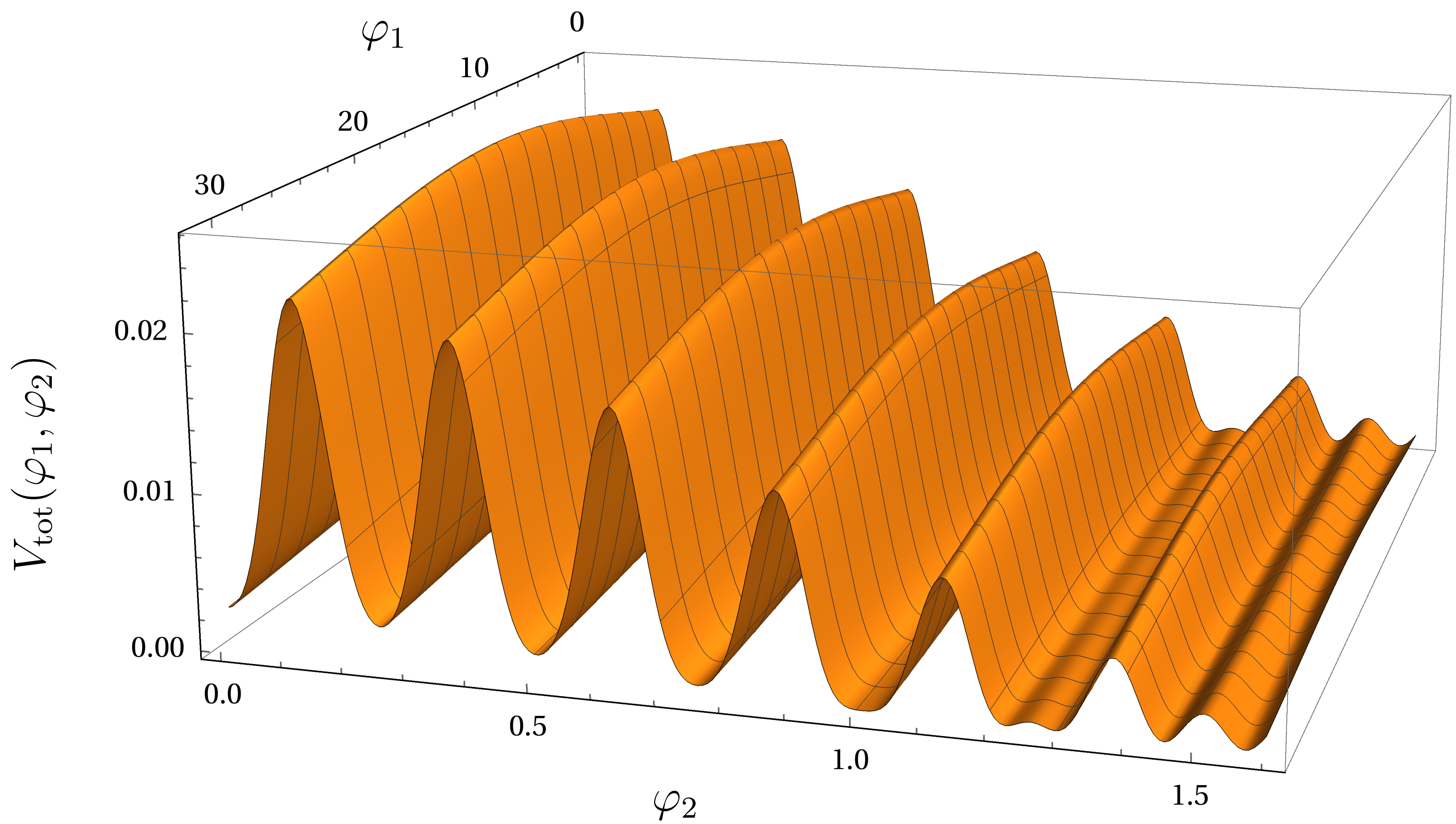
- by looking for different hierarchies among GV invariants, can change potential along shallow valley:



- the last version gives uplift mechanism to de Sitter, if full string model also has negative potential energy contributions (which string vacua always have)

[Hebecker & Leonhardt '20]

- if we look at a CY with 4 moduli, we can combine both sectors!



[Carta, Mininno, Righi & AW '21]