Axion in chiral perturbation

References:

$K^{\pm} \to \pi^{\pm}\pi^{0}$
- hep-ph/0310351, Cirigliano, Ecker, Neufeld, Pich
- 1107.6001, Cirigliano, Ecker, Neufeld, Pich, Portoles
- 2102.09308, Pich, Rodriguez-Sanchez

etc

$K^{\pm} \to \pi^{\pm}a$
- 1710.03764, Alves, Weiner
- 2005.05170, Gori, Perez, Tobioka
- 2102.13112, Bauer, Neubert, Rnner, Schnubel, Thamm

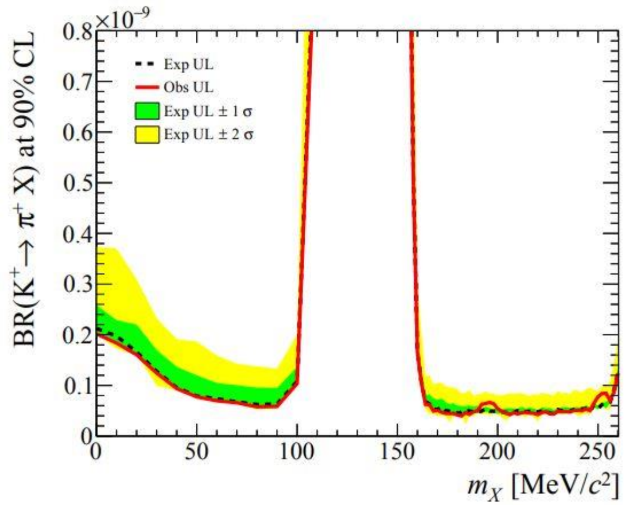
etc

2021. 3. 18 @ DESY journal club

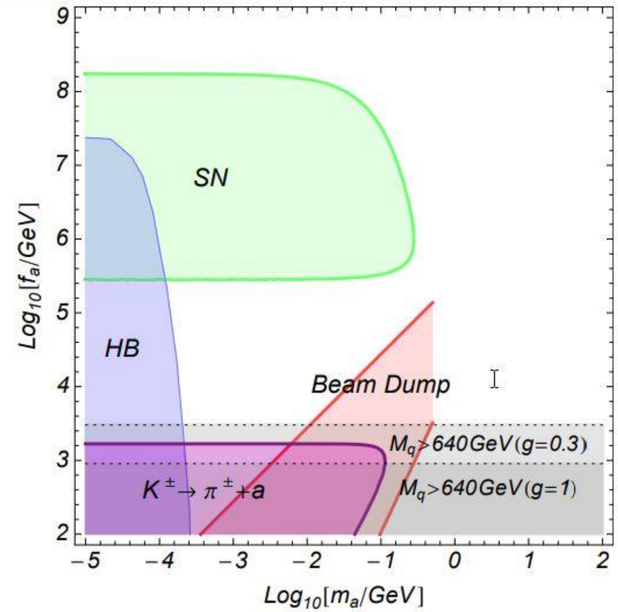
$K^+ \to \pi^+ X$

$$\mathrm{Br}(K^+ \to \pi^+ a) \; < \sim 2 \times 10^{-10}$$

Kaon constraint is important for QCD axion and ALPs.



[NA62 (2011.11329)]



[Fukuda, Harigaya, Ibe, Yanagida (1504.06084)]

2102.13112

Consistent treatment of axions in the weak chiral Lagrangian

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We present a consistent implementation of weak decays involving an axion or axion-like particle in the context of an effective chiral Lagrangian. We argue that previous treatments of such processes have used an incorrect representation of the flavor-changing quark currents in the chiral theory. As an application, we derive model-independent results for the decays $K^- \to \pi^- a$ and $\pi^- \to e^- \bar{\nu}_e a$ at leading order in the chiral expansion and for arbitrary axion couplings and mass. In particular, we find that the $K^- \to \pi^- a$ branching ratio is almost 40 times larger than previously estimated.

40 !?

Naïve formula

$$M(K^+ \to \pi^+ a) \simeq M(K^+ \to \pi^+ \pi^0) \times \theta_{a\pi} \text{ ???}$$

!!! THIS NAÏVE FORMULA IS WRONG !!!

Two things we should know:

1. Octet enhancement ($\Delta I = 1/2$ rule)
   - $(\overline{s_L}\gamma^\mu u_L)(\overline{u_L}\gamma_\mu d_L)$ gives octet (adjoint) and 27-plet operators in ChPT.
   - Octet contributes to $K^+ \to \pi^+ a$
   - Only 27-plet contribute to $K^+ \to \pi^+ \pi^0$

2. Mixing angle is "basis-dependent". Not physical!

1. $K \to \pi\pi$ in ChPT
2. $K \to \pi a$ in ChPT

Chiral Perturbation Theory

- Spontaneously symmetry breaking

$$SU(3)_L \times SU(3)_R \to SU(3)_V \qquad U \to g_L U g_R^\dagger$$

Chiral symmetry Meson field

$$L_0 = \frac{f_\pi^2}{4} \mathrm{tr}[\partial_\mu U \partial^\mu U^\dagger]$$

$$L_1 = a_1 \left(\mathrm{tr}\left[\partial_\mu U \partial^\mu U^\dagger\right]\right)^2 + a_2 \mathrm{tr}\left[\partial_\mu U \partial_\nu U^\dagger\right] \mathrm{tr}\left[\partial^\mu U \partial^\nu U^\dagger\right] + a_3 \mathrm{tr}\left[\partial_\mu U \partial^\mu U^\dagger \partial_\nu U \partial^\nu U^\dagger\right]$$

$$L_2 = \cdots$$

- Explicit breaking of $SU(3)_L \times SU(3)_R$
  - Light quark mass
  - Four-Fermi operator from weak interaction
  - (Electromagnetic interaction)

Those can be treated as VEV of spurion.

Light quark mass in ChPT

- Spontaneously symmetry breaking

$$SU(3)_L \times SU(3)_R \to SU(3)_V \qquad U \to g_L U g_R^\dagger$$

Chiral symmetry   Meson field

$$L_0 = \frac{f_\pi^2}{4}\mathrm{tr}[\partial_\mu U \partial^\mu U^\dagger]$$

- Light quark mass term

UV $\quad L_{mass} = m_{ij} q_{Li}^* q_{Rj} + h.c. \qquad m_{ij}$: $(3, \bar{3})$ of $SU(3)_L \times SU(3)_R$

ChPT $\quad L_{mass} = \frac{f_\pi^2}{2} \mathrm{B}_0 \mathrm{tr}[m^\dagger U + m U^\dagger]$ $\qquad$ Normalization B_0 is determined by meson mass

$$\Rightarrow \quad \begin{aligned} m_\pi^2 &= B_0(m_u + m_d) \\ m_{K^{\pm,0}}^2 &= B_0\left(m_{u,d} + m_s\right) \\ m_\eta^2 &= B_0(m_u + m_d + 4m_s)/3 \end{aligned}$$

4-Fermi operator in ChPT

- Spontaneously symmetry breaking

$$SU(3)_L \times SU(3)_R \to SU(3)_V \qquad U \to g_L U g_R^\dagger$$

Chiral symmetry　　Meson field

$$L_0 = \frac{f_\pi^2}{4}\mathrm{tr}[\partial_\mu U \partial^\mu U^\dagger]$$

- 4-Fermi operator

UV $$L_{4\mathrm{Fermi}} = \frac{4G_F}{\sqrt{2}} V_{ud} V_{us}^* \left(\overline{s_L}\gamma_\mu u_L\right)\left(\overline{u_L}\gamma^\mu d_L\right) + h.c.$$

ChPT $$L_{4\mathrm{Fermi}} = ??$$

Group theory for 4-Fermi operator

See, e.g., 2102.09308

$$O^{ij}_{kl} \equiv (\bar{q}_{Lk}\gamma^{\mu}q_{Li})(\bar{q}_{Ll}\gamma^{\mu}q_{Lj})$$

$i, j, k, l = u, d, s$

Fierz identity : $O^{ij}_{kl} = O^{ji}_{kl} = O^{ij}_{lk} = O^{ji}_{lk}$

ij and kl are symmetric indices. Possible choices are 11, 12, 13, 22, 23, 33.

6-plet

Total number of independent O^{ij}_{kl} : 36

O^{ij}_{kl} are in reducible representation of $SU(3)_L$

$$\longrightarrow \quad 6 \times \bar{6} \rightarrow 27 + 8 + 1$$

Group theory for 4-Fermi operator

See, e.g., 2102.09308

$$(\overline{s_L}\gamma_\mu d_L)(\overline{u_L}\gamma^\mu u_L) = (\overline{s_L}\gamma_\mu d_L)\left(\frac{1}{5}(\overline{u_L}\gamma^\mu u_L)+\frac{1}{5}(\overline{d_L}\gamma^\mu d_L)+\frac{1}{5}(\overline{s_L}\gamma^\mu s_L)\right)$$

8-plet

$$+(\overline{s_L}\gamma_\mu d_L)\left(\frac{4}{5}(\overline{u_L}\gamma^\mu u_L)-\frac{1}{5}(\overline{d_L}\gamma^\mu d_L)-\frac{1}{5}(\overline{s_L}\gamma^\mu s_L)\right)$$

27-plet

$$-\frac{G_F}{\sqrt{2}}V_{ud}V_{us}g_8 f_\pi^4 (L_\mu L^\mu)_{32} - \frac{G_F}{\sqrt{2}}V_{ud}V_{us}g_{27} f_\pi^4 \left(L_{23}L^\mu_{11}+\frac{2}{3}L_{\mu 21}L^\mu_{13}\right)+h.c.$$

$$L_\mu \equiv iU\partial_\mu U^\dagger$$

Kaon decay measurement → $g_8 \simeq 4.99,\quad g_{27}\simeq 0.253$ [1107.6001]

$\Gamma(K_S\to\pi^+\pi^-, 2\pi^0) \gg \Gamma(K^+\to\pi^+\pi^0)$, octet enhancement a.k.a. $\Delta I = 1/2$ rule

1. $K \to \pi\pi$ in ChPT
2. $K \to \pi a$ in ChPT

Simplest case (KSVZ axion)

EFT with light quarks

$$L = i\bar{q}\gamma^\mu\partial_\mu q - m_q\bar{q}q + \left[\frac{4G_F}{\sqrt{2}}V_{ud}V_{us}(\overline{s_L}\gamma^\mu d_L)(\overline{u_L}\gamma^\mu u_L) + h.c.\right]$$

Standard model

Simplest case (KSVZ axion)

EFT with light quarks and axion

$$
\begin{aligned}
L \quad = \quad & i\bar{q}\gamma^{\mu}\partial_{\mu}q - m_q\bar{q}q \\
& + \left[\frac{4G_F}{\sqrt{2}}V_{ud}V_{us}(\overline{s_L}\gamma^{\mu}d_L)(\overline{u_L}\gamma^{\mu}u_L) + h.c.\right] \\
& + \frac{1}{2}(\partial_{\mu}a)^2 + \frac{c_{\gamma\gamma}\alpha}{4\pi}\frac{a}{f}F_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{\alpha_s}{4\pi}\frac{a}{f}G_{\mu\nu}\tilde{G}^{\mu\nu}
\end{aligned}
$$

Standard model (first two lines)

KSVZ axion (third line)

In general, we can add $(\partial_{\mu}a/f)(\bar{q}\gamma^{\mu}\gamma_5 q)$ coupling in this basis.
See, e.g., DFSZ, variant axion, flaxion, ….

Simplest case (KSVZ axion)

EFT with light quarks and axion

$$L = i\bar{q}\gamma^\mu\partial_\mu q - m_q\bar{q}q + \left[\frac{4G_F}{\sqrt{2}}V_{ud}V_{us}(\overline{s_L}\gamma^\mu d_L)(\overline{u_L}\gamma^\mu u_L) + h.c.\right]$$

} Standard model

$$+\frac{1}{2}(\partial_\mu a)^2 + \frac{c_{\gamma\gamma}\alpha}{4\pi}\frac{a}{f}F_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{\alpha_s}{4\pi}\frac{a}{f}G_{\mu\nu}\tilde{G}^{\mu\nu}$$

} KSVZ axion

In general, we can add $(\partial_\mu a/f)(\bar{q}\gamma^\mu\gamma_5 q)$ coupling in this basis.
See, e.g., DFSZ, variant axion, flaxion, ….

Let us take a new basis :

$$q \to \exp\left(-\frac{i\kappa_q a}{f}\gamma_5\right)q \qquad \kappa_u + \kappa_d + \kappa_s = 1$$

Simplest case (KSVZ axion)

EFT with light quarks and axion in a new basis

$$L = i\bar{q}\gamma^{\mu}\partial_{\mu}q - m_q\bar{q}\exp\left(-\frac{2i\kappa_q a}{f}\gamma_5\right)q + \frac{\kappa_q}{f}(\partial_\mu a)\,\bar{q}\gamma^{\mu}\gamma^5 q$$

$$+\left[\frac{4G_F}{\sqrt{2}}V_{ud}V_{us}\exp\left(\frac{i(\kappa_d-\kappa_s)a}{f}\right)(\overline{s_L}\gamma^{\mu}d_L)(\overline{u_L}\gamma^{\mu}u_L) + h.c.\right]$$

$$+\frac{1}{2}(\partial_\mu a)^2 + \frac{\tilde{c}_{\gamma\gamma}\alpha}{4\pi}\frac{a}{f}F_{\mu\nu}\tilde{F}^{\mu\nu}$$

Simplest case (KSVZ axion)

EFT with mesons and axion

$$
\begin{aligned}
L \quad = \quad & \frac{f_\pi^2}{4}\mathrm{tr}\left[D_\mu U D^\mu U^\dagger\right] + \frac{f_\pi^2}{2}B_0 \mathrm{tr}\left[\hat{m}^\dagger U + U^\dagger \hat{m}\right] \\
& + \left[\frac{G_F}{\sqrt{2}}V_{ud}V_{us}g_8 \exp\left(\frac{i(\kappa_d - \kappa_s)a}{f}\right)\mathrm{tr}\left[\lambda_{sd}\left(D_\mu U^\dagger\right)(D^\mu U)\right] + h.c.\right] \\
& + \frac{1}{2}\left(\partial_\mu a\right)^2 + \frac{\tilde{c}_{\gamma\gamma}\alpha}{4\pi}\frac{a}{f}F_{\mu\nu}\tilde{F}^{\mu\nu}
\end{aligned}
$$

Note that $D_\mu U = \partial_\mu U + i\frac{\partial_\mu a}{f}(\kappa U + U\kappa)$

Simplest case (KSVZ axion)

EFT with mesons and axion

$$L = \frac{f_\pi^2}{4}\mathrm{tr}\left[D_\mu U D^\mu U^\dagger\right] + \frac{f_\pi^2}{2}B_0\mathrm{tr}\left[\widehat{m}^\dagger U + U^\dagger \widehat{m}\right]$$

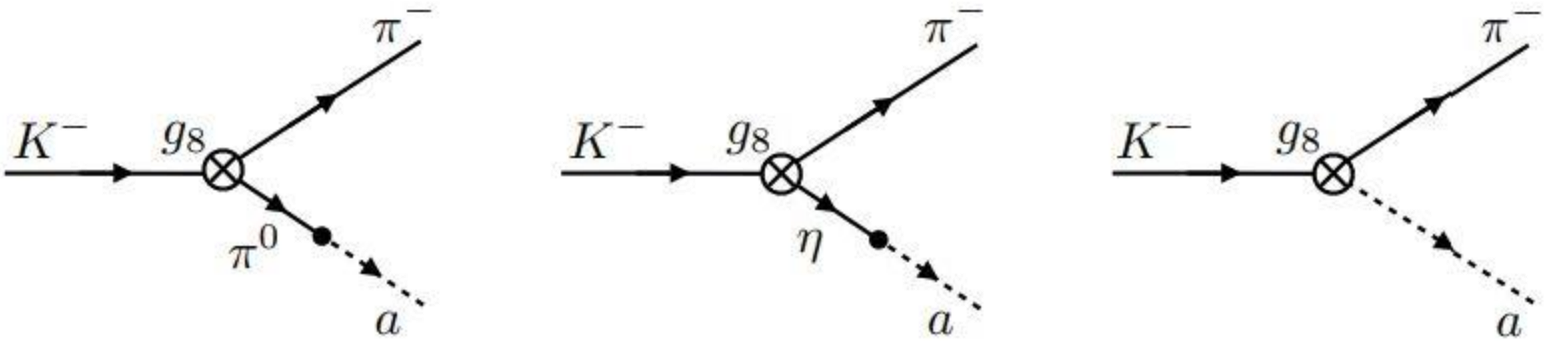
$$+\left[\frac{G_F}{\sqrt{2}}V_{ud}V_{us}g_8 \exp\left(\frac{i(\kappa_d-\kappa_s)a}{f}\right)\mathrm{tr}\left[\lambda_{sd}\left(D_\mu U^\dagger\right)(D^\mu U)\right] + h.c.\right]$$

$$+\frac{1}{2}\left(\partial_\mu a\right)^2 + \frac{\tilde{c}_{\gamma\gamma}\alpha}{4\pi}\frac{a}{f}F_{\mu\nu}\tilde{F}^{\mu\nu}$$

Note that $D_\mu U = \partial_\mu U + i\frac{\partial_\mu a}{f}(\kappa U + U\kappa)$

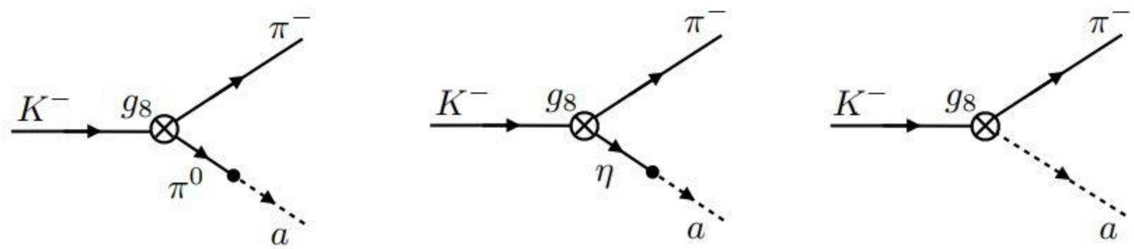
	1/f	$G_F V_{ud} V_{us} g_8$
• Mass mixing between a and π^0, η	✓	
• Kinetic mixing between a and π^0, η	✓	
• $K^\pm\pi^\mp\pi^0,\ K^\pm\pi^\mp\eta$ vertex		✓
• $K^\pm\pi^\mp a$ vertex	✓	✓

diagrams



2102.13112 pointed out $\partial_\mu a$ in $\mathrm{tr}\left[\lambda_{sd}\left(D_\mu U^\dagger\right)(D^\mu U)\right]$

Scheme dependence term

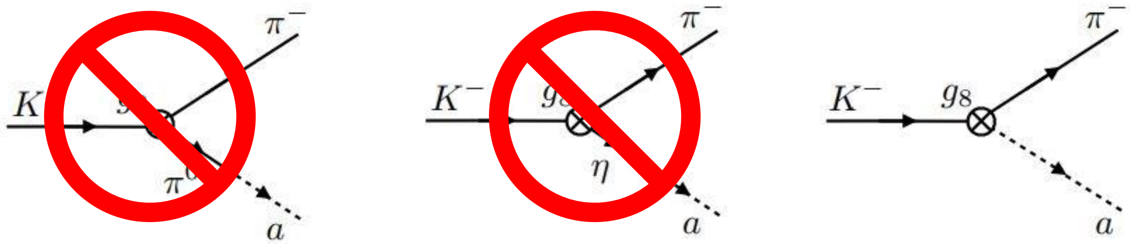


$$-\frac{G_F}{\sqrt{2}}\frac{f_\pi^2}{f_a}m_K^2 V_{ud}V_{us}g_8 \times \left[\quad O\left(\frac{m_\pi^2}{m_K^2}\right) \quad -\frac{1}{3}\kappa_u - \frac{1}{3}\kappa_d + \frac{2}{3}\kappa_s \quad +\kappa_u + \kappa_d \quad \right]$$

- Each diagram is “scheme-dependent”.
- Total amplitude is “scheme-independent”.

A simplified case

$$m_K^2 \gg m_\pi^2, m_a^2$$
$$\kappa_s = 0,$$
$$\kappa_u + \kappa_d = 1$$



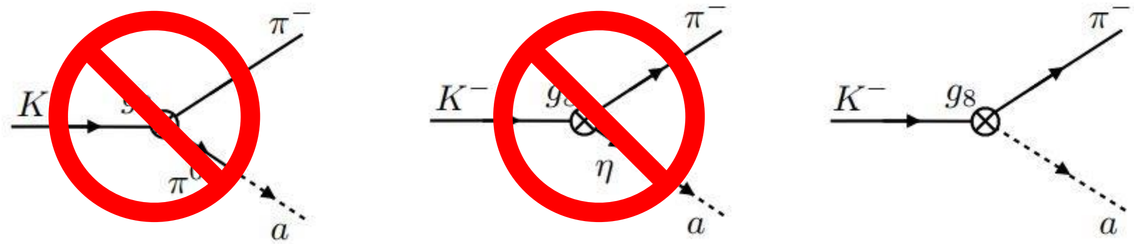
$$i\frac{G_F}{\sqrt{2}} g_8 V_{ud} V_{us} \frac{1}{f_\pi^2 f_a} [\ \kappa_d (\partial K^-)(\partial \pi^+) a \ + \ \kappa_u (\partial K^-) \pi^+ (\partial a) \ - \ K^- (\partial \pi^+)(\partial a)] + h.c.$$

$$\Rightarrow \quad A = \frac{iG_F}{\sqrt{2}} g_8 V_{ud} V_{us} \frac{1}{f_\pi^2 f_a} (\kappa_d + \kappa_u + 1) \frac{m_K^2}{2}$$

This result does not depend on the value of $\kappa_u(\kappa_d)$

A simplified case

$$m_K^2 \gg m_\pi^2, m_a^2$$
$$\kappa_s = 0,$$
$$\kappa_u + \kappa_d = 1$$



$$+ \left[\frac{G_F}{\sqrt{2}} V_{ud} V_{us} g_8 \exp\left(\frac{i(\kappa_d - \kappa_s)a}{f} \right) \mathrm{tr}\left[\lambda_{sd} \left(D_\mu U^\dagger \right) (D^\mu U) \right] + h.c. \right]$$

$$i \frac{G_F}{\sqrt{2}} g_8 V_{ud} V_{us} \frac{1}{f_\pi^2 f_a} [\ \kappa_d (\partial K^-)(\partial \pi^+) a \ + \ \kappa_u (\partial K^-) \pi^+ (\partial a) \ - \ K^-(\partial \pi^+)(\partial a)] + h.c.$$

$$\Rightarrow \quad A = \frac{i G_F}{\sqrt{2}} g_8 V_{ud} V_{us} \frac{1}{f_\pi^2 f_a} (\kappa_d + \kappa_u + 1) \frac{m_K^2}{2}$$

This result does not depend on the value of $\kappa_u (\kappa_d)$

Comparison

$$A = -i\frac{G_F}{\sqrt{2}}\frac{f_\pi^2}{f_a}V_{ud}V_{us}g_8 \times \frac{1}{2}m_K^2$$

[Bauer, Neubert, Renner, Schnubel, Thamm (2021)]

$$A = -i\frac{G_F}{\sqrt{2}}\frac{f_\pi^2}{f_a}V_{ud}V_{us}g_8 \times \frac{1}{2}m_K^2 \times \underbrace{\frac{m_u}{2(m_u+m_d)}}_{0.16}$$

[Georgi, Kaplan, Randall (1986)]

The correct A^2 is 37 times larger than Gerogi-Kaplan-Randall.

Backup

Chiral Perturbation Theory

$$SU(3)_L \times SU(3)_R \to SU(3)_V \qquad U \to g_L U g_R^* \qquad M \to g_L M g_R^*$$

$$\mathcal{L}_{\mathrm{meson}} = \frac{f_\pi^2}{4}\mathrm{tr}\left[D_\mu U D^\mu U^\dagger\right] + \frac{f_\pi^2}{2} B_0 \mathrm{tr}\left[M^\dagger U + U^\dagger M\right], \qquad f_\pi = 93\ \mathrm{MeV}$$

$$U = \exp\left(-\frac{\sqrt{2} i \phi}{f_\pi}\right), \qquad \phi = \begin{pmatrix} \pi^0/\sqrt{2} + \eta^0/\sqrt{6} & \pi^+ & K^+ \\ \pi^- & -\pi^0/\sqrt{2} + \eta^0/\sqrt{6} & K^0 \\ K^- & \bar{K}^0 & -2\eta^0/\sqrt{6} \end{pmatrix},$$

Group theory for 4 Fermi operators

Tensor product

$\mathfrak{su}_3$ tensor products		
$\mathbf{3}\otimes\mathbf{3}$	$=$	$(\overline{\mathbf{6}})_{\mathrm{S}}\oplus(\overline{\mathbf{3}})_{\mathrm{A}}$
$\overline{\mathbf{3}}\otimes\mathbf{3}$	$=$	$(\mathbf{8})\oplus(\mathbf{1})$
$\overline{\mathbf{3}}\otimes\overline{\mathbf{3}}$	$=$	$(\mathbf{6})_{\mathrm{S}}\oplus(\mathbf{3})_{\mathrm{A}}$
$\mathbf{6}\otimes\mathbf{3}$	$=$	$(\overline{\mathbf{15}})\oplus(\overline{\mathbf{3}})$
$\mathbf{6}\otimes\overline{\mathbf{3}}$	$=$	$(\overline{\mathbf{10}})\oplus(\mathbf{8})$
$\mathbf{6}\otimes\mathbf{6}$	$=$	$(\overline{\mathbf{15'}})_{\mathrm{S}}\oplus(\overline{\mathbf{6}})_{\mathrm{S}}\oplus(\overline{\mathbf{15}})_{\mathrm{A}}$
$\overline{\mathbf{6}}\otimes\mathbf{3}$	$=$	$(\mathbf{10})\oplus(\mathbf{8})$
$\overline{\mathbf{6}}\otimes\overline{\mathbf{3}}$	$=$	$(\mathbf{15})\oplus(\mathbf{3})$
$\overline{\mathbf{6}}\otimes\mathbf{6}$	$=$	$(\mathbf{27})\oplus(\mathbf{1})\oplus(\mathbf{8})$
$\overline{\mathbf{6}}\otimes\overline{\mathbf{6}}$	$=$	$(\mathbf{15'})_{\mathrm{S}}\oplus(\mathbf{6})_{\mathrm{S}}\oplus(\mathbf{15})_{\mathrm{A}}$

[Table 1423 (page 10748) in 1511.08771v2]

Fierz identity

$$\sigma^{\mu}_{\alpha\dot{\alpha}}\sigma_{\mu\,\beta\dot{\beta}} = -\sigma^{\mu}_{\alpha\dot{\beta}}\sigma_{\mu\,\beta\dot{\alpha}}\,,$$

$$\overline{\sigma}^{\mu\,\dot{\alpha}\alpha}\overline{\sigma}^{\dot{\beta}\beta}_{\mu} = -\sigma^{\mu\,\dot{\alpha}\beta}\sigma^{\dot{\beta}\alpha}_{\mu}\,,$$

[appendix B.1 in 0812.1594]

Octet enhancement, $\Delta I = ½$ rule

$$\frac{\Gamma(K_S^0 \to \pi\pi)}{\Gamma(K^+ \to \pi^+\pi^0)} \approx 668\,.$$

$$L \sim g_8 V_{ud} V_{us} G_F \left(\partial_\mu K^-\right)(\pi^0 \partial_\mu \pi^+ - \pi^+ \partial_\mu \pi^0)$$

In the limit of $m_{\pi^0} = m_{\pi^\pm}$,
there is no contribution to on-shell amplitude of kaon dacay

$$8_{SU(3)} = (3 + 2 + 2 + 1)_{SU(2)}$$

Short summary of kaon decay

QCD + EW :

$$L_{4\mathrm{Fermi}} = \frac{4G_F}{\sqrt{2}} V_{ud} V_{us}^{*} \left(\overline{s_L}\gamma_\mu u_L\right)\left(\overline{u_L}\gamma^\mu d_L\right)$$

Chiral perturbation

8 $\gg$ **27**

$\Delta I = 1/2$ $\gg$ $\Delta I = 3/2$

$K_S^0 \to \pi^+\pi^-, \pi^0\pi^0$ $\quad$ $K^\pm \to \pi^\pm\pi^0$

$$\Gamma\left(K_S^0 \to \pi^+\pi^-, \pi^0\pi^0\right) = 1.1 \times 10^{10} s^{-1} \gg \Gamma\left(K^+ \to \pi^+\pi^0\right) = 1.8 \times 10^{7} s^{-1}$$