Relating quarks and lepton mass without unification

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Work in progress in colaboration with:

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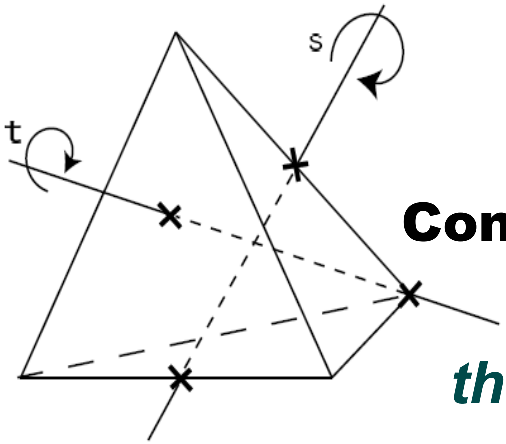
18th International Conference on Supersymmetry and Unification of Fundamental Interactions
Physikalisches Institut, Bonn, Germany
23rd August 2010 - 28th August 2010

A4

the discrete group even permutations four objects



$$S^2 = T^3 = (ST)^3 = 1$$

Contains 4 Irreducible representations

three singlets 1, 1', 1''

one triplet 3

$$S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \qquad T = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

A4

Products of Irreducible representations:

$$1' \times 1' = 1'', \; 1' \times 1'' = 1, \; 1'' \times 1'' = 1'$$

$$1 = a_1 b_1 + a_2 b_2 + a_3 b_3$$

$$1' = a_1 b_1 + \omega^2 a_2 b_2 + \omega a_3 b_3$$

$$1'' = a_1 b_1 + \omega a_2 b_2 + \omega^2 a_3 b_3$$

$$3 \sim (a_2 b_3, a_3 b_1, a_1 b_2)$$

$$3 \sim (a_3 b_2, a_1 b_3, a_2 b_1)$$

$\omega^3 = 1$

Problem of vevs alignments in A4

In general A4 models need two different triplets with different alignments

Babu, Ma and Valle
Phys.Lett.B552:207-213, 2003

$$(1\quad 0\quad 0) \qquad\qquad (1\quad 1\quad 1)$$

To obtain this, Extra dim or Susy with other extra auxiliary symmetries

Altarelli arXiv:0705.0860

Altarelli, Feruglio,
Nucl.Phys.B741:215-235,2006

It is possible to generate correct neutrino mixing angles and fermion masses with only one or two triplets with the same alignment?

Alternative

$$\begin{pmatrix} r & 1 & 1 \end{pmatrix}$$

Quarks

Lavoura and Kuhbock, Eur. Phys. J. C. 55, 303 (2008)

fields	Q_i	u_R / d_R	c_R / s_R	t_R / b_R	ϕ_i
$SU(2)_L$	2	1	1	1	2
A_4	3	1	$1'$	$1''$	3

$$\begin{pmatrix} r & e^{i\alpha} & e^{-i\alpha} \end{pmatrix}$$

Leptons

Morisi and Peinado, Phys. Rev. D80, 113011 (2009)

fields	L_i	l_i^c	ϕ_i
$SU(2)_L$	2	1	2
A_4	3	3	3

Quarks and leptons incompatible with the same r

Possibilities

Two Higgs triplets of A4 with alignments (r,1,1) and (s,1,1)

How to couple one to quarks and the other one to leptons?

Supersymmetric extension:

one couples to Up quarks and neutrinos

The other couples to Down quarks and charged leptons

Does it gives the correct hierarchies in quarks and charged leptons?

Which assignment of irreps to choose?

The Lavoura and khuboch assignment needs the phase and the CP violation is zero.

Let's start with the Unifiable one: everything in triplets

The Model

fields	$\hat{L}$	$\hat{E}^c$	$\hat{Q}$	$\hat{U}^c$	$\hat{D}^c$	$\hat{H}^u$	$\hat{H}^d$
$SU(2)_L$	2	1	2	1	1	2	2
A_4	3	3	3	3	3	3	3

The Model

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$SU(2)_L$	2	1	2	1	1	2	2
A_4	3	3	3	3	3	3	3

All matter fields in triplet irreducible representation

The Model

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$SU(2)_L$	2	1	2	1	1	2	2
A_4	3	3	3	3	3	3	3

Good phenomenology for leptons

Morisi and Peinado, Phys. Rev. D80, 113011 (2009)

The Model

fields	$\hat{L}$	$\hat{E}^c$	$\hat{Q}$	$\hat{U}^c$	$\hat{D}^c$	$\hat{H}^u$	$\hat{H}^d$
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A_4	3	3	3	3	3	3	3

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Morisi and Peinado, Phys. Rev. D80, 113011 (2009)

Same textures as charged leptons

The Model

fields	$\hat{L}$	$\hat{E}^c$	$\hat{Q}$	$\hat{U}^c$	$\hat{D}^c$	$\hat{H}^u$	$\hat{H}^d$
$SU(2)_L$	2	1	2	1	1	2	2
A_4	3	3	3	3	3	3	3

Good phenomenology for leptons

Morisi and Peinado, Phys. Rev. D80, 113011 (2009)

Same textures as charged leptons

Possible relation between Down quarks and Charged leptons

Alignment in the theory

	$H_u = (H_1^u, H_2^u, H_3^u)$	$H_d = (H_1^d, H_2^d, H_3^d)$
$SU(2)$	2	2
A_4	3	3

In the soft susy breaking terms, explicit breaking of the A4 symmetry gives the alignment

$$\langle H^u \rangle = (v^u, \epsilon, \epsilon)$$

$$\langle H^d \rangle = (v^d, \delta, \delta)$$

As a solution of a minimum for the scalar potential

If those A4 soft breaking terms are not introduced the solutions are:

(1 0 0) (1 1 1)

Dirac Fermions

$$
\begin{aligned}
L_{\text{Yukawa}} &= y_1^l \left(\hat{L}_1 \hat{H}_3^d \hat{E}_2^c + \hat{L}_2 \hat{H}_1^d \hat{E}_3^c + \hat{L}_3 \hat{H}_2^d \hat{E}_1^c\right) + y_2^l \left(\hat{L}_1 \hat{H}_2^d \hat{E}_3^c + \hat{L}_2 \hat{H}_3^d \hat{E}_1^c + \hat{L}_3 \hat{H}_1^d \hat{E}_2^c\right) + \\
&+ y_1^d \left(\hat{Q}_1 \hat{H}_3^d \hat{D}_2^c + \hat{Q}_2 \hat{H}_1^d \hat{D}_3^c + \hat{Q}_3 \hat{H}_2^d \hat{D}_1^c\right) + y_2^l \left(\hat{Q}_1 \hat{H}_2^d \hat{D}_3^c + \hat{Q}_2 \hat{H}_3^d \hat{D}_1^c + \hat{Q}_3 \hat{H}_1^d \hat{D}_2^c\right) + \\
&+ y_1^u \left(\hat{Q}_1 \hat{H}_3^u \hat{U}_2^c + \hat{Q}_2 \hat{H}_1^u \hat{U}_3^c + \hat{Q}_3 \hat{H}_2^u \hat{U}_1^c\right) + y_2^u \left(\hat{Q}_1 \hat{H}_2^u \hat{U}_3^c + \hat{Q}_2 \hat{H}_3^u \hat{U}_1^c + \hat{Q}_3 \hat{H}_1^u \hat{U}_2^c\right)
\end{aligned}
$$

Morisi and EP
Phys. Rev. D80,
113011 (2009)

Dirac Fermions

$$L_{\rm Yukawa} = y_1^l \left(\hat{L}_1 \hat{H}_3^d \hat{E}_2^c + \hat{L}_2 \hat{H}_1^d \hat{E}_3^c + \hat{L}_3 \hat{H}_2^d \hat{E}_1^c\right) + y_2^l \left(\hat{L}_1 \hat{H}_2^d \hat{E}_3^c + \hat{L}_2 \hat{H}_3^d \hat{E}_1^c + \hat{L}_3 \hat{H}_1^d \hat{E}_2^c\right) +$$
$$+ y_1^d \left(\hat{Q}_1 \hat{H}_3^d \hat{D}_2^c + \hat{Q}_2 \hat{H}_1^d \hat{D}_3^c + \hat{Q}_3 \hat{H}_2^d \hat{D}_1^c\right) + y_2^l \left(\hat{Q}_1 \hat{H}_2^d \hat{D}_3^c + \hat{Q}_2 \hat{H}_3^d \hat{D}_1^c + \hat{Q}_3 \hat{H}_1^d \hat{D}_2^c\right) +$$
$$+ y_1^u \left(\hat{Q}_1 \hat{H}_3^u \hat{U}_2^c + \hat{Q}_2 \hat{H}_1^u \hat{U}_3^c + \hat{Q}_3 \hat{H}_2^u \hat{U}_1^c\right) + y_2^u \left(\hat{Q}_1 \hat{H}_2^u \hat{U}_3^c + \hat{Q}_2 \hat{H}_3^u \hat{U}_1^c + \hat{Q}_3 \hat{H}_1^u \hat{U}_2^c\right)$$

Morisi and EP
Phys. Rev. D80, 113011 (2009)

$$M_f = \begin{pmatrix} 0 & y_1^f \langle H_3^f \rangle & y_2^f \langle H_2^f \rangle \\ y_2^f \langle H_3^f \rangle & 0 & y_1^f \langle H_1^f \rangle \\ y_1^f \langle H_2^f \rangle & y_2^f \langle H_1^f \rangle & 0 \end{pmatrix}$$

$$\begin{pmatrix} r & 1 & 1 \end{pmatrix}$$

$$M_f = \begin{pmatrix} 0 & a^f & b^f \\ b^f & 0 & a^f r^f \\ a^f & b^f r^f & 0 \end{pmatrix}$$

Dirac Fermions

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&+ y_1^u \left(\hat{Q}_1 \hat{H}_3^u \hat{U}_2^c + \hat{Q}_2 \hat{H}_1^u \hat{U}_3^c + \hat{Q}_3 \hat{H}_2^u \hat{U}_1^c\right) + y_2^u \left(\hat{Q}_1 \hat{H}_2^u \hat{U}_3^c + \hat{Q}_2 \hat{H}_3^u \hat{U}_1^c + \hat{Q}_3 \hat{H}_1^u \hat{U}_2^c\right)
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M_f = \begin{pmatrix} 0 & y_1^f \langle H_3^f \rangle & y_2^f \langle H_2^f \rangle \\ y_2^f \langle H_3^f \rangle & 0 & y_1^f \langle H_1^f \rangle \\ y_1^f \langle H_2^f \rangle & y_2^f \langle H_1^f \rangle & 0 \end{pmatrix}
\qquad
\begin{pmatrix} r & 1 & 1 \end{pmatrix}
\qquad
M_f = \begin{pmatrix} 0 & a^f & b^f \\ b^f & 0 & a^f r^f \\ a^f & b^f r^f & 0 \end{pmatrix}
$$

$$
r^f \approx \frac{m_3}{\sqrt{m_1 m_2}} \qquad a^f \approx \frac{m_2}{m_3}\sqrt{m_1 m_2} \qquad b^f \approx \sqrt{m_1 m_2}
$$

Dirac Fermions

$$
\begin{aligned}
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&+ y_1^d \left(\hat{Q}_1 \hat{H}_3^d \hat{D}_2^c + \hat{Q}_2 \hat{H}_1^d \hat{D}_3^c + \hat{Q}_3 \hat{H}_2^d \hat{D}_1^c\right) + y_2^l \left(\hat{Q}_1 \hat{H}_2^d \hat{D}_3^c + \hat{Q}_2 \hat{H}_3^d \hat{D}_1^c + \hat{Q}_3 \hat{H}_1^d \hat{D}_2^c\right) + \\
&+ y_1^u \left(\hat{Q}_1 \hat{H}_3^u \hat{U}_2^c + \hat{Q}_2 \hat{H}_1^u \hat{U}_3^c + \hat{Q}_3 \hat{H}_2^u \hat{U}_1^c\right) + y_2^u \left(\hat{Q}_1 \hat{H}_2^u \hat{U}_3^c + \hat{Q}_2 \hat{H}_3^u \hat{U}_1^c + \hat{Q}_3 \hat{H}_1^u \hat{U}_2^c\right)
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$$

$$
\begin{pmatrix} r & 1 & 1 \end{pmatrix}
$$

$$
M_f = \begin{pmatrix} 0 & a^f & b^f \\ b^f & 0 & a^f r^f \\ a^f & b^f r^f & 0 \end{pmatrix}
$$

$$
r^f \approx \frac{m_3}{\sqrt{m_1 m_2}} \qquad a^f \approx \frac{m_2}{m_3}\sqrt{m_1 m_2} \qquad b^f \approx \sqrt{m_1 m_2}
$$

$$
r^l = r^d \quad \longrightarrow \quad \frac{m_\tau}{\sqrt{m_e m_\mu}} \approx \frac{m_b}{\sqrt{m_d m_s}}
$$

From the squared mass matrix

$$M_f M_f^T = \begin{pmatrix} a_f^2 + b_f^2 & a_f b_f r_f & a_f b_f r_f \\ a_f b_f r_f & b_f^2 + a_f^2 r_f^2 & a_f b_f \\ a_f b_f r_f & a_f b_f & a_f^2 + b_f^2 r_f^2 \end{pmatrix}$$

Compute the mixing matrices

$$O_{f_{12}} \approx \frac{b}{a} r^{-1}, \quad O_{f_{13}} \approx \frac{a}{b} r^{-1}, \quad O_{f_{23}} \approx \frac{a}{b} r^{-2}$$

$$O_{f_{12}} \approx \sqrt{\frac{m_1}{m_2}}$$

This term gives Correct Cabbibo angle and is responsible also for the small deviations from

$$\theta_{13} = 0$$

The other two CKM mixing angles are close to zero

Neutrinos

Majorana neutrinos and the masses generated by Dim-5 op.

$$\mathcal{L}_{5d} = \beta\left(\hat{L}\hat{L}\right)_3\left(\hat{H}^u\hat{H}^u\right)_3 + k\left(\hat{L}\hat{L}\right)_1\left(\hat{H}^u\hat{H}^u\right)_1 + \alpha'\left(\hat{L}\hat{L}\right)_{1'}\left(\hat{H}^u\hat{H}^u\right)_{1''} + \alpha''\left(\hat{L}\hat{L}\right)_{1''}\left(\hat{H}^u\hat{H}u\right)_{1'} +$$

$$+ \left[a\left(\hat{L}\hat{H}^u\right)_{3^a}\left(\hat{L}\hat{H}^u\right)_{3^a} + b\left(\hat{L}\hat{H}^u\right)_{3^a}\left(\hat{L}\hat{H}^u\right)_{3^b} + c\left(\hat{L}\hat{H}^u\right)_{3^b}\left(\hat{L}\hat{H}^u\right)_{3^a} + d\left(\hat{L}\hat{H}^u\right)_{3^b}\left(\hat{L}\hat{H}^u\right)_{3^b}\right] +$$

$$+ l\left(\hat{L}\hat{H}^u\right)_1\left(\hat{L}\hat{H}^u\right)_1 + l'\left[\left(\hat{L}\hat{H}^u\right)_{1'}\left(\hat{L}\hat{H}^u\right)_{1''} + \left(\hat{L}\hat{H}^u\right)_{1''}\left(\hat{L}\hat{H}^u\right)_{1'}\right],$$

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Phys. Rev. D80, 113011 (2009)

$$M_\nu = \begin{pmatrix} xr^2+y+z & \kappa r^u & \kappa r^u \\ \kappa r^u & zr_u^2+x+y & \kappa \\ \kappa r^u & \kappa & yr_u^2+z+x \end{pmatrix}$$

The neutrino mass matrix in the limit r>>1

$$M_\nu = \begin{pmatrix} xr^2 & \kappa r & \kappa r \\ \kappa r & zr^2 & \kappa \\ \kappa r & \kappa & yr^2 \end{pmatrix}$$

Neutrinos

$$M_\nu = \begin{pmatrix} xr^2 & \kappa r & \kappa r \\ \kappa r & zr^2 & \kappa \\ \kappa r & \kappa & yr^2 \end{pmatrix}$$

z=y

$\mu \longleftrightarrow \tau$

$$V = \begin{pmatrix} -\cos\theta & \sin\theta & 0 \\ \sin\theta/\sqrt{2} & \cos\theta/\sqrt{2} & -1/\sqrt{2} \\ \sin\theta/\sqrt{2} & \cos\theta/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$$

Corrections of order $\sqrt{\frac{m_e}{m_\mu}}$

And corrections from the breaking

z=y

$$\tan 2\theta = -\frac{2\sqrt{2}\kappa r}{(r^2(x-y)-\kappa)}$$

Predictions for the neutrinos:

5 parameters

x, y
relative phase of xy
k and the breaking of y=z

9 observables

$\Delta m^2_{12},\ \Delta m^2_{13},\ m_{ee}$

θ_{13} θ_{23} θ_{12}

CP Jarlskog invariant

2 Majorana phases

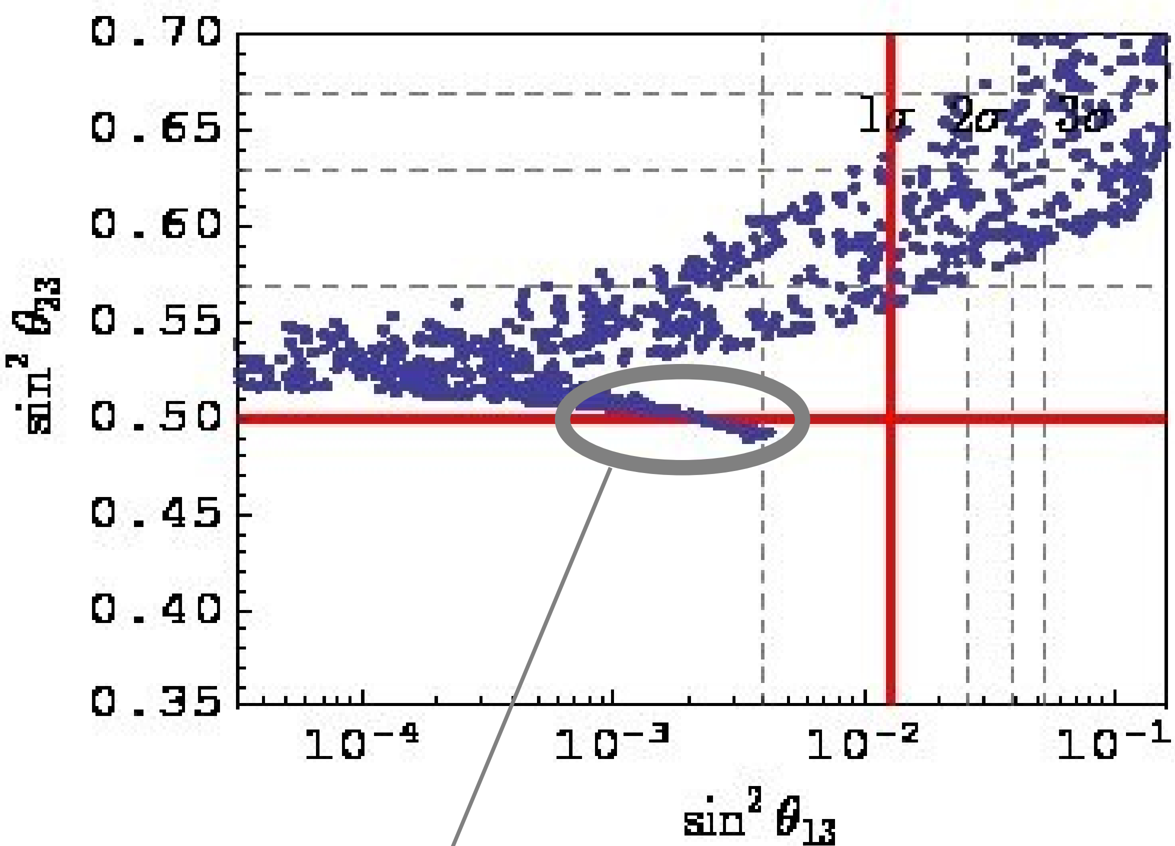
Inverted neutrino mass spectrum

Nearly maximal atmospheric angle

Small deviations for $\theta_{13} = 0$

$$\sqrt{\frac{m_e}{m_\mu}}$$

Mu-Tau breaking



Small y=z breaking zone

Quarks and charged leptons

From the form of the mass matrices:

$$M_f = \begin{pmatrix} 0 & a^f & b^f \\ b^f & 0 & a^f r^f \\ a^f & b^f r^f & 0 \end{pmatrix}$$

$$r^f \approx \frac{m_3}{\sqrt{m_1 m_2}}$$

$$a^f \approx \frac{m_2}{m_3}\sqrt{m_1 m_2} \qquad b^f \approx \sqrt{m_1 m_2}$$

$$\frac{m_\tau}{\sqrt{m_e m_\mu}} \approx \frac{m_b}{\sqrt{m_d m_s}}$$

For quark mixing we obtain a Cabbibo angle prediction:

$$V_{ds} \approx \sqrt{\frac{m_d}{m_s}}$$

The other two mixing angles are small

Conclusions

- A4 Susy extension (possibly unifiable) to explain masses and neutrino mixings with small numbers of free parameters in the low energy sector.
- Small deviations from the max atmospheric and zero reactor angles from the charged leptons $\sqrt{\frac{m_e}{m_\mu}}$
- Prediction of a mass relation among the masses of Down quarks and leptons $\frac{m_\tau}{\sqrt{m_e m_\mu}} \approx \frac{m_b}{\sqrt{m_d m_s}}$
- Prediction of the Cabbibo angle $V_{ds} \approx \sqrt{\frac{m_d}{m_s}}$
- We need to compute the Higgs mass spectrum and pheno, and a mechanism to generate the other two CKM mixing angles.