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Non-supersymmetric Extremal RN-AdS Black Holes in $\mathcal{N} = 2$ Gauged Supergravity

based on arXiv:1005.4607 [hep-th]

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Introduction

Motivation: search Black Hole solutions in 4D $\mathcal{N}=2$ Gauged SUGRA

- WHY $\mathcal{N}=2$ (8-SUSY charges)?
  - ✓ Scalar fields living in highly symmetric spaces
  - ✓ (Flux) compactification scenarios in string/M-theory

- WHY Gauged?
  - ✓ Non-trivial scalar potential giving the cosmological constant

- WHY Black Holes?
  - ✓ Attractive in the study of solutions in 4D $\mathcal{N}=2$ SUGRA
  - ✓ Application to AdS$_4$/CFT$_3$ (or AdS$_4$/CMP$_3$)

Well-known: Extremal RN-BHs in Ungauged SUGRA

BHs in Gauged SUGRA have also been studied in asymptotically non-flat spacetime

Λ: given by bare constant (pure AdS-SUGRA) or by FI parameters

(Notice: Naked singularity appears in SUSY solution unless BH is rotating.)

Romans [hep-th/9203018], Caldarelli-Klemm [hep-th/9808097] etc.

Questions

How can we obtain non-SUSY solutions without FI parameters
in asymptotically non-flat spacetime?

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Action (grav. const. κ; gauge coupling const. g; indices $\Lambda = 0, 1, \ldots, n_{\mathsf{V}}$):

$$
\begin{aligned}
S \;=\; \int \mathrm{d}^4 x \sqrt{-g} \Big\{ & \frac{1}{2\kappa^2} R - G_{a\overline{b}}(z, \overline{z}) \partial_{\mu} z^a \partial^{\mu} \overline{z}^{\overline{b}} - h_{uv}(q) \nabla_{\mu} q^u \nabla^{\mu} q^v \\
& + \frac{1}{4} \mu_{\Lambda\Sigma}(z, \overline{z}) F_{\mu\nu}^{\Lambda} F^{\Sigma \mu\nu} + \frac{1}{4} \nu_{\Lambda\Sigma}(z, \overline{z}) F_{\mu\nu}^{\Lambda} (*F^{\Sigma})^{\mu\nu} \\
& - \mathrm{g}^2 V(z, \overline{z}, q) \\
& + \text{(fermionic terms)} \Big\}
\end{aligned}
$$

$\mu_{\Lambda\Sigma} = \mathrm{Im}\mathcal{N}_{\Lambda\Sigma}$ (generalized $-1/\mathrm{g}^2$) , $\nu_{\Lambda\Sigma} = \mathrm{Re}\mathcal{N}_{\Lambda\Sigma}$ (generalized θ-angle)

Here we do not consider hypermultiplets seriously

Reduce the gauge symmetry to abelian

Equations of Motion (abbreviate κ and g; set fermionic fields to be zero):

$$g_{\mu\nu}: \quad \Big(R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu}\Big) - 2 G_{a\bar{b}}\, \partial_{(\mu} z^a \partial_{\nu)} \overline{z}^{\bar{b}} + G_{a\bar{b}}\, \partial_\rho z^a \partial^\rho \overline{z}^{\bar{b}}\, g_{\mu\nu} \; = \; T_{\mu\nu} - V g_{\mu\nu}$$

$$T_{\mu\nu} \; = \; -\mu_{\Lambda\Sigma}\, F^\Lambda_{\mu\rho}\, F^\Sigma_{\nu\sigma}\, g^{\rho\sigma} + \frac{1}{4} \mu_{\Lambda\Sigma}\, F^\Lambda_{\rho\sigma}\, F^{\Sigma\rho\sigma}\, g_{\mu\nu}$$

(energy-momentum tensor)

$$z^a: \quad -\frac{G_{a\bar{b}}}{\sqrt{-g}} \partial_\mu \Big(\sqrt{-g} g^{\mu\nu} \partial_\nu \overline{z}^{\bar{b}}\Big) - \frac{\partial G_{a\bar{b}}}{\partial \overline{z}^{\bar{c}}}\, \partial_\rho \overline{z}^{\bar{b}}\, \partial^\rho \overline{z}^{\bar{c}}$$

$$= \; \frac{1}{4} \frac{\partial \mu_{\Lambda\Sigma}}{\partial z^a} F^\Lambda_{\mu\nu} F^{\Sigma\mu\nu} + \frac{1}{4} \frac{\partial \nu_{\Lambda\Sigma}}{\partial z^a} F^\Lambda_{\mu\nu} (*F^\Sigma)^{\mu\nu} - \frac{\partial V}{\partial z^a}$$

$$A^\Lambda_\mu: \quad \varepsilon^{\mu\nu\rho\sigma} \partial_\nu G_{\Lambda\rho\sigma} \; = \; 0\,, \qquad G_{\Lambda\rho\sigma} \; = \; \nu_{\Lambda\Sigma} F^\Sigma_{\rho\sigma} - \mu_{\Lambda\Sigma} (*F^\Sigma)_{\rho\sigma}$$

electric charge $q_\Lambda \; \equiv \; \frac{1}{4\pi} \int_{S^2} G_\Lambda\,,$ magnetic charge $p^\Lambda \; \equiv \; \frac{1}{4\pi} \int_{S^2} F^\Lambda$

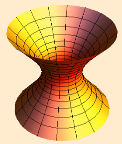
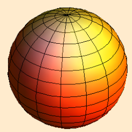
Introduce a metric ansatz for RN(-AdS) BH: "charged", "static", "spherically symmetric"

$$ds^2 \;=\; -e^{2A(r)}dt^2 + e^{2B(r)}dr^2 + e^{2C(r)}r^2\big(d\theta^2 + \sin^2\theta\, d\phi^2\big)$$

$\mathrm{AdS}_2 \times S^2$ as near horizon geometry (radii: r_A and r_H)

$$A(r) \;=\; \log\frac{r - r_\mathsf{H}}{r_\mathsf{A}}\,, \qquad B(r) \;=\; -A(r)\,, \qquad C(r) \;=\; \log\frac{r_\mathsf{H}}{r}$$

$$R(\mathrm{AdS}_2 \times S^2) \;=\; 2\Big(-\frac{1}{r_\mathsf{A}^2} + \frac{1}{r_\mathsf{H}^2}\Big)$$

 × 

$$\rightarrow \quad ds^2(\text{near horizon}) \;=\; -\left(\frac{r - r_\mathsf{H}}{r_\mathsf{A}}\right)^2 dt^2 + \left(\frac{r_\mathsf{A}}{r - r_\mathsf{H}}\right)^2 dr^2 + r_\mathsf{H}^2\big(d\theta^2 + \sin^2\theta\, d\phi^2\big)$$

$$= -\frac{e^{2\tau}}{r_\mathsf{A}^2}dt^2 + r_\mathsf{A}^2 d\tau^2 + r_\mathsf{H}^2\big(d\theta^2 + \sin^2\theta\, d\phi^2\big) \qquad (\tau = \log(r - r_\mathsf{H}))$$

Area of horizon is $A_\mathsf{H} = 4\pi r_\mathsf{H}^2$

If the attractor mechanism works (via extremality), the scalar fields behave as

$$z^{a\prime}\Big|_{\text{horizon}} = 0\,, \qquad z^{a\prime\prime}\Big|_{\text{horizon}} = 0$$

The EoM are drastically reduced to Bellucci et.al. [arXiv:0802.0141]

$$g_{tt}, g_{rr} : \quad \frac{1}{r_{\text{H}}^2} = \frac{1}{r_{\text{H}}^4} I_1 + V\Big|_{\text{horizon}} \quad \Rightarrow \quad r_{\text{H}}^2 = \frac{1 - \sqrt{1 - 4 I_1 V}}{2V}\Big|_{\text{horizon}}$$

$$g_{\theta\theta}, g_{\phi\phi} : \quad \frac{1}{r_{\text{A}}^2} = \frac{1}{r_{\text{H}}^4} I_1 - V\Big|_{\text{horizon}} \quad \Rightarrow \quad r_{\text{A}}^2 = \frac{r_{\text{H}}^2}{\sqrt{1 - 4 I_1 V}}\Big|_{\text{horizon}}$$

$$z^a : \quad 0 = \frac{1}{r_{\text{H}}^4} \frac{\partial I_1}{\partial z^a} - \frac{\partial V}{\partial z^a}\Big|_{\text{horizon}} \quad \Rightarrow \quad 0 = \frac{1}{r_{\text{H}}^4} (1 - 2 r_{\text{H}}^2 V) \frac{\partial}{\partial z^a} r_{\text{H}}^2 \Big|_{\text{horizon}}$$

$$\begin{aligned} I_1(z, \overline{z}, p, q) &= -\frac{1}{2} \begin{pmatrix} p^{\Lambda} & q_{\Lambda} \end{pmatrix} \begin{pmatrix} \mu_{\Lambda\Sigma} + \nu_{\Lambda\Gamma} (\mu^{-1})^{\Gamma\Delta} \nu_{\Delta\Sigma} & -\nu_{\Lambda\Gamma} (\mu^{-1})^{\Gamma\Sigma} \\ -(\mu^{-1})^{\Lambda\Gamma} \nu_{\Gamma\Sigma} & (\mu^{-1})^{\Lambda\Sigma} \end{pmatrix} \begin{pmatrix} p^{\Sigma} \\ q_{\Sigma} \end{pmatrix} \\ &\equiv -\frac{1}{2} \Gamma^{\text{T}} \mathbb{M} \, \Gamma \end{aligned}$$

1st symplectic invariant

Black Hole Entropy is given as the Area of the horizon as in the case of RN-BH:

$$S_{\mathsf{BH}}(p,q) \;=\; \frac{A_{\mathsf{H}}}{4\pi} \;=\; r_{\mathsf{H}}^2\Big|_{\mathsf{horizon}} \;\equiv\; V_{\mathsf{eff}}(z,\overline{z},p,q)\Big|_{\mathsf{horizon}}$$

$$V_{\mathsf{eff}}(z,\overline{z},p,q) \;=\; \frac{1-\sqrt{1-4I_1 V}}{2V}$$

$V_{\mathsf{eff}} \to I_1$ (if $V \to 0$)

$$0 \;=\; \frac{1}{r_{\mathsf{H}}^4}(1-2r_{\mathsf{H}}^2 V)\frac{\partial}{\partial z^a}V_{\mathsf{eff}}\Big|_{\mathsf{horizon}}$$

We read the "cosmological constant Λ" from the scalar curvature:

$$R(\mathrm{AdS}_2\times S^2) \;=\; 2\Big(-\frac{1}{r_{\mathsf{A}}^2}+\frac{1}{r_{\mathsf{H}}^2}\Big) \;=\; 4V$$

$$V\Big|_{\mathsf{horizon}} \;\equiv\; \Lambda(\text{"cosmological constant"})$$

The "attractor equation" which we have to solve is $\quad 0 \;=\; \frac{\partial}{\partial z^a}V_{\mathsf{eff}}(z,\overline{z},p,q)\Big|_{\mathsf{horizon}}$

(If r_{H} is finite and if Λ is non-positive)

The "attractor equation" which we have to solve is

$$0 = \frac{\partial}{\partial z^a} V_{\text{eff}}(z, \overline{z}, p, q)\Big|_{\text{horizon}}$$

$$= \frac{1}{2V^2\sqrt{1 - 4I_1 V}} \left\{ 2V^2 \frac{\partial I_1}{\partial z^a} - \left(\sqrt{1 - 4I_1 V} + 2I_1 V - 1\right) \frac{\partial V}{\partial z^a} \right\} \Big|_{\text{horizon}}$$

Evaluate I_1 and V: Description in terms of the central charge Z

Useful when we consider (non-)SUSY solutions

Def. of Z comes from the SUSY variation of gravitini:

$$\delta\psi_{A\mu} = D_\mu \varepsilon_A + \epsilon_{AB} T^-_{\mu\nu} \gamma^\nu \varepsilon^B + ig \mathcal{S}_{AB} \gamma_\mu \varepsilon^B + (\text{fermionic fields})$$

$$Z = -\frac{1}{2}\left(\frac{1}{4\pi}\int_{S^2} T^-\right), \qquad \mathcal{S}_{AB} = \frac{i}{2}(\sigma_x)_{AB} \mathcal{P}^x$$

Use the property of the Special Kähler geometry

Mainly we use the followings (The basic variables are X^Λ and $\mathcal{F}_\Lambda$):

$$\mathcal{F}_\Lambda = \frac{\partial \mathcal{F}}{\partial X^\Lambda}, \qquad z^a = \frac{X^a}{X^0}$$

$$K = -\log \left[i(\overline{X}^\Lambda \mathcal{F}_\Lambda - X^\Lambda \overline{\mathcal{F}}_\Lambda) \right], \qquad G_{a\bar{b}} = \frac{\partial}{\partial z^a}\frac{\partial}{\partial \overline{z}^{\bar{b}}} K$$

$$\Pi = \mathrm{e}^{K/2} \begin{pmatrix} X^\Lambda \\ \mathcal{F}_\Lambda \end{pmatrix} = \begin{pmatrix} L^\Lambda \\ M_\Lambda \end{pmatrix}, \qquad D_a \Pi = \left(\frac{\partial}{\partial z^a} + \frac{1}{2}\frac{\partial K}{\partial z^a} \right) \Pi = \begin{pmatrix} f_a^\Lambda \\ h_{\Lambda a} \end{pmatrix}$$

$$M_\Lambda = \mathcal{N}_{\Lambda\Sigma} L^\Sigma, \qquad h_{\Lambda a} = \overline{\mathcal{N}}_{\Lambda\Sigma} f_a^\Sigma, \qquad G^{a\bar{b}} f_a^\Lambda f_{\bar{b}}^\Sigma = -\frac{1}{2}\mathrm{Im}(\mathcal{N}^{-1})^{\Lambda\Sigma} - \overline{L}^\Lambda L^\Sigma$$

Write down Z, I_1 and V in terms of $(L^\Lambda, M_\Lambda) = \mathrm{e}^{K/2}(X^\Lambda, \mathcal{F}_\Lambda)$:

$$Z = L^\Lambda q_\Lambda - M_\Lambda p^\Lambda$$

$$I_1 = |Z|^2 + G^{a\bar{b}} D_a Z \overline{D_b Z}$$

$$V = \sum_{x=1}^{3} \Big(-3|\mathcal{P}^x|^2 + G^{a\bar{b}} D_a \mathcal{P}^x \overline{D_b \mathcal{P}^x} \Big) + 4 h_{uv} k^u \bar{k}^v$$

$\mathcal{P}^x_\Lambda$, $\widetilde{\mathcal{P}}^{x\Lambda}$: $SU(2)$ triplet of Killing prepotentials in $\mathcal{N}=2$ SUGRA

$$\mathcal{P}^x = \mathcal{P}^x_\Lambda L^\Lambda - \widetilde{\mathcal{P}}^{x\Lambda} M_\Lambda \quad \text{in } \mathcal{S}_{AB} \qquad (x = 1,2,3)$$

If no hypermultiplets, only $\mathcal{P}^3 = \mathcal{P}^3_\Lambda L^\Lambda - \widetilde{\mathcal{P}}^{3\Lambda} M_\Lambda$ contributes to the potential.

Further, we could identify $(\mathcal{P}^3_\Lambda, \widetilde{\mathcal{P}}^{3\Lambda}) = (q_\Lambda, p^\Lambda) \rightsquigarrow \mathcal{P}^3 \equiv Z$ Cassani et.al. [arXiv:0911.2708]

$$V = -3|Z|^2 + G^{a\bar{b}} D_a Z \overline{D_b Z}$$

Rewrite the "attractor equation" in terms of the central charge:

$$
\begin{aligned}
0 &= \frac{\partial}{\partial z^a} V_{\text{eff}}(z, \overline{z}, p, q) \Big|_{\text{horizon}} \\
&= \frac{1}{2V^2\sqrt{1-4I_1 V}} \Big\{ 2V^2 \frac{\partial I_1}{\partial z^a} - \big(\sqrt{1-4I_1 V} + 2I_1 V - 1 \big) \frac{\partial V}{\partial z^a} \Big\} \Big|_{\text{horizon}} \\
&= \frac{1+V_{\text{eff}}^2}{\sqrt{1-4I_1 V}} \Big\{ 2 G_{\text{V}} \overline{Z} D_a Z + i C_{abc} G^{b\overline{b}} G^{c\overline{c}} \, \overline{D_b Z} \, \overline{D_c Z} \Big\} \Big|_{\text{horizon}}
\end{aligned}
$$

A Non-trivial factor $G_{\text{V}} = \dfrac{1 - V_{\text{eff}}^2}{1 + V_{\text{eff}}^2}$

If $\Lambda < 0$ and $D_a Z = 0$ (SUSY) $\rightarrow$ Naked Singularity $\rightarrow$ Search non-SUSY sol. $D_a Z \neq 0$

If $\partial_a I_1 = 0$ or $\partial_a V = 0$ $\rightarrow$ $V|_{\text{horizon}} = \Lambda = 0$, or Empty Hole $Z|_{\text{horizon}} = 0$

If $G_{\text{V}} = 0$ $\rightarrow$ $S_{\text{BH}} = 1$ (strange!)

Rewrite the "attractor equation" in terms of the central charge:

$$
\begin{aligned}
0 &= \frac{\partial}{\partial z^a} V_{\text{eff}}(z, \overline{z}, p, q) \Big|_{\text{horizon}} \\
&= \frac{1}{2V^2 \sqrt{1 - 4I_1 V}} \left\{ 2V^2 \frac{\partial I_1}{\partial z^a} - \left(\sqrt{1 - 4I_1 V} + 2I_1 V - 1 \right) \frac{\partial V}{\partial z^a} \right\} \Bigg|_{\text{horizon}} \\
&= \frac{1 + V_{\text{eff}}^2}{\sqrt{1 - 4I_1 V}} \left\{ 2G_{\mathsf{V}} \overline{Z} D_a Z + i C_{abc} G^{b\overline{b}} G^{c\overline{c}} \, \overline{D_b Z} \, \overline{D_c Z} \right\} \Bigg|_{\text{horizon}}
\end{aligned}
$$

Solve the equation $0 = 2G_{\mathsf{V}} \overline{Z} D_a Z + i C_{abc} G^{b\overline{b}} G^{c\overline{c}} \, \overline{D_b Z} \, \overline{D_c Z} \Big|_{\text{horizon}}$

under the condition $V < 0$, $1 - 4I_1 V > 0$, $\partial_a I_1 \neq 0$, $\partial_a V \neq 0$, $D_a Z \neq 0$

Contents

- Consider the single modulus model w/ charges $\Gamma = (0, p, 0, q_0)$ ("D0-D4" system):

Holomorphic central charge $W = \mathrm{e}^{-K/2} Z$ and its discriminant $\Delta(W)$ are

$$\mathcal{F} = \frac{(X^1)^3}{X^0}, \qquad t = \frac{X^1}{X^0}; \qquad W = q_0 - 3p\, t^2, \qquad \Delta(W) = 12 p q_0$$

The attractor equation and its solution ($t = 0 + iy$, $y < 0$):

$$p(y^2)^3 + (q_0 - 18p^3 q_0^2)(y^2)^2 - 12p^2 q_0^3 (y^2) - 2pq_0^4 = 0$$

$$y^2 = A + B \quad \text{or} \quad A + \omega^{\pm} B \qquad (\omega^3 = 1)$$

$$A = \frac{q_0}{3p}(18p^3 q_0 - 1), \qquad B = \frac{1}{3p}\left(C^{1/3} + \frac{q_0^2}{4} \frac{1 + (18p^3 q_0)^2}{C^{1/3}} \right)$$

$$C = -q_0^3 \left[1 - 27p^3 q_0 - (18p^3 q_0)^3 - 3\sqrt{3}\sqrt{-2p^3 q_0 - 9(p^3 q_0)^2 - 432(p^3 q_0)^3} \right]$$

with $pq_0 < 0$

Various values at the horizon are

$$Z\Big|_{\text{horizon}} = \frac{q_0 + 3p\,y^2}{2}\sqrt{-\frac{1}{2y^3}} \neq \mathbf{0}\,, \qquad D_t Z\Big|_{\text{horizon}} = \frac{3i(q_0 - p\,y^2)}{4y}\sqrt{-\frac{1}{2y^3}} \neq \mathbf{0}$$

$$I_1 = \frac{q_0^2 + 3p^2 y^4}{-2y^3} > \mathbf{0}$$

$$\Lambda = \frac{6(pq_0)^2(q_0 + 3p\,y^2)^2}{y^5} < \mathbf{0}$$

$$S_{\text{BH}} = \frac{-y}{12(pq_0)^2(q_0 + 3p\,y^2)^2}\Big\{-y^4 + \sqrt{y^8 + 12(pq_0)^2(q_0 + 3p\,y^2)^2(q_0^2 + 3p^2\,y^4)}\Big\} > \mathbf{0}$$

Focus on the Large q_0 limit:

The dominant part ot the Modulus $t = 0 + iy$ $(y < 0)$ is

$$y \;\sim\; pq_0 + \text{(sub-leading orders)}$$

The dominant parts of various values are

$$Z\Big|_{\text{horizon}} \;\sim\; \sqrt{-p^3 q_0} + \ldots \;\neq\; \mathbf{0}\,, \qquad D_t Z\Big|_{\text{horizon}} \sim \frac{-i}{pq_0}\sqrt{-p^3 q_0} + \ldots \;\neq\; \mathbf{0}$$

$$I_1 \;\sim\; -p^3 q_0 + \ldots \;>\; \mathbf{0}$$

$$\Lambda \;\sim\; p^3 q_0 + \ldots \;<\; \mathbf{0} \qquad \text{(up to overall factors)}$$

$$S_{\text{BH}} \;\sim\; \mathcal{O}(1) + \ldots \;>\; \mathbf{0} \qquad ?$$

Strange behaviors of Λ and S_{BH}: incorrect expansions?

Look at the Small q_0 limit:

The dominant part of the Modulus $t = 0 + iy$ $(y < 0)$ is

$$y \sim -\sqrt{-\frac{q_0}{p}} + \text{(sub-leading orders)}$$

The dominant parts of various values are

$$Z\Big|_{\text{horizon}} \sim q_0 \left(-\frac{p^3}{q_0^3}\right)^{1/4} + \ldots \neq \mathbf{0}, \qquad D_t Z\Big|_{\text{horizon}} \sim ip \left(-\frac{p}{q_0}\right)^{1/4} + \ldots \neq \mathbf{0}$$

$$I_1 \sim \sqrt{-p^3 q_0} + \ldots > \mathbf{0}$$

$$\Lambda \sim -\sqrt{(-p^3 q_0)^3} + \ldots < \mathbf{0}$$

(up to overall factors)

$$S_{\text{BH}} \sim \sqrt{-p^3 q_0} + \ldots > \mathbf{0}$$

Very small $|\Lambda|$ compared to others: similar to the non-BPS RN-BH sol.

Comparison: the values at the attractor point of RN-BH w/ $\Lambda = 0$:

- non-BPS solution is given as

$$t = 0 + iy \,, \qquad y = -\sqrt{-\frac{q_0}{p}}$$

$$Z\Big|_{\text{horizon}} = -\frac{q_0}{\sqrt{2}}\Big(-\frac{p^3}{q_0^3}\Big)^{1/4} \neq 0 \,, \qquad D_t Z\Big|_{\text{horizon}} = -3ip\Big(-\frac{p}{q_0}\Big)^{1/4} \neq 0$$

$$S_{\text{BH}} = I_1 = |Z|^2 + G^{t\bar{t}} D_t Z \overline{D_t Z} = 4|Z|^2 = \sqrt{-4p^3 q_0} > 0 \,, \qquad \Lambda = 0$$

- 1/2-BPS solution is given as

$$t = 0 + iy \,, \qquad y = -\sqrt{\frac{q_0}{p}}$$

$$Z\Big|_{\text{horizon}} = \sqrt{2} q_0 \Big(\frac{p^3}{q_0^3}\Big)^{1/4} \neq 0 \,, \qquad D_t Z\Big|_{\text{horizon}} = 0$$

$$S_{\text{BH}} = I_1 = |Z|^2 = \sqrt{4p^3 q_0} > 0 \,, \qquad \Lambda = 0$$

Contents

☑ Studied Extremal RN-AdS Black Hole solutions in Abelian gauged SUGRA

☑ Described the non-SUSY solution of the D0-D4 system in the T^3-model
(see the D2-D6 system in Appendix)

⇨ Different behavior of the modulus, BH entropy, etc.

☞ Description in all region in the asymptotically non-flat spacetime?

☞ Include (charged) hypermultiplets?

Hristov-Looyestijn-Vandoren [arXiv:1005.3650] (constant sol. of Behrndt-Lüst-Sabra–type, etc.)

Cassani-Ferrara-Marrani-Morales-Samtleben [arXiv:0911.2708] (nongeometric flux compactifications)

Fin

Appendix

Study charged Black Hole solutions

in "4D", "Asymptotically (non-)flat", "Static", "Spherically Symmetric" spacetime:

$$ds^2 = -V(r)dt^2 + \frac{1}{V(r)}dr^2 + r^2\big(d\theta^2 + \sin^2\theta\, d\phi^2\big)$$

$$V(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{\Lambda r^2}{3}, \qquad Q^2 = \underset{\text{(ele.)}}{q^2} + \underset{\text{(mag.)}}{p^2}, \qquad \Lambda = \text{(cosmological constant)}$$

"flat Minkowski" :	$M = Q = \Lambda = 0$
Schwarzschild :	$M \neq 0,\ Q = \Lambda = 0$
Schwarzschild-AdS :	$M \neq 0,\ Q = 0,\ \Lambda = -\frac{3}{\ell^2} < 0$
Reissner-Nordström (RN) :	$M \neq 0,\ Q \neq 0,\ \Lambda = 0$
RN-AdS :	$M \neq 0,\ Q \neq 0,\ \Lambda = -\frac{3}{\ell^2} < 0$

Supersymmetric multilpets in 4D $\mathcal{N}=2$ SUGRA:

1 graviton multiplet: $\{g_{\mu\nu}, A_\mu^0, \psi_{A\mu}\}$ — $\mu = 0,1,2,3$ (4D, curved), $A = 1,2$ ($SU(2)$ R-symmetry)

n_{V} vector multiplets: $\{A_\mu^a, z^a, \lambda^{aA}\}$ — $a = 1, \ldots, n_{\mathsf{V}}$

z^a in special Kähler geometry $\mathcal{SM}$

$n_{\mathsf{H}}+1$ hypermultiplets: $\{q^u, \zeta^\alpha\}$ — $u = 1, \ldots, 4n_{\mathsf{H}}+4$, $\alpha = 1, \ldots, 2n_{\mathsf{H}}+2$

q^u in quaternionic geometry $\mathcal{HM}$

Gauging: PROMOTE global symmetries from isometry groups on $\mathcal{SM}$ and $\mathcal{HM}$ to local symmetries

Ref.: Andrianopoli et.al. [hep-th/9605032]

Identity

A useful formula among the BH charges $\Gamma = (p^\Lambda, q_\Lambda)^{\rm T}$ and the invariant $I_1(z, \overline{z}, p, q)$

$$\Gamma^{\rm T} + i\frac{\partial I_1}{\partial \widetilde{\Gamma}} \;=\; 2i\overline{Z}\Pi^{\rm T} + 2iG^{a\overline{b}} D_a Z \overline{D_b \Pi}^{\rm T}$$

$$\widetilde{\Gamma} = \begin{pmatrix} 0 & \mathbb{1} \\ -\mathbb{1} & 0 \end{pmatrix} \Gamma \,, \qquad \Pi = \begin{pmatrix} L^\Lambda \\ M_\Lambda \end{pmatrix} \,, \qquad Z = L^\Lambda q_\Lambda - M_\Lambda p^\Lambda = \widetilde{\Gamma}^{\rm T} \Pi$$

Kallosh et.al. [hep-th/0606263]

This does not (explicitly) depend on the scalar potential $-{\rm g}^2 V$.

This can be applied to any points in the spcatime.

$$G^{a\overline{b}} D_a \Pi \otimes \overline{D_b \Pi}^{\rm T} \;=\; -\overline{\Pi} \otimes \Pi^{\rm T} - \frac{i}{2} \begin{pmatrix} 0 & \mathbb{1} \\ -\mathbb{1} & 0 \end{pmatrix} - \frac{1}{2} \widetilde{\mathbb{M}}_{\rm V}$$

$$\widetilde{\mathbb{M}}_{\rm V} \;\equiv\; \begin{pmatrix} (\mu^{-1})^{\Lambda\Sigma} & (\mu^{-1})^{\Lambda\Gamma} \nu_{\Gamma\Sigma} \\ \nu_{\Lambda\Gamma} (\mu^{-1})^{\Gamma\Sigma} & \mu_{\Lambda\Sigma} + \nu_{\Lambda\Gamma} (\mu^{-1})^{\Gamma\Delta} \nu_{\Delta\Sigma} \end{pmatrix} = \begin{pmatrix} 0 & \mathbb{1} \\ -\mathbb{1} & 0 \end{pmatrix} \mathbb{M}_{\rm V} \begin{pmatrix} 0 & -\mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}$$

$$I_1 \;=\; -\frac{1}{2} \Gamma^{\rm T} \mathbb{M}_{\rm V} \Gamma \;=\; -\frac{1}{2} \widetilde{\Gamma}^{\rm T} \widetilde{\mathbb{M}}_{\rm V} \widetilde{\Gamma} \,, \qquad \frac{\partial I_1}{\partial \widetilde{\Gamma}} \;=\; -\widetilde{\Gamma}^{\rm T} \widetilde{\mathbb{M}}_{\rm V}$$

- Single modulus model ($a = 1$): $\mathcal{F} = \dfrac{(X^1)^3}{X^0}$

$$Z = \mathrm{e}^{K/2}\Big(q_0 + q\,t - 3p\,t^2 + p^0\,t^3\Big)\,, \qquad t = \frac{X^1}{X^0}$$

$$\mathrm{e}^K = \frac{i}{(t-\bar{t})^3}\,, \qquad G_{t\bar{t}} = -\frac{3}{(t-\bar{t})^2} \equiv e_t{}^{\widehat{1}}\, e_{\bar{t}}{}^{\bar{\widehat{1}}}\, \delta_{\widehat{1}\bar{\widehat{1}}}\,, \qquad C_{ttt} = \frac{6i}{(t-\bar{t})^3}$$

Search the sol. w/ $V = -3|Z|^2 + |D_{\widehat{1}}Z|^2 < 0 \to Z \neq 0$

Consider non-SUSY sol. $\to D_{\widehat{1}}Z \neq 0$

$\Downarrow$

The generic forms of the central charge and its derivative:

$$Z \equiv -i\rho\,\mathrm{e}^{i(\alpha-3\phi)}\,, \qquad D_{\widehat{1}}Z \equiv \sigma\,\mathrm{e}^{-i\phi} \qquad (\rho, \sigma > 0)$$

[hep-th/0606263]

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The volume factors ρ and σ are related via the attractor equation.

$$\sigma = -\frac{\rho}{3} \mathrm{e}^{-i\alpha} G_{\mathsf{V}} \qquad (G_{\mathsf{V}} \neq 0)$$

The formula leads to the following two equations: $(\Gamma = (p^0, p, q, q_0)^{\mathrm{T}})$:

$$p + \frac{\partial I_1}{\partial q} = -\frac{2\rho}{3\sqrt{3}} \mathrm{e}^{-i\alpha} \mathrm{e}^{K/2} \Big[\big(3\sqrt{3} - 2G_{\mathsf{V}}\big)\, t - G_{\mathsf{V}}\, \bar{t} \Big]$$

$$p^0 + \frac{\partial I_1}{\partial q_0} = -\frac{2\rho}{3\sqrt{3}} \mathrm{e}^{-i\alpha} \mathrm{e}^{K/2} \big(3\sqrt{3} - G_{\mathsf{V}}\big)$$

$$\rightarrow \qquad t = \frac{3\sqrt{3} - 2G_{\mathsf{V}}}{3\sqrt{3} - G_{\mathsf{V}}} \left[\frac{p + i\frac{\partial I_1}{\partial q}}{p^0 + i\frac{\partial I_1}{\partial q_0}} \right] + \frac{G_{\mathsf{V}}}{3\sqrt{3} - G_{\mathsf{V}}} \left[\frac{p - i\frac{\partial I_1}{\partial q}}{p^0 - i\frac{\partial I_1}{\partial q_0}} \right]$$

"generic sol."

Difficult to evaluate the explicit sol. caused by the complicated functions G_{V} and I_1

- Three Moduli model called the STU-model: $\mathcal{F} = \dfrac{X^1 X^2 X^3}{X^0}$

  (Cartan part of 4D $\mathcal{N} = 8$ $SO(8)$ gauged SUGRA $\leftarrow$ IIA/IIB/Heterotic string triality)

$$Z \;=\; \mathrm{e}^{K/2}\Big(q_0 + q_a z^a - p^1 z^2 z^3 - p^2 z^3 z^1 - p^3 z^1 z^2 + p^0 z^1 z^2 z^3\Big)\,, \qquad z^a \;=\; \frac{X^a}{X^0}$$

$$K \;=\; -\log\Big[-i(z^1 - \overline{z}^1)(z^2 - \overline{z}^2)(z^3 - \overline{z}^3)\Big]$$

$$G_{a\overline{b}} \;=\; -\frac{\delta_{ab}}{(z^a - \overline{z}^a)^2} \;=\; e_a{}^{\widehat{a}}\, e_{\overline{b}}{}^{\widehat{\overline{b}}}\, \delta_{\widehat{a}\widehat{\overline{b}}}\,, \qquad C_{\widehat{1}\widehat{2}\widehat{3}} \;=\; 1$$

Search the sol. w/ $V = -3|Z|^2 + |D_{\widehat{a}} Z|^2 < 0 \to Z \neq 0$

Consider non-SUSY sol. $\to D_{\widehat{a}} Z \neq 0$

$\Downarrow$

The generic forms: $Z \;\equiv\; -i\rho\, \mathrm{e}^{i(\alpha - 3\phi)}\,, \qquad D_{\widehat{a}} Z \;\equiv\; \sigma\, \mathrm{e}^{-i\phi} \qquad (\rho, \sigma > 0)$

[hep-th/0606263]

The generic forms: $Z \equiv -i\rho\, \mathrm{e}^{i(\alpha - 3\phi)}\,, \qquad D_{\widehat{a}} Z \equiv \sigma\, \mathrm{e}^{-i\phi} \qquad (\rho, \sigma > 0)$

The volume factors ρ and σ are related via the attractor equation.

$$\sigma = -\rho\, \mathrm{e}^{-i\alpha} G_{\mathsf{V}} \qquad (G_{\mathsf{V}} \neq 0)$$

The formula leads to the following two equations:

$$\begin{aligned} p^a + \frac{\partial I_1}{\partial q_a} &= -2\rho\, \mathrm{e}^{-i\alpha} \mathrm{e}^{K/2} \Big[\big(1 - G_{\mathsf{V}}\big)\, z^a - 2 G_{\mathsf{V}}\, \overline{z}^{\overline{a}} \Big] \\ p^0 + \frac{\partial I_1}{\partial q_0} &= -2\rho\, \mathrm{e}^{-i\alpha} \mathrm{e}^{K/2} \big(1 - 3 G_{\mathsf{V}}\big) \end{aligned}$$

$$\rightarrow \qquad z^a = V_{\mathsf{eff}}^2 \left[\frac{p^a + i \frac{\partial I_1}{\partial q_a}}{p^0 + i \frac{\partial I_1}{\partial q_0}} \right] + \big(1 - V_{\mathsf{eff}}^2\big) \left[\frac{p^a - i \frac{\partial I_1}{\partial q_a}}{p^0 - i \frac{\partial I_1}{\partial q_0}} \right] \qquad \text{“generic sol.”}$$

Neither $V_{\mathsf{eff}} = 1$ nor $V_{\mathsf{eff}} = 0$

Difficult to evaluate the explicit sol. caused by the complicated functions G_{V} and I_1

Another Example in T^3-model

- Study the "D2-D6 system" w/ charges $\Gamma = (p^0, 0, q, 0)$

The holomorphic central charge $W = \mathrm{e}^{-K/2} Z$ and its discriminant are

$$W = q\,t + p^0\,t^3\,, \qquad \Delta(W) = -4p^0 q^3$$

The "attractor equation" is reduced to the cubic equation of $t = 0 + iy$ $(y < 0)$:

$$f(y^2) = 2(p^0)^4 q (y^2)^3 - 4(p^0)^3 q^2 (y^2)^2 + p^0(3 + 2p^0 q^3)(y^2) - q = 0$$

$$g(y^2) = \frac{\partial f}{\partial y^2} = 6(p^0)^4 (y^2)^2 - 8(p^0)^3 q^2 (y^2) + p^0(3 + 2p^0 q^3)$$

$$\Delta(f) = -\frac{(p^0 q^3)^4}{q^{10}} \Big[\Big(8(p^0 q^3)^2 - \frac{9}{4} \Big)^2 + 3375 \Big]$$

$$\Delta(g) = 8(p^0)^5 q \Big[-9 + 2p^0 q^3 \Big]$$

The various values at the attractor point are

$$Z\Big|_{\text{horizon}} = -i(q - p^0 y^2)\sqrt{-\frac{1}{8y}} \neq \mathbf{0}\,, \qquad D_t Z\Big|_{\text{horizon}} = -(q + 3p^0\, y^2)\sqrt{-\frac{1}{32y^3}} \neq \mathbf{0}$$

$$I_1 = \frac{q^2 + 3(p^0)^2 y^4}{-6y} > \mathbf{0}$$

$$\Lambda = \frac{2q^2 y}{3}\big((p^0)^2 y^2 - q\big)^2 < \mathbf{0}$$

$$S_{\text{BH}} = \frac{-3 + \sqrt{9 + 4q^2\big(q - (p^0)^2 y^2\big)^2\big(q^2 + 3(p^0)^2 y^4\big)}}{-4q^2(q - (p^0)^2 y^2)^2 y} > \mathbf{0}$$

The solution of the Modulus $t = 0 + iy$ $(y < 0)$ is given as

$$y^2 = A + B \quad \text{or} \quad A + \omega^{\pm} B\,, \qquad \omega^3 = 1$$

$$A = \frac{2q}{3p^0}\,, \qquad B = \frac{1}{6(p^0)^3 q}\left(C^{1/3} + \frac{1}{4(p^0)^2}\frac{\Delta(g)}{C^{1/3}}\right)$$

$$C = -54(p^0)^5 q^3 - 8(p^0)^6 q^6 + 3\sqrt{3}p^0\sqrt{-q^2\Delta(f)}\,, \qquad \text{with} \quad p^0 q^3 > 0$$

Compare our result to the (non-)SUSY solution of the RN-BH w/ $\Lambda = 0$

- non-BPS solution:

$$t \;=\; 0 + iy\,, \qquad y \;=\; -\sqrt{\frac{q}{3p^0}}$$

$$Z\Big|_{\text{horizon}} \;=\; \frac{iq}{3\sqrt{2}}\Big(\frac{3p^0}{q}\Big)^{1/4} \;\neq\; 0\,, \qquad D_t Z\Big|_{\text{horizon}} \;=\; -\frac{q}{2\sqrt{2}}\Big(\frac{3p^0}{q}\Big)^{3/4} \;\neq\; 0$$

$$S_{\text{BH}} \;=\; I_1 \;=\; |Z|^2 + G^{t\bar{t}} D_t Z \overline{D_t Z} \;=\; 4|Z|^2 \;=\; \frac{2}{3}\sqrt{\frac{p^0 q^3}{3}} \;>\; 0\,, \qquad \Lambda \;=\; 0$$

- 1/2-BPS solution:

$$t \;=\; 0 + iy\,, \qquad y \;=\; -\sqrt{-\frac{q}{3p^0}}$$

$$Z\Big|_{\text{horizon}} \;=\; \frac{-i\sqrt{2}q}{3}\Big(-\frac{3p^0}{q}\Big)^{1/4} \;\neq\; 0\,, \qquad D_t Z\Big|_{\text{horizon}} \;=\; 0$$

$$S_{\text{BH}} \;=\; I_1 \;=\; |Z|^2 \;=\; \frac{2}{3}\sqrt{-\frac{p^0 q^3}{3}} \;>\; 0\,, \qquad \Lambda \;=\; 0$$

Hypermultiplets

Action including hypermultiplets:

$$S = \int d^4x\sqrt{-g}\Big\{\frac{1}{2\kappa^2}R - G_{a\bar{b}}(z,\bar{z})\partial_\mu z^a \partial^\mu \bar{z}^{\bar{b}} - h_{uv}(q)\nabla_\mu q^u \nabla^\mu q^v$$

$$+ \frac{1}{4}\mu_{\Lambda\Sigma}(z,\bar{z})F^{\Lambda}_{\mu\nu}F^{\Sigma\mu\nu} + \frac{1}{4}\nu_{\Lambda\Sigma}(z,\bar{z})F^{\Lambda}_{\mu\nu}(*F^{\Sigma})^{\mu\nu}$$

$$- \mathrm{g}^2 V(z,\bar{z},q)$$

$$+ \text{(fermionic terms)}\Big\}$$

Moduli space of hypermultiplets = quaternionic geometry

We borrow the description in (non)geometric flux compactifications scenarios

arXiv:0911.2708 etc.

$$\underset{4n_{\mathsf{H}}+4}{\{q^u\}} = \underset{2n_{\mathsf{H}}\text{(SKG)}}{\{\mathsf{z}^i, \bar{\mathsf{z}}^{\bar{\jmath}}\}} + \underset{2n_{\mathsf{H}}}{\{\xi^i, \widetilde{\xi}_i\}} + \underset{4\text{ (universal)}}{\{\varphi, a, \xi^0, \widetilde{\xi}_0\}}$$

(special quaternionic geometry)

Contribution of hypermultiplets to the kinematics and potential:

$$h_{uv}\,\mathrm{d}q^u\mathrm{d}q^v \;=\; \underbrace{G_{i\bar{\jmath}}\,\mathrm{dz}^i\,\mathrm{d}\bar{\mathrm{z}}^{\bar{\jmath}}}_{\text{SKG}_\mathsf{H}} + \underbrace{(\mathrm{d}\varphi)^2}_{\text{4D dilaton}} + \frac{1}{4}\mathrm{e}^{4\varphi}\big(\underbrace{\mathrm{d}a}_{\text{axion}} - \xi^{\mathrm{T}}\,\mathbb{C}_\mathsf{H}\,\mathrm{d}\xi\big)^2 - \frac{1}{2}\mathrm{e}^{2\varphi}\,\underbrace{\mathrm{d}\xi^{\mathrm{T}}\,\mathbb{M}_\mathsf{H}\,\mathrm{d}\xi}_{\text{scalars from RR}}$$

$$\nabla_\mu q^u \;=\; \partial_\mu q^u + \mathrm{g}\,k^u_\Lambda A^\Lambda_\mu\,, \qquad k_\Lambda \;=\; -\big[2q_\Lambda + e_\Lambda{}^{\mathbb{I}}(\mathbb{C}_\mathsf{H}\xi)_{\mathbb{I}}\big]\frac{\partial}{\partial a} - e_\Lambda{}^{\mathbb{I}}\frac{\partial}{\partial \xi^{\mathbb{I}}}$$

$$\mathcal{P}^+ \;\equiv\; \mathcal{P}^1 + i\mathcal{P}^2 \;=\; 2\mathrm{e}^{\varphi}\,\Pi^{\mathrm{T}}_\mathsf{V}\,Q\,\mathbb{C}_\mathsf{H}\,\Pi_\mathsf{H}$$

$$\mathcal{P}^- \;\equiv\; \mathcal{P}^1 - i\mathcal{P}^2 \;=\; 2\mathrm{e}^{\varphi}\,\Pi^{\mathrm{T}}_\mathsf{V}\,Q\,\mathbb{C}_\mathsf{H}\,\overline{\Pi}_\mathsf{H}$$

$$\mathcal{P}^3 \;=\; \mathrm{e}^{2\varphi}\,\Pi^{\mathrm{T}}_\mathsf{V}\,\mathbb{C}_\mathsf{V}(c + \widetilde{Q}\xi)$$

$$\mathbb{M}_{\mathsf{V,H}} \;=\; \begin{pmatrix} \mu + \nu\mu^{-1}\nu & -\nu\mu^{-1} \\ -\mu^{-1}\nu & \mu^{-1} \end{pmatrix}_{\mathsf{V,H}}\,, \qquad Q_{\boldsymbol{\Lambda}}{}^{\mathbb{I}} \;=\; \begin{pmatrix} e_\Lambda{}^I & e_{\Lambda I} \\ m^{\Lambda I} & m^\Lambda{}_I \end{pmatrix}\,, \qquad \mathbb{C}_{\mathsf{V,H}} \;=\; \begin{pmatrix} 0 & \mathbb{1} \\ -\mathbb{1} & 0 \end{pmatrix}$$

$$\mu_{\mathsf{V,H}} \;=\; \mathrm{Im}\mathcal{N}_{\mathsf{V,H}}\,, \quad \nu_{\mathsf{V,H}} \;=\; \mathrm{Re}\mathcal{N}_{\mathsf{V,H}} \qquad\qquad \widetilde{Q}^{\boldsymbol{\Lambda}}{}_{\mathbb{I}} \;=\; \mathbb{C}^{\mathrm{T}}_\mathsf{V}\,Q\,\mathbb{C}_\mathsf{H}$$

$\Pi_\mathsf{H} = \mathrm{e}^{\mathcal{K}_\mathsf{H}/2}(Z^I, \mathcal{G}_I)^{\mathrm{T}}$, $\mathrm{z}^i = Z^i/Z^0$: SKG variables in hypermoduli

$\Pi_\mathsf{V} = \mathrm{e}^{\mathcal{K}_\mathsf{V}/2}(X^\Lambda, \mathcal{F}_\Lambda)^{\mathrm{T}}$: SKG variables in vector moduli

$c = (p^\Lambda, q_\Lambda)^{\mathrm{T}}$ can also be regarded as the BH charges

$$h_{uv}\nabla_\mu q^u \nabla^\mu q^v = (\partial_\mu \varphi)^2 + \frac{1}{4}\mathrm{e}^{4\varphi}\big(\nabla_\mu a - \xi^0 \nabla_\mu \widetilde{\xi}_0 + \widetilde{\xi}^0 \nabla_\mu \xi^0\big)^2$$

$$\nabla_\mu a = \partial_\mu a - \mathrm{g}(2q_\Lambda + e_\Lambda{}^0 \widetilde{\xi}_0 - e_{\Lambda 0}\xi^0) A_\mu^\Lambda$$

$$\nabla_\mu \xi^0 = \partial_\mu \xi^0 - \mathrm{g}(e_\Lambda{}^0) A_\mu^\Lambda \, , \quad \nabla_\mu \widetilde{\xi}_0 = \partial_\mu \widetilde{\xi}_0 - \mathrm{g}(e_{\Lambda 0}) A_\mu^\Lambda$$

$$V(z,\overline{z},q) = G^{a\overline{b}} D_a \mathcal{P}^3 \overline{D_b \mathcal{P}^3} - 3|\mathcal{P}^3|^2 \, , \quad \mathcal{P}^3 = \mathrm{e}^{2\varphi}\big(Z + Z_\xi\big)$$

$$Z \equiv L^\Lambda q_\Lambda - M_\Lambda p^\Lambda \, , \quad Z_\xi \equiv L^\Lambda (e_\Lambda{}^0 \widetilde{\xi}_0 - e_{\Lambda 0}\xi^0) - M_\Lambda (m^\Lambda{}_0 \xi^0 - m^{\Lambda 0}\widetilde{\xi}_0)$$

Very complicated even when we focus only on the Universal hypermultiplet

compared to the system only with Vector multiplets

arXiv:1005.3650

SUSY BH-sol. in stationary, axisymmetric, asymptotically flat spacetime
has constant universal hypermoduli
and vector multiplets which follow the ordinary attractor mechanism

How is non-SUSY RN(-AdS) BH-sol. in the presence of Universal hypermoduli?

$\longrightarrow$ work in progress