Globally and Locally Supersymmetric Effective Theories for Light Fields

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Overview

- Subject

  Study of the conditions for supersymmetrically integrating out heavy fields in global SUSY and SUGRA

- Motivations

  Relevant in string-inspired SUGRA models where there are too many fields (moduli) and some of them can be supersymmetrically stabilized at a large mass scale $M \gg M_{SUSY}$

- Results

  Simple procedure to construct two-derivatives supersymmetric low-energy effective theories:

Chiral multiplets	$\partial_h W = 0$	Stationarity of Superpotential
Vector multiplets	$\partial_x K = 0$	Stationarity of Kahler Potential

Valid in the same form for the global and local SUSY case!!

Introduction

- Effective theories with minimal number of derivatives. We restrict to 2 deriv. for scalars and 1 deriv. for fermions.

- How to integrate out heavy fields in supersymmetric theories?
  - Two options, depending on the ratio M_{SB}/M

- Standard non-supersymmetric approach, in components
  We can procede as for standard non-supersymmetric theories solving for each heavy field the approximate algebraic e.o.m obtained by neglecting derivatives terms

$$\Box\phi_0^h + V_h(\phi^l, \phi_0^h) = 0 \quad \Rightarrow \quad V_h(\phi^l, \phi_0^h) \approx 0$$

$$\mathcal{L}_{eff} = Z(\phi^l, \phi_0^h)_{ll'}\,\partial^\mu\phi^l\,\partial_\mu\phi^{l'} - V(\phi^l, \phi_0^h) + \mathcal{O}\left(\frac{\Box^2}{M^4}\phi^l\right)$$

- The effective theory will contain a maximum of 2 deriv. for scalars and 1 for fermions BUT any power of auxiliary fields and fermions!

Introduction

- This derivative expansion DOES NOT preserve SUSY and in general the effective theory will be non-supersymmetric
- A SUSY theory with minimal number of derivatives is at most quadratic in auxiliary fields and fermion-bilinears
- This is a consequence of the fact that no covariant derivative dependence is allowed in the Kähler and superpotential
- Integrating heavy fields in components is the best we can do when the SUSY breaking scale $M_{SB} \sim M$ and heavy multiplets are stabilized with large auxiliary fields.

Introduction

- In the limit in which $M_{SB} \ll M$ and all the auxiliary fields and fermions are small the effective theory is approximately supersymmetric

- There is a more efficient procedure to integrate heavy fields:

- Manifestly supersymmetric approach, in superfields
  Solve approximate algebraic superfield e.o.m obtained by neglecting covariant derivatives

- The effective theory is exactly supersymmetric containing the minimal number of derivatives, auxiliary fields and fermions

- The two procedures coincide in the region of light field space in which:

$$\boxed{m, \mu \ll M, \quad F \ll M^2, \quad \psi \ll M^{3/2}}$$

Rigid SUSY - Chiral Multiplets

Affleck, Dine, Sieberg. - Intrilligator, Sieberg

$$\mathcal{L} = \int d^4\theta K(\Phi^i, \bar{\Phi}^{\bar{\imath}}) + \int d^2\theta W(\Phi^i) + \int d^2\bar{\theta}\bar{W}(\bar{\Phi}^{\bar{\imath}})$$

- By construction $n \leqslant 2$ with: $n = n_\partial + \frac{1}{2}n_\psi + n_F$
- If $K(\Phi^i, \bar{\Phi}^{\bar{\imath}}, D^2\Phi^i, \bar{D}^2\bar{\Phi}^{\bar{\imath}})$ and $W(\Phi^i, \bar{D}^2\bar{\Phi}^{\bar{\imath}}) \Rightarrow n \geqslant 3$
- Exact e.o.m. for heavy superfields:

$$W_h - \frac{1}{4}\bar{D}^2 K_h = 0$$

- At leading order in covariant derivatives:

$$\Phi_0^h = \Phi_0^h(\Phi^l) + \mathcal{O}(D^2\Phi^l/M), \qquad \boxed{W_h(\Phi_0^h) = 0}$$

- The supersymmetric low-energy effective theory is obtained by:

$$K_{eff} = K\left(\Phi^l, \bar{\Phi}^{\bar{l}}, \Phi_0^h(\Phi^l), \bar{\Phi}_0^{\bar{h}}(\bar{\Phi}^{\bar{l}})\right), \qquad W_{eff} = W\left(\Phi^l, \Phi_0^h(\Phi^l)\right)$$

Rigid SUSY - Chiral Multiplets

- Sub-leading corrections in $D^2\Phi$ give an eff. action with $n \geqslant 3$
  - Obvious for $\int d^4\theta$ part which has already $n = 2$;
  - For $\int d^2\theta$ which has $n = 1$ they vanish because $\propto W_h = 0$

- The approx. e.o.m $W_h = 0$ can also be obtained in components by the exact one by keeping only the leading contributions in n:

$$W_h = 0 + \mathcal{O}(n = 1)$$
$$W_{hi}\psi^i = 0 + \mathcal{O}(n = 3/2)$$
$$W_{hi}F^i - \frac{1}{2}W_{hij}\psi^i\psi^j = 0 + \mathcal{O}(n = 2)$$

- These eq. can be manifestly combined into an algebraic superfield equation since they are the components of $W_h = 0$. This is not the case if we neglect only space-time deriv. without restrictions on auxiliary fields and fermions

SUGRA - Chiral Multiplets

Kaku, Townsend. - Kugo, Uehara

- Gravity DOES NOT introduce new complications

- It's useful to work in Superconformal superspace formalism:

  - Extend super-Poincaré to super-Conformal

  $G = (g_{\mu\nu}, \psi^{\mu}_{\alpha}, B_{\mu}, R_{\mu})$ Conformal gravitational multip.

  $\Phi = (\phi, \psi, F_{\phi})$ Conformal Compensator Chiral multip.

- Minimal SUGRA in Einstein frame is obtained by the gauge fixing:

$$\Phi = \Lambda(1, 0, \tilde{F}_{\phi}), \qquad R_{\mu} = 0 \qquad \text{with} \quad \Lambda = e^{K/6}$$

- Ordinary rigid superspace tensor calculus and SUSY transformations are deformed in a simple way

SUGRA - Chiral Multiplets

- The superconformal invariant Lagrangian is given by

$$\mathcal{L} = \int d^4\theta \Big(-3e^{-K/3} \Big) \bar{\Phi}\Phi + \int d^2\theta \, W\Phi^3 + \int d^2\bar{\theta} \, \bar{W}\bar{\Phi}^3$$

- Tensor calculus is slightly modified with respect to rigid SUSY:

$$\int d^2\theta \, W\Phi^3 = e\Big[\underbrace{W|_{\theta^2} \, \phi^3 + 3\phi^2 W| \, F_\phi}_{\text{rigid SUSY tensor calculus}} + \text{gravitational sector} \Big]$$

- We'll not keep trace of the gravitational sector which is uniquely determined
- Manipulate superspace quantities exactly as in the rigid case
- The counting of number of derivatives, auxiliary fields and fermions is unchanged

SUGRA - Chiral Multiplets

De Alwis. - Gallego, Serone.

- Exact e.o.m for heavy superfields:

$$W_h - \frac{1}{4}\bar{D}^2\left(K_h\, e^{-K/3}\bar{\Phi}\right)\Phi^{-2} = 0$$

- At two-derivatives level we need to neglect also supercovariant deriv. of compensator superfield
- As in the rigid SUSY case: $\boxed{W_h(\Phi_0^h) = 0}$
- Not invariant under general Kähler transformations:

$$(\Phi, K, W) \to (\Phi\, e^{X/3}, K + X + \bar{X}, W\, e^{-X}), \qquad X = X(\Phi^i)$$

  but invariant for X depending only on light fields. Why?
- Because in approximating e.o.m as $W_h = 0$ we assume that the large masses of heavy fields come only from W and not from K. This assumption selects a particular subclass of Kähler gauges suitable for deriving the effective theory.

SUGRA - Chiral Multiplets

De Alwis

- We obtain the same result working in components and taking leading terms in n
- Neglecting covariant derivatives of compensator implies $F_\phi \ll M$
- Is it really justified to neglect $D^2\Phi$? What about neglecting only derivatives of matter chiral field?

- Heavy Fields approx. e.o.m $\quad W_h - \frac{1}{4}\Phi^{-2}K_h e^{-K/3}\bar{D}^2\bar{\Phi} = 0$
- Compensator approx. e.o.m $\quad -\frac{1}{4}\bar{D}^2\bar{\Phi}\Phi^{-2} = e^{K/3}W$

- We obtain $\quad W_h + K_h W = 0$
- It's NOT a chiral equation: more component equations than components field. It cannot be solved as a superfield equation!

SUGRA - Chiral Multiplets

- Physical meaning of the constraints $F^i \ll M^2, \quad F_\phi \ll M$:

- Small gravitino mass: $m_{3/2} = |W|e^{K/2} = |e^{-K/6} F_\phi - \frac{1}{3} K_i F^i| \ll M$
- Small cosmological constant: $V = g_{i\bar{\jmath}} F^i \bar{F}^{\bar{\jmath}} - 3m_{3/2}^2 \ll M^4$

- The reason for demanding also $F_\phi \ll M$ is then two-fold depending whether SUSY is broken or not.

- In order to have broken SUSY in SUGRA:

$$< \delta_\zeta \psi^i >= -\sqrt{2}\,\zeta < e^{K/2} g^{i\bar{\jmath}} (\bar{W}_{\bar{\jmath}} + K_{\bar{\jmath}} \bar{W}) >= -\sqrt{2}\zeta < F^i > \neq 0$$

- SUSY is unbroken Iff $< F^i >= 0$; the space-time can only be AdS ($m_{3/2} \neq 0$) or Minkowski ($m_{3/2} = 0$).
  Remark: No inconsistency in having massive gravitino, massless graviton and unbroken SUSY since in AdS $[P^\mu, Q_\alpha] \neq 0$ and fields in the same multipl. has not the same mass.

SUGRA - Chiral Multiplets

- $F_\phi \neq 0$ represents an extra mass splitting Δm for fields in the same multiplet.
- If SUSY is unbroken Δm coincides with the splitting required by AdS algebra and it's proportional to the inverse of curvature radius

$$\Delta m \propto \frac{1}{L_{AdS}}$$

In this case we need $F_\phi \ll M$ because two-derivatives expansion in the gravity sector is justified only for large curvature radius (small cosmological constant)

- If SUSY is broken and space-time is Minkowski Δm corresponds to an ordinary splitting in the masses of light multiplets and we need $\Delta m \ll M$ in order to avoid large masses for scalar and pseudo-scalar components

SUGRA - Chiral Multiplets

- $m_{3/2}$ or equivalently $W \approx 0$ represent an extra constraint on the region of the light field space in which the two procedures coincide

$$\boxed{m, \mu \ll M, \quad F \ll M^2, \quad \psi \ll M^{3/2}, \quad W \ll M^3}$$

- In general these conditions require a tuning of the parameters in the Lagrangian
- This is not a severe restriction because for phenomenological applications it's anyhow necessary to tune the cosmological constant to an even smaller value

Vector Multiplets

- The most general 2-derivatives Lagrangian we consider is:

$$\mathcal{L} = \int d^4\theta\, K(\Phi, \bar{\Phi}, V) + \int d^2\theta\, W(\Phi) + \frac{1}{4}\int d^2\theta\, H_{ab}(\Phi)\mathcal{W}^a \cdot \mathcal{W}^b + c.c$$

- To properly integrate out the heavy multiplets we need to completely fix the gauge freedom.
  - Fix a charged chiral field to a reference scale.
  - All the components of vector multiplets become dynamical

- Neglecting covariant deriv. in superfield e.o.m:

$$K_x + \frac{1}{8}D^\alpha(H_{xa}\bar{D}^2 D_\alpha V^a) + c.c. = 0$$

$$V_0^x(\Phi, V^a) = V_0^x + \mathcal{O}\left(\bar{D}^2 D^2 \Phi/M^2, \bar{D}^2 D^2 V^a/M^2\right) \qquad \boxed{K_x(\Phi, \bar{\Phi}, V_0^x) = 0}$$

Freezing Out Heavy Fields

Achucarro, Hardeman, Sousa.

- In general heavy fields induce relevant corrections on the dynamics of light fields ad they should be properly integrated out
- It can be a non-trivial task due to non linearities or large number of fields
- In particular cases it may be possible to freeze heavy superfields to constant values:
- Chiral multiplets: $\boxed{W = W_L(\Phi^l) + W_H(\Phi^h)}$
  No light-heavy mixing in the superpotential; allowed in Kähler
- Vector multiplets: $\boxed{K = K_L(\Phi, \bar{\Phi}, V^l) + K_H(V^h)}$
  No light-heavy mixing at all