Dissecting the SUSY Breaking Mechanism Using Renormalization Group Invariants

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Outline

- Introduction.
- RGEs for sparticle masses and gauge couplings in the MSSM .
- 1-Loop RG Invariants constructed.
- 1-Loop RG Invariants used to constrain and extract information about SUSY breaking.
- 2-Loop effects invariants analyzed and always included.
- Highs scale extraction of parameters using RGIs:
  - Generic Flavor Blind Models
  - GGM
  - MGM
- Numerical simulation: scan over model space of GGM,
  - Certain invariants may be used to test FB/GGM hypothesis.
  - If data consistent with model, RGIs may be used to extract information about soft SUSY breaking parameters.
  - Expected determination of parameters depends on experimental errors at LHC in measuring the physical sparticle masses.
- Outlook and Conclusions.

- Assumptions:
  - No new physics alters 1-loop MSSM β functions below messenger scale, at which SUSY breaking is transmitted to visible sector.
- MSSM:
  - Soft SUSY breaking parameters governing sparticle masses unknown.
    - Highly dependent on SUSY breaking scheme.
  - If sparticles light, flavor physics strongly constrains structure of m_{soft}
    - Flavor Blind Models of SUSY breaking favored.

Could LHC measurements determine:

Messenger scale?

Soft SUSY breaking parameters ?

- TOOL:
  - 1-Loop RG Invariants in the MSSM.
    - Do 2-loop effects spoil invariance?
    - Effect on extraction of high scale parameters?
    - Experimental constraints need to be satisfied to extract information?

The Minimal Supersymmetric Standard Model.

1-Loop RG Evolution

- Sfermion mass evolution:
  - Soft sfermion masses flavor diagonal.
  - 1st and 2nd generation masses degenerate at the messenger scale.
  - Neglect 1st and 2nd generation yukawa and trilinear couplings.

$$16\pi^2 \frac{dm_i^2}{dt} = \sum_{jk} y_{ijk}^* y^{ijk} (m_i^2 + m_j^2 + m_k^2 + A_{ijk}^* A^{ijk}) - 8 \sum_a C_a(i) g_a^2 |M_a|^2 + \frac{6}{5} Y_i g_a^2 D_Y \,,$$

  - 1st sum:
    - D.o.f running in self-energy loop.
  - 2nd sum:
    - Gauge groups.
  - Tr(D_Y):
    - All chiral multiplets.

$$\begin{aligned} D_Y &\equiv \mathrm{Tr}_i (Y_i m_i^2) \\ &= \sum_{gen} \left(m_{\tilde{Q}}^2 - 2m_{\tilde{u}}^2 + m_{\tilde{d}}^2 - m_{\tilde{L}}^2 + m_{\tilde{e}}^2 \right) + m_{H_u}^2 - m_{H_d}^2 . \end{aligned}$$

- Gauge couplings: homogenous RGEs at 1-loop:

$$16\pi^2 \frac{dg_r}{dt} = g_r^3 \left(\mathrm{Tr}_n \, I_r(n) - 3C_r(G) \right) ,$$

- Soft gaugino mass evolution:

$$16\pi^2 \partial_t M_r = g_r^2 M_r \left(2 \, \mathrm{Tr}_n \, I_r(n) - 6C_r(G) \right) .$$

Constructing 1-Loop Renormalization Group Invariants

Sfermion Dependant Invariants.

- Construct linear combinations of soft masses, D_i, evolving only with D_Y
- Six combinations:

$$D_i \equiv \mathrm{Tr}(Q_i m^2), \qquad \frac{dD_i}{dt} = a_i D_Y\,,$$

  - For yukawa terms to vanish,
    - Q_is must correspond to charges of global symmetry of classical yukawa potential.
  - For gaugino terms to cancel,
    - Symmetry must have vanishing mixed anomalies with SM gauge groups.
  - Above conditions supply three independent constraints each on the Q_is.
- Can construct basis in which 5 of 6 combinations also satisfy Tr(QY)=0,
  - Cancels D_Y dependence.
  - Promotes them to 1-loop RG Invariants, independent of vanishing of D_Y.

Invariants Testing Flavor Structure

- Baryon Number ($Q = B$) and Lepton Number ($Q = L$)
  - Classical symmetries, anomalous and flavor independent in the MSSM.
  - Difference between the 1st (2nd) and 3rd generation anomaly free.
- Can then generate two invariants:

$$D_{L_{13}} \equiv D_{L_1} - D_{L_3}$$

$$D_{B_{13}} \equiv D_{B_1} - D_{B_3}$$

Anomalous U(1)s and Invariants involving g_r / M_r

- Use similar idea for *Y* as with *B* & *L*.
  - Must include Higgs doublet with 3rd generation, since evolution linked with yukawas.
- The RG Invariant is given by:

$$D_{Y_{13H}} \equiv D_{Y_1} - \frac{10}{13} D_{Y_{3H}}$$

- For (*B-L*), even restricted to one generation, D_Y and $D_{(B-L)}$ evolve only with D_Y
  - Construct RGI depending only on 1st generation:

$$D_{\chi 1} \equiv 4D_{Y_1} - 5D_{(B-L)1}$$

  - Identified with $U(1)_\chi$ generated in breaking E_6 to $SU(5)xU(1)_\chi xU(1)$.
- Anomalous combination of both *U(1)*s, (setting 1st generation left handed slepton charges to zero):
  - Obtain anomaly free $U(1)_Z$:

$$D_Z \equiv 3m^2_{\tilde{d}_3} + 2m^2_{\tilde{L}_3} - 2m^2_{H_d} - 3m^2_{\tilde{d}_1}.$$

- D_Y vanishes only in minimal GGM.
  - Using RGE for g_1:

$$I_{Y\alpha} \equiv \frac{D_Y}{g_1^2}.$$

- From RGEs for gauge couplings, we can further obtain:

$$I_{g_2} \equiv \frac{1}{g_1^2} - \frac{33}{5g_2^2} \qquad I_{g_3} \equiv \frac{1}{g_1^2} + \frac{33}{15g_3^2},$$

- From RGEs for gaugino masses, can construct:

$$I_{B_r} \equiv M_r/g_r^2.$$

- 3 invariants mixing sfermion and gaugino masses can be obtained from the 1st generation:

$$I_{M_1} \equiv M_1^2 - \frac{33}{8}(m^2_{\tilde{d}_1} - m^2_{\tilde{e}_1} - m^2_{\tilde{u}_1})$$

$$I_{M_2} \equiv M_2^2 - \frac{1}{24}(-9m^2_{\tilde{d}_1} - 16m^2_{\tilde{L}_1} + m^2_{\tilde{e}_1} + 9m^2_{\tilde{u}_1})$$

$$I_{M_3} \equiv M_3^2 - \frac{3}{16}(5m^2_{\tilde{d}_1} - m^2_{\tilde{e}_1} + m^2_{\tilde{u}_1}).$$

2-Loop Effects?

- RGIs have vanishing β-functions only at 1-loop.
- Can easily check invariance not preserved at 2-loops.
  - Important to estimate 2-loop effects.
    - How do they compare to expected experimental errors in measurements of invariants?
    - How do these constrain experimental accuracy required to determine any high scale SUSY breaking model parameters?
- Implemented full 2-loop RGEs for evolution of soft SUSY breaking parameters, gauge and Yukawa couplings when performing numerical simulations in Mathematica.
- Compared our mass spectrum to one obtained from SUSPECT and obtained excellent agreement.

Extraction of High Scale Parameters using RGIs.

Invariant	Definition in Terms of Soft Masses	Value at M in GGM	Value at M in MGM
$D_{B_{13}}$	$2(m^2_{\tilde{Q}_1} - m^2_{\tilde{Q}_3}) - m^2_{\tilde{u}_1} + m^2_{\tilde{u}_3} - m^2_{\tilde{d}_1} + m^2_{\tilde{d}_3}$	0	0
$D_{L_{13}}$	$2(m^2_{\tilde{L}_1} - m^2_{\tilde{L}_3}) - m^2_{\tilde{e}_1} + m^2_{\tilde{e}_3}$	0	0
D_{χ_1}	$3(3m^2_{\tilde{d}_1} - 2(m^2_{\tilde{Q}_1} - m^2_{\tilde{L}_1}) - m^2_{\tilde{u}_1}) - m^2_{\tilde{e}_1}$	0	0
$D_{Y_{13H}}$	$m^2_{\tilde{Q}_1} - 2m^2_{\tilde{u}_1} + m^2_{\tilde{d}_1} - m^2_{\tilde{L}_1} + m^2_{\tilde{e}_1}$ $-\frac{10}{13}\left(m^2_{\tilde{Q}_3} - 2m^2_{\tilde{u}_3} + m^2_{\tilde{d}_3} - m^2_{\tilde{L}_3} + m^2_{\tilde{e}_3} + m^2_{H_u} - m^2_{H_d}\right)$	$-\frac{10}{13}(\delta_u - \delta_d)$	$-\frac{10}{13}(\delta_u - \delta_d)$
D_Z	$3(m^2_{\tilde{d}_3} - m^2_{\tilde{d}_1}) + 2(m^2_{\tilde{L}_3} - m^2_{H_d})$	$-2\delta_d$	$-2\delta_d$
$I_{Y\alpha}$	$\left(m^2_{H_u} - m^2_{H_d} + \sum_{gen}(m^2_{\tilde{Q}} - 2m^2_{\tilde{u}} + m^2_{\tilde{d}} - m^2_{\tilde{L}} + m^2_{\tilde{e}})\right)/g_1^2$	$(\delta_u - \delta_d)/g_1^2$	$(\delta_u - \delta_d)/g_1^2$
I_{B_r}	M_r/g_r^2	B_r	B
I_{M_1}	$M_1^2 - \frac{33}{8}(m^2_{\tilde{d}_1} - m^2_{\tilde{u}_1} - m^2_{\tilde{e}_1})$	$g_1^4\left(B_1^2 + \frac{33}{10}A_1\right)$	$\frac{38}{5}g_1^4 B^2$
I_{M_2}	$M_2^2 + \frac{1}{24}\left(9(m^2_{\tilde{d}_1} - m^2_{\tilde{u}_1}) + 16m^2_{\tilde{L}_1} - m^2_{\tilde{e}_1}\right)$	$g_2^4\left(B_2^2 + \frac{1}{2}A_2\right)$	$2g_2^4 B^2$
I_{M_3}	$M_3^2 - \frac{3}{16}(5m^2_{\tilde{d}_1} + m^2_{\tilde{u}_1} - m^2_{\tilde{e}_1})$	$g_3^4\left(B_3^2 - \frac{3}{2}A_3\right)$	$-2g_3^4 B^2$
I_{g_2}	$1/g_1^2 - 33/(5g_2^2)$	≈ -10.9	≈ -10.9
I_{g_3}	$1/g_1^2 + 33/(15g_3^2)$	≈ 6.2	≈ 6.2

Measure only 1st generation + Gaugino masses and gauge couplings:

Can calculate subset of RGIs: Constrain parameter space of GGM

Differentiate MGM from within GGM

High Scale Parameters

3 gauge couplings, $g_r(M)$

- Gauge couplings must satisfy:

$$0.05 \lesssim g_1^4(M) \lesssim 0.25$$
$$0.2 \lesssim g_2^4(M) \lesssim 0.25$$
$$0.25 \lesssim g_3^4(M) \lesssim 1.0$$

Flavor Blind Models

- 5 sfermion masses
- 3 Gaugino masses
- 2 Higgs Parameters

Under-constrained:

- have 1 undefined parameter

GGM:

- A_r: defining sfermion masses
- B_r: defining Gaugino masses
- 2 Higgs Mass Parameters

Exactly right number of parameters.

MGM:

- Subset of GGM
  - $A_r = A = 2B^2 = 2B_r^2$

Over-constrained:

- Consistency relations.

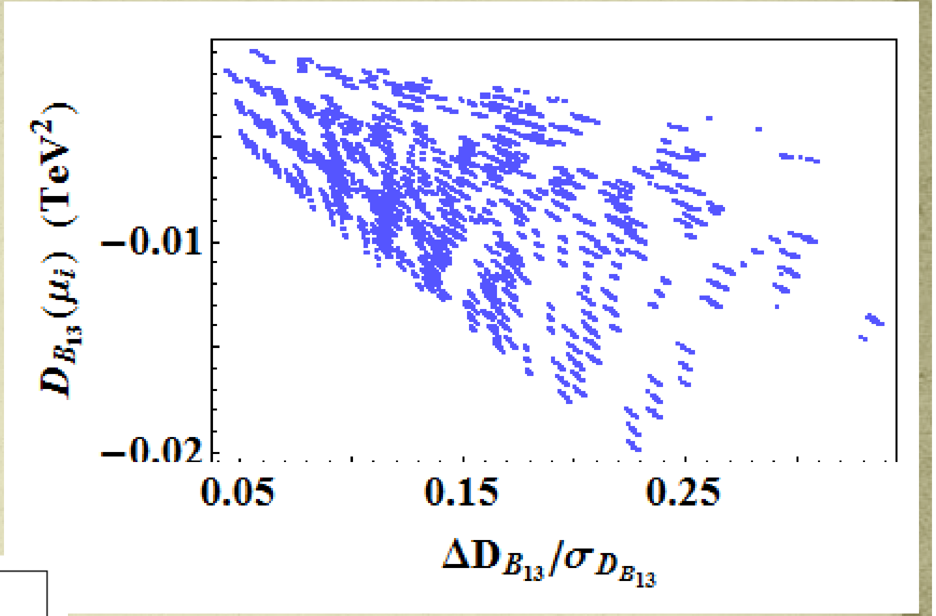
Numerical Simulations and Results.

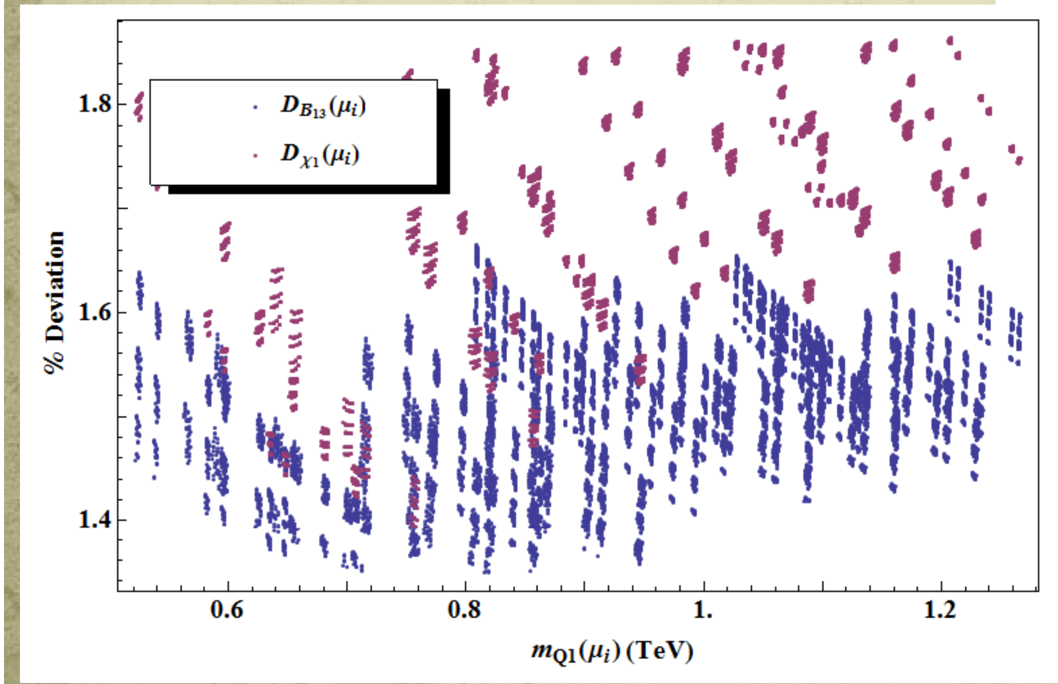
Procedure

- Scan messenger scale parameter space of models for GGM.
  - A_r: 0.1, 0.55, 1 (TeV)^2
  - B_r: 0.1, 0.55, 1 (TeV)
  - δ_u: 0, 0.5, 1 (TeV)^2
  - δ_d: 0, 0.5, 1 (TeV)^2
  - Log[μ_f/m_Z]: 12, 21, 30
  - Tan[β]: 2, 9, 16, 26, 50
- Compute invariants, soft masses and gauge/yukawa couplings at messenger scale.
- Using 2-loop RGEs, run down to TeV scale.
- Compute invariants, soft masses and physical masses at TeV scale.
- Assume each point in model space maybe an experimental measurement for the soft masses at TeV scale, with error of 1%:
  - Test hypothesis of flavor blindness using first 2 invariants.
  - Test GGM using 3rd Invariant
  - Extract messenger scale parameters from the rest.
  - Assume measurement of a subset: (1st generation and Gaugino masses + g_r).
    - Constrain parameter space of GGM
    - Using constraint equations, test whether one can distinguish MGM models.
- Considered flat 1% experimental error in measurement of all soft masses at TeV scale.
  - Probably highly optimistic at the LHC (lepton colliders, ILC ?)
  - In reality would be highly dependant on exact decays chains depending on mass hierarchy, etc used to measure masses experimentally.
  - Since we assume flat % errors, easy to see from plots what change in % error would imply.

$\sigma_{\text{soft mass}} = 1\%$: 2-loop Running Preserves Invariance within Error.

D_{B13}, expected to be zero in FBMs, plotted as function of ratio of 2-loop running and expected experimental errors.
Can invert relationship to extract % deviation in soft masses that could be detected when any of them are non-zero within error.





Treatment of 2-loop running: Approximate β-function, and subtract expected value from Invariant calculated. All RGIs 2-loop running then ~< 1 σ. Hence, after shifting invariants, can ignore 2-loop effects.

Determination of g_r at the messenger scale:

$$g_1^2(\mu_f) = -13/10\ D_{Y13H}\ /I_{Y\alpha}$$

If the difference between the corrections to the Higgs up and down sector are determined to be zero within error, there will be large errors associated with the determination of the gauge couplings at the messenger scale.

However, actual value of $g_i^2(\mu_f)$ within **0.6 σ** of the calculated value of $g_i^2(\mu_f)$:

Can determine range for gauge couplings at high scale.

Determination of GGM parameters A_r

A_r extracted using B_r and g_r:

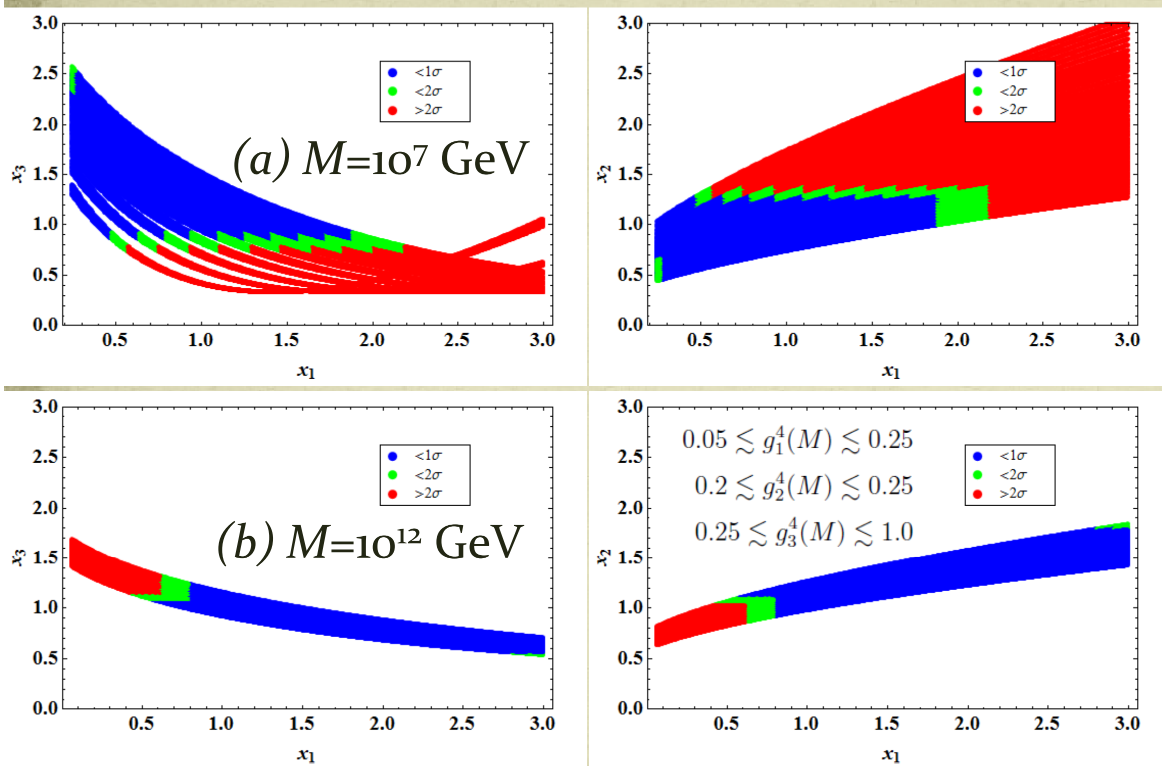
Indeterminacy for g_r transmits to A_r.

Actual value of A_r within **1.25 σ** of the calculated value of A_r :

Can extract a range of consistent A_r.

Measure 1st generation + M_r and g_r: GGM parameters?

$$x_1 \equiv A_1/2B_1^2$$
$$x_2 \equiv A_2/2B_2^2$$
$$x_3 \equiv A_3/2B_3^2.$$



$$0 \equiv \sqrt{\frac{38 I_{B_1}^2}{5 I_{M_1}}\left(1-\frac{33}{38}(1-x_1)\right)} - \frac{33}{5}\sqrt{\frac{2 I_{B_2}^2}{I_{M_2}}\left(1-\frac{1}{2}(1-x_2)\right)} - I_{g2}$$

$$0 \equiv \sqrt{\frac{38 I_{B_1}^2}{5 I_{M_1}}\left(1-\frac{33}{38}(1-x_1)\right)} + \frac{11}{5}\sqrt{\frac{-2 I_{B_3}^2}{I_{M_3}}\left(1-\frac{3}{2}(1-x_3)\right)} - I_{g3}.$$

Calculate:

I_{Br}, I_{Mr}, I_{g2} and I_{g3}

GGM parameters:

- Obtain B_r directly from I_{Br}
- Constraint must be satisfied.
- Reconstructed $g_r(M)$ must satisfy inequality.

x_i points (Blue) satisfying constraints for 2 sample mass spectra.

High messenger scale M:

- x_2 and x_3 dominate constraints.
- Small window for x_2 and x_3
- Largely insensitive to x_1.

Low messenger scale M:

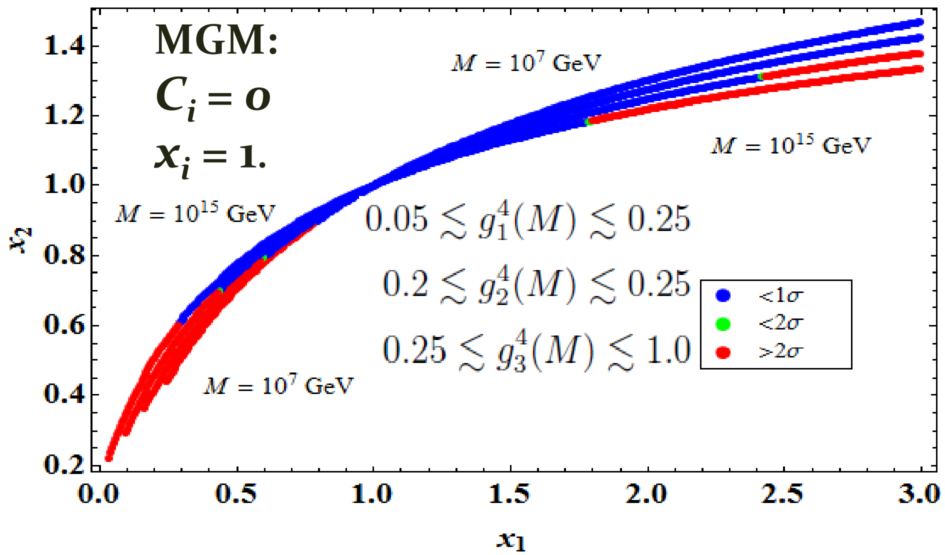
- More sensitivity to x_1.
- x_2 and x_3 not as sensitive:
  - Larger range allowed.

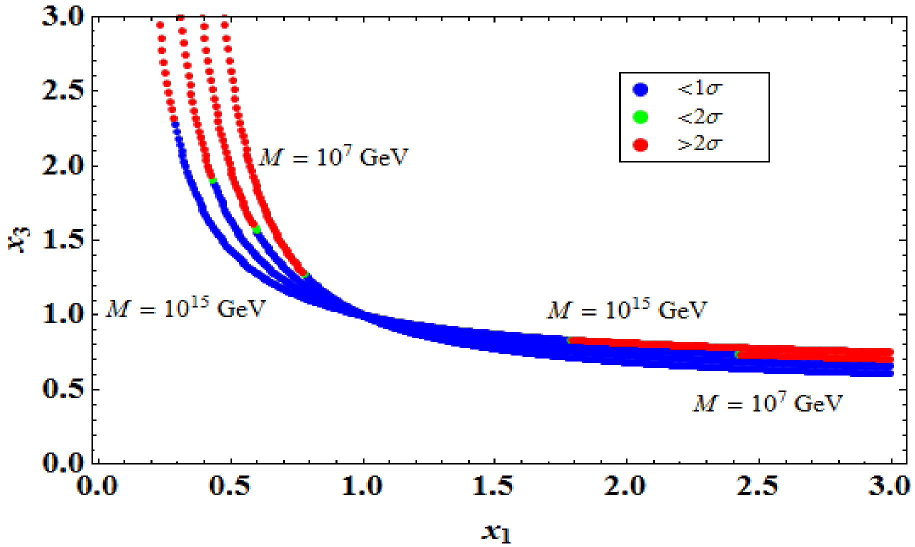
If (1,1,1) in allowed region and B_r equal

- MGM?

$$x_1 \equiv A_1/2B^2 = 1$$
$$x_2 \equiv A_2/2B^2 = 1$$
$$x_3 \equiv A_3/2B^2 = 1.$$

Example: GGM models that can never mimic MGM





$$C_1 \equiv \sqrt{\frac{38 I_B^2}{5 I_{M_1}}} - \frac{33}{5}\sqrt{\frac{2 I_B^2}{I_{M_2}}} - I_{g2} \qquad C_1 = \frac{1}{g_1^2(M)}\left(1 - \frac{33}{38}(1-x_1)\right)^{-1/2} - \frac{33}{5}\frac{1}{g_2^2(M)}\left(1 - \frac{1}{2}(1-x_2)\right)^{-1/2} - I_{g2}$$

$$C_2 \equiv \sqrt{\frac{38 I_B^2}{5 I_{M_1}}} + \frac{11}{5}\sqrt{\frac{-2 I_B^2}{I_{M_3}}} - I_{g3} \qquad C_2 = \frac{1}{g_1^2(M)}\left(1 - \frac{33}{38}(1-x_1)\right)^{-1/2} + \frac{11}{5}\frac{1}{g_3^2(M)}\left(1 - \frac{3}{2}(1-x_3)\right)^{-1/2} - I_{g3}$$

At any M, certain $x_i \neq 1$ can satisfy constraint.
Apply inequality on the reconstructed $g_i(M)$.

In above plots, Red points in the x_i space can never mimic MGM when this condition is imposed.

Without RGIs:

- Need hypercharge *D*-term
- Need messenger scale, *M*.
- Need to evolve errors and parameters.

Outlook and Conclusions

- If SUSY discovered at the LHC: raises the question of SUSY breaking scheme at some high scale.
- One would like to be able to probe this high scale phenomenon using TeV scale measurements.
- 1-loop RG Invariants provide a powerful tool to probe the high energy parameter space of models.
- Assuming an optimistic estimate of 1% measurement for the soft masses at the TeV scale, we checked that the 2-loop running does not destroy the invariance within experimental errors.
- These RGIs may determine the high energy scale.
- We can also check generic features like flavor blindness.
- Additionally, consistency with model parameter space can be probed.
- We ran numerical simulations scanning the parameter space of GGM, demonstrating the power of using these invariants to probe physics beyond the reach of the LHC.
- This methodology may be used for other models, hence building subset of SUSY breaking scenarios which satisfy an experimentally measured low energy spectrum.

Back Up Slides

Particle Content

Names		spin 0	spin 1/2	$SU(3)_C,\ SU(2)_L,\ U(1)_Y$
squarks, quarks	Q	$(\widetilde{u}_L\ \ \widetilde{d}_L)$	$(u_L\ \ d_L)$	$(\mathbf{3},\ \mathbf{2},\ \frac{1}{6})$
(×3 families)	$\overline{u}$	$\widetilde{u}_R^*$	$u_R^\dagger$	$(\overline{\mathbf{3}},\ \mathbf{1},\ -\frac{2}{3})$
	$\overline{d}$	$\widetilde{d}_R^*$	$d_R^\dagger$	$(\overline{\mathbf{3}},\ \mathbf{1},\ \frac{1}{3})$
sleptons, leptons	L	$(\widetilde{\nu}\ \widetilde{e}_L)$	$(\nu\ \ e_L)$	$(\mathbf{1},\ \mathbf{2},\ -\frac{1}{2})$
(×3 families)	$\overline{e}$	$\widetilde{e}_R^*$	$e_R^\dagger$	$(\mathbf{1},\ \mathbf{1},\ 1)$
Higgs, higgsinos	H_u	$(H_u^+\ \ H_u^0)$	$(\widetilde{H}_u^+\ \ \widetilde{H}_u^0)$	$(\mathbf{1},\ \mathbf{2},\ +\frac{1}{2})$
	H_d	$(H_d^0\ \ H_d^-)$	$(\widetilde{H}_d^0\ \ \widetilde{H}_d^-)$	$(\mathbf{1},\ \mathbf{2},\ -\frac{1}{2})$

- Chiral supermultiplets in MSSM:
  - Spin-0 fields are complex scalars,
  - Spin-1/2 fields are left-handed two-component Weyl fermions.

Names	spin 1/2	spin 1	$SU(3)_C,\ SU(2)_L,\ U(1)_Y$
gluino, gluon	$\widetilde{g}$	g	$(\mathbf{8},\ \mathbf{1},\ 0)$
winos, W bosons	$\widetilde{W}^\pm\ \ \widetilde{W}^0$	$W^\pm\ \ W^0$	$(\mathbf{1},\ \mathbf{3},\ 0)$
bino, B boson	$\widetilde{B}^0$	B^0	$(\mathbf{1},\ \mathbf{1},\ 0)$

- Gauge supermultiplets in the MSSM.

Table 1: U(1) Representations in the MSSM

Particle	Y	B	L
$Q=\begin{pmatrix} u \\ d \end{pmatrix}_L$	1/6	1/3	0
$L=\begin{pmatrix} \nu \\ l \end{pmatrix}_L$	-1/2	0	1
$u=u_R^C$	-2/3	-1/3	0
$d=d_R^C$	1/3	-1/3	0
$e=l_R^C$	1	0	-1
H_u	1/2	0	0
H_d	-1/2	0	0

Soft SUSY Breaking Masses

- GGM provides class of models in which perhaps flavor blindness is most natural.

At the Messenger Scale

- Soft sfermion masses are parameterized in terms of three numbers A_r originating from hidden sector current-current correlation functions.

$$m_{\tilde{f}}^2 = g_1^2 Y_{\tilde{f}} \xi + \sum_{r=1}^{3} g_r^4 C_r(f) A_r$$

- Assume Fayet Iliopoulos term is zero.
- Gaugino masses given in terms of three more numbers B_r:

$$M_r = g_r^2 M B_r,$$

- To generate Higgsino mass parameter, μ, may need supplemental SUSY breaking in the Higgs sector, modifying Higgs mass parameters:

$$m_{H_u}^2 = m_{\tilde{L}_3}^2 + \delta_u$$
$$m_{H_d}^2 = m_{\tilde{L}_3}^2 + \delta_d.$$

Invariant	Generic Flavor Blind Model	GGM
$D_{B_{13}}$	0	0
$D_{L_{13}}$	0	0
D_{χ_1}	$9m_{\tilde d}^2 - m_{\tilde e}^2 + 6m_{\tilde L}^2 - 6m_{\tilde Q}^2 - 3m_{\tilde u}^2$	0
$D_{Y_{13H}}$	$\frac{1}{13}\left(3(m_{\tilde d}^2 + m_{\tilde e}^2 - m_{\tilde L}^2 + m_{\tilde Q}^2 - 2m_{\tilde u}^2) + 10(m_{H_d}^2 - m_{H_u}^2)\right)$	$-\frac{10}{13}(\delta_u - \delta_d)$
D_Z	$2\left(m_{\tilde L}^2 - m_{H_d}^2\right)$	$-2\delta_d$
$I_{Y\alpha}$	$\frac{1}{g_1^2}\left(3(m_{\tilde d}^2 + m_{\tilde e}^2 - m_{\tilde L}^2 + m_{\tilde Q}^2 - 2m_{\tilde u}^2) - m_{H_d}^2 + m_{H_u}^2\right)$	$\frac{1}{g_1^2}(\delta_u - \delta_d)$
I_{B_r}	M_r/g_r^2	MB_r
I_{M_1}	$M_1^2 + \frac{33}{8}\left(m_{\tilde e}^2 + m_{\tilde u}^2 - m_{\tilde d}^2\right)$	$g_1^4\left(\frac{33}{10}A_1 + (MB_1)^2\right)$
I_{M_2}	$M_2^2 + \frac{1}{24}\left(9m_{\tilde d}^2 - m_{\tilde e}^2 + 16m_{\tilde L}^2 - 9m_{\tilde u}^2\right)$	$g_2^4\left(\frac{1}{2}A_2 + (MB_2)^2\right)$
I_{M_3}	$M_3^2 - \frac{3}{16}\left(5m_{\tilde d}^2 - m_{\tilde e}^2 + m_{\tilde u}^2\right)$	$g_3^4\left((MB_3)^2 - \frac{3}{2}A_3\right)$
I_{g_2}	≈ -10.9	≈ -10.9
I_{g_3}	≈ 6.2	≈ 6.2

$$\delta_u = -\frac{1}{2}D_Z - \frac{13}{10}D_{Y_{13H}}$$

$$\delta_d = -\frac{1}{2}D_Z.$$

$$MB_r = I_{B_r}.$$

$$A_1 = \frac{10}{33}\left(\frac{I_{M_1}}{g_1^4} - I_{B_1}^2\right)$$

$$A_2 = 2\left(\frac{I_{M_2}}{g_2^4} - I_{B_2}^2\right)$$

$$A_3 = \frac{2}{3}\left(\frac{-I_{M_3}}{g_3^4} + I_{B_3}^2\right)$$

If $I_{Y\alpha}$ and D_{Y13H} non-zero, from their ratio we can extract value of g_1, at the high scale, μ_f.

Hence, using the running of g_1, one could determine μ_f .

When errors in the determination of A_r large, can still determine certain correlations between the A_r and g_r with high accuracy:

$$g_1^4\frac{33}{10}A_1 + g_2^4\frac{11}{2}A_2 = I_{M_{12}} - g_1^4 I_{B_1}^2 - 11 g_2^4 I_{B_2}^2$$

Example LHC Accuracy

basic par.	2-loop RGE +full R.C.	1-loop RGE +approx. R.C.	relevant pole masses	2-loop RGE +full R.C.	1-loop RGE +approx. R.C.
Q_{EWSB}	465.5	468.2			
M_1	101.5	108.8	$m_{\tilde{N}_1}$	97.2	105.1
M_2	191.6	208.9	$m_{\tilde{N}_2}$	180.8	189.9
M_3	586.6	603.8	$m_{\tilde{g}}$	606.1	603.8
μ	356.9	340.6	$m_{\tilde{N}_4}$	381.8	369.6
$\tan\beta$	9.74	9.75			
$m^2_{H_d}$	$(179.9)^2$	$(187.3)^2$	m_h	110.85	111.28
$m^2_{H_u}$	$-(358.1)^2$	$-(341.7)^2$			
m_{e_L}	195.5	201.5			
m_{τ_L}	194.7	200.6			
m_{e_R}	136	138.6	$m_{\tilde{e}_2}$	142.8	145.4
m_{τ_R}	133.5	136.2			
$m_{Q_L 1,2}$	545.8	554.1	$m_{\tilde{u}_1}$	562.3	551.6
$m_{Q_L 3}$	497	502.9	$m_{\tilde{b}_1}$	516.2	502.1
m_{u_R}	527.8	531.6			
m_{t_R}	421.5	421.6			
m_{d_R}	525.7	528.7			
m_{b_R}	522.4	525.4	$m_{\tilde{b}_2}$	546.3	530.1
$-A_t$	494.5	501.0			
$-A_b$	795.2	791.3			
$-A_\tau$	251.7	255.0			
$-A_u$	677.3	686.6			
$-A_d$	859.4	857.2			
$-A_e$	253.4	256.7			

Soft and other basic parameters, plus sparticle pole masses for SPS1a input (with m_{top} = 175 GeV), calculated with SuSpect ver 2.41, for two illustrative optional choices:

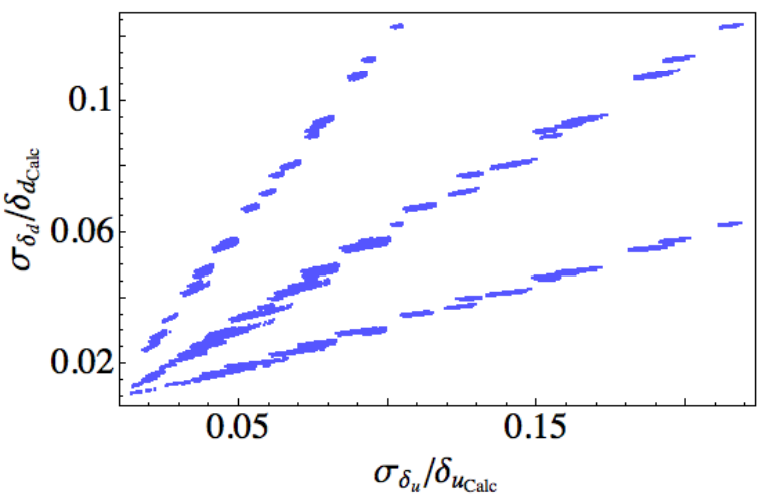
- full two-loop in RGE and full radiative corrections to sparticle masses (second and fifth columns);
- one-loop RGE, no radiative corrections to squarks, gluino, neutralinos, charginos masses, simple approximation for m_h radiative corrections (third and sixth columns).

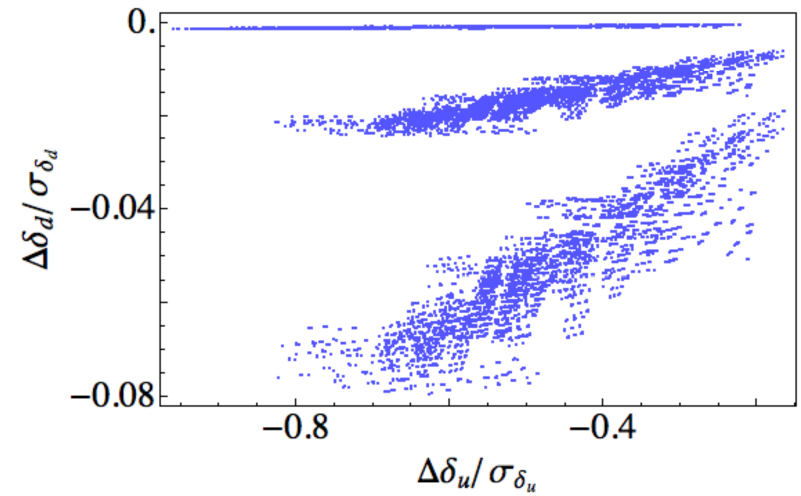
Experimental accuracies on mass determinations from LHC gluino cascade and other decays.

mass	expected LHC accuracy (GeV)	decay or process
$m_{\tilde{g}}$	7.2	$\tilde{g}$ cascade decay
$m_{\tilde{N}_1}$	3.7	" "
$m_{\tilde{N}_2}$	3.6	" "
$m_{\tilde{q}_L}$	3.7	" "
$m_{\tilde{l}_R}$	6.0	" "
$m_{\tilde{N}_4}$	5.1	$\tilde{q}_L \to \tilde{\chi}^0_4 + ..$ cascade
$m_{\tilde{b}_1}$	7.5	$\tilde{g}$ cascade decay
$m_{\tilde{b}_2}$	7.9	" "
m_h	0.25 (exp)–2 (th)	$h \to \gamma\gamma$ (mainly)

J.-L. Kneur, N. Sahoury, Phys.Rev.D79:075010,2009.
B.Allanach, C.Lester, M.Parker and B.Webber, JHEP 0009 (2000) 004.
G. Weiglein et al, Phys. Rept. 426 (2006) 47.

Corrections to the Higgs mass parameters and their expected experimental determination





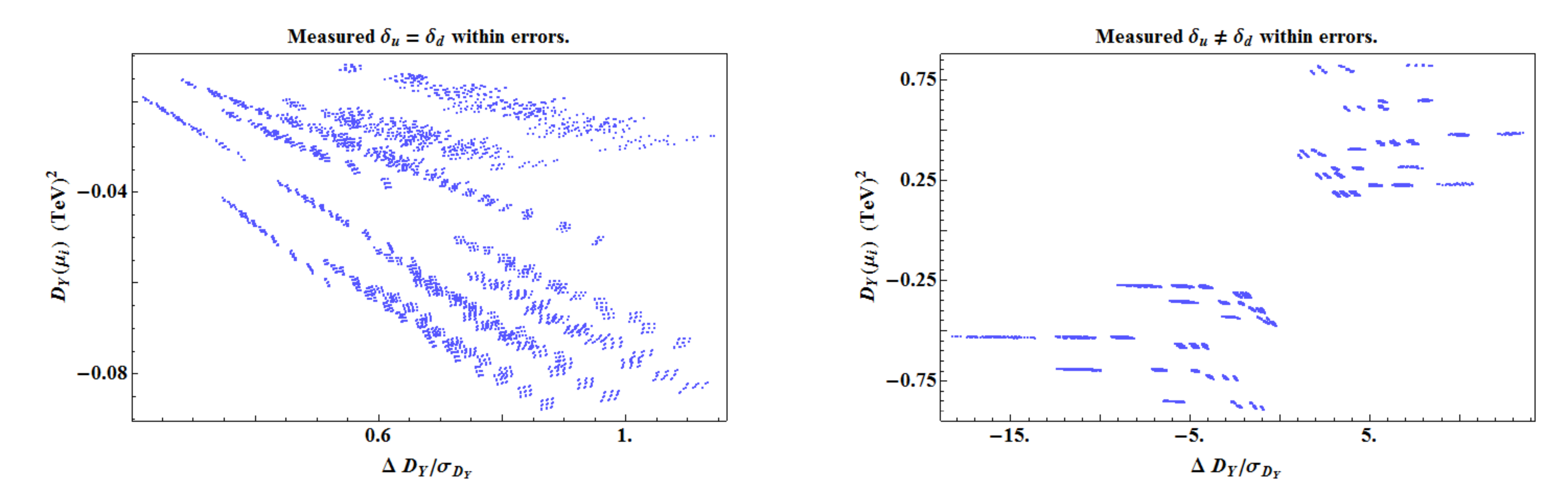
Large % errors in corrections to Higgs mass parameters expected when either is zero at input scale.

In above plots, Higgs mass parameter corrections are different with in error.

As seen from plot on the right, 2-loop contributions can be ignored for the extraction of these parameters.

This is true for both zero and non-zero corrections.

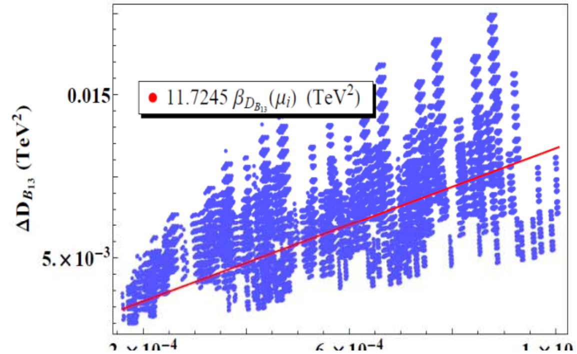
D_Y vs. the ratio of the running of D_Y and its expected error in measurements

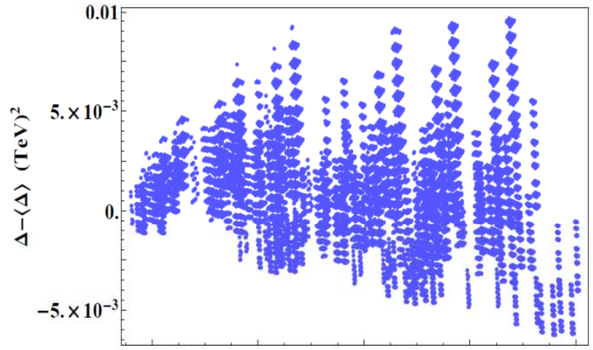


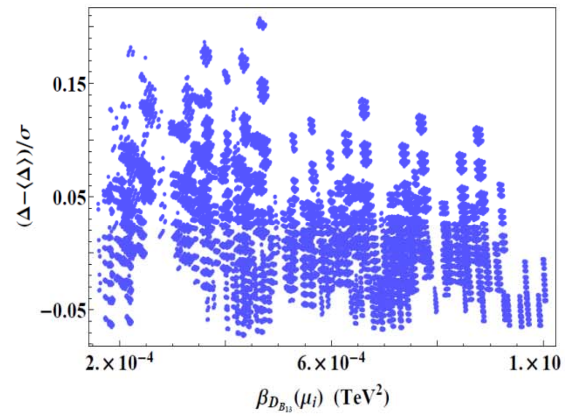
If minimal GGM, i.e., the Higgs mass parameters don't get corrections from supplemental SUSY breaking, D_Y running is smaller than the expected error in its measurement.

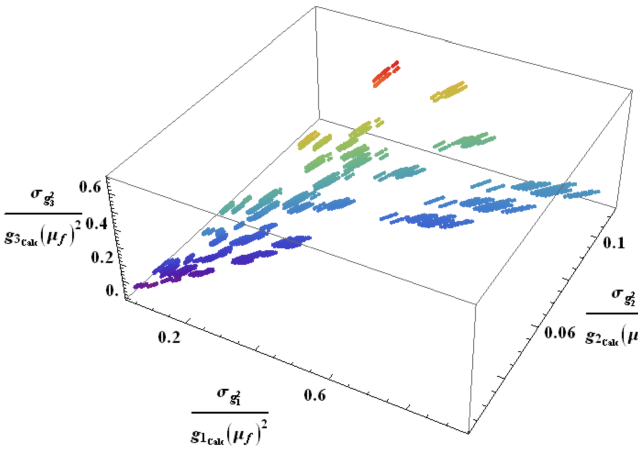
In non-minimal GGM, D_Y is not an invariant, hence as can be seen from the plot on the right, the running is comparable to or larger than the expected error. However, this would still give us information about whether it is consistent with zero or not.

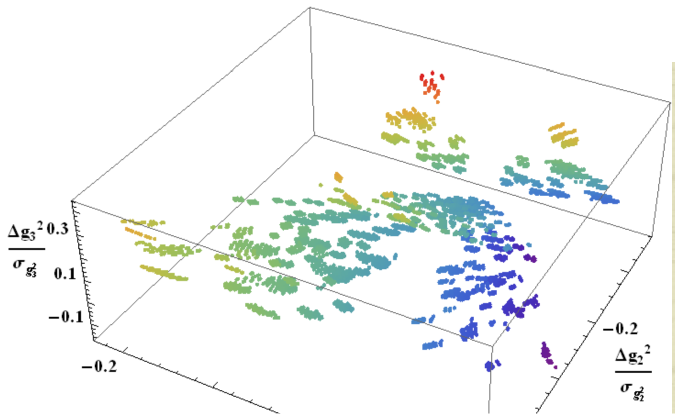
2-Loop beta function, A and gauge couplings

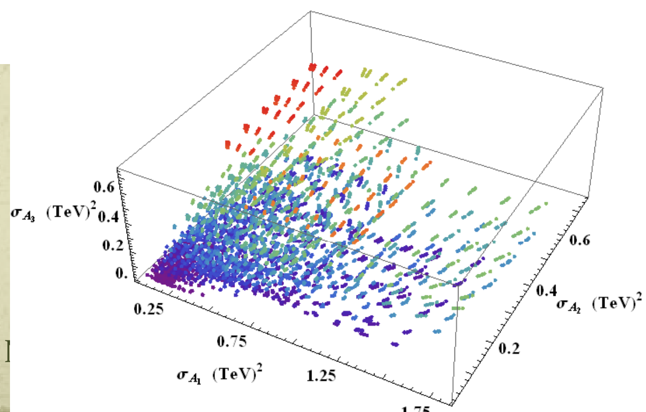


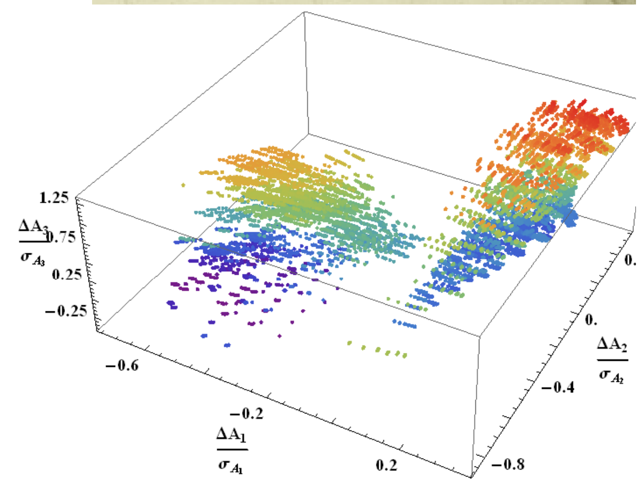




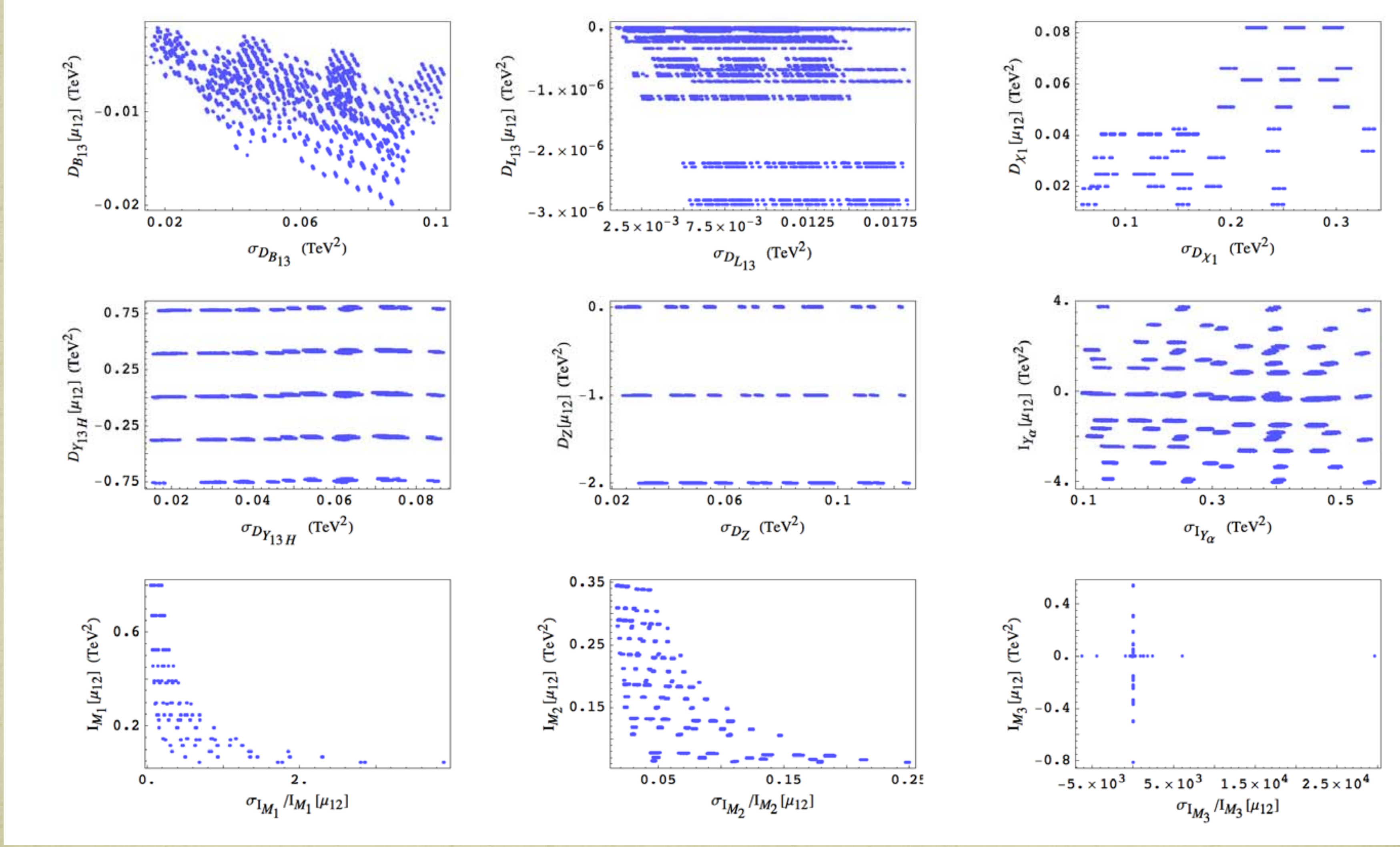




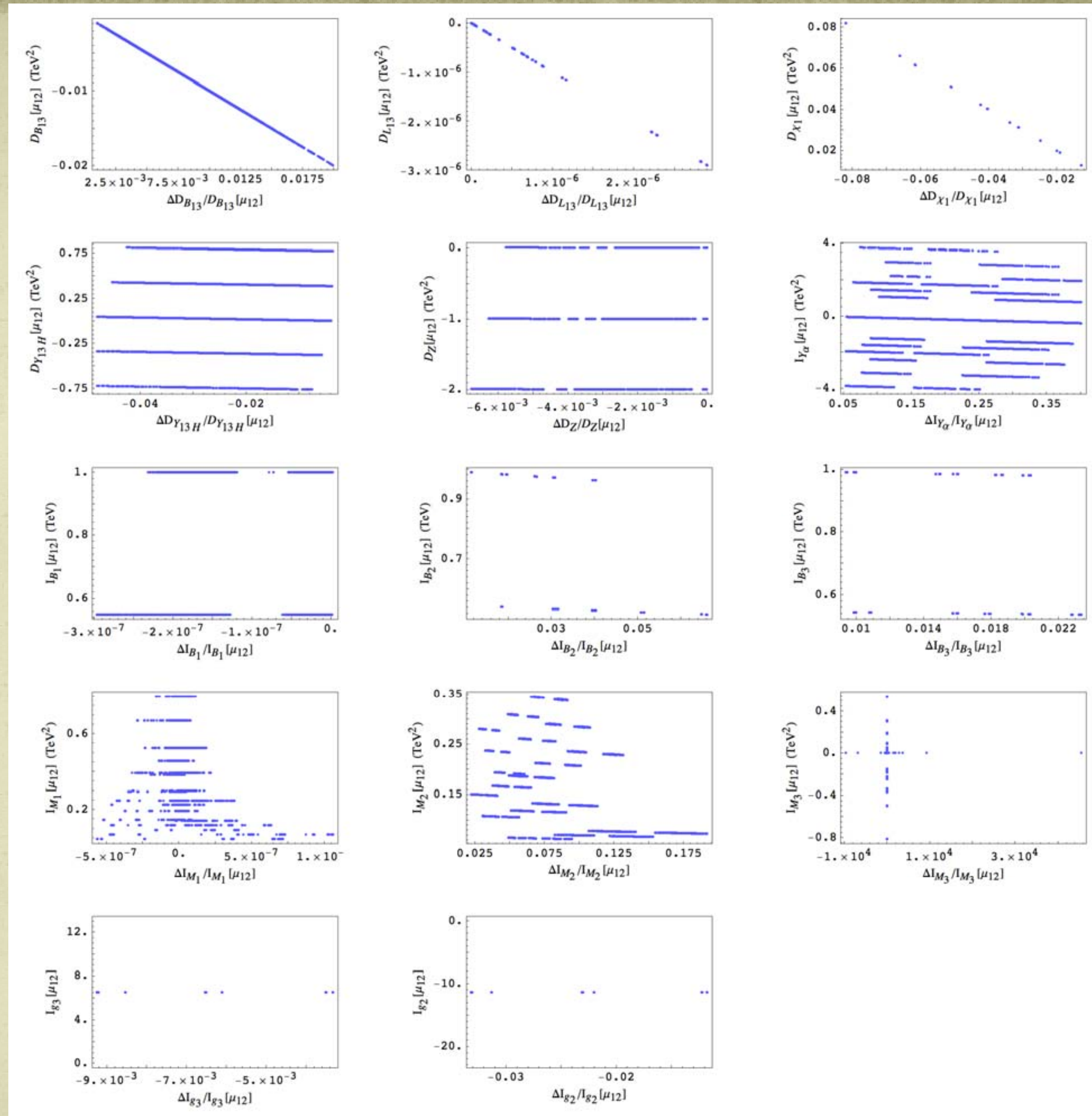




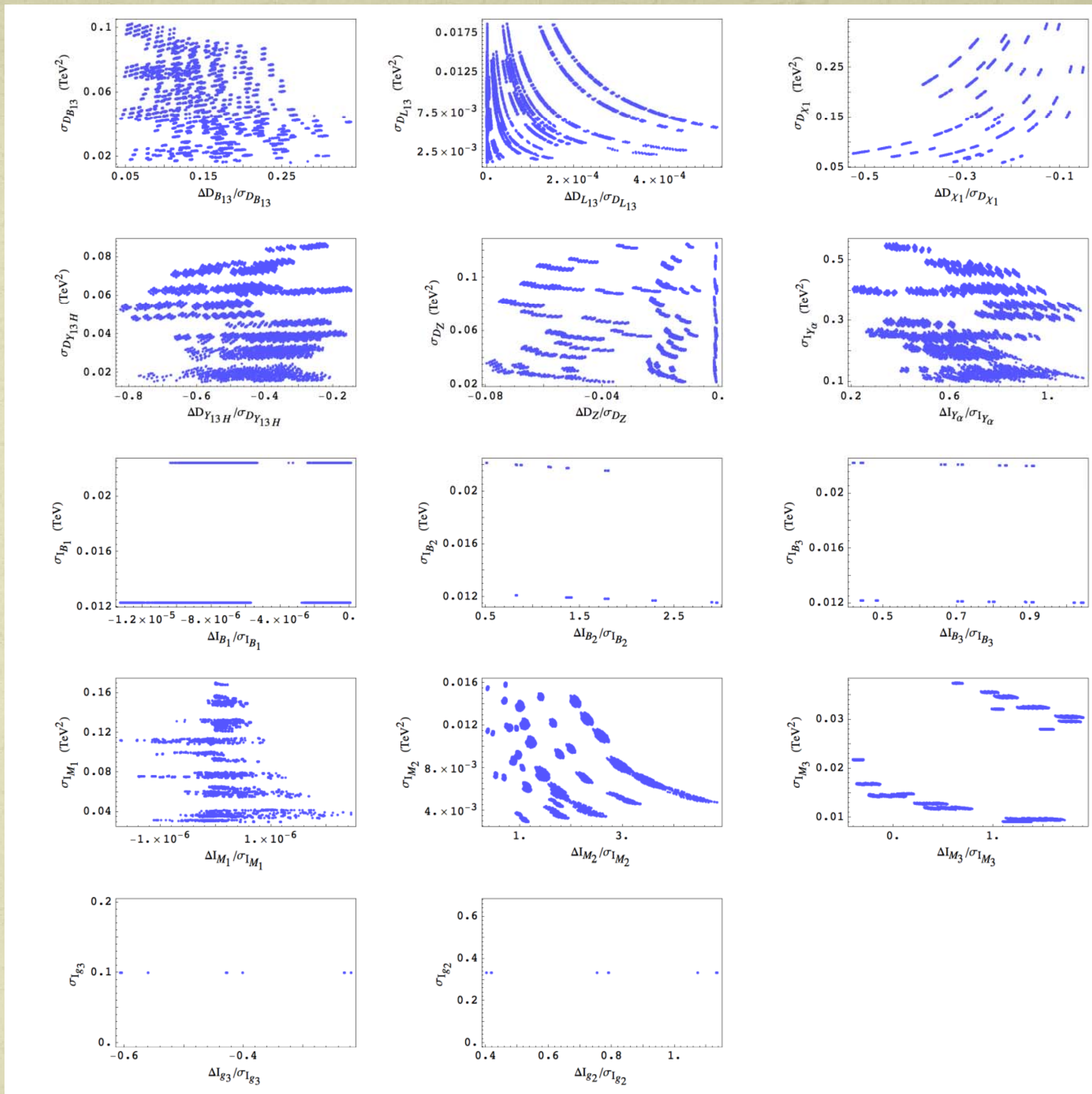
Invariants vs. σ



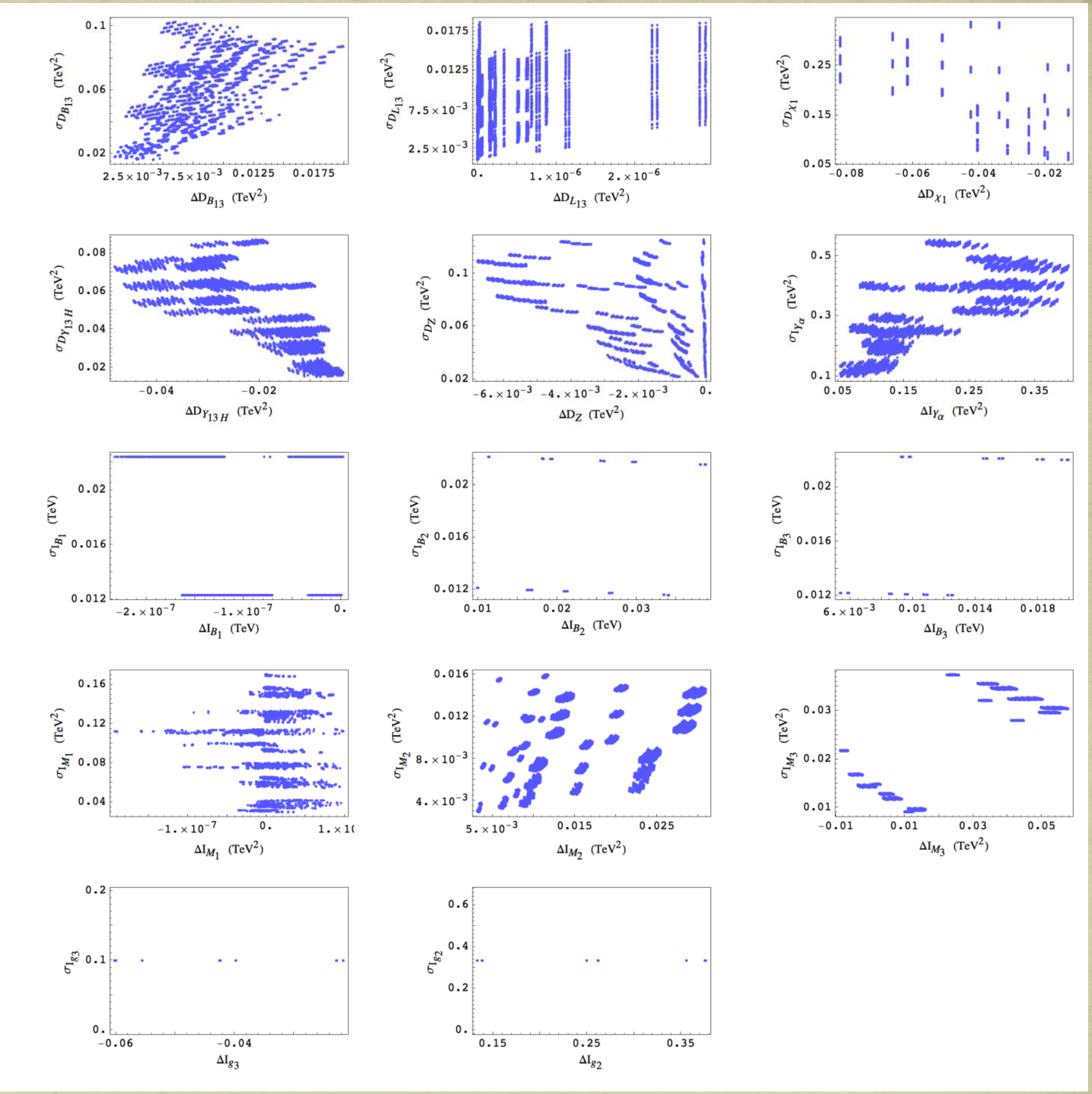
Invariants vs. 2-loop

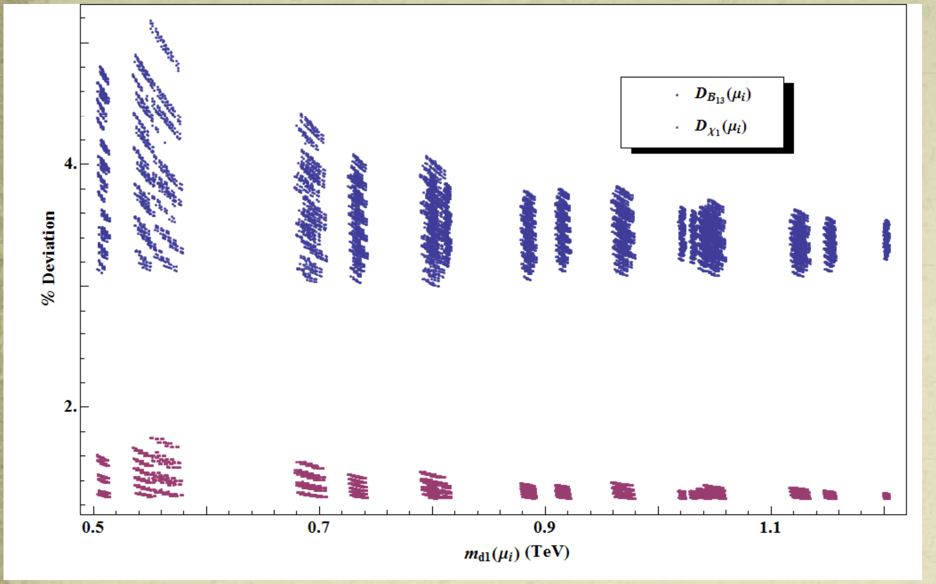


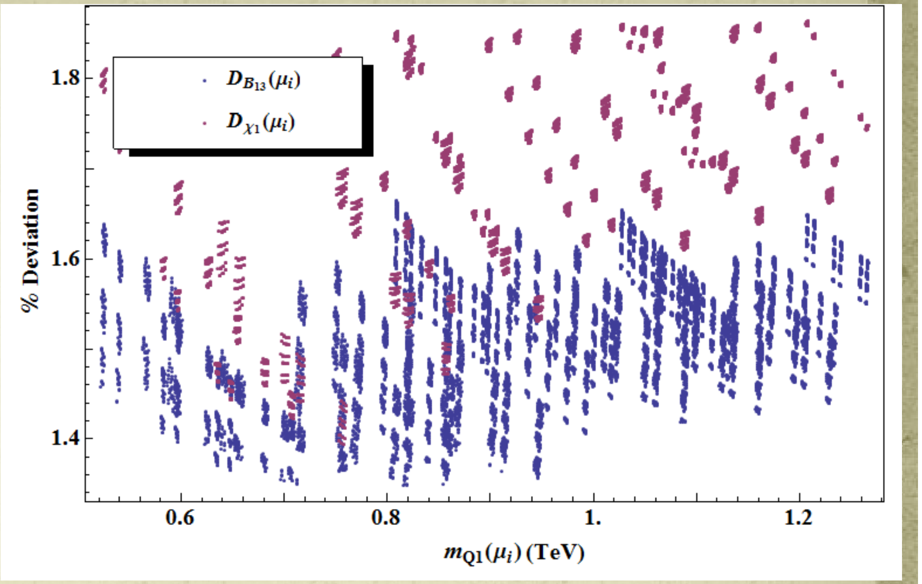
σ vs. Δ/σ

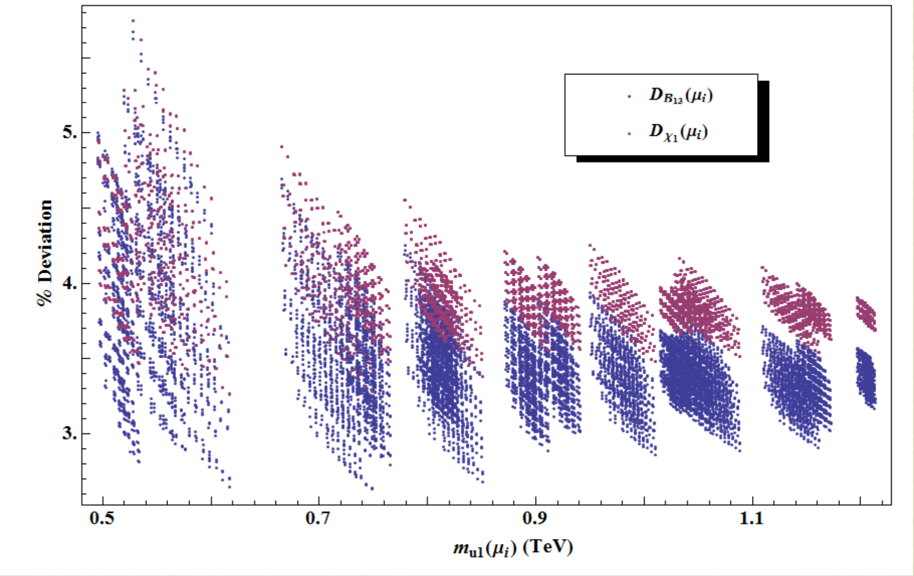


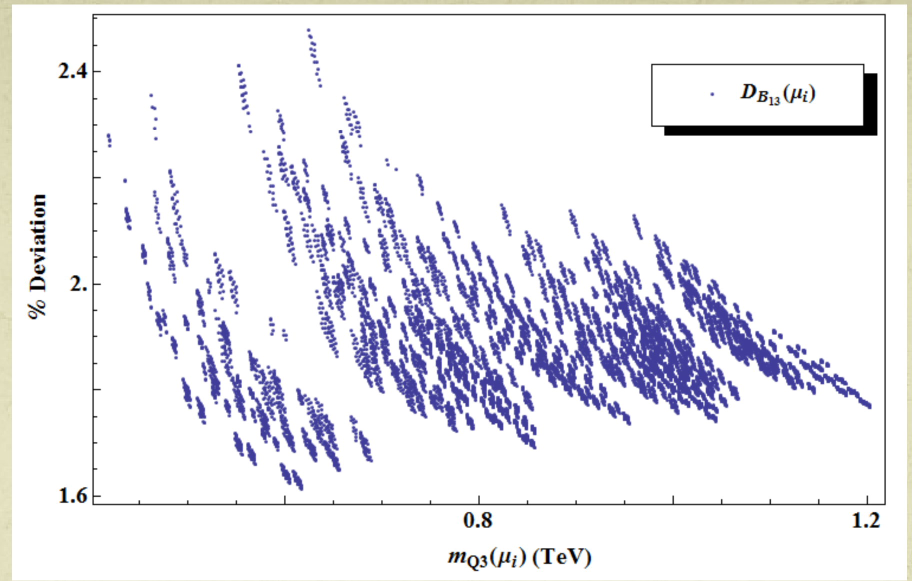
σ vs. Δ

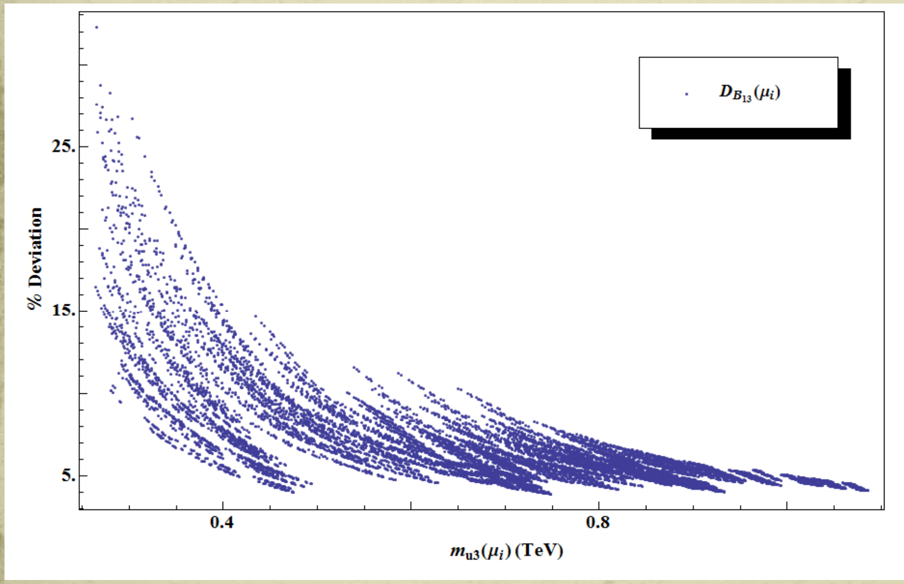


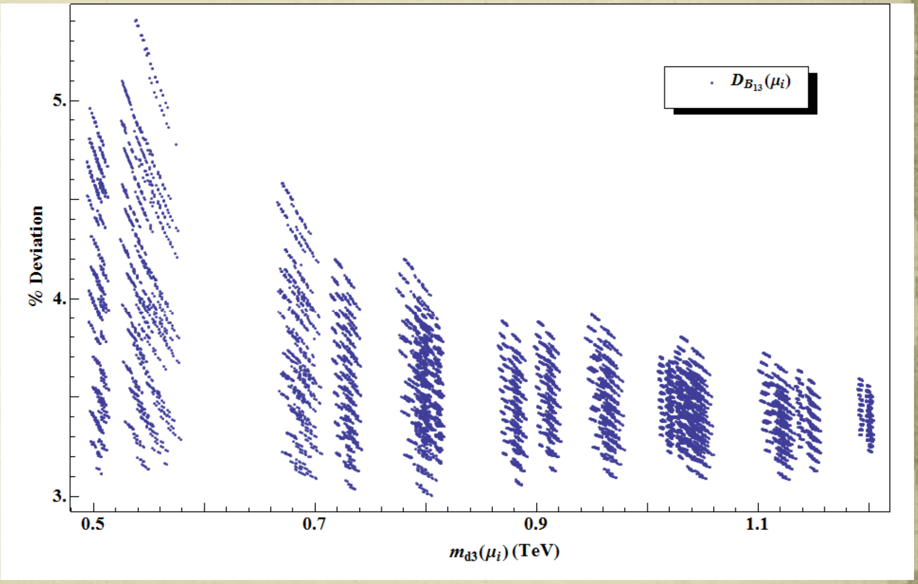

$D_{B_{13}}(\mu_i)$
$D_{\chi 1}(\mu_i)$
% Deviation
$m_{d1}(\mu_i)$ (TeV)

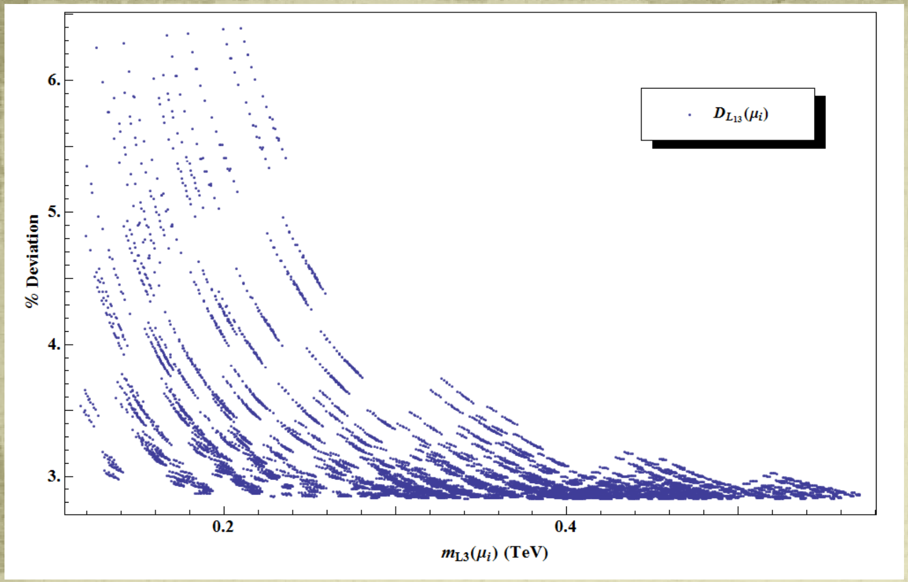

$D_{B_{13}}(\mu_i)$
$D_{\chi 1}(\mu_i)$
% Deviation
$m_{Q1}(\mu_i)$ (TeV)


$D_{B_{13}}(\mu_i)$
$D_{\chi 1}(\mu_i)$
% Deviation
$m_{u1}(\mu_i)$ (TeV)

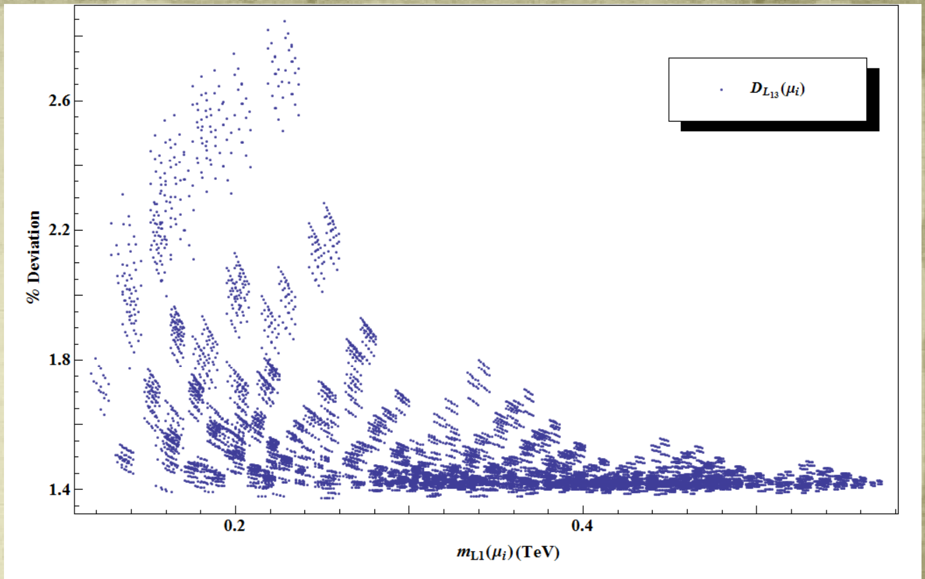

$D_{B_{13}}(\mu_i)$
% Deviation
2.4
2.
1.6
0.8
1.2
$m_{Q3}(\mu_i)$ (TeV)


$D_{B_{13}}(\mu_i)$
% Deviation
25.
15.
5.
0.4
0.8
$m_{u3}(\mu_i)$ (TeV)

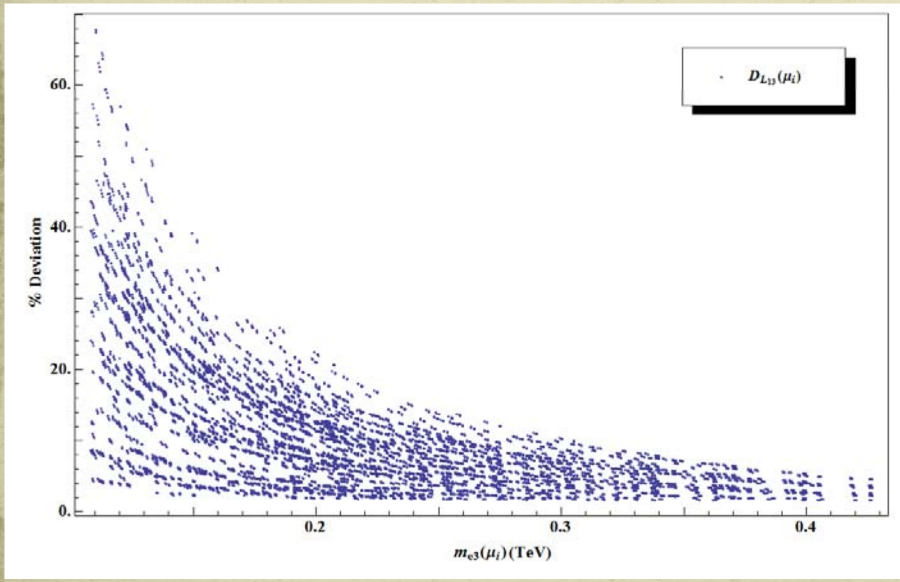

$D_{B_{13}}(\mu_i)$
% Deviation
5.
4.
3.
0.5
0.7
0.9
1.1
$m_{d3}(\mu_i)$ (TeV)



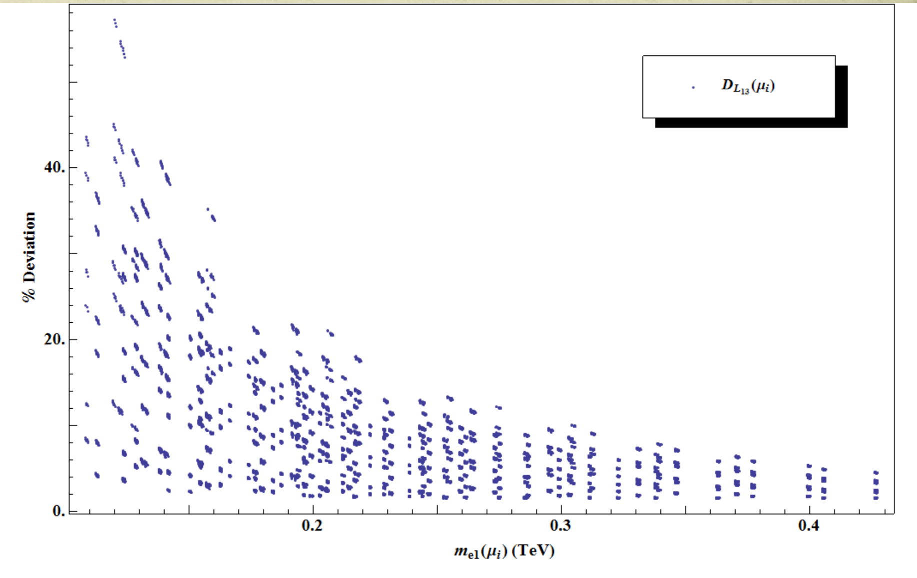
$D_{L_{13}}(\mu_i)$
% Deviation
6.
5.
4.
3.
0.2
0.4
$m_{L3}(\mu_i)$ (TeV)



$D_{L_{13}}(\mu_i)$
% Deviation
2.6
2.2
1.8
1.4
0.2
0.4
$m_{L1}(\mu_i)$ (TeV)



$D_{L_{13}}(\mu_i)$
% Deviation
60.
40.
20.
0.
0.2
0.3
0.4
$m_{e3}(\mu_i)$ (TeV)



$D_{L_{13}}(\mu_i)$
% Deviation
40.
20.
0.
0.2
0.3
0.4
$m_{e1}(\mu_i)$ (TeV)

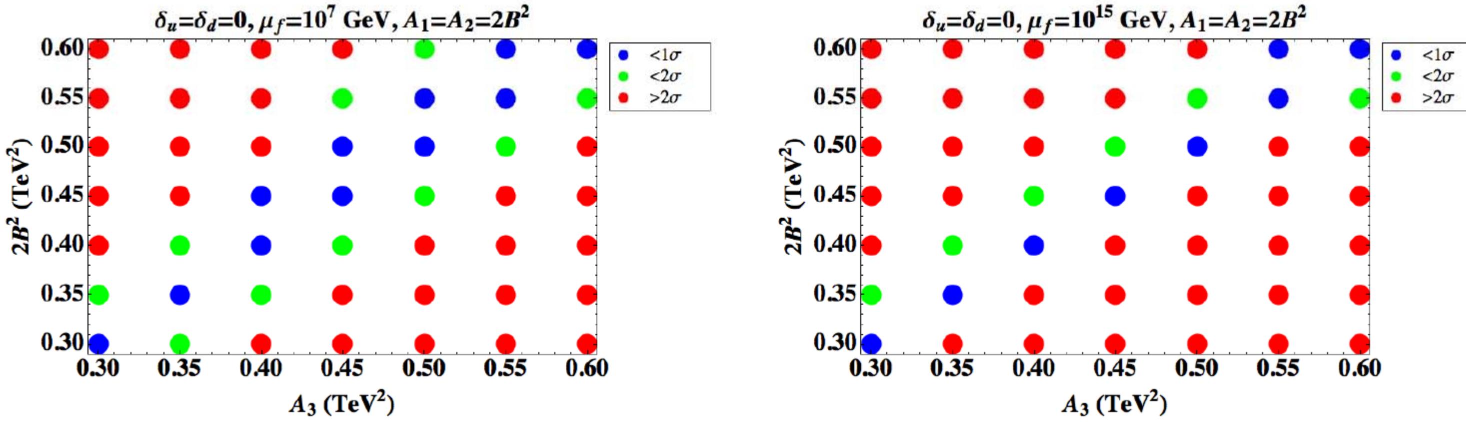


Figure 1: Ability to rule out MGM in the presence of a deformation in the A_3 direction in GGM parameter space. *Left*: $M = 10^7$ GeV; *Right*: $M = 10^{15}$ GeV.

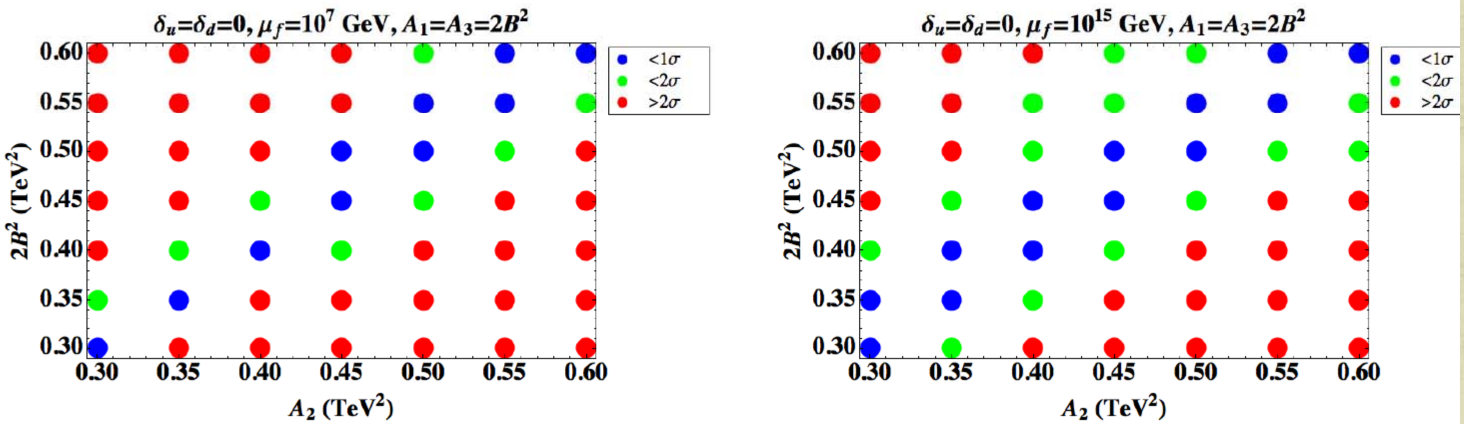


Figure 2: Ability to rule out MGM in the presence of a deformation in the A_2 direction in GGM parameter space. *Left*: $M = 10^7$ GeV; *Right*: $M = 10^{15}$ GeV.

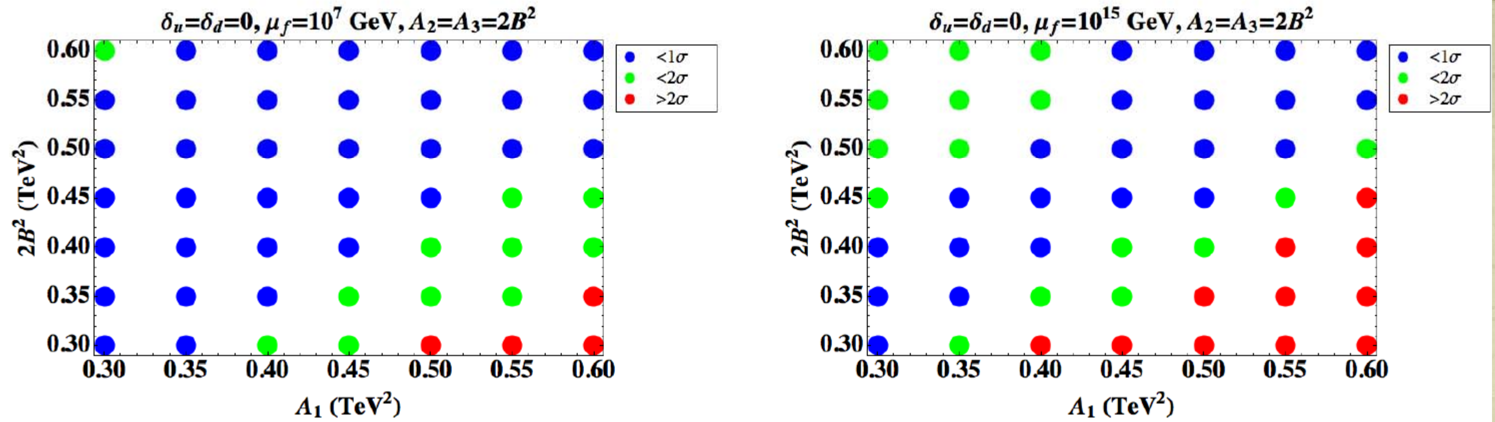


Figure 3: Ability to rule out MGM in the presence of a deformation in the A_1 direction in GGM parameter space. *Left*: $M = 10^7$ GeV; *Right*: $M = 10^{15}$ GeV.

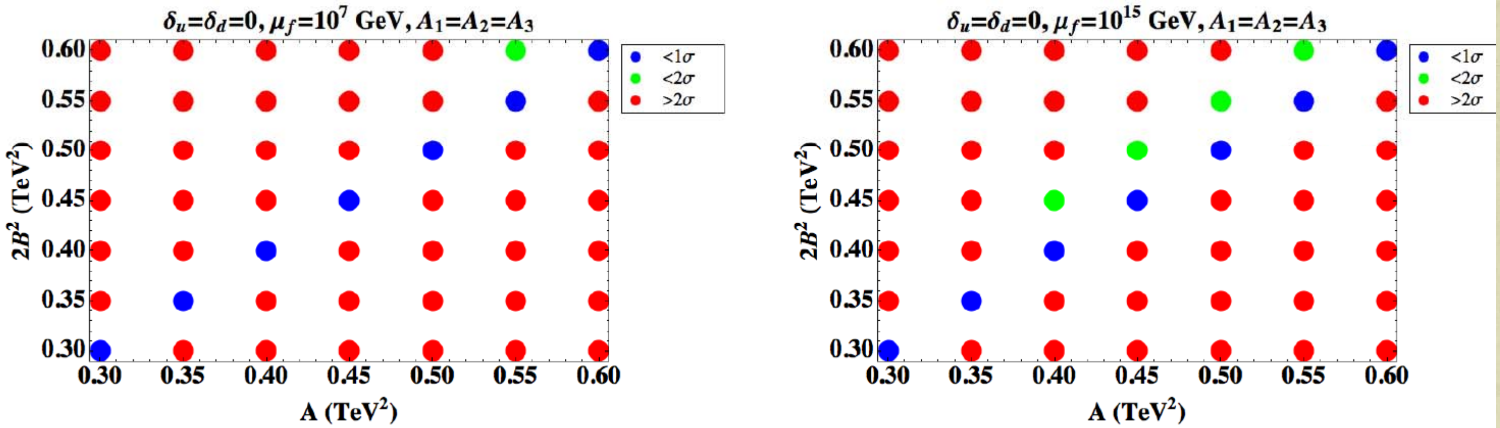


Figure 4: Ability to rule out MGM in the presence of a deformation in the A direction in GGM parameter space. *Left*: $M = 10^7$ GeV; *Right*: $M = 10^{15}$ GeV.

Data points used in producing constraint plots for GGM

(a) $M = 10^7$ GeV

Measured Spectrum:

- m_{L1} = 0.222575 TeV,
- m_{e1} = 0.113619 TeV,
- m_{Q1} = 0.629064 TeV,
- m_{u1} = 0.596435 TeV,
- m_{d1} = 0.59288 TeV,
- M_1 = 0.084916 TeV,
- M_2 = 0.157498 TeV,
- M_3 = 0.433696 TeV,

GGM paramters at M:

- $g1^4\ (M)$ = 0.0683364 (0.0663455),
- $g2^4\ (M)$ = 0.192089 (0.193213),
- $g3^4\ (M)$ = 0.685081 (0.671717)
- Tan[β] = 10,
- A =0.3,
- B = 0.387298.

(b) $M = 10^{12}$ GeV

Measured Spectrum:

- m_{L1} = 0.276946 TeV,
- m_{e1} = 0.193308 TeV,
- m_{Q1} = 0.555331 TeV,
- m_{u1} = 0.508198 TeV,
- m_{d1} = 0.496052 TeV,
- M_1 = 0.084916 TeV,
- M_2 = 0.154209 TeV,
- M_3 = 0.429292 TeV

GGM paramters at M:

- $g1^4\ (M)$ = 0.185671 (0.180322),
- $g2^4\ (M)$ = 0.245644(0.244866),
- $g3^4\ (M)$ = 0.284403 (0.281248)
- Tan[β] = 10,
- A =0.3,
- B = 0.387298.