

# More on Dimension-4 Proton Decay Problem in F-theory

SUSY 10, Physikalisches Institut, Bonn

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August 23<sup>th</sup>, 2010

Based on:

H.H., T. Kawano, Y. Tsuchiya & T. Watari : Nucl. Phys. B **840** pp. 304-348 (2010)  
arXiv: 1004.3870 [hep-th]

# Why F-theory?

A Candidate for beyond Standard Model

→ Minimal Supersymmetric Standard Model (MSSM)

→ Grand Unified Theory (GUT) around  $2 \times 10^{16}$  GeV

↳ Flavor structures, Proton decays, ...  
might be able to be answered from **string theory**.

Focus:

Constructing a supersymmetric SU(5) GUT model from string compactifications

□ Necessary components for a supersymmetric SU(5) GUT model

☆ Matter Contents

Matter

$$10_M \rightarrow (3, 2)_{\frac{1}{6}} + (\bar{3}, 1)_{-\frac{2}{3}} + (1, 1)_1$$

$$\bar{5}_M \rightarrow (\bar{3}, 1)_{\frac{1}{3}} + (1, 2)_{-\frac{1}{2}}$$

Higgs

$$5_H \rightarrow (3, 1)_{-\frac{1}{3}} + (1, 2)_{\frac{1}{2}}$$

$$\bar{5}_H \rightarrow (\bar{3}, 1)_{\frac{1}{3}} + (1, 2)_{-\frac{1}{2}}$$

☆ Yukawa Couplings

$$W_{GUT} \supset y_u 10_M 10_M 5_H + y_d \bar{5}_M 10_M \bar{5}_H$$

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☆ Yukawa Couplings

$$W_{GUT} \supset y_u 10_M 10_M 5_H + y_d \bar{5}_M 10_M \bar{5}_H \quad \leftarrow \text{F-theory}$$

# Dimension-4 Proton Decay Problem

Phenomenological Target:  
Dimension-4 Proton Decay Operator

$$W_{GUT} \supset y_u 10_M 10_M 5_H + y_d \bar{5}_M 10_M \bar{5}_H + \lambda \bar{5}_M 10_M \bar{5}_M$$

Necessary Yukawa interactions

Dimension-4 Proton  
Decay Operator!

□ In order to prohibit such a operator, It is important to distinguish...

$$\bar{5}_M \leftrightarrow \bar{5}_H$$

**Ex.** In MSSM, dimension-4 proton decay operators can be eliminated by imposing Matter Parity or R-Parity.

How is such a kind of symmetry achieved in F-theory compactifications?

# Proposed Solutions so far

## ☆ Proposed Solutions so far

(i) global compactifications with  $\mathbf{Z}_2$  symmetry Tatar, Tsuchiya, Watari 0905  
H.H. et al 0910, ...

(ii) rank 5- GUT scenario with U(1) Flux Tatar, Watari 0602, ...

(iii) factorized spectral surface scenario

(using an unbroken U(1) symmetry obtained by taking a special configuration of adjoint higgs vev)

Beasley, Heckman, Vafa 0806

Tatar, Tsuchiya, Watari 0905

Marsano, Saulina, Schafer-Nameki 0906, 0912

Grimm, Krause, Weigand 0912,

Chen et al 1005, ...

(iv) spontaneous R-parity violation

Tatar, Watari 0602, Tatar, Tsuchiya, Watari 0905

Blumenhagen, Grimm, Jurke, Weigand 0908, ...

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Chen et al 1005, ...

(iv) spontaneous R-parity violation

Tatar, Watari 0602, Tatar, Tsuchiya, Watari 0905  
Blumenhagen, Grimm, Jurke, Weigand 0908, ...

The (iii) scenario is not without a theoretical concern.  
We analyzed carefully whether this U(1) is actually unbroken or not.

# Outline

1. Motivation
2. Factorized Spectral Surface Scenario
3. Monodromy and  $U(1)$  symmetry
4. Summary

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# What is F-theory?

Vafa 96

F-theory ~ including strong coupling effects of Type IIB string theory

Basic components → **7-branes**  
(not only D7 branes but also mutually non-local ones)

Gauge Fields → Localized on 7-branes ( $S$ )

Matters → Localized along intersections ( $\Sigma$ ) of 7-branes ( $S$ )

Yukawas → Localized at intersection points ( $p$ ) of matter curves ( $\Sigma$ )

$$\begin{array}{c} E \rightarrow CY_4 \\ \downarrow \\ B_3 \supset S \supset \Sigma (= S \cdot S') \supset p (= \Sigma \cdot \Sigma') \\ \updownarrow \\ G_{\text{GUT}} = G_S \subset G_\Sigma \subset G_p \\ \uparrow \end{array}$$

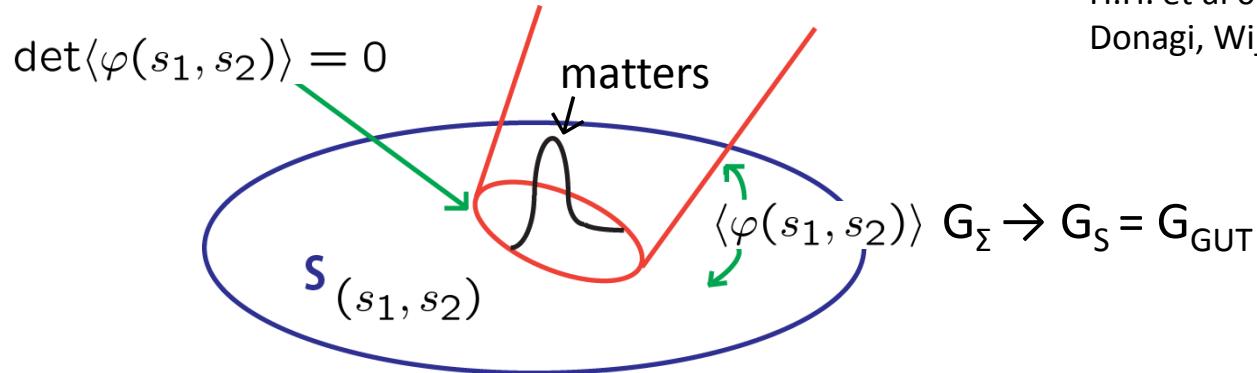
Donagi, Wijnholt 0802  
Beasley, Heckman, Vafa 0802  
H.H. et al 0805

Enhancement of singularities (vanishing 2-cycles)  
~ gauge symmetry enhancement

# 8D Gauge Theory on 7-branes

## ○ 8D gauge theory on 7-branes (“Higgs bundle”)

Beasley, Heckman, Vafa 0802,  
H.H. et al 0901,  
Donagi, Wijnholt 0904



## ○ $E_8$ Higgs bundle: incorporating all the necessary Yukawa couplings

$$E_8 \supset SU(5)_{GUT} \times \langle SU(5)_{broken} \rangle$$

$\uparrow$   $\langle \varphi(s_1, s_2) \rangle$

a parent gauge group

Marsano, Saulina, Schafer-Nameki 0906, 0912  
Blumenhagen, Grimm, Jurke, Weigand 0908  
Grimm, Krause, Weigand 0912,  
Chen, Chung 1005, Chen et al 1005, Choi 1007,...

Spectral surface

$$\det(\xi I_{5 \times 5} - \langle \varphi \rangle) = a_0 \xi^5 + a_2 \xi^3 + a_3 \xi^2 + a_4 \xi + a_5 = 0$$

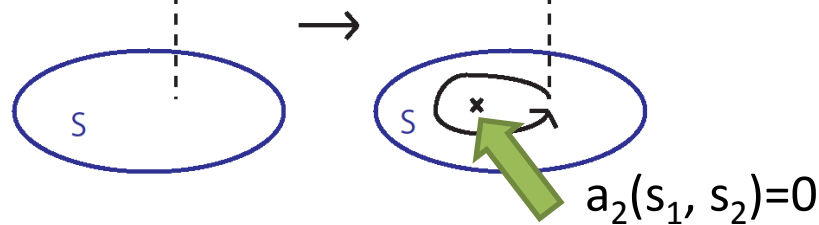

  
sections over the surface on which GUT 7-branes wrap

# Factorized Spectral Surface Scenario

Ex. Rank 2 case :  $\langle \text{SU}(2) \rangle \rightarrow$  naively we expect an  $\text{U}(1)$  symmetry unbroken...

$$a_0(\det(\xi I_{2 \times 2} - \langle \varphi \rangle)) = a_0 \xi^2 + a_2 = 0 \rightarrow \xi = \pm \sqrt{\frac{a_2}{a_0}}$$

$$\langle \varphi(s_1, s_2) \rangle = \begin{pmatrix} \sqrt{\frac{a_2}{a_0}} & 0 \\ 0 & -\sqrt{\frac{a_2}{a_0}} \end{pmatrix} - \begin{pmatrix} \sqrt{\frac{a_2}{a_0}} & 0 \\ 0 & -\sqrt{\frac{a_2}{a_0}} \end{pmatrix}$$



This monodromy kills the  $\text{U}(1)$  symmetry.

➔ Factorization:  $a_0 \xi^2 + a_2 = (c_0 \xi + c_1)(d_0 \xi + d_1) \rightarrow \langle \varphi(s_1, s_2) \rangle = \begin{pmatrix} \frac{c_1}{c_0} & 0 \\ 0 & -\frac{c_1}{c_0} \end{pmatrix}$

Then, we have no monodromy  
and there seems to be an unbroken  $\text{U}(1)$  symmetry.

This kind of  $\text{U}(1)$  symmetry seems to be able to be used for prohibiting dimension-4 proton decay operators when applying to a  $\text{SU}(5)$  model.

# Factorized Spectral Surface Scenario

## Caveats

1. 8D gauge theory description is only an approximation of the global compactification of F-theory. There might be some monodromy missed by this approximation.
2. At  $a_0 \sim 0$ , higgs vev becomes infinity, then effective gauge theory description fails around this region. There might be another monodromy from the loop in this region.

$$\det(\xi I_{5 \times 5} - \langle \varphi \rangle) = a_0 \xi^5 + a_2 \xi^3 + a_3 \xi^2 + a_4 \xi + a_5 = 0$$

→ We have to go **beyond 8D gauge theory** descriptions in order to find out whether the U(1) symmetry in factorized spectral surface scenario is unbroken or not.

Since we cannot rely on 8D gauge theory descriptions of F-theory, we have to go back to the origin of gauge fields in **global** compactifications of F-theory.

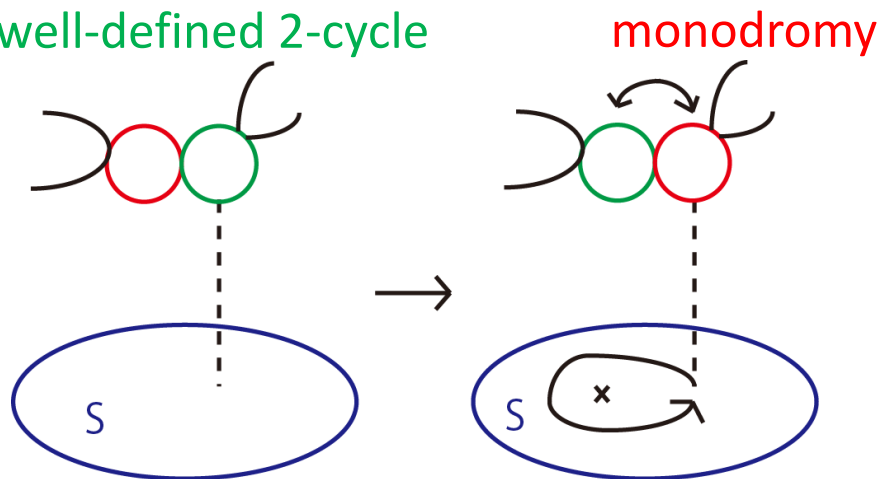
# Outline

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3. **Monodromy and  $U(1)$  symmetry**
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# Globally well-defined 2-cycles

The origin of gauge fields in F-theory  
→  $(p, q)$  strings ending on 7-branes or  
M2-branes wrapping on **globally well-defined** 2-cycles.

× Not globally well-defined 2-cycle



What we did:

Figure out whether there is a **globally well-defined** 2-cycle,  
or a  $(p, q)$  string, in the factorized spectral surface scenario  
by considering a **global** compactification manifold.


# Globally well-defined 2-cycles

Ex. K3 fib.  $CY_4$  with  $E_6$  singularity ( $SU(3)$  bdl)

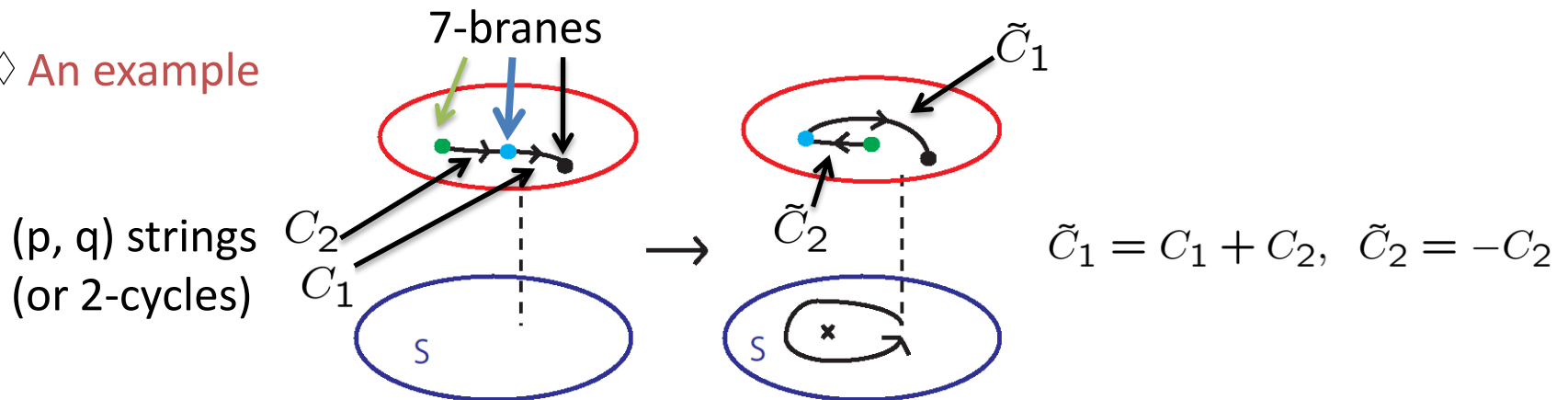
→ take (2+1) factorization of a rank 3 spectral surface

$E \rightarrow K3$

↓  
 $P^1$

22 2-cycles in K3  an elliptic fiber and a zero section  
20 2-cycles ← **Not** all of them are globally well-defined

◇ An example



Monodromy of 2-cycles  $\leftrightarrow$  The change of the locations of 7-branes

This can be analyzed by calculating the discriminant of the elliptic fibration

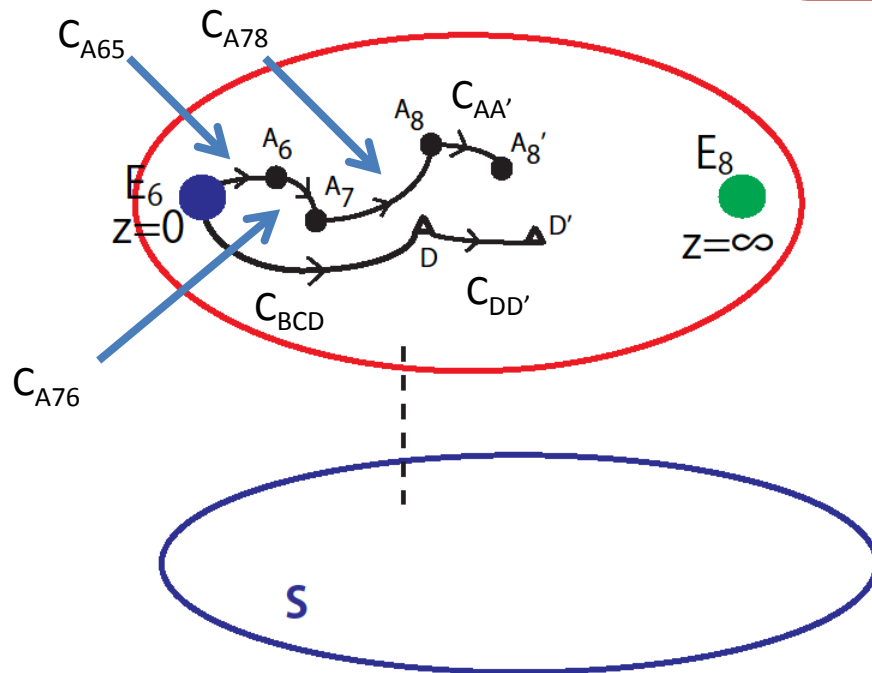
# 7-brane configurations of a K3 surface

$$\text{K3 fib. CY}_4: y^2 = x^3 + \underbrace{(a_2 z^3 + f_0 z^4)}_F x + \underbrace{\left(\frac{1}{4} a_3^2 z^4 + a_0 z^5 + g_0 z^6 + a_0'' z^7\right)}_G$$

$$\Delta = 4F^3 + 27G^2 = z^8 \times (\text{degree 6 polynomial})$$

8 7-branes at  $z=0$   $\uparrow$   
( $E_6$  singularity)

$\uparrow$  6 other 7-branes  
at  $z=z_i$  ( $i=1, \dots, 6$ )



- [1, 0]-brane (A-brane)
- [1, -1]-brane (B-brane)
- [1, 1]-brane (C-brane)
- △ [3, 1]-brane ("D"-brane)



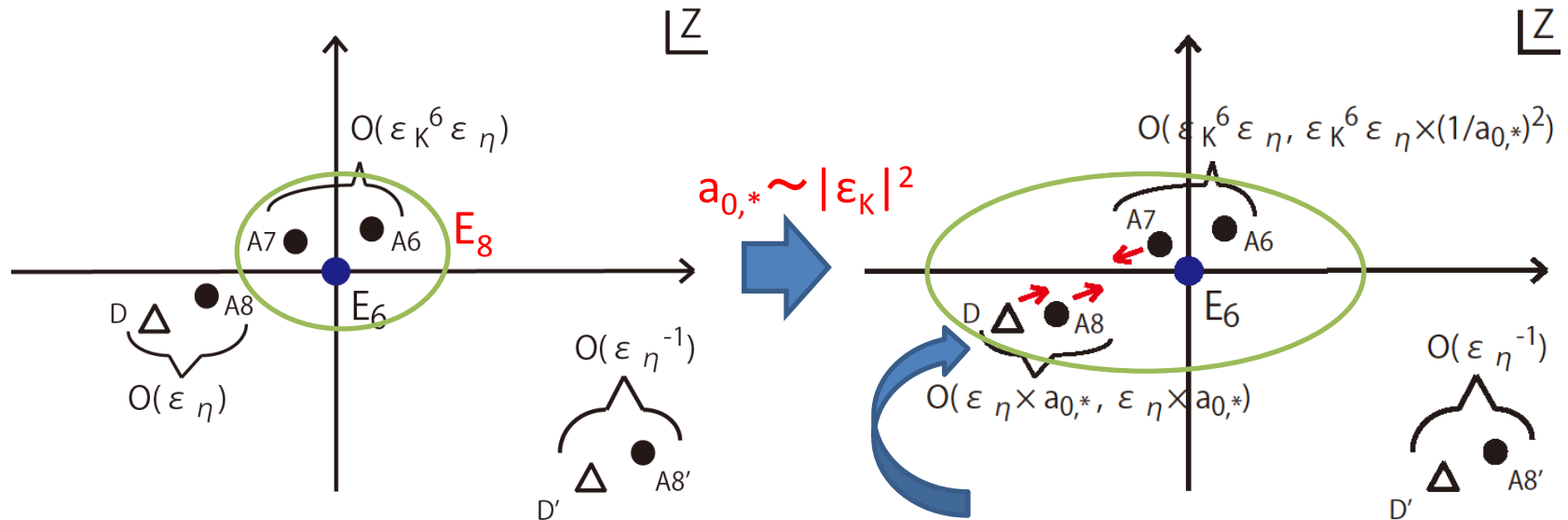
# 8D Gauge Theory Region and Beyond

$$\text{CY}_4: y^2 = x^3 + (a_2 z^3 + f_0 z^4)x + \left(\frac{1}{4}a_3^2 z^4 + a_0 z^5 + g_0 z^6 + a_0'' z^7\right)$$

8D gauge theory region:

$$a_0 = a_{0,*} \epsilon_\eta, \quad a_2 = a_{2,*} \epsilon_K^2 \epsilon_\eta, \quad a_3 = a_{3,*} \epsilon_K^3 \epsilon_\eta$$

with  $|a_{r,*}| \sim \mathcal{O}(1)$ ,  $0 \neq |\epsilon_K| \ll 1$ ,  $0 \neq |\epsilon_\eta| < 1$



We have to take into account another two 7-branes

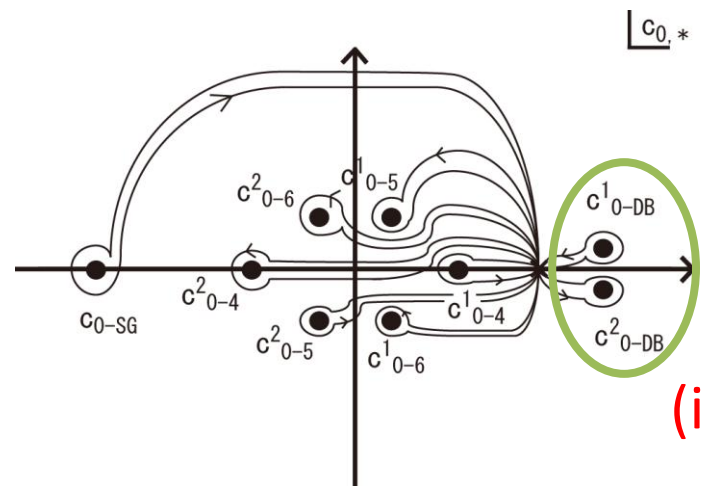
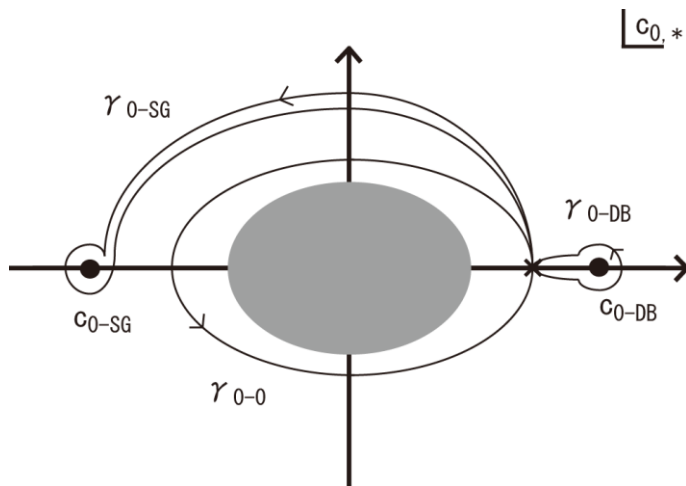
Beyond 8D gauge theory region

# Monodromy Locus

(2+1) Factorization:  $a_0\xi^3 + a_2\xi + a_3 = (c_0\xi^2 + c_1\xi + c_2)(d_0\xi + d_1)$

(8D gauge theory region)

(Beyond 8D gauge theory region)

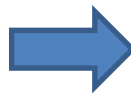


(i)  $c_{0-DB}$  splits into two  $c_{0-DB}^{1,2}$

$$\rho(\gamma_{0-SG}) = W_{C_{A76}+C_{-\theta}}$$

$$\rho(\gamma_{0-DB}) = id$$

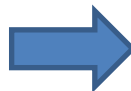
$$\rho(\gamma_{0-0}) \sim id$$



$$\rho(\gamma_{0-SG}) = W_{C_{A76}+C_{-\theta}}$$

$$\rho(\gamma_{0-DB}^{1,2}) = \rho(\gamma_{0-out}) = W_{C_{A76}}$$

$$\mathbb{Z}_2 \subset SU(3)$$



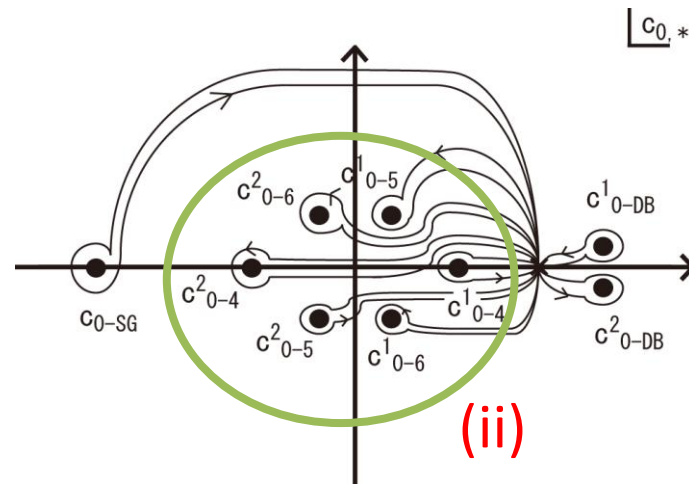
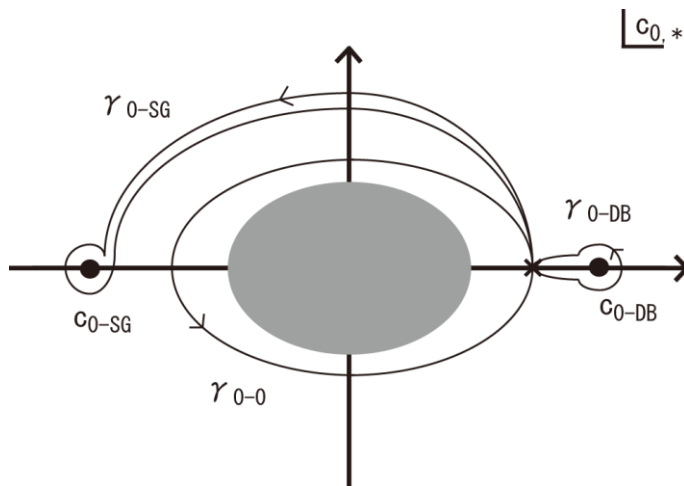
$$S_3 \subset SU(3)$$

# Monodromy Locus

(2+1) Factorization:  $a_0\xi^3 + a_2\xi + a_3 = (c_0\xi^2 + c_1\xi + c_2)(d_0\xi + d_1)$

(8D gauge theory region)

(Beyond 8D gauge theory region)



(ii) New 6  $c_{0-4,5,6}^{1,2}$  points

$\rho(\gamma_{0-4,5,6}^{1,2})$

→ mix the 2-cycles within  $E_8$  with the ones without  $E_8$ !  
(i.e. **beyond  $S_3$ !**)

Even after the factorization, monodromy is **NOT** reduced.  
**i.e. NO UNBROKEN U(1) SYMMETRY!**

# Loopholes

1. To avoid the mixing of 2-cycles within  $E_8$  and those without  $E_8$ , the easiest way is to take  $a_0$  as a section of a trivial line bundle. Then,  $a_0$  do not have zeros and every region can be characterized by gauge theory descriptions. However, in this case also, we can not avoid the higher order terms which cause the splitting in the previous example.
2. Other solution is that to impose the factorization condition more globally. In a simple factorization limit, we only impose the factorization condition on the leading terms of the defining equation of  $CY_4$ . But we can also extend the condition to other higher order terms.

$$y^2 = x^3 + (a_0 z^5 + g_0 z^6 + a_0'' z^7 + \mathcal{O}(z^8)) + (a_2 z^3 + f_0 z^4 + \mathcal{O}(z^5))x + (a_3 z^2 + \mathcal{O}(z^3))y + \dots$$

Imposing conditions further  
on higher order terms

Grimm, Weigand 1006  
Marsano, Saulina, Schafer-Nameki 1006

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# Summary

We revisited the dimension-4 proton decay problem in F-theory and we explicitly computed **globally well-defined 2-cycles in  $CY_4$**  in order to find out how many U(1) symmetries are left unbroken. Then we found that a supposed unbroken U(1) symmetry in a simple factorization limit is indeed **broken**.

To avoid the problem, we need **to tune more parameters** of the internal space of compactifications.

## Discussion

Although we showed that dimension-4 proton decay operators are likely to be generated in a factorized spectral surface scenario, it would be interesting **to compute the coefficients** of those operators. There may be a chance to suppress the dangerous operators