FCNC in non-Abelian discrete flavor symmetry with SUSY

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Introduction

- The experimental data indicate Tri-bimaximal mixing.

$$U_{\text{Tri-bimaximal}} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}.$$

- Non-Abelian discrete symmetry can lead to tri-bimaximal mixing, since non-Abelian discrete symmetry connects different generations.

Our model:

S_4 flavor model in $SU(5)$ SUSY GUT!!

- The Cabibbo angle of quarks is 15°, while the lepton mixing is tri-bimaximal.
- Flavor symmetry constraints the slepton structure, which predicts

FCNC ($\mu \to e\gamma$).

S_4 group

- S_4 group is the symmetry group of octahedron or permutation of four elements. Number of elements is 24.

- Irreducible representations of S_4 are $\mathbf{1}$, $\mathbf{1}'$, $\mathbf{2}$, $\mathbf{3}$, and $\mathbf{3}'$.
- Multiplication rules are

$$\mathbf{3} \otimes \mathbf{3} = \mathbf{1} \oplus \mathbf{2} \oplus \mathbf{3} \oplus \mathbf{3}'$$
$$\mathbf{3} \otimes \mathbf{3}' = \mathbf{1}' \oplus \mathbf{2} \oplus \mathbf{3} \oplus \mathbf{3}'$$
$$\mathbf{2} \otimes \mathbf{2} = \mathbf{1} \oplus \mathbf{1}' \oplus \mathbf{2}$$
$$\vdots$$

etc.

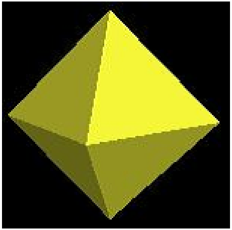

Figure: S_4 symmetry : Octahedron

S_4 flavor model in $SU(5)$ SUSY GUT

H. Ishimori, K. Saga, Y. S, and M. Tanimoto, Phys.Rev.D81:115009,2010.

	(T_1, T_2)	T_3	(F_1, F_2, F_3)	(N_e^c, N_μ^c)	N_τ^c	H_5	$H_{\bar{5}}$	H_{45}	Θ
$SU(5)$	10	10	$\bar{5}$	1	1	5	$\bar{5}$	45	1
S_4	**2**	**1**	**3**	**2**	$\mathbf{1}'$	**1**	**1**	**1**	**1**
Z_4	$-i$	-1	i	1	1	1	1	-1	1
$U(1)_{FN}$	ℓ	0	0	m	0	0	0	0	-1

$$T_1 = (q_1, u^c, e^c),\ F_1 = (d^c, l),\ N_e^c, N_\mu^c, N_\tau^c\text{: RH Majorana neutrinos}$$

- We introduce new scalars χ_i, which are $SU(5)$ gauge singlets.

	Up	Majorana	Dirac	Charged leptons + Down-type quarks		
	(χ_1, χ_2)	(χ_3, χ_4)	(χ_5, χ_6, χ_7)	$(\chi_8, \chi_9, \chi_{10})$	$(\chi_{11}, \chi_{12}, \chi_{13})$	χ_{14}
$SU(5)$	1	1	1	1	1	1
S_4	**2**	**2**	$\mathbf{3}'$	**3**	**3**	**1**
Z_4	$-i$	1	$-i$	-1	i	i
$U(1)_{FN}$	$-\ell$	$-n$	0	0	0	$-\ell$

- We add Z_4 and $U(1)_{FN}$ which control these scalar couplings.

- $SU(5) \times S_4 \times Z_4 \times U(1)_{FN}$ invariant superpotential of Yukawa sector.

$$
\begin{aligned}
w = {} & y_1^u (T_1, T_2) \otimes T_3 \otimes (\chi_1, \chi_2) \otimes H_5/\Lambda + y_2^u T_3 \otimes T_3 \otimes H_5 \\
& + y_1^N (N_e^c, N_\mu^c) \otimes (N_e^c, N_\mu^c) \otimes \Theta^{2m}/\bar{\Lambda}^{2m-1} + M N_\tau^c \otimes N_\tau^c \\
& + y_2^N (N_e^c, N_\mu^c) \otimes (N_e^c, N_\mu^c) \otimes (\chi_3, \chi_4) \otimes \Theta^{2m-n}/\bar{\Lambda}^{2m-n} \\
& + y_1^D (N_e^c, N_\mu^c) \otimes (F_1, F_2, F_3) \otimes (\chi_5, \chi_6, \chi_7) \otimes H_5 \otimes \Theta^m/(\Lambda\bar{\Lambda}^m) \\
& + y_2^D N_\tau^c \otimes (F_1, F_2, F_3) \otimes (\chi_5, \chi_6, \chi_7) \otimes H_5/\Lambda \\
& + y_1 (F_1, F_2, F_3) \otimes (T_1, T_2) \otimes (\chi_8, \chi_9, \chi_{10}) \otimes H_{45} \otimes \Theta^\ell/(\Lambda\bar{\Lambda}^\ell) \\
& + y_2 (F_1, F_2, F_3) \otimes T_3 \otimes (\chi_{11}, \chi_{12}, \chi_{13}) \otimes H_{\bar{5}}/\Lambda,
\end{aligned}
$$

- Λ and $\bar{\Lambda}$: S_4 and $U(1)_{FN}$ breaking scales.
- M: Mass parameter for right-handed Majorana neutrinos.
- y_i^a and y_i: Yukawa coupling constants are order 1.

By using w, we can discuss mass matrices of quarks and leptons.

Lepton sector

Vacuum alignment (Define VEVs: $\langle\chi_i\rangle/\Lambda \equiv \alpha_i$)

- $(\langle\chi_8\rangle, \langle\chi_9\rangle, \langle\chi_{10}\rangle) = \langle\chi_9\rangle(0,1,0)$, $(\langle\chi_{11}\rangle, \langle\chi_{12}\rangle, \langle\chi_{13}\rangle) = \langle\chi_{13}\rangle(0,0,1)$,

$$M_l = \begin{pmatrix} 0 & -3y_1\lambda^\ell\alpha_9 v_{45}/\sqrt{2} & 0 \\ 0 & -3y_1\lambda^\ell\alpha_9 v_{45}/\sqrt{6} & 0 \\ 0 & 0 & y_2\alpha_{13}v_d \end{pmatrix},$$

$$M_l^\dagger M_l = v_d^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 6|\bar{y}_1\lambda^\ell\alpha_9|^2 & 0 \\ 0 & 0 & |y_2|^2\alpha_{13}^2 \end{pmatrix}.$$

- Charged lepton masses:

$$m_e^2 = 0\ ,\ \ m_\mu^2 = 6|\bar{y}_1\lambda^\ell\alpha_9|^2 v_d^2\ ,\ \ m_\tau^2 = |y_2|^2\alpha_{13}^2 v_d^2\ .$$

The electron mass appears at the next-to-leading order.

- No mixing in the left-handed !
- $\theta_{12}^d = 60^\circ$ in the right-handed ! $\longrightarrow$ We will show after.

- Vacuum alignment
  $(\langle\chi_3\rangle,\langle\chi_4\rangle)=\langle\chi_4\rangle(0,1)$, $(\langle\chi_5\rangle,\langle\chi_6\rangle,\langle\chi_7\rangle)=\langle\chi_5\rangle(1,1,1)$.
  Right-handed Majorana mass matrix of neutrinos turns to

$$M_N=\begin{pmatrix}\lambda^{2m-n}(y_1^N\lambda^n\bar\Lambda+y_2^N\alpha_4\Lambda) & 0 & 0\\ 0 & \lambda^{2m-n}(y_1^N\lambda^n\bar\Lambda-y_2^N\alpha_4\Lambda) & 0\\ 0 & 0 & M\end{pmatrix}.$$

- Dirac mass matrix of neutrinos turns to

$$M_D=y_1^D\lambda^m v_u\begin{pmatrix}2\alpha_5/\sqrt6 & -\alpha_5/\sqrt6 & -\alpha_5/\sqrt6\\ 0 & \alpha_5/\sqrt2 & -\alpha_5/\sqrt2\\ 0 & 0 & 0\end{pmatrix}+y_2^D v_u\begin{pmatrix}0&0&0\\0&0&0\\ \alpha_5&\alpha_5&\alpha_5\end{pmatrix}.$$

- After using the seesaw mechanism, we get the tri-bimaximal mixing.

$$M_\nu=\frac{b+c}{2}\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}+\frac{3a-b}{3}\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix}+\frac{b-c}{2}\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix}.$$

$$a=\frac{(y_2^D\alpha_5 v_u)^2}{M},\ b=\frac{(y_1^D\alpha_5 v_u\lambda^m)^2}{\lambda^{2m-n}(y_1^N\lambda^n\bar\Lambda+y_2^N\alpha_4\Lambda)},\ c=\frac{(y_1^D\alpha_5 v_u\lambda^m)^2}{\lambda^{2m-n}(y_1^N\lambda^n\bar\Lambda-y_2^N\alpha_4\Lambda)}.$$

$$m_1=b\ ,\quad m_2=3a\ ,\quad m_3=c\ .$$

Magnitudes of VEVs

- $\ell = 1$, $m = 1$, and $n = 2$ as Frogatt-Nielsen charges:

$$\alpha_3 = \alpha_8 = \alpha_{10} = \alpha_{11} = \alpha_{12} = 0,$$

$$\alpha_1 = \alpha_2 \simeq \sqrt{\frac{m_c}{2\left|y^u_{\Delta_{a_2}} - \frac{y_1^{u2}}{y_2^u}\right| v_u}},$$

$$\alpha_4 = \frac{(y_1^D \lambda)^2 (m_3 - m_1) m_2 M}{6 y_2^N y_2^{D^2} m_1 m_3 \Lambda}, \qquad \alpha_5 = \alpha_6 = \alpha_7 = \frac{\sqrt{m_2 M}}{\sqrt{3} y_2^D v_u},$$

$$\alpha_9 = \frac{m_\mu}{\sqrt{6}|\bar{y}_1|\lambda v_d}, \qquad \alpha_{13} = \frac{m_\tau}{y_2 v_d}.$$

- Putting typical values of quark and lepton masses (GUT scale), $M = 10^{12}$ GeV, $\lambda = 0.1$, and $\tan\beta = 3$:

$$\alpha_1 \sim 3 \times 10^{-2}, \qquad \alpha_4 \sim 10^{-2}, \qquad \alpha_5 \sim 10^{-2},$$
$$\alpha_9 \sim 5 \times 10^{-3}, \qquad \alpha_{13} \sim 2 \times 10^{-2}.$$

- The magnitudes of all VEVs $\tilde{\alpha} \simeq \mathcal{O}\left(10^{-2}\right)$.

Deviation from the tri-bimaximal mixing

In neutrino sector, if including the next-to-leading order:

- Majorana neutrinos: $y^N_{\Delta_1}(N^c_e, N^c_\mu)(N^c_e, N^c_\mu)(\chi_1, \chi_2)\chi_{14}H_5/\Lambda$.

$$\Delta M_N = \Lambda \times \begin{pmatrix} y^N_{\Delta_1}\alpha_1\alpha_{14} & y^N_{\Delta_1}\alpha_1\alpha_{14} & -\frac{\lambda}{\sqrt{6}}y^N_{\Delta_2}\alpha_5\alpha_{13} + \frac{\lambda}{\sqrt{2}}y^N_{\Delta_3}\alpha_9^2 \\ y^N_{\Delta_1}\alpha_1\alpha_{14} & -y^N_{\Delta_1}\alpha_1\alpha_{14} & -\frac{\lambda}{\sqrt{2}}y^N_{\Delta_2}\alpha_5\alpha_{13} + \frac{\lambda}{\sqrt{6}}y^N_{\Delta_3}\alpha_9^2 \\ -\frac{\lambda}{\sqrt{6}}y^N_{\Delta_2}\alpha_5\alpha_{13} + \frac{\lambda}{\sqrt{2}}y^N_{\Delta_3}\alpha_9^2 & -\frac{\lambda}{\sqrt{2}}y^N_{\Delta_2}\alpha_5\alpha_{13} + \frac{\lambda}{\sqrt{6}}y^N_{\Delta_3}\alpha_9^2 & y^N_{\Delta_4}\alpha_9^2 \end{pmatrix},$$

$$U_{e3} \sim \frac{y^N_{\Delta_1}\alpha_1\alpha_{14}}{y^N_2\alpha_4} \sim \mathcal{O}(\tilde{\alpha}) \ .$$

- Dirac neutrinos: $y^D_\Delta(N^c_e, N^c_\mu)(F_1, F_2, F_3)(\chi_8, \chi_9, \chi_{10})(\chi_{11}, \chi_{12}, \chi_{13})H_5\Theta/(\Lambda^2\bar{\Lambda})$.

$$\Delta M_D = \begin{pmatrix} * & * & * \\ y^D_\Delta\lambda\alpha_9\alpha_{13}v_u & * & * \\ * & * & * \end{pmatrix}, \quad U_{e3} \sim -\frac{\sqrt{6}y^D_\Delta\alpha_9\alpha_{13}}{3y^D_1\alpha_5} \sim \mathcal{O}(\tilde{\alpha}) \ .$$

Numerical results

- Random plots on $\sin^2\theta_{12}$ - $\sin\theta_{13}$ and $\sin^2 2\theta_{23}$ - $\sin\theta_{13}$ plane:

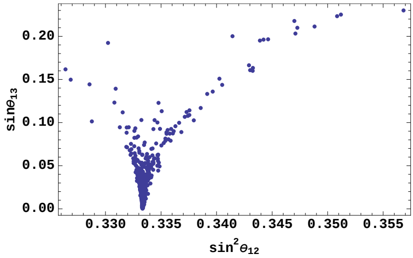

Figure: The allowed region on $\sin^2\theta_{12}$ - $\sin\theta_{13}$ plane.

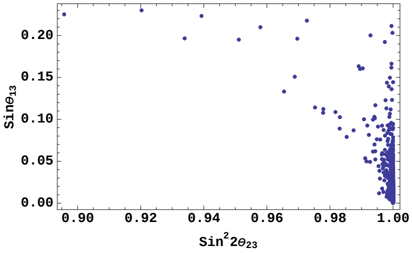

Figure: The allowed region on $\sin^2 2\theta_{23}$ - $\sin\theta_{13}$ plane.

Quark sector

- For down-type quarks, putting $\alpha_8 = \alpha_{10} = \alpha_{11} = \alpha_{12} = 0$, which is the condition in the charged lepton sector, we have M_d as

$$M_d = \begin{pmatrix} 0 & 0 & 0 \\ y_1\lambda^\ell \alpha_9 v_{45}/\sqrt{2} & y_1\lambda^\ell \alpha_9 v_{45}/\sqrt{6} & 0 \\ 0 & 0 & y_2\alpha_{13}v_d \end{pmatrix},$$

$$M_d^\dagger M_d = v_d^2 \begin{pmatrix} \frac{1}{2}|\bar{y}_1\lambda^\ell\alpha_9|^2 & \frac{1}{2\sqrt{3}}|\bar{y}_1\lambda^\ell\alpha_9|^2 & 0 \\ \frac{1}{2\sqrt{3}}|\bar{y}_1\lambda^\ell\alpha_9|^2 & \frac{1}{6}|\bar{y}_1\lambda^\ell\alpha_9|^2 & 0 \\ 0 & 0 & |y_2|^2\alpha_{13}^2 \end{pmatrix}.$$

- The mass matrix is diagonalized by the orthogonal matrix $U_d^{(0)}$:

$$U_d^{(0)} = \begin{pmatrix} \cos 60^\circ & \sin 60^\circ & 0 \\ -\sin 60^\circ & \cos 60^\circ & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

- $\theta_{12}^d = 60^\circ$ comes from Clebsch Gordan coefficients of S_4

- For up-type quarks, vacuum alignment: $(\langle\chi_1\rangle, \langle\chi_2\rangle) = \langle\chi_1\rangle(1,1)$,

$$M_u = v_u \begin{pmatrix} 2y^u_{\Delta_{a_1}}\alpha_1^2 + y^u_{\Delta_b}\alpha_{14}^2 & y^u_{\Delta_{a_2}}\alpha_1^2 & y_1^u\alpha_1 \\ y^u_{\Delta_{a_2}}\alpha_1^2 & 2y^u_{\Delta_{a_1}}\alpha_1^2 + y^u_{\Delta_b}\alpha_{14}^2 & y_1^u\alpha_1 \\ y_1^u\alpha_1 & y_1^u\alpha_1 & y_2^u + y^u_{\Delta_c}\alpha_9^2 \end{pmatrix}.$$

- After rotating M_u by the orthogonal matrix $U_u^{(0)}$ as

$$U_u^{(0)} = \begin{pmatrix} \cos 45^\circ & \sin 45^\circ & 0 \\ -\sin 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$\hat{M}_u = U_u^{(0)\dagger} M_u U_u^{(0)} = v_u \times$$
$$\begin{pmatrix} (2y^u_{\Delta_{a_1}} - y^u_{\Delta_{a_2}})\alpha_1^2 + y^u_{\Delta_b}\alpha_{14}^2 & 0 & 0 \\ 0 & (2y^u_{\Delta_{a_1}} + y^u_{\Delta_{a_2}})\alpha_1^2 + y^u_{\Delta_b}\alpha_{14}^2 & \sqrt{2}y_1^u\alpha_1 \\ 0 & \sqrt{2}y_1^u\alpha_1 & y_2^u + y^u_{\Delta_c}\alpha_9^2 \end{pmatrix}.$$

- Mass eigenvalues of up-type quarks:

$$m_u = \left[(2y^u_{\Delta_{a_1}} - y^u_{\Delta_{a_2}})\alpha_1^2 + y^u_{\Delta_b}\alpha_{14}^2\right] v_u,$$

$$m_c \simeq \frac{y_2^u\left[(2y^u_{\Delta_{a_1}} + y^u_{\Delta_{a_2}})\alpha_1^2 + y^u_{\Delta_b}\alpha_{14}^2\right] - 2y_1^{u2}\alpha_1^2}{y_2^u} v_u, \qquad m_t \simeq y_2^u v_u.$$

- We obtain CKM matrix ($V^0 = U_u^\dagger U_d$).

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-i\rho} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad V_u = \begin{pmatrix} 1 & 0 & 0 \\ 0 & r_t & r_c \\ 0 & -r_c & r_t \end{pmatrix}, \quad r_c = \sqrt{\frac{m_c}{m_c + m_t}}, \quad r_t = \sqrt{\frac{m_t}{m_c + m_t}},$$

$$U_u \simeq U_u^{(0)} P V_u = \begin{pmatrix} \cos 45^\circ & \sin 45^\circ & 0 \\ -\sin 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-i\rho} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & r_t & r_c \\ 0 & -r_c & r_t \end{pmatrix},$$

$$U_d \simeq \begin{pmatrix} \cos 60^\circ & \sin 60^\circ & 0 \\ -\sin 60^\circ & \cos 60^\circ & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & \theta_{12}^d & \theta_{13}^d \\ -\theta_{12}^d - \theta_{13}^d \theta_{23}^d & 1 & \theta_{23}^d \\ -\theta_{13}^d + \theta_{12}^d \theta_{23}^d & -\theta_{23}^d - \theta_{12}^d \theta_{13}^d & 1 \end{pmatrix},$$

where $\theta_{12}^d = \mathcal{O}\left(\frac{m_d}{m_s}\right), \quad \theta_{13}^d = \mathcal{O}\left(\frac{m_d}{m_b}\right), \quad \theta_{23}^d = \mathcal{O}\left(\frac{m_d}{m_b}\right).$

In the leading order, we predict

- $V_{us}^0 \simeq \sin 15^\circ \simeq 0.26, \quad V_{cb}^0 \simeq \sqrt{m_c/m_t} \simeq 0.048, \quad V_{ub}^0 \simeq 0.$

Slepton sector

Flavor symmetry constrains not only quark/lepton mass matrices, but also mass matrices of their superpartner, i.e. squark/slepton.

We consider soft SUSY breaking term in supergravity.

- The S_4 invariant Kähler potential:

$$K = Z^{(L)}(\Phi) \sum_{i=e,\mu,\tau} |L_i|^2 + Z^{(R)}_{(1)}(\Phi) \sum_{i=e,\mu} |e_i|^2 + Z^{(R)}_{(2)}(\Phi)|e_\tau|^2 \ .$$

- Soft scalar masses:

$$(m^2_{\tilde{L}})_{ij} = \begin{pmatrix} m_L^2 & 0 & 0 \\ 0 & m_L^2 & 0 \\ 0 & 0 & m_L^2 \end{pmatrix}, \qquad (m^2_{\tilde{R}})_{ij} = \begin{pmatrix} m^2_{R(1)} & 0 & 0 \\ 0 & m^2_{R(1)} & 0 \\ 0 & 0 & m^2_{R(2)} \end{pmatrix}.$$

Masses of three generations are degenerated. Masses of 1, 2 generations are degenerated.

- For left-handed sleptons, Kähler potential at next-to-leading order:

$$\Delta K_L = \sum_{i=1,3} Z^{(L)}_{\Delta_{a_i}}(\Phi)(L_e, L_\mu, L_\tau) \otimes (L^c_e, L^c_\mu, L^c_\tau) \otimes (\chi_i, \chi_{i+1}) \otimes (\chi^c_i, \chi^c_{i+1})/\Lambda^2$$
$$+ \sum_{i=5,8,11} Z^{(L)}_{\Delta_{b_i}}(\Phi)(L_e, L_\mu, L_\tau) \otimes (L^c_e, L^c_\mu, L^c_\tau) \otimes (\chi_i, \chi_{i+1}, \chi_{i+2}) \otimes (\chi^c_i, \chi^c_{i+1}, \chi^c_{i+2})/\Lambda^2$$
$$+ Z^{(L)}_{\Delta_c}(\Phi)(L_e, L_\mu, L_\tau) \otimes (L^c_e, L^c_\mu, L^c_\tau) \otimes \chi_{14} \otimes \chi^c_{14}/\Lambda^2$$
$$+ Z^{(L)}_{\Delta_d}(\Phi)(L_e, L_\mu, L_\tau) \otimes (L^c_e, L^c_\mu, L^c_\tau) \otimes \Theta \otimes \Theta^c/\bar{\Lambda}^2.$$

- Left-handed slepton mass matrix:

$$(m^2_{\tilde{L}})_{ij} = \begin{pmatrix} m^2_L + \mathcal{O}(\tilde{\alpha}^2 m^2_{3/2}) & \mathcal{O}(\tilde{\alpha}^2 m^2_{3/2}) & \mathcal{O}(\tilde{\alpha}^2 m^2_{3/2}) \\ \mathcal{O}(\tilde{\alpha}^2 m^2_{3/2}) & m^2_L + \mathcal{O}(\tilde{\alpha}^2 m^2_{3/2}) & \mathcal{O}(\tilde{\alpha}^2 m^2_{3/2}) \\ \mathcal{O}(\tilde{\alpha}^2 m^2_{3/2}) & \mathcal{O}(\tilde{\alpha}^2 m^2_{3/2}) & m^2_L + \mathcal{O}(\tilde{\alpha}^2 m^2_{3/2}) \end{pmatrix},$$

where $\tilde{\alpha}$ is a linear combination of α_i's.

- For right-handed slepton mass matrix at next-to-reading order:

$$\begin{aligned}
\Delta K_R = & \sum_{i=1,3} Z^{(R)}_{\Delta_{a_i}}(\Phi)(R_e, R_\mu)\otimes(R_e^c, R_\mu^c)\otimes(\chi_i,\chi_{i+1})\otimes(\chi_i^c,\chi_{i+1}^c)/\Lambda^2 \\
& + \sum_{i=5,8,11} Z^{(R)}_{\Delta_{b_i}}(\Phi)(R_e, R_\mu)\otimes(R_e^c, R_\mu^c)\otimes(\chi_i,\chi_{i+1},\chi_{i+2})\otimes(\chi_i^c,\chi_{i+1}^c,\chi_{i+2}^c)/\Lambda^2 \\
& + Z^{(R)}_{\Delta_c}(\Phi)(R_e, R_\mu)\otimes(R_e^c, R_\mu^c)\otimes\chi_{14}\otimes\chi_{14}^c/\Lambda^2 \\
& + Z^{(R)}_{\Delta_d}(\Phi)(R_e, R_\mu)\otimes R_\tau^c\otimes(\chi_1,\chi_2)/\Lambda^2 + Z^{(R)}_{\Delta_e}(\Phi)(R_e^c, R_\mu^c)\otimes R_\tau\otimes(\chi_1^c,\chi_2^c)/\Lambda^2 \\
& + \sum_{i=1,3} Z^{(R)}_{\Delta_{f_i}}(\Phi)R_\tau\otimes R_\tau^c\otimes(\chi_i,\chi_{i+1})\otimes(\chi_i^c,\chi_{i+1}^c)/\Lambda^2 \\
& + \sum_{i=5,8,11} Z^{(R)}_{\Delta_{g_i}}(\Phi)R_\tau\otimes R_\tau^c\otimes(\chi_i,\chi_{i+1},\chi_{i+2})\otimes(\chi_i^c,\chi_{i+1}^c,\chi_{i+2}^c)/\Lambda^2 \\
& + Z^{(R)}_{\Delta_h}(\Phi)R_\tau\otimes R_\tau^c\otimes\chi_{14}\otimes\chi_{14}^c/\Lambda^2 \\
& + Z^{(R)}_{\Delta_i}(\Phi)(R_e, R_\mu)\otimes(R_e^c, R_\mu^c)\otimes\Theta\otimes\Theta^c/\bar{\Lambda}^2 \\
& + Z^{(R)}_{\Delta_j}(\Phi)R_\tau\otimes R_\tau^c\otimes\Theta\otimes\Theta^c/\bar{\Lambda}^2 .
\end{aligned}$$

- Right-handed slepton mass matrix:

$$(m^2_{\tilde{R}})_{ij} = \begin{pmatrix} m^2_{R(1)} + \mathcal{O}(\tilde{\alpha}^2 m^2_{3/2}) & \mathcal{O}(\tilde{\alpha}^2 m^2_{3/2}) & \mathcal{O}(\alpha_1 m^2_{3/2}) \\ \mathcal{O}(\tilde{\alpha}^2 m^2_{3/2}) & m^2_{R(1)} + \mathcal{O}(\tilde{\alpha}^2 m^2_{3/2}) & \mathcal{O}(\alpha_1 m^2_{3/2}) \\ \mathcal{O}(\alpha_1 m^2_{3/2}) & \mathcal{O}(\alpha_1 m^2_{3/2}) & m^2_{R(2)} + \mathcal{O}(\tilde{\alpha}^2 m^2_{3/2}) \end{pmatrix} .$$

In order to estimate magnitude of FCNC, we move to a super-CKM basis by diagonalizing charged lepton mass matrix.

- For left-handed slepton mass matrix: (U_E is left-handed mixing.)

$$(m^2_{\tilde{L}})^{(SCKM)}_{ij} = U_E^\dagger (m^2_{\tilde{L}})_{ij} U_E$$
$$\simeq \begin{pmatrix} m_L^2 + \mathcal{O}(\frac{\tilde{\alpha}^2}{\lambda^2} m^2_{3/2}) & \mathcal{O}(\tilde{\alpha}^2 m^2_{3/2}) & \mathcal{O}(\tilde{\alpha}^2 m^2_{3/2}) \\ \mathcal{O}(\tilde{\alpha}^2 m^2_{3/2}) & m_L^2 + \mathcal{O}(\frac{\tilde{\alpha}^2}{\lambda^2} m^2_{3/2}) & \mathcal{O}(\tilde{\alpha}^2 m^2_{3/2}) \\ \mathcal{O}(\tilde{\alpha}^2 m^2_{3/2}) & \mathcal{O}(\tilde{\alpha}^2 m^2_{3/2}) & m_L^2 + \mathcal{O}(\tilde{\alpha}^2 m^2_{3/2}) \end{pmatrix}.$$

- For right-handed slepton mass matrix: (V_E is right-handed mixing.)

$$(m^2_{\tilde{R}})^{(SCKM)}_{ij} = V_E^\dagger (m^2_{\tilde{R}})_{ij} V_E$$
$$\simeq \begin{pmatrix} m^2_{R(1)} + \mathcal{O}(\tilde{\alpha}^2 m^2_{3/2}) & \mathcal{O}(\tilde{\alpha}^2 m^2_{3/2}) & \mathcal{O}(\alpha_1 m^2_{3/2}) \\ \mathcal{O}(\tilde{\alpha}^2 m^2_{3/2}) & m^2_{R(1)} + \mathcal{O}(\tilde{\alpha}^2 m^2_{3/2}) & \mathcal{O}(\alpha_1 m^2_{3/2}) \\ \mathcal{O}(\alpha_1 m^2_{3/2}) & \mathcal{O}(\alpha_1 m^2_{3/2}) & m^2_{R(2)} + \mathcal{O}(\tilde{\alpha}^2 m^2_{3/2}) \end{pmatrix}.$$

- In mass insertion parameters, we predict

$$(\Delta_{LL(RR)})_{12} \equiv \frac{(m^2_{\tilde{L}(\tilde{R})})^{(SCKM)}_{12}}{m^2_{\text{SUSY}}} = \mathcal{O}(\tilde{\alpha}^2) = \mathcal{O}(10^{-4}).$$

- Experimental constraints:

$(\Delta_{LL(RR)})^{\text{exp}}_{12} \leq \mathcal{O}(10^{-3})$, when $m_{\text{SUSY}} \simeq 100$ GeV.

F. Gabbiani, E. Gabrielli, A. Masiero and L. Silvestrini, Nucl. Phys. B **477**, 321 (1996)

W. Altmannshofer, A. J. Buras, G. Gori, P. Paradisi, and D. M. Straub, Nucl. Phys. B **830**, 17 (2010).

- Our model is consistent with FCNC constraints!!

- A-term: Moving to the super-CKM basis, we have

$$(m_{LR}^2)_{ij}^{SCKM} = U_E^\dagger (m_{LR}^2)_{ij} V_E \simeq m_{3/2} \begin{pmatrix} \mathcal{O}\left(\tilde{\alpha}^2 v_d\right) & \mathcal{O}\left(\tilde{\alpha}^2 v_d\right) & \mathcal{O}\left(\tilde{\alpha}^2 v_d\right) \\ \mathcal{O}\left(\tilde{\alpha}^2 v_d\right) & \mathcal{O}(m_\mu) & \mathcal{O}\left(\tilde{\alpha}^2 v_d\right) \\ \mathcal{O}\left(\tilde{\alpha}^2 v_d\right) & \mathcal{O}\left(\tilde{\alpha}^2 v_d\right) & \mathcal{O}(m_\tau) \end{pmatrix}$$

$$\simeq m_{3/2} \begin{pmatrix} \mathcal{O}\left(m_e\right) & \mathcal{O}\left(m_e\right) & \mathcal{O}\left(m_e\right) \\ \mathcal{O}\left(m_e\right) & \mathcal{O}(m_\mu) & \mathcal{O}\left(m_e\right) \\ \mathcal{O}\left(m_e\right) & \mathcal{O}\left(m_e\right) & \mathcal{O}(m_\tau) \end{pmatrix}.$$

- The FCNC measure

$$(\Delta_{LR})_{12} \equiv \frac{\left(\tilde{m}_{LR}^2\right)_{12}}{m_{\text{SUSY}}^2} = \frac{\mathcal{O}(m_e)}{m_{\text{SUSY}}},$$

is predicted to be of order 5×10^{-6} for $m_{\text{SUSY}} = 100$ GeV.

- Experimental constraints:
  $(\Delta_{LR})_{12}^{\text{exp}} \leq \mathcal{O}(10^{-6})$, when $m_{\text{SUSY}} \simeq 100$ GeV.

- $\mu \longrightarrow e\gamma$ may be observed soon.

Summary

- We have presented a successful flavor model with S_4 symmetry to unify quarks and leptons in the framework of $SU(5)$ SUSY GUT.
- Our model predicts:

  The tri-bimaximal mixing of leptons.
  The Cabibbo angle to be around 15°.
- FCNC is enough suppressed.

Typical non-Abelian discrete group

H. Ishimori, T. Kobayashi, H. Ohki, H. Okada, Y. S. and M. Tanimoto, PTP supplement No. 183, 2010.

- Symmetry group S_n (number of group elements is $n!$)

	Number of elements	Geometry	Irreducible representations
S_3	6	Regular triangle	$1_S, 1_A, 2$
S_4	24	Octahedron	$1_1, 1_2, 2, 3_1, 3_2$

- Even permutation group A_n (number of group elements is $n!/2$)

	Number of elements	Geometry	Irreducible representations
A_4	12	Tetrahedron	$1, 1', 1'', 3$

- T' group

	Number of elements	Geometry	Irreducible representations
T'	24	Double tetrahedron	$1, 1', 1'', 2, 2', 2'', 3$

- A_4 and T' flavor symmetry are successful to get tri-bimaximal mixing.
- It is found that S_4 flavor symmetry can lead to tri-bimaximal mixing.

Typical one: A_4 flavor symmetry

E. Ma and G. Rajasekaran, Phys. Rev. D 64, 113012 (2001).

G. Altarelli and F. Feruglio, Nucl. Phys. B 720, 64 (2005); Nucl. Phys. B 741, 215 (2006).

A_4 group is the tetrahedral symmetry or even permutation of four elements. Number of elements is 12.

- Irreducible representations of A_4 are $\mathbf{1}$, $\mathbf{1}'$, $\mathbf{1}''$, and $\mathbf{3}$.
- Multiplication rules:
  $$\mathbf{3} \otimes \mathbf{3} = \mathbf{1} \oplus \mathbf{1}' \oplus \mathbf{1}'' \oplus \mathbf{3} \oplus \mathbf{3}$$
- A_4 invariant representation is $\mathbf{1}$.

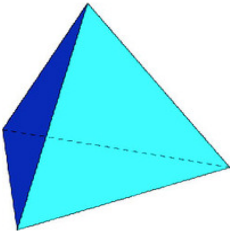

Figure: A_4 symmetry : Tetrahedron

Let us show how to get the tri-bimaximal in A_4 model.

(G. Altarelli and F. Feruglio, Nucl. Phys. B 741, 215 (2006))

	(l_e, l_μ, l_τ)	$(\nu_e^c, \nu_\mu^c, \nu_\tau^c)$	e^c	μ^c	τ^c	h_u	h_d	ξ	$\tilde{\xi}$	φ_T	φ_S
A_4	3	3	1	$1''$	$1'$	1	1	1	1	3	3
Z_3	ω	ω^2	ω^2	ω^2	ω^2	1	1	ω^2	ω^2	1	ω^2

- The superpotential of lepton sector

$$w_l = y_e e^c(\varphi_T l) + y_\mu \mu^c(\varphi_T l)' + y_\tau \tau^c(\varphi_T l)'' + y(\nu^c l) + (x_A\xi + \tilde{x}_A\tilde{\xi})(\nu^c\nu^c) + x_B(\varphi_S\nu^c\nu^c) + \text{h.c.} + \dots$$

- Vacuum alignment: $\langle\varphi_T\rangle = (\langle\varphi_1^T\rangle, \langle\varphi_2^T\rangle, \langle\varphi_3^T\rangle) = (v_T, 0, 0)$,
  Charged lepton mass matrix:

$$M_l = \begin{pmatrix} y_e\varphi_1^T & y_e\varphi_3^T & y_e\varphi_2^T \\ y_\mu\varphi_2^T & y_\mu\varphi_1^T & y_\mu\varphi_3^T \\ y_\tau\varphi_3^T & y_\tau\varphi_2^T & y_\tau\varphi_1^T \end{pmatrix} . \quad \rightarrow \quad M_l = \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix} v_T \ .$$

- Mass matrix of charged leptons is diagonal.

Vacuum alignment: $\langle \varphi_S \rangle = (v_S, v_S, v_S)$, $(\langle \xi \rangle = u,\ \langle \tilde{\xi} \rangle = 0)$

- The neutrino mass matrix

$$M = B \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} u - \frac{B}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} u + A \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} u$$

$$m_\nu^D = y v_u \mathbf{1}, \quad A \equiv 2x_A, \quad B \equiv 2x_B \frac{v_S}{u}$$

- By using seesaw mechanism $m_\nu = (m_\nu^D)^T M^{-1} m_\nu^D$, Neutrino mixing is tri-bimaximal.

In order to reproduce the tri-bimaximal mixing, we need

- Non-Abelian discrete symmetry (A_4, T', S_4, $\cdots$).
- Vacuum alignment of flavons (Dynamics of flavons).

Realization of vacuum alignment

- Vacuum alignment are summarized as:

$$(\chi_1,\chi_2)=(1,1),\quad (\chi_3,\chi_4)=(0,1),$$
$$(\chi_5,\chi_6,\chi_7)=(1,1,1),\quad (\chi_8,\chi_9,\chi_{10})=(0,1,0),\quad (\chi_{11},\chi_{12},\chi_{13})=(0,0,1).$$

- We introduce driving fields with R charge 2 under $U(1)_R$ symmetry:

	(χ_1,χ_2)	(χ_3,χ_4)	(χ_5,χ_6,χ_7)	$(\chi_8,\chi_9,\chi_{10})$	$(\chi_{11},\chi_{12},\chi_{13})$	χ_{14}
$SU(5)$	1	1	1	1	1	1
S_4	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{3}'$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$
Z_4	$-i$	1	$-i$	-1	i	i
$U(1)_{FN}$	$-\ell$	$-n$	0	0	0	$-\ell$
$U(1)_R$	0	0	0	0	0	0

	$(\chi_{15},\chi_{16},\chi_{17})$	χ_1^0	χ_2^0	χ_3^0	(χ_4^0,χ_5^0)
$SU(5)$	1	1	1	1	1
S_4	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$
Z_4	-1	-1	i	-1	$-i$
$U(1)_{FN}$	$-z$	$2\ell+n$	0	2ℓ	z
$U(1)_R$	0	2	2	2	2

- The $SU(5) \times S_4 \times Z_4 \times U(1)_{FN} \times U(1)_R$ invariant superpotential:

$$\begin{aligned} w' = & \kappa_1 (\chi_1, \chi_2) \otimes (\chi_1, \chi_2) \otimes (\chi_3, \chi_4) \otimes \chi_1^0 / \Lambda \\ & + \eta_1 (\chi_8, \chi_9, \chi_{10}) \otimes (\chi_{11}, \chi_{12}, \chi_{13}) \otimes \chi_2^0 \\ & + \eta_2 (\chi_1, \chi_2) \otimes (\chi_1, \chi_2) \otimes \chi_3^0 + \eta_3 \chi_{14} \otimes \chi_{14} \otimes \chi_3^0 \\ & + \eta_4 (\chi_5, \chi_6, \chi_7) \otimes (\chi_{15}, \chi_{16}, \chi_{17}) \otimes \left(\chi_4^0, \chi_5^0\right), \end{aligned}$$

- The scalar potential:

$$\begin{aligned} V = & \left| \frac{\kappa_1}{\Lambda} \left[2\chi_1\chi_2\chi_3 + \left(\chi_1^2 - \chi_2^2\right) \chi_4 \right] \right|^2 + \left| \eta_1 \left(\chi_8\chi_{11} + \chi_9\chi_{12} + \chi_{10}\chi_{13} \right) \right|^2 \\ & + \left| \eta_2 (\chi_1^2 + \chi_2^2) + \eta_3 \chi_{14}^2 \right|^2 + \left| \frac{1}{\sqrt{2}} \eta_4 \left(\chi_6\chi_{16} - \chi_7\chi_{17} \right) \right|^2 \\ & + \left| \frac{1}{\sqrt{6}} \eta_4 \left(-2\chi_5\chi_{15} + \chi_6\chi_{16} + \chi_7\chi_{17} \right) \right|^2 . \end{aligned}$$

- We obtain the vacuum alignment:

$$\chi_1 = \chi_2, \quad \chi_3 = 0, \quad \chi_5 = \chi_6 = \chi_7, \quad \chi_8 = \chi_{10} = \chi_{11} = \chi_{12} = 0,$$
$$\chi_{14}^2 = -\frac{2\eta_2}{\eta_3} \chi_1^2, \quad \chi_{15} = \chi_{16} = \chi_{17} .$$

Multiplication rules of S_4

- Multiplication of $\mathbf{2} \otimes \mathbf{2}$ is given by

$$(a_1, a_2)_{\mathbf{2}} \otimes (b_1, b_2)_{\mathbf{2}} = (a_1 b_1 + a_2 b_2)_{\mathbf{1}} \oplus (-a_1 b_2 + a_2 b_1)_{\mathbf{1}'} \oplus \begin{pmatrix} a_1 b_2 + a_2 b_1 \\ a_1 b_1 - a_2 b_2 \end{pmatrix}_{\mathbf{2}}$$

- Multiplication of $\mathbf{3} \otimes \mathbf{3}$ is

$$(a_1, a_2, a_3)_{\mathbf{3}} \otimes (b_1, b_2, b_3)_{\mathbf{3}} = \left(\sum_{j=1}^{3} a_j b_j \right)_{\mathbf{1}} \oplus \begin{pmatrix} \frac{1}{\sqrt{2}}(a_2 b_2 - a_3 b_3) \\ \frac{1}{\sqrt{6}}(-2a_1 b_1 + a_2 b_2 + a_3 b_3) \end{pmatrix}_{\mathbf{2}} \oplus \begin{pmatrix} a_2 b_3 + a_3 b_2 \\ a_1 b_3 + a_3 b_1 \\ a_1 b_2 + a_2 b_1 \end{pmatrix}_{\mathbf{3}} \oplus \begin{pmatrix} a_3 b_2 - a_2 b_3 \\ a_1 b_3 - a_3 b_1 \\ a_2 b_1 - a_1 b_2 \end{pmatrix}_{\mathbf{3}'}$$

- $\mathbf{3}' \times \mathbf{3}'$ is the same to $\mathbf{3} \times \mathbf{3}$.

- Multiplication of $\mathbf{2} \otimes \mathbf{3}$ is

$$(a_1, a_2)_{\mathbf{2}} \otimes (b_1, b_2, b_3)_{\mathbf{3}}$$
$$= \begin{pmatrix} a_2 b_1 \\ -\frac{1}{2}(\sqrt{3}a_1 b_2 + a_2 b_2) \\ \frac{1}{2}(\sqrt{3}a_1 b_3 - a_2 b_3) \end{pmatrix}_{\mathbf{3}} \oplus \begin{pmatrix} a_1 b_1 \\ \frac{1}{2}(\sqrt{3}a_2 b_2 - a_1 b_2) \\ -\frac{1}{2}(\sqrt{3}a_2 b_3 - a_1 b_3) \end{pmatrix}_{\mathbf{3'}}$$

- Multiplication of $\mathbf{3} \otimes \mathbf{3'}$ is

$$(a_1, a_2, a_3)_{\mathbf{3}} \otimes (b_1, b_2, b_3)_{\mathbf{3'}}$$
$$= \left(\sum_{j=1}^{3} a_j b_j\right)_{\mathbf{1'}} \oplus \begin{pmatrix} \frac{1}{\sqrt{6}}(2a_1 b_1 - a_2 b_2 - a_3 b_3) \\ \frac{1}{\sqrt{2}}(a_2 b_2 - a_3 b_3) \end{pmatrix}_{\mathbf{2}}$$
$$\oplus \begin{pmatrix} a_3 b_2 - b_2 a_3 \\ a_1 b_3 - a_3 b_1 \\ a_2 b_1 - a_1 b_2 \end{pmatrix}_{\mathbf{3}} \oplus \begin{pmatrix} a_2 b_3 + b_3 a_2 \\ a_1 b_3 + a_3 b_1 \\ a_1 b_2 + a_2 b_1 \end{pmatrix}_{\mathbf{3'}}$$

Next-to-leading order

The superpotential at the next-to-leading order:

$$\Delta w_l = y_{\Delta_a}(T_1, T_2) \otimes (F_1, F_2, F_3) \otimes (\chi_1, \chi_2) \otimes (\chi_{11}, \chi_{12}, \chi_{13}) \otimes H_{\bar{5}}/\Lambda^2$$
$$+ y_{\Delta_b}(T_1, T_2) \otimes (F_1, F_2, F_3) \otimes (\chi_5, \chi_6, \chi_7) \otimes \chi_{14} \otimes H_{\bar{5}}/\Lambda^2$$
$$+ y_{\Delta_c}(T_1, T_2) \otimes (F_1, F_2, F_3) \otimes (\chi_1, \chi_2) \otimes (\chi_5, \chi_6, \chi_7) \otimes H_{45}/\Lambda^2$$
$$+ y_{\Delta_d}(T_1, T_2) \otimes (F_1, F_2, F_3) \otimes (\chi_{11}, \chi_{12}, \chi_{13}) \otimes \chi_{14} \otimes H_{45}/\Lambda^2$$
$$+ y_{\Delta_e} T_3 \otimes (F_1, F_2, F_3) \otimes (\chi_5, \chi_6, \chi_7) \otimes (\chi_8, \chi_9, \chi_{10}) \otimes H_{\bar{5}} \otimes /\Lambda^2$$
$$+ y_{\Delta_f} T_3 \otimes (F_1, F_2, F_3) \otimes (\chi_8, \chi_9, \chi_{10}) \otimes (\chi_{11}, \chi_{12}, \chi_{13}) \otimes H_{45} \otimes /\Lambda^2 \ ,$$
$$\Delta w_N = y^N_{\Delta_1}(N^c_e, N^c_\mu) \otimes (N^c_e, N^c_\mu) \otimes (\chi_1, \chi_2) \otimes \chi_{14}/\Lambda$$
$$+ y^N_{\Delta_2}(N^c_e, N^c_\mu) \otimes N^c_\tau \otimes (\chi_5, \chi_6, \chi_7) \otimes (\chi_{11}, \chi_{12}, \chi_{13}) \otimes \Theta/(\Lambda\bar{\Lambda})$$
$$+ y^N_{\Delta_3}(N^c_e, N^c_\mu) \otimes N^c_\tau \otimes (\chi_8, \chi_9, \chi_{10}) \otimes (\chi_8, \chi_9, \chi_{10}) \otimes \Theta/(\Lambda\bar{\Lambda})$$
$$+ y^N_{\Delta_4} N^c_\tau \otimes N^c_\tau \otimes (\chi_8, \chi_9, \chi_{10}) \otimes (\chi_8, \chi_9, \chi_{10})/\Lambda,$$
$$\Delta w_D = y^D_{\Delta}(N^c_e, N^c_\mu) \otimes (F_1, F_2, F_3) \otimes (\chi_8, \chi_9, \chi_{10}) \otimes (\chi_{11}, \chi_{12}, \chi_{13}) \otimes H_5 \otimes \Theta/(\Lambda^2\bar{\Lambda}).$$

The charged lepton mass matrix elements ϵ_{ij} at the next-to-leading order:

$$\epsilon_{11} = y_{\Delta_b}\alpha_5\alpha_{14}v_d - 3\bar{y}_{\Delta_{c_2}}\alpha_1\alpha_5 v_d,$$

$$\epsilon_{12} = -\frac{1}{2}y_{\Delta_b}\alpha_5\alpha_{14}v_d + 3\left[\frac{\sqrt{3}}{4}(\sqrt{3}-1)\bar{y}_{\Delta_{c_1}} - \frac{1}{4}(\sqrt{3}+1)\bar{y}_{\Delta_{c_2}}\right]\alpha_1\alpha_5 v_d,$$

$$\epsilon_{13} = \left[\left\{\frac{\sqrt{3}}{4}(\sqrt{3}-1)y_{\Delta_{a_1}} + \frac{1}{4}(\sqrt{3}+1)y_{\Delta_{a_2}}\right\}\alpha_1\alpha_{13} - \frac{1}{2}y_{\Delta_b}\alpha_5\alpha_{14}\right]v_d$$
$$- 3\left[\left\{-\frac{\sqrt{3}}{4}(\sqrt{3}+1)\bar{y}_{\Delta_{c_1}} - \frac{1}{4}(\sqrt{3}-1)\bar{y}_{\Delta_{c_2}}\right\}\alpha_1\alpha_5 + \frac{\sqrt{3}}{2}\bar{y}_{\Delta_d}\alpha_{13}\alpha_{14}\right]v_d,$$

$$\epsilon_{21} = -3\bar{y}_{\Delta_{c_1}}\alpha_1\alpha_5 v_d,$$

$$\epsilon_{22} = \frac{\sqrt{3}}{2}y_{\Delta_b}\alpha_5\alpha_{14}v_d + 3\left[\frac{1}{4}(\sqrt{3}-1)\bar{y}_{\Delta_{c_1}} + \frac{\sqrt{3}}{4}(\sqrt{3}+1)\bar{y}_{\Delta_{c_2}}\right]\alpha_1\alpha_5 v_d,$$

$$\epsilon_{23} = \left[\left\{-\frac{1}{4}(\sqrt{3}-1)y_{\Delta_{a_1}} + \frac{\sqrt{3}}{4}(\sqrt{3}+1)y_{\Delta_{a_2}}\right\}\alpha_1\alpha_{13} - \frac{\sqrt{3}}{2}y_{\Delta_b}\alpha_5\alpha_{14}\right]v_d$$
$$- 3\left[\left\{\frac{1}{4}(\sqrt{3}+1)\bar{y}_{\Delta_{c_1}} - \frac{\sqrt{3}}{4}(\sqrt{3}-1)\bar{y}_{\Delta_{c_2}}\right\}\alpha_1\alpha_5 - \frac{1}{2}\bar{y}_{\Delta_d}\alpha_{13}\alpha_{14}\right]v_d,$$

$$\epsilon_{31} = -y_{\Delta_e}\alpha_5\alpha_9 v_d - 3\bar{y}_{\Delta_f}\alpha_9\alpha_{13}v_d,$$

$$\epsilon_{33} = y_{\Delta_e}\alpha_5\alpha_9 v_d.$$

- The charged lepton mass matrix $(M_l^\dagger M_l)$ is written as

$$M_l^\dagger M_l \simeq \begin{pmatrix} |\epsilon_{11}|^2+|\epsilon_{21}|^2+|\epsilon_{31}|^2 & \frac{1}{2}(\sqrt{3}\epsilon_{11}^*+\epsilon_{21}^*)m_\mu & \epsilon_{31}^* m_\tau \\ \frac{1}{2}(\sqrt{3}\epsilon_{11}+\epsilon_{21})m_\mu & m_\mu^2 & \frac{1}{2}(\sqrt{3}\epsilon_{13}+\epsilon_{23})m_\mu \\ \epsilon_{31} m_\tau & \frac{1}{2}(\sqrt{3}\epsilon_{13}^*+\epsilon_{23}^*)m_\mu & m_\tau^2 \end{pmatrix},$$

where $m_\mu = \mathcal{O}(\tilde{\alpha}/\lambda)$, $m_\tau = \mathcal{O}(\tilde{\alpha})$, and $\epsilon_{ij} = \mathcal{O}(\alpha_i \alpha_j v_d) = \mathcal{O}(m_e)$.

$$U_E = \begin{pmatrix} 1 & \mathcal{O}\left(\frac{m_e}{m_\mu}\right) & \mathcal{O}\left(\frac{m_e}{m_\tau}\right) \\ \mathcal{O}\left(\frac{m_e}{m_\mu}\right) & 1 & \mathcal{O}\left(\frac{m_e m_\mu}{m_\tau^2}\right) \\ \mathcal{O}\left(\frac{m_e}{m_\tau}\right) & \mathcal{O}\left(\frac{m_e m_\mu}{m_\tau^2}\right) & 1 \end{pmatrix}.$$

- Since the lepton mixing is given as $U_{\rm MNS} = U_E^\dagger U_{\text{tri-bimaximal}}$, we have

$$|U_{e3}| \sim \frac{1}{\sqrt{2}}\left(\mathcal{O}\left(\frac{m_e}{m_\mu}\right)\right), \quad |U_{e2}| \sim \frac{1}{\sqrt{3}}\left(1+\mathcal{O}\left(\frac{m_e}{m_\mu}\right)\right),$$

$$|U_{\mu 3}| \sim \frac{1}{\sqrt{2}}\left(1-\mathcal{O}\left(\frac{m_e m_\mu}{m_\tau^2}\right)\right) .$$

The electron (down quark) mass

- The determinant of $M_l^\dagger M_l$:

$$\det\left[M_l^\dagger M_l\right] \simeq \frac{3}{2} m_\mu^2 m_\tau^2 \left(\frac{1}{6}\epsilon_{11}^2 - \frac{1}{\sqrt{3}}\epsilon_{11}\epsilon_{21} + \frac{1}{2}\epsilon_{21}^2\right) .$$

- The electron mass:

$$m_e^2 \simeq \frac{3}{2}\left(\frac{1}{6}\epsilon_{11}^2 - \frac{1}{\sqrt{3}}\epsilon_{11}\epsilon_{21} + \frac{1}{2}\epsilon_{21}^2\right)$$
$$\simeq \frac{3}{2}\left[\frac{1}{6}y_{\Delta_d}^2\alpha_5^2\alpha_{14}^2 + y_{\Delta_d}(\sqrt{3}\bar{y}_{\Delta_{e_1}} - \bar{y}_{\Delta_{e_2}})\alpha_1\alpha_5^2\alpha_{14} + \frac{1}{2}\left(3\bar{y}_{\Delta_{e_1}} - \sqrt{3}\bar{y}_{\Delta_{e_2}}\right)^2\alpha_1^2\alpha_5^2\right] v_d^2 .$$

- The down quark mass:

$$m_d^2 \simeq \frac{3}{2}\left[\frac{1}{6}y_{\Delta_d}^2\alpha_5^2\alpha_{14}^2 - \frac{1}{3}y_{\Delta_d}(\sqrt{3}\bar{y}_{\Delta_{e_1}} - \bar{y}_{\Delta_{e_2}})\alpha_1\alpha_5^2\alpha_{14} + \frac{1}{18}\left(3\bar{y}_{\Delta_{e_1}} - \sqrt{3}\bar{y}_{\Delta_{e_2}}\right)^2\alpha_1^2\alpha_5^2\right] v_d^2 .$$

- The condition of the ratio $m_e^2 : m_d^2 = 1 : 9$ is

$$\alpha_{14} = -\frac{5\left(\sqrt{3}\bar{y}_{\Delta_{e_1}} - \bar{y}_{\Delta_{e_2}}\right)}{y_{\Delta_d}}\alpha_1, \quad \text{or} \quad \alpha_{14} = -\frac{2\left(\sqrt{3}\bar{y}_{\Delta_{e_1}} - \bar{y}_{\Delta_{e_2}}\right)}{y_{\Delta_d}}\alpha_1.$$

- Including next-to-leading order corrections, we get

$$
\begin{aligned}
V^0_{ud} &\simeq \cos 15^\circ - (\theta^d_{12} + \theta^d_{13}\theta^d_{23}) \sin 15^\circ, \\
V^0_{us} &\simeq \theta^d_{12} \cos 15^\circ + \sin 15^\circ, \\
V^0_{ub} &\simeq \theta^d_{13} \cos 15^\circ + \theta^d_{23} \sin 15^\circ, \\
V^0_{cd} &\simeq -r_t e^{i\rho} \sin 15^\circ - r_t(\theta^d_{12} + \theta^d_{13}\theta^d_{23}) e^{i\rho} \cos 15^\circ + r_c(\theta^d_{13} - \theta^d_{12}\theta^d_{23}) \ , \\
V^0_{cs} &\simeq -r_t \theta^d_{12} e^{i\rho} \sin 15^\circ + r_t e^{i\rho} \cos 15^\circ + r_c(\theta^d_{23} + \theta^d_{12}\theta^d_{13}) \ , \\
V^0_{cb} &\simeq -r_t \theta^d_{13} e^{i\rho} \sin 15^\circ + r_t \theta^d_{23} e^{i\rho} \cos 15^\circ - r_c \ , \\
V^0_{td} &\simeq -r_c \sin 15^\circ e^{i\rho} - r_c(\theta^d_{12} + \theta^d_{13}\theta^d_{23}) e^{i\rho} \cos 15^\circ + r_t(-\theta^d_{13} + \theta^d_{12}\theta^d_{23}) \ , \\
V^0_{ts} &\simeq -r_c \theta^d_{12} \sin 15^\circ e^{i\rho} + r_c e^{i\rho} \cos 15^\circ - r_t(\theta^d_{23} + \theta^d_{12}\theta^d_{23}) \ , \\
V^0_{tb} &\simeq -r_c \theta^d_{12} \sin 15^\circ e^{i\rho} + r_c \theta^d_{23} e^{i\rho} \cos 15^\circ + r_t \ .
\end{aligned}
$$

- We obtain a parameter set

$$
\rho = 123^\circ, \quad \theta^d_{12} = -0.0340, \quad \theta^d_{13} = 0.00626, \quad \theta^d_{23} = -0.00880.
$$

- Values of parameters are consistent with our mass matrices.

$$
\theta^d_{12} = \mathcal{O}\left(\frac{m_d}{m_s}\right) = \mathcal{O}(0.05) \, , \ \theta^d_{13} = \mathcal{O}\left(\frac{m_d}{m_b}\right) = \mathcal{O}(0.005) \, , \ \theta^d_{23} = \mathcal{O}\left(\frac{m_d}{m_b}\right) = \mathcal{O}(0.005) \, .
$$

- $|J_{CP}| \equiv |\mathrm{Im}\{V_{us}V_{cs}^* V_{ub}V_{cb}^*\}|$.
- $\phi_1 \equiv \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right), \quad \phi_2 \equiv \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right), \quad \phi_3 \equiv \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{ccb}^*}\right).$

- Taking account next leading terms and another phase σ, we can adjust the CKM mixing angles and CP violation.

$$|V_{\mathrm{CKM}}| \simeq \begin{pmatrix} 0.974 & 0.226 & 0.0036 \\ 0.226 & 0.973 & 0.042 \\ 0.0087 & 0.041 & 0.999 \end{pmatrix}, \qquad \begin{array}{l} \sin 2\phi_1 \simeq 0.693, \\ \phi_2 \simeq 89.4^\circ,\ \phi_3 \simeq 68.7^\circ, \\ J_{\mathrm{CP}} \simeq 3.06 \times 10^{-5}. \end{array}$$

Experimental data (PDG 2008)

$$|V_{\mathrm{CKM}}^{\mathrm{exp}}| \simeq \begin{pmatrix} 0.974 & 0.226 & 0.0036 \\ 0.226 & 0.973 & 0.042 \\ 0.0087 & 0.041 & 0.999 \end{pmatrix}, \qquad \begin{array}{l} \sin 2\phi_1 \sim 0.681 \pm 0.025, \\ \phi_2 \sim (88^{+6}_{-5})^\circ,\ \phi_3 \sim (77^{+30}_{-32})^\circ, \\ J_{\mathrm{CP}} \simeq (3.05^{+0.19}_{-0.20}) \times 10^{-5}. \end{array}$$