Seven-Dimensional Super-Yang–Mills Theory in $\mathcal{N} = 1$ Superfields

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Motivation

- Superfield convenient for SUSY model building
- For higher SUSY (i.e. $\mathcal{N} = 2$ or $\mathcal{N} = 4$), superspace not so useful anymore
- Idea: Single out one SUSY to be manifest, arrange component fields into suitable superfields
- Even in higher-dimensional models, end up with $\mathcal{N} = 1$ in 4D, e.g. by compactification or coupling to lower-dimensional subspaces which preserve some, but not all SUSY
- This talk: Seven-dimensional case (e.g. D6 branes in IIA, ADE singularities in M theory on G_2 manifolds)

Contents

Superfields for Higher-Dimensional SUSY

- Choosing embedding into $\mathcal{N} = 1$ superfields singles out one manifest supersymmetry – remaining SUSY enforced by Lorentz invariance
- Ten-dimensional SYM: [Marcus, Sagnotti, Siegel '83]
- Renewed interest: [Arkani-Hamed et al. '01]; [Marti, Pomarol '01] in 5D, including radion
- Supergravity: [Linch, Luty, Phillips '02] for linearised 5D SUGRA; [Schmidt, Paccetti Correia, Tavartkiladze '04-06] in 5D, including compactification
- Gauge-covariant formulation in 5D: [Hebecker '01]
- Here: Extend this to seven dimensions

General Dimensions

- Higher-dimensional SUSY is either $\mathcal{N} = 2$ or $\mathcal{N} = 4$ from 4D perspective
- Degree-of-freedom-wise, SYM multiplet corresponds to one vector and one/three chiral multiplets of $\mathcal{N} = 1$
- However, theory has more structure than just 4D $\mathcal{N}$ =4 SUSY: higher-dimensional Lorentz symmetry, different for different dimensions
- In general, action cannot be written in terms of field strengths and covariant derivatives only
- Action contains V explicitly (i.e. not just e^V and W_α) $\rightsquigarrow$ extra term to maintain gauge invariance (vanishes in WZ gauge)
- Reason: non-Abelian V transforms nonpolynomially

5D Case: Covariant Description

[Hebecker '01]

- 5d gauge multiplet: Vector A_M, scalar B, symplectic Majorana spinor pair ψ_I
- Off-shell component description known – includes $SU(2)$ triplet of auxiliary fields [Mirabelli, Peskin '97]
- Identify vector and chiral superfields V, $\Phi = A_5 + \mathrm{i}B + \cdots$
- Crucial point: Define covariant derivative $\nabla_5 = \partial_5 + \Phi$
- Define covariantly transforming field strength $Z = e^{-2V}\nabla_5 e^{2V}$
- Simplest possible action, symbolically

$$\mathscr{L} \sim \mathrm{tr}\, W^\alpha W_\alpha + \mathrm{tr}\, Z^2\,,$$

reproduces component action (already off-shell)

Symmetry Argument: 5 and 7 OK

Approach does not work in any dimension, for a symmetry reason: In 4D language, we have $\mathcal{N} = 2$ or $\mathcal{N} = 4$ with extra structure, so naïve R symmetry is broken to the geometrical symmetry $SO(d)$.

- In five or six dimensions, choosing one supersymmetry manifest breaks $SU(2)_R \to$ nothing. Geometrical symmetry is nothing for 5D, $SO(2)$ for 6D
- In seven to ten dimensions, naïve manifest R symmetry is $SU(3)$. Only $SO(3)$ is subgroup, hence only 7D works.
- Form technical point of view, want scalar components of chiral multiplets to be $A_i + \mathrm{i}B_i$, so need equal number of extra gauge field components and non-gauge scalars.

7D SYM: Symmetries, Field Content

Symmetries of seven dimensional theory:

- Symmetries of the seven-dimensional theory
  - Lorentz symmetry $SO(1,6)$
  - Supersymmetry
  - R symmetry $SU(2)$ – this is the remnant Lorentz $SO(3)$ which reappears as automorphism of the superalgebra, even for minimal 7D supersymmetry [Strathdee '86]
- Fields:
  - Vector $A_M \sim (\mathbf{7};\mathbf{1})$
  - $SU(2)$ triplet of adjoint scalars $B_i \sim (\mathbf{1};\mathbf{3})$
  - $SU(2)$ doublet of spinors $\Psi_I \sim (\mathbf{8};\mathbf{2})$ with symplectic Majorana condition

$$\Psi_I = \varepsilon_{IJ} C \overline{\Psi}_J^T$$

7D SYM: Action, SUSY Transformations

The action can be obtained e.g. by dimensional reduction from 10D:

$$\mathscr{L} = -\frac{1}{4}\operatorname{tr} F_{MN}F^{MN} - \frac{1}{2}\operatorname{tr} D_M B_i D^M B^i + \frac{1}{4}g^2 \operatorname{tr}\left[B_i, B_j\right]\left[B^i, B^j\right] + \frac{\mathrm{i}}{2}\operatorname{tr}\bar{\Psi}_I \Gamma^M D_M \Psi_I + \frac{\mathrm{i}}{2} g \operatorname{tr}\bar{\Psi}_I\left[B_i \sigma^i_{IJ}, \Psi_J\right]$$

SUSY transformations:

$$\delta A_M = -\frac{\mathrm{i}}{2}\bar{\varepsilon}_I \Gamma_M \Psi_I\,, \qquad \delta B_i = -\frac{1}{2}\sigma^i_{IJ}\bar{\varepsilon}_I \Psi_J\,,$$

$$\delta\Psi_I = -\frac{1}{4}F_{MN}\Gamma^{MN}\varepsilon_I + \frac{\mathrm{i}}{2}\Gamma^M D_M \left(B_i\sigma_i\right)_{IJ}\varepsilon_J + \frac{1}{4}g\varepsilon_{ijk}\left[B_i, B_j\right]\left(\sigma_k\right)_{IJ}\varepsilon_J$$

4D Degrees of Freedom

4D description reduces manifest symmetry:
$SO(1,6) \times SU(2) \to SO(1,3) \times SO(3) \times SU(2)$
In 4D language, the fields are:

- gauge vector $A_\mu \sim (\mathbf{4}; \mathbf{1}, \mathbf{1})$
- three "gauge scalars" $A_i \sim (\mathbf{1}; \mathbf{3}, \mathbf{1})$
- three adjoint scalars $B_i \sim (\mathbf{1}; \mathbf{1}, \mathbf{3})$
- four (complex) Weyl spinors $\lambda_r \sim (\mathbf{2}; \mathbf{2}, \mathbf{2})$

In full dimensional reduction, R symmetry and extra-dimensional Lorentz symmetry would form $SU(2) \times SU(2) = SO(4)$, enhanced to $SU(4)$ – the scalars in a $(\mathbf{3}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{3}) = \mathbf{6}$, spinors in a $\mathbf{4}$.
Singling out one supersymmetry breaks $SU(4) \to SU(3)$ – scalars now in $\mathbf{3}$, spinors in $\mathbf{1} \oplus \mathbf{3}$.
Here, however, the A_i are different from the B_i – keep the diagonal $SO(3)$.

Superfield Embedding

Scalars and three spinors form triplet of chiral multiplets,

$$\boxed{\Phi_i = A_i + \mathrm{i}B_i + 2\theta\psi_i + \theta^2 F_i}$$

$$\delta\left(A_i + \mathrm{i}B_i\right) \sim \epsilon\psi_i\,, \qquad \delta\psi_i \sim F_{\mu i}\sigma^\mu\overline{\epsilon} - F_i\epsilon\,.$$

The other spinor and the vector form vector multiplet

$$\boxed{V = -\theta\sigma^\mu\bar{\theta}A_\mu + \mathrm{i}\theta^2\bar{\theta}\overline{\chi} - \mathrm{i}\bar{\theta}^2\theta\chi + \frac{1}{2}\theta^4 D}$$

$$\delta A_\mu \sim \epsilon\sigma_\mu\overline{\chi} + \text{H.c.}\,, \qquad \delta\chi \sim F_{\mu\nu}\sigma^{\mu\nu}\epsilon - D\epsilon\,.$$

Note: Still on-shell description – auxiliary fields fixed to be

$$F_i = \varepsilon_{ijk}\left(\partial_j\Phi_k + \tfrac{1}{2}\mathrm{i}\left[\Phi_j, \Phi_k\right]\right) = \frac{1}{2}\varepsilon_{ijk}\left(F_{jk} + 2\mathrm{i}D_jB_k - \mathrm{i}\left[B_j, B_k\right]\right)\,,$$

$$D = -D_iB_i\,.$$

SUSY $\leftrightarrow$ Gauge mixing

As usual, closure of SUSY transformations on the vector multiplet requires additional field-dependent gauge transformation:

$$[\delta_\varepsilon, \delta_\eta]\, A_\mu = -2\mathrm{i}\left(\varepsilon\sigma^\nu\overline{\eta} - \eta\sigma^\nu\overline{\varepsilon}\right)\partial_\nu A_\mu + \delta_{\text{gauge}}$$

where δ_{gauge} is a gauge transformation with parameter

$$\Lambda = 2\mathrm{i}\left(\varepsilon\sigma^\mu\overline{\eta} - \eta\sigma^\mu\overline{\varepsilon}\right)A_\mu$$

Here, the same happens for the chiral multiplets,

$$[\delta_\varepsilon, \delta_\eta]\, \phi_i = -2\mathrm{i}\left(\varepsilon\sigma^\mu\overline{\eta} - \eta\sigma^\mu\overline{\varepsilon}\right)\partial_\mu \phi_i + \delta_{\text{gauge}}\,,$$

with the same Λ – makes right-hand side gauge covariant.

Superfield Gauge Transformations, Field Strengths

The superfield gauge transformations are

$$e^{2V} \longrightarrow e^{-\mathrm{i}\bar{\Lambda}} e^{2V} e^{\mathrm{i}\Lambda}\,, \qquad \Phi_i \longrightarrow e^{-\mathrm{i}\Lambda} \left(\Phi_i - \mathrm{i}\partial_i\right) e^{\mathrm{i}\Lambda}\,.$$

We can define a covariant derivative in the extra dimensions as

$$\nabla_i = \partial_i + \mathrm{i}\Phi_i\,, \qquad \nabla_i \rightarrow e^{-\mathrm{i}\Lambda} \nabla_i e^{\mathrm{i}\Lambda}$$

This allows to define two field strength superfields:

$$W_\alpha = -\frac{1}{4}\overline{D}^2 e^{-2V} D_\alpha e^{2V}\,,$$
$$Z_i = e^{-2V} \nabla_i e^{2V}\,,$$

which both transform as

$$Z_i \rightarrow e^{-\mathrm{i}\Lambda} Z_i e^{\mathrm{i}\Lambda}\,, \qquad W_\alpha \rightarrow e^{-\mathrm{i}\Lambda} W_\alpha e^{\mathrm{i}\Lambda}\,.$$

Action — $W^2 + Z^2$

The action contains three pieces, each gauge invariant and supersymmetric. The coefficients are fixed by Lorentz symmetry (also ensures $\mathcal{N} = 4$ SUSY):

- The usual four-dimensional gauge kinetic term,

$$\frac{1}{16} \int \mathrm{d}^2\theta \; \mathrm{tr}\, W^\alpha W_\alpha + \mathrm{H.c.} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \mathrm{i}\chi\sigma^\mu D_\mu \overline{\chi} + \frac{1}{2} D^2$$

- The kinetic term of the chiral multiplets,

$$\frac{1}{4} \int \mathrm{d}^4\theta \; \mathrm{tr}\, Z_i Z_i = -\frac{1}{2} F_{\mu i} F^{\mu i} - \frac{1}{2} D_\mu B_i D^\mu B_i + D D_i B_i + 2 F_i \overline{F}_i - \mathrm{i}\psi_i \sigma^\mu D_\mu \overline{\psi}_i - \left(\chi D_i \psi_i - \chi \left[B_i, \psi_i\right] + \mathrm{H.c.}\right)$$

Note: $\mathrm{tr}\, Z_i Z_i$ is Hermitean although Z_i itself is not!

Action — Chern–Simons Term

The purely extra-dimensional piece of the action is provided by a superpotential-like term

$$\frac{1}{4}\int \mathrm{d}^2\theta\ \mathrm{tr}\,\varepsilon_{ijk}\Phi_i\left(\partial_j\Phi_k + \frac{\mathrm{i}}{3}\left[\Phi_j,\Phi_k\right]\right) + \mathrm{H.c.} =$$

$$\frac{1}{4}\varepsilon_{ijk}F_i\left(F_{jk} + 2\mathrm{i}D_jB_k - \mathrm{i}\left[B_j,B_k\right]\right)$$

$$-\frac{1}{2}\varepsilon_{ijk}\psi_iD_j\psi_k + \frac{1}{2}\varepsilon_{ijk}\psi_i\left[B_j,\psi_k\right] + \mathrm{H.c.}$$

This is a superfield Chern–Simons term $\sim A\wedge dA + \frac{2}{3}A^3$, so it is gauge invariant up to a “winding number” due to $\pi_3(G) = \mathbb{Z}$:

$$\delta\mathscr{L}_{\mathrm{CS}} \sim \int \mathrm{d}^2\theta\ \mathrm{tr}\,\varepsilon_{ijk}\left(U^{-1}\partial_iU\right)\left(U^{-1}\partial_jU\right)\left(U^{-1}\partial_kU\right)$$

Complete Action

The full action

$$\mathscr{L} = \mathscr{L}_{W^2} + \mathscr{L}_{Z^2} + \mathscr{L}_{\mathsf{CS}} \, ,$$

reproduces the auxiliary fields

$$F_i = \frac{1}{2}\varepsilon_{ijk}\left(F_{jk} + 2\mathrm{i}D_j B_k - \mathrm{i}\left[B_j, B_k\right]\right) \, , \qquad D = -D_i B_i \, .$$

After eliminating, recover original action expressed in 4D fields:

$$\begin{aligned}\mathscr{L}_{\mathsf{SF}} = & -\frac{1}{4}F_{MN}F^{MN} - \frac{1}{2}D_M B_i D^M B_i + \frac{1}{4}\left[B_i, B_j\right]\left[B_i, B_j\right] \\ & - \mathrm{i}\chi\sigma^\mu D_\mu\bar{\chi} - \mathrm{i}\psi_i\sigma^\mu D_\mu\bar{\psi}_i \\ & - \left[\chi\left(D_i\psi_i - \left[B_i, \psi_i\right]\right) + \frac{1}{2}\varepsilon_{ijk}\psi_i\left(D_j\psi_k - \left[B_j, \psi_k\right]\right) + \mathrm{H.c.}\right]\end{aligned}$$

Flux SUSY Breaking

(still work in progress)

Internal profiles $\langle\phi_i\rangle$

$$F_i = \frac{1}{2}\varepsilon_{ijk}\left(F_{jk} + 2\mathrm{i}D_j B_k - \mathrm{i}\left[B_j, B_k\right]\right) \sim \mathrm{Curl}\,\vec{\phi}\,,$$
$$D = -D_i B_i \sim \mathrm{Grad}\,\vec{B}\,.$$

with "covariant curl" and gradient. This induces masses for scalars and fermions of the form

$$\sim A_i^2 F + A_i B_i D + \chi\vec{\psi}\cdot\vec{\phi} + \vec{\psi}\cdot\vec{\psi}\times\vec{\phi}\,.$$

Masses appear only for non-Abelian groups!

Higher-Dimensional Operators in 7D

(still work in progress)

- Application of the formalism: Study of supersymmetric higher-dimensional operators
- Once three dimensions are compactified, full Lorentz symmetry is not required anymore
- For $U(1)$ gauge groups, W_α and Z_i are already gauge invariant
- Example: Gaugino masses from internal flux via

$$\int \mathrm{d}^4\theta \, Z_i Z_i W^\alpha W_\alpha \sim F_{ij} F^{ij} \chi^\alpha \chi_\alpha$$

- For string theory compactification, coupling to supergravity required – should at least include relevant moduli

Summary

- Gauge-covariant $\mathcal{N} = 1$ superspace description of seven-dimensional SYM theory
- Possible in 5D and 7D
- Construction of Lagrangean straightforward – coefficients determined by higher-dimensional Lorentz symmetry
- Applies e.g. to D6 branes, M theory on G_2 manifolds with ADE singularities (still requires coupling to supergravity or relevant moduli)
- Facilitates systematic study of higher-dimensional operators
- Application: Coupling to lower-dimensional subspaces, as e.g. in brane intersections – $\mathcal{N} = 1$ SUSY is manifest
- still work in progress...