Constraints on the split-UED mass spectrum from flavor violation

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Outline

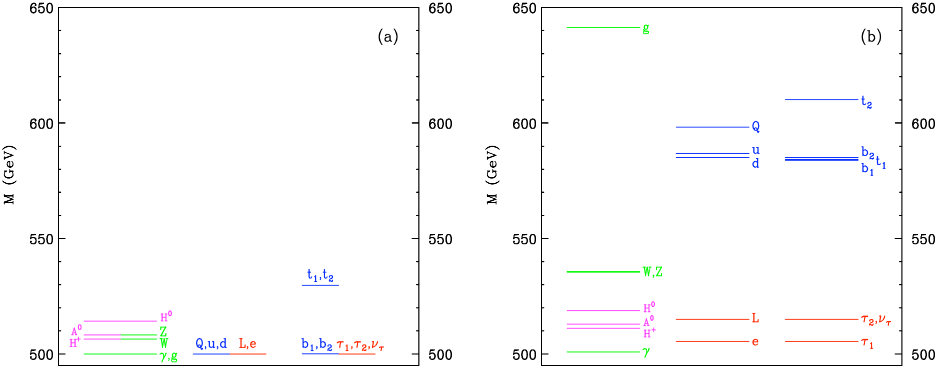
- UED review
- Split UED
  - Flavor violation in sUED
  - Implications for the sUED mass spectrum
- Conclusions and Outlook

UED: The basic setup

- UED models are models with flat, compact extra dimensions in which *all* fields propagate. [Appelquist, Cheng, Dobrescu,(2001)]
- The Standard Model (SM) particles are identified with the lowest-lying modes of the respective Kaluza-Klein (KK) towers.
- Compactification on S^1/Z_2 allows for boundary conditions on the fermion and gauge fields such that
  - half of the fermion zero mode is projected out $\Rightarrow$ chiral fermions
  - $A_5^{(0)}$ is projected out $\Rightarrow$ no additional massless scalar
- The presence of orbifold fixed points breaks 5D translational invariance.

  $\Rightarrow$ KK-number conservation is violated, *but*
  a discrete Z_2 parity (KK-parity) remains.
  $\Rightarrow$ The lightest KK mode (LKP) is stable.

UED - the spectrum



[Cheng, Matchev, Schmaltz, PRD **66** (2002) 036005, hep-ph/0204342]

UED - Constraints

- Phenomenological constraints on R^{-1}
  - lower bound $\Rightarrow$ $R^{-1} \gtrsim 650\,\mathrm{GeV}$
    - no detection of KK-modes

      [Appelquist *et al.* (2001); Rizzo (2001); Macesanu *et al.* (2002); Lin (2005)]

      $R^{-1} > 280\,\mathrm{GeV}$ at 95% cl.
    - FCNCs [Buras, Weiler *et al.* (2003); Weiler, Haisch (2007)]

      $R^{-1} > 600(330)\,\mathrm{GeV}$ at 95% (99%) cl.
    - Electroweak Precision Constraints [Appelquist, Yee (2002);Gogoladze, Macesanu (2006)]

      $R^{-1} > 600(300)\,\mathrm{GeV}$ for $m_H = 115(800)\,\mathrm{GeV}$ at 95% cl.
  - upper bound: preventing over closure of the Universe by $B^{(1)}$ dark matter

    $\Rightarrow$ $R^{-1} \lesssim 1.5\mathrm{TeV}$ [Servant, Tait (2002); Matchev, Kong (2005); Burnell, Kribs (2005)]

Why are the bounds on UED so weak?

- KK parity guarantees that no single-production of a BSM state occurs.
- In "standard" UED, the KK basis $\{f_n(y)\}$ is identical *for all fields*.

  $\Rightarrow$ no triple-vertices of two zero modes with a higher KK mode exist.

  $\Rightarrow$ KK mode exchange does not influence 4-fermi operators at tree-level.

  Relevant for

  Quark flavor physics: E.g. no $s\bar{d} \to \bar{s}d$

  Lepton flavor physics: E.g. no $\mu \to ee\bar{e}$

  Electroweak Precision Tests: No $\mu\bar{\nu}_\mu \to e\bar{\nu}_e$ or $\mu\bar{\mu} \to e\bar{e}$ corrections.
- The UED GIM mechanism guarantees that loop contributions to FCNCs are not only loop- but also CKM-suppressed.

Extensions of UED with minimal field content

Even without extending the field content, the spectrum and/or the interactions of the UED model can by modified by the inclusion of additional operators.
Three classes are

1. Bulk mass terms for fermions (dimension 4 operators),
2. kinetic and mass terms at the orbifold fixed points
   (dimension 5; radiatively induced in MUED),
3. bulk or boundary localized interactions (dimension 6 or higher)

The former two modify the free field equations and thereby the spectrum and the KK bases $\{f_n^{\psi}(y)\}$.
Today, we focus on bulk mass terms: “split UED”

Bulk mass terms for fermions

[Park, Shu, *et al.* (2009); for earlier work, see Csaki (2003)]

A plain bulk mass term for fermions of the form

$$S \supset \int d^5x \; - M\overline{\Psi}\Psi$$

is forbidden by KK parity, but

it can be allowed if realized by a KK-parity odd background field

$$S \supset \int d^5x - \lambda\Phi\overline{\Psi}\Psi,$$

where $\Phi(-y) = -\Phi(y)$

(Orbifold fixed points are at $\pm L = \pm\pi R/2$)

In the simplest case $M = \mu_5\theta(y)$

(similar to the bulk fermion mass term in Randall-Sundrum models)

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Structural consequences:

[Park, Shu, *et al.* (2009); Kong, Park, Rizzo (2010)]

- The chiral fermion zero modes remain massless,

  but the profile of the zero mode becomes exponentially localized towards to or away from the orbifold fixed points:

$$f_0^{\psi_{L,R}} = \sqrt{\frac{\pm\mu_5}{1-e^{\mp 2\mu_5 L}}}\, e^{\mp 2\mu_5 |y|}$$

- The KK mode masses are $m^{(n)} = \sqrt{(\mu_5)^2 + k_n^2}$ with k_n^2 determined from

  $0 = \cot(k_n \pi R/2)$ for even-numbered modes and

  $(\mu_5)^2 = k_n^2 \cot^2(k_n \pi R/2)$ for odd-numbered modes.

- Wave functions of the fermion and gauge KK modes are not orthogonal:

$$g_{002n} = g^{SM} \mathcal{F}^{\psi\psi}_{002m}(\mu_5 L) = g^{SM} \frac{(\mu_5 L)^2(-1 + (-1)^n e^{2\mu_5 L}(\coth(\mu_5 L) - 1)}{\sqrt{2(1+\delta_{0n}}((\mu_5 L)^2 + n^2\pi^2/4)}$$

sUED - known bounds

Bounds studied so far (all for uniform $M^Q = M^U = M^D = M \times \mathbb{1}$):

- Modifications of di-quark channels at Tevatron:

$$R^{-1} \gtrsim .6 \text{ TeV for } \mu_5 L = 10$$

[Park, Shu, *et al.* (2009)]

- Modifications of contact interactions at LEP2:

$$R^{-1} \gtrsim .75 \text{ TeV for } \mu_5 L = 10$$

[Kong, Park, Rizzo (2010)]

The sUED fermion action

The most general action for fermions reads

$$S = \int d^5x \mathcal{L}_f + \mathcal{L}_{Yuk}$$

with

$$\begin{aligned}
\mathcal{L}_f &= \sum_{ij} \left\{ \frac{i}{2}\delta_{ij} \left(D_M \overline{\Psi}_i \Gamma^M \Psi_j - \overline{\Psi}_\Gamma^M D_M \Psi_j \right) - M_{ij}^\Psi(y) \overline{\Psi}_i \Psi_j \right\}, \\
\mathcal{L}_{Yuk} &= \sum_{ij} \left\{ \lambda_{ij}^U \overline{Q}_i \tilde{H} U_j + \lambda_{ij}^D \overline{Q}_i H D_j + \lambda_{ij}^E \overline{L}_i H E_j \right\} + \text{h.c.}.
\end{aligned}$$

$M^{Q,u,d,L,e}$ are 3×3 hermitian matrices in flavor space,

$\lambda^{U,D,E}$ are 3×3 matrices in flavor space.

Compared to UED, sUED contains 18 new moduli plus 9 new phases in the quark sector and 12 new moduli plus 6 new phases in the lepton sector.

Calculating the 4D effective action

With a set of $\left(M^\psi, \lambda^{U,D,E}\right)$, we perform the KK decomposition and then integrate out the heavy modes.

Via field redefinitions, the mass matrices M^ψ can be diagonalized, and the fermion zero mode Lagrangian in the zero mode approximation reads

$$
\begin{aligned}
\mathcal{L}_{kin} &= \overline{\psi}^{(0)} i\gamma^\mu \partial_\mu \psi^{(0)} \\
\mathcal{L}_{f,g} &= \sum_{n=0} \left[\overline{\psi}^{(0)} i\gamma^\mu (D_\mu - \partial_\mu)^{(2n)} \psi^{(0)} \mathcal{F}^{\psi,\psi}_{002n}\right] \\
\mathcal{L}_{Yuk} &= \overline{u^q}^{(0)}_{L,i} \frac{\lambda'^U_{ij}}{\sqrt{2}} v_5 \mathcal{F}^{q^i_L, u^j_R}_{000} u^{(0)}_{R,j} + \overline{d^q}^{(0)}_{L,i} \frac{\lambda'^D_{ij}}{\sqrt{2}} v_5 \mathcal{F}^{q^i_L, d^j_R}_{000} d^{(0)}_{R,j} + \overline{e^l}^{(0)}_{L,i} \frac{\lambda'^E_{ij}}{\sqrt{2}} v_5 \mathcal{F}^{l^i_L, e^j_R}_{000} e^{(0)}_{R,j} \\
&\quad + \text{h.c.}
\end{aligned}
$$

The basis in which the M^ψ are diagonal thus signifies the gauge eigenbasis.

Transformation to the quark mass eigenbasis by bi-unitary transformations:

$$u_L = S_u^\dagger (u^q)_L^{(0)}, \quad d_L = S_d^\dagger (d^q)_L^{(0)}, \quad e_L = S_e^\dagger (e^l)_L^{(0)}$$
$$u_R = T_u^\dagger u_R^{(0)}, \quad d_R = T_d^\dagger d_R^{(0)}, \quad e_R = T_e^\dagger e_R^{(0)}.$$

In the fermion mass eigenbasis, the couplings to the gauge bosons read

$$\mathcal{L}_{q,eff} \subset \sum_{n=0} \eta^{\mu\nu} \left[g_3 G_\mu^{A(2n)} J_{q\nu}^{A(2n)} + \left(\frac{g_2}{\sqrt{2}} W_\mu^{+(2n)} J_{q\nu}^{+(2n)} + \text{h.c.} \right) + e A_\mu^{(2n)} J_{q\nu}^{em,(2n)} + \frac{g_2}{\cos(\theta_W)} Z_\mu^{(2n)} J_{q\nu}^{0(2n)} \right],$$

where *e.g.*

$$J_{q\nu}^{A(2n)} = \left(V_{L,ij}^{u(2n)} \overline{u}_{L,i} T^A \gamma_\nu u_{L,j} + V_{L,ij}^{d(2n)} \overline{d}_{L,i} T^A \gamma_\nu d_{L,j} \right) + (L \leftrightarrow R)$$
$$J_{q\nu}^{em(2n)} = \left(\frac{2}{3} V_{L,ij}^{u(2n)} \overline{u}_{L,i} \gamma_\nu u_{L,j} - \frac{1}{3} V_{L,ij}^{d(2n)} \overline{d}_{L,i} \gamma_\nu d_{L,j} \right) + (L \leftrightarrow R)$$

with

$$V_{L,ij}^{u(2n)} = (S_u^\dagger \mathcal{F}_{002n}^{q_L,q_L} S_u)_{ij} \quad , \quad V_{R,ij}^{u(2n)} = (T_u^\dagger \mathcal{F}_{002n}^{u_R,u_R} T_u)_{ij}$$
$$V_{L,ij}^{d(2n)} = (S_d^\dagger \mathcal{F}_{002n}^{q_L,q_L} S_d)_{ij} \quad , \quad V_{R,ij}^{d(2n)} = (T_d^\dagger \mathcal{F}_{002n}^{d_R,d_R} T_d)_{ij}$$

Integrating out all but the zero modes leads the low-energy effective action

$$S_{eff,4q} = -\sum_{n=1} \eta^{\mu\nu} \left[\frac{g_3^2}{2m^2_{G^{(2n)}}} J^{A(2n)}_{q\mu} J^{A(2n)}_{q\nu} + \frac{g_2^2}{2m^2_{W^{(2n)}}} J^{+(2n)}_{q\nu} J^{-(2n)}_{q\nu} + \frac{e^2}{2m^2_{A^{(2n)}}} J^{em(2n)}_{q\mu} J^{em(2n)}_{q\nu} + \frac{g_2^2}{2\cos^2\theta_W^{(2n)} m^2_{Z^{(2n)}}} J^{0(2n)}_{q\mu} J^{0(2n)}_{q\nu} \right],$$

which in particular contain the $\Delta F = 2$ effective operators of sUED, which can be parameterized according to

$$\mathcal{H}_{int}^{\Delta F=2} = \sum_{i=1}^{5} C^i_{q_i q_j} Q_i^{q_i q_j} + \sum_{i=1}^{3} \tilde{C}^i_{q_i q_j} \tilde{Q}_i^{q_i q_j}$$

with

$$Q_1^{q_i,q_j} = \left(\overline{q}^a_{L,j}\gamma_\mu q^a_{L,i}\right)\left(\overline{q}^b_{L,j}\gamma^\mu q^b_{L,i}\right)$$

$$Q_2^{q_i,q_j} = \left(\overline{q}^a_{R,j} q^a_{L,i}\right)\left(\overline{q}^b_{R,j} q^b_{L,i}\right), \quad Q_3^{q_i,q_j} = \left(\overline{q}^a_{R,j} q^b_{L,i}\right)\left(\overline{q}^b_{R,j} q^a_{L,i}\right)$$

$$Q_4^{q_i,q_j} = \left(\overline{q}^a_{R,j} q^a_{L,i}\right)\left(\overline{q}^b_{L,j} q^b_{R,i}\right), \quad Q_5^{q_i,q_j} = \left(\overline{q}^a_{R,j} q^b_{L,i}\right)\left(\overline{q}^b_{L,j} q^a_{R,i}\right),$$

and $\tilde{Q}_{1,2,3} = Q_{1,2,3}(L \leftrightarrow R)$.

Experimental constraints: [Bona *et al.* (UTfit Collaboration, 2007)]

Parameter	95% allowed $[TeV^{-2}]$	Parameter	95% allowed $[TeV^{-2}]$
$\text{Re}C_K^1$	$[-9.6, 9.6] \cdot 10^{-7}$	$\text{Im}C_K^1$	$[-4.4, 2.8] \cdot 10^{-9}$
$\text{Re}C_K^2$	$[-1.8, 1.9] \cdot 10^{-8}$	$\text{Im}C_K^2$	$[-5.1, 9.3] \cdot 10^{-11}$
$\text{Re}C_K^3$	$[-6.0, 5.6] \cdot 10^{-8}$	$\text{Im}C_K^3$	$[-3.1, 1.7] \cdot 10^{-10}$
$\text{Re}C_K^4$	$[-3.6, 3.6] \cdot 10^{-9}$	$\text{Im}C_K^4$	$[-1.8, 0.9] \cdot 10^{-11}$
$\text{Re}C_K^5$	$[-1.0, 1.0] \cdot 10^{-8}$	$\text{Im}C_K^5$	$[-5.2, 2.8] \cdot 10^{-11}$
$\lvert C_{B_d}^1 \rvert$	$< 2.3 \cdot 10^{-5}$	$\lvert C_{B_s}^1 \rvert$	$< 1.1 \cdot 10^{-3}$
$\lvert C_{B_d}^2 \rvert$	$< 7.2 \cdot 10^{-7}$	$\lvert C_{B_s}^2 \rvert$	$< 5.6 \cdot 10^{-5}$
$\lvert C_{B_d}^3 \rvert$	$< 2.8 \cdot 10^{-6}$	$\lvert C_{B_s}^3 \rvert$	$< 2.1 \cdot 10^{-4}$
$\lvert C_{B_d}^4 \rvert$	$< 2.1 \cdot 10^{-7}$	$\lvert C_{B_s}^4 \rvert$	$< 1.6 \cdot 10^{-5}$
$\lvert C_{B_d}^5 \rvert$	$< 6.0 \cdot 10^{-7}$	$\lvert C_{B_s}^5 \rvert$	$< 4.5 \cdot 10^{-5}$
$\lvert C_D^1 \rvert$	$< 7.2 \cdot 10^{-7}$		
$\lvert C_D^2 \rvert$	$< 1.6 \cdot 10^{-7}$		
$\lvert C_D^3 \rvert$	$< 3.9 \cdot 10^{-6}$		
$\lvert C_D^4 \rvert$	$< 4.8 \cdot 10^{-8}$		
$\lvert C_D^5 \rvert$	$< 4.8 \cdot 10^{-7}$		

Calculating the Wilson coefficients in sUED, we find

$$
\begin{aligned}
C_K^1 &= \sum_n \left(\frac{g_3^2}{3m^2_{G^{(2n)}}} + \frac{g_2^2}{2\cos^2\theta_W^{(2n)} m^2_{Z^{(2n)}}} + \frac{e^2}{2m^2_{A^{(2n)}}} \right) V^{d(2n)}_{L,ds} V^{d(2n)}_{L,ds} \\
\tilde{C}_K^1 &= \sum_n \left(\frac{g_3^2}{3m^2_{G^{(2n)}}} + \frac{g_2^2}{2\cos^2\theta_W^{(2n)} m^2_{Z^{(2n)}}} + \frac{e^2}{2m^2_{A^{(2n)}}} \right) V^{d(2n)}_{R,ds} V^{d(2n)}_{R,ds} \\
C_K^4 &= = \sum_n - \left(\frac{g_3^2}{m^2_{G^{(2n)}}} + \frac{g_2^2}{\cos^2\theta_W^{(2n)} m^2_{Z^{(2n)}}} + \frac{e^2}{m^2_{A^{(2n)}}} \right) V^{d(2n)}_{R,ds} V^{d(2n)}_{L,ds} \\
C_K^5 &= \sum_n \frac{g_3^2}{3m^2_{G^{(2n)}}} V^{d(2n)}_{R,ds} V^{d(2n)}_{L,ds} .
\end{aligned}
$$

The analogous expressions for C^i_{D,B_d,B_s} follow from these by the replacements $(ds) \to (uc), (db), (sb)$.

How can FCNCs be avoided in the quark sector?

The constraints on the C^i's imply, that the products $\frac{1}{m^2_{G(2n)}} V^{u/d(2n)}_{L/R,ij} V^{u/d(2n)}_{L/R,ij}$ have to be very small. This can be achieved in three ways

1. High compactification scale: $R^{-1} \gtrsim 10^5\,\mathrm{TeV}$ satisfies all constraints for $S_{ij}, T_{ij}, \mathcal{F}_{ij}$ of $\mathcal{O}(1)$. [*c.f.* talk by Johannes Heinonen].
2. Degenerate mass matrices: $M^{Q,U,D} = \mu_5^{q,u,d}\theta(y)\mathbb{1}$ with $\mu_5^{q,u,d} \in \mathbb{R}$. *This setup is not flavor blind*. In particular, it implies chiral couplings of quarks to KK gluons.
3. Alignment: For $S_d = T_d = T_u = \mathbb{1}$, $S_u = U^\dagger_{CKM}$, all flavor constraints but the constraint for C^1_D is satisfied. The C^1_D constraint implies $\left|\left(\mathcal{F}^Q_{002}\right)_{22} - \left(\mathcal{F}^Q_{002}\right)_{11}\right| \times R \lesssim 3 \times 10^{-3}\,\mathrm{TeV}^{-1}$ with no other constraints on $M^{Q,U,D}$.
   Example: $R^{-1} = 1\,\mathrm{TeV}$, $(\mu_5^Q)_{11} = 1\,\mathrm{TeV} \Rightarrow (\mu_5^Q)_{22} = (1 \pm .005)\,\mathrm{TeV}$

Implications for the sUED mass spectrum

The masses of the first KK mode fermions are $m^{(n)} = \sqrt{(\mu_5)^2 + k_n^2} + \delta_{HM}$ where k_n^2 is determined from $(\mu_5)^2 = k_n^2 \cot^2(k_n \pi R/2)$. δ_m is a small correction $\mathcal{O}(m_\psi / m_\psi^{(1)})$ from mixing via the Yukawas.

The implications of the three solutions to the FCNC problem are therefore:

1. High complactification scale $\Rightarrow$ No new physics at the TeV scale.
2. Degenerate mass matrices $\Rightarrow$ First KK mode quarks come in three mass degenerate sets $\left(u_1^{(1)}, c_1^{(1)}, t_1^{(1)}\right)$, $\left(d_1^{(1)}, s_1^{(1)}, b_1^{(1)}\right)$, $\left(u_2^{(1)}, d_2^{(1)} c_2^{(1)}, s_2^{(1)}, b_2^{(1)}, t_2^{(1)}\right)$.
3. Alignment $\Rightarrow$ The mass degenerate first KK mode sets are $\left(u_2^{(1)}, d_2^{(1)} c_2^{(1)}, s_2^{(1)}\right)$ and $\left(b_2^{(1)}, t_2^{(1)}\right)$, but the remaining first KK quark masses are not constrained by flavor physics

Conclusions

- Fermionic bulk mass terms are thought to arise from couplings of the fermions to a KK-odd background field.
  In the absence of a flavor symmetry, there is no reason to assume the 5D mass matrices to be uniform.
- We showed that the absence of FCNCs strongly constrains the allowed Yukawa couplings and have some implications for the 5D mass matrices. Solutions in which some of the degeneracies in the fermion KK mass spectrum are lifted exist and have been presented here.

Outlook

- We currently perform a numerical analysis to make sure that the two scenarios described here are the only parameter sets which avoid the sUED flavor problem. Results are about to be published.
- We performed an analogous study for lepton flavor violation which shows that in the lepton sector, FCNCs are generically present, but an aligned solution exists in which no constraints are imposed on the lepton mass splittings at the first KK level.
- So far, all results are calculated at tree-level, only. A one-loop analysis of flavor and electroweak constraints is needed because tree-level constraints of sUED and loop level constraints in standard UED are of the same order. (work in progress)
- The analysis presented here can be extended to nUED models with boundary localized kinetic and mass terms (work in progress).