Nonlinear SUSY and General Relativity Theory

-NL/L SUSY Structure and Physical Meanings-

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1. Motivation

@SUSY and its spontaneous breakdown are essentially related to
the space-time symmetry, therefore, to be studied
in the low energy particle physics and in the cosmology as well.

$\Longrightarrow$ S(N) superPoincaré(sP) group gives a natural framework.

@We have found group theoretically:

The SM with just three generations emerges in
one irreducible representation of SO(10) sP and
SO(10) is unique among all SO(N) sP,
provided SO(10) sP with $\underline{10} = \underline{5} + \underline{5}^*, 5_{SU(5)GUT}$ for $SO(10) \supset SU(5)$ is preserved.

SO(N>8) Linear(L) SUSY $\Longrightarrow$ no-go theorem in S-matrix !

A way to field theoretical breakthrough:

We show in this talk:

- The nonlinear(NL) SUSY invariant coupling of spin $\frac{1}{2}$ fermion with spin 2 graviton is crucial to circumvent the no-go theorem of S-matrix arguments for SO(N>8) Linear SUSY.

- This is attributed to the geometrical structure of particular (empty) unstable space-time unifying two notions:

the fundamental object(spin $\frac{1}{2}$ NLSUSY) and the background space-time manifold(general relativity).

- We may be tempted to imagine that there may be a certain composite structure beyond the SM.

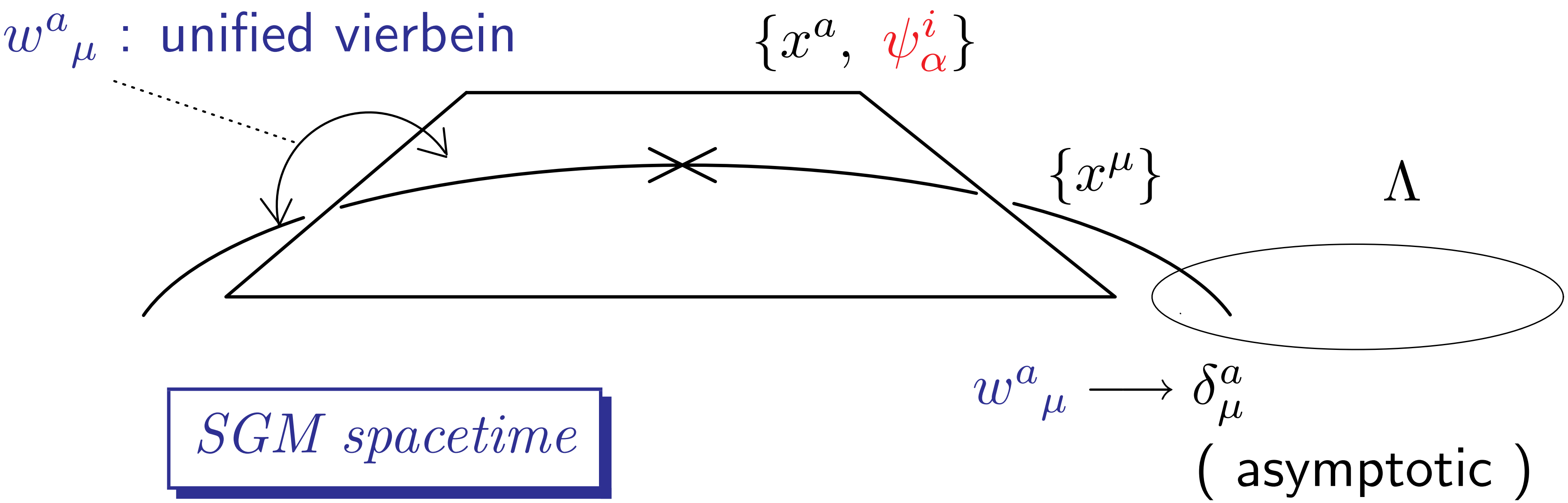
2.Nonlinear Supersymmetric General Relativity (NLSUSY GR)

We extend geometrical arguments of Einstein general relativity(EGR) on Riemann space-time to new space-time inspired by nonlinear(NL) SUSY :

The tangent space-time of new space-time is specified by the SL(2,C) Grassman coordinates ψ_α of NLSUSY besides the ordinary SO(1,3) Minkowski coordinate x^a,

i.e. ψ_α of the NLSUSY d.o.f turning subsequently to the NG fermion d.o.f. (called *superon* hereafter) of the coset space $\frac{superGL(4,R)}{GL(4,R)}$ and x^a are attached at every curved space-time point.

- Ultimate shape of nature $\Longleftrightarrow$ (empy) unstable spacetime:



(Homomorphic non-compact groups SO(1,3) and SL(2,C) for space-time d.o.f. are analogous to compact groups SO(3) and SU(2) for gauge d.o.f. of 't Hooft-Polyakov monopole.)

- Note that $SO(1, D-1) \cong SL(d, C)$ holds only for $D = 4, d = 2$.

4 dimensional spacetime is predicted.

A brief review of NLSUSY:

- Take flat space-time specified by x^a and ψ_α.
- Consider one form $\omega_a = dx_a - \frac{\kappa^2}{2i}(\bar{\psi}\gamma^a d\psi - d\bar{\psi}\gamma^a\psi)$,
- $\delta\omega_a = 0$ under $\delta x_a = \frac{i\kappa^2}{2}(\bar{\zeta}\gamma^a\psi - \bar{\psi}\gamma^a\zeta)$ and $\delta\psi = \zeta$ with a global spinor parameter ζ and κ is a dimensionfull arbitrary constant.
- An invariant acction($\sim$ invariant volume) is obtained:

$$S = -\frac{1}{2\kappa^2}\int \omega_0 \wedge \omega_1 \wedge \omega_2 \wedge \omega_3 = \int d^4x L_{VA}$$

- N=1 Volkov-Akulov model of NLSUSY is given by

$$L_{\rm VA} = -\frac{1}{2\kappa^2}|w_{VA}| = -\frac{1}{2\kappa^2}\left[1 + t^a{}_a + \frac{1}{2}(t^a{}_a t^b{}_b - t^a{}_b t^b{}_a) + \cdots\right],$$

$$|w_{VA}| = \det w^a{}_b = \det(\delta^a_b + t^a{}_b),$$

$$t^a{}_b = -i\kappa^2(\bar{\psi}\gamma^a\partial_b\psi - \bar{\psi}\gamma^a\partial_b\psi),$$

which is invariant under N=1 NLSUSY transformation,

$$\delta_\zeta\psi = \frac{1}{\kappa}\zeta - i\kappa(\bar{\zeta}\gamma^a\psi - \bar{\zeta}\gamma^a\psi)\partial_a\psi. \longleftrightarrow \text{NG fermioon for SB SUSY}$$

- ψ is NG fermion (the coset space coordinate) of $\frac{Super-Poincare}{Poincare}$.

We have found that parallel arguments to the Einstein genaral relativity(EG) theory on Riemann space-time is possible on new (SGM) space-time as well.

- Unified vierbein of new space-time

$$w^a{}_\mu(x)(= e^a{}_\mu + t^a{}_\mu(\psi)),$$

$$w_a{}^\mu(x)(= e_a{}^\mu - t^\mu{}_a + t^\mu{}_\rho t^\rho{}_a - t^\mu{}_\sigma t^\sigma{}_\rho t^\rho{}_a + t^\mu{}_\kappa t^\kappa{}_\sigma t^\sigma{}_\rho t^\rho{}_a),$$

$$w^a{}_\mu(x) w_b{}^\mu(x) = \delta^a{}_b$$

We have obtained the following N-extended NLSUSY GR action of the vacuum EH-type in new empty space-time,

N-extended NLSUSY GR action:

$$L_{NLSUSYGR}(w) = -\frac{c^4}{16\pi G}|w|(\Omega(w) + \Lambda), \qquad (1)$$

$$|w| = \det w^a{}_\mu = \det(e^a{}_\mu + t^a{}_\mu(\psi)), \qquad (2)$$

$$t^a{}_\mu(\psi) = \frac{\kappa^2}{2i}(\bar{\psi}^I \gamma^a \partial_\mu \psi^I - \partial_\mu \bar{\psi}^I \gamma^a \psi^I), (I = 1, 2, .., N) \qquad (3)$$

- $w^a{}_\mu(x)(= e^a{}_\mu + t^a{}_\mu(\psi))$: the unified vierbein of new space-time,
- $\Omega(w)$: the unified scalar curvature of new space-time,
- $e^a{}_\mu(x)$: the ordinary vierbein for the local SO(1,3) of EGR,
- $t^a{}_\mu(\psi(x))$: the mimic vierbein for the local SL(2,C) composed of the stress-energy-momentum of NG fermion $\psi(x)^I$(called superons),

- $s_{\mu\nu} \equiv w^a{}_\mu \eta_{ab} w^b{}_\nu$ and $s^{\mu\nu}(x) \equiv w^\mu{}_a(x) w^{\nu a}(x)$ are unified metric tensors of new space-time.

- G is the (Newton) gravitational constant.

- Λ : $(small)$ cosmological constant indicating the NLSUSY structure of new space-time.

- No-go theorem has been circumvented in a sense that

SO(N>8) SUSY with the non-trivial gravitational interaction and with $\Delta J = \frac{3}{2}$ has been constructed

by using NLSUSY, i.e. the vacuum degeneracy.

- Note that $SO(1, D-1) \cong SL(d, C)$, i.e. $\frac{D(D-1)}{2} = 2(d^2 - 1)$, holds only for $D = 4, d = 2$.

NLSUSYGR(SGM) scenario predicts **the 4 dimensional spacetime**

• Remarkably the constatnt κ^2 with the dimension $(length)^4$, which is arbitrary in NLSUSY model so far, is now fixed to

$$\kappa^2 = (\frac{c^4 \Lambda}{8\pi G})^{-1}$$

by NLSUSY GR scenario.

• Also the plus sign of Λ in the action is now fixed uniuely to give the correct sign to the kinetic term of $\psi(x)$, which indicates

(i) the positive potential minimum for ${w^a}_\mu(x)$
and
(ii) the dark energy density interpretation for Λ
for the present universe acceleration (in Sec.4).

Symmetries of NLSUSY GR(N-extended SGM action)

• NLSUSY GR action is invariant at least under the following space-time symmetries which is isomorophic to SP:

$$[\mathrm{NLSUSY}] \otimes [\mathrm{local\ GL}(4,\mathrm{R})] \otimes [\mathrm{local\ Lorentz}] \otimes [\mathrm{local\ spinor\ translation}] \tag{4}$$

and the following internal symmetries for N-extended NLSUSY GR (with N-superons ψ^I $(I = 1, 2, ..N)$) :

$$[\mathrm{global\ SO(N)}] \otimes [\mathrm{local\ U}(1)^{\mathrm{N}}] \otimes [\mathrm{chiral}]. \tag{5}$$

For example:

- NLSUSY GR (1) is invariant under the new NLSUSY transformation;

$$\delta_{\zeta^I}\psi = \frac{1}{\kappa}\zeta^I - i\kappa\bar{\zeta}^J\gamma^\rho\psi^J\partial_\rho\psi^I, \quad \delta_\zeta e^a{}_\mu = i\kappa\bar{\zeta}^J\gamma^\rho\psi^J\partial_{[\mu}e^a{}_{\rho]}, \tag{6}$$

which induce remarkably GL(4,R) transformations on $w^a{}_\mu$ and the unified metric $s_{\mu\nu}$

$$\delta_\zeta w^a{}_\mu = \xi^\nu\partial_\nu w^a{}_\mu + \partial_\mu\xi^\nu w^a{}_\nu, \quad \delta_\zeta s_{\mu\nu} = \xi^\kappa\partial_\kappa s_{\mu\nu} + \partial_\mu\xi^\kappa s_{\kappa\nu} + \partial_\nu\xi^\kappa s_{\mu\kappa}, \tag{7}$$

where ζ is a constant spinor parameter, $\partial_{[\rho}e^a{}_{\mu]} = \partial_\rho e^a{}_\mu - \partial_\mu e^a{}_\rho$ and $\xi^\rho = -i\kappa\bar{\zeta}^I\gamma^\rho\psi^I$).

The commutators of two new NLSUSY transformations (6) on ψ^I and $e^a{}_\mu$ are GL(4,R),

$$[\delta_{\zeta_1}, \delta_{\zeta_2}]\psi^I = \Xi^\mu \partial_\mu \psi^I, \quad [\delta_{\zeta_1}, \delta_{\zeta_2}]e^a{}_\mu = \Xi^\rho \partial_\rho e^a{}_\mu + e^a{}_\rho \partial_\mu \Xi^\rho, \tag{8}$$

where $\Xi^\mu = 2i\bar{\zeta}^I{}_1 \gamma^\mu \zeta^I{}_2 - \xi_1^\rho \xi_2^\sigma e_a{}^\mu \partial_{[\rho} e^a{}_{\sigma]}$. *Q.E.D.*

i.e. new NLSUSY (6) is the square-root of GL(4,R);

$$[\delta_1, \delta_2] = \delta_{GL(4R)}, \quad \text{i.e. } \delta_1 \sim \sqrt{\delta_{GL(4R)}}.$$

(The ordinary GL(4R) invariance of $L(w(x))$ is trivial by the construction.)

c.f. SUGRA

$$[\delta_1, \delta_2] = \delta_P \underline{+\delta_L + \delta_g}$$

- NLSUSY GR (1) is invariant under the local Lorentz transformation;

$$\delta_L w^a{}_\mu = \epsilon^a{}_b w^b{}_\mu \tag{9}$$

with the local parameter $\epsilon_{ab} = (1/2)\epsilon_{[ab]}(x)$

or equivalently on ψ^i and $e^a{}_\mu$

$$\delta_L \psi^I = -\frac{i}{2}\epsilon_{ab}\sigma^{ab}\psi^I, \quad \delta_L e^a{}_\mu = \epsilon^a{}_b e^b{}_\mu + \frac{\kappa^4}{4}\varepsilon^{abcd}\bar{\psi}^I)\gamma_5\gamma_d\psi^I(\partial_\mu\epsilon_{bc}). \tag{10}$$

The local Lorentz transformation forms a closed algebra, for example, on $e^a{}_\mu(x)$

$$[\delta_{L_1}, \delta_{L_2}]e^a{}_\mu = \beta^a{}_b e^b{}_\mu + \frac{\kappa^4}{4}\varepsilon^{abcd}\bar{\psi}^j\gamma_5\gamma_d\psi^j(\partial_\mu\beta_{bc}), \tag{11}$$

where $\beta_{ab} = -\beta_{ba}$ is defined by $\beta_{ab} = \epsilon_{2ac}\epsilon_1{}^c{}_b - \epsilon_{2bc}\epsilon_1{}^c{}_a$. $Q.E.D.$

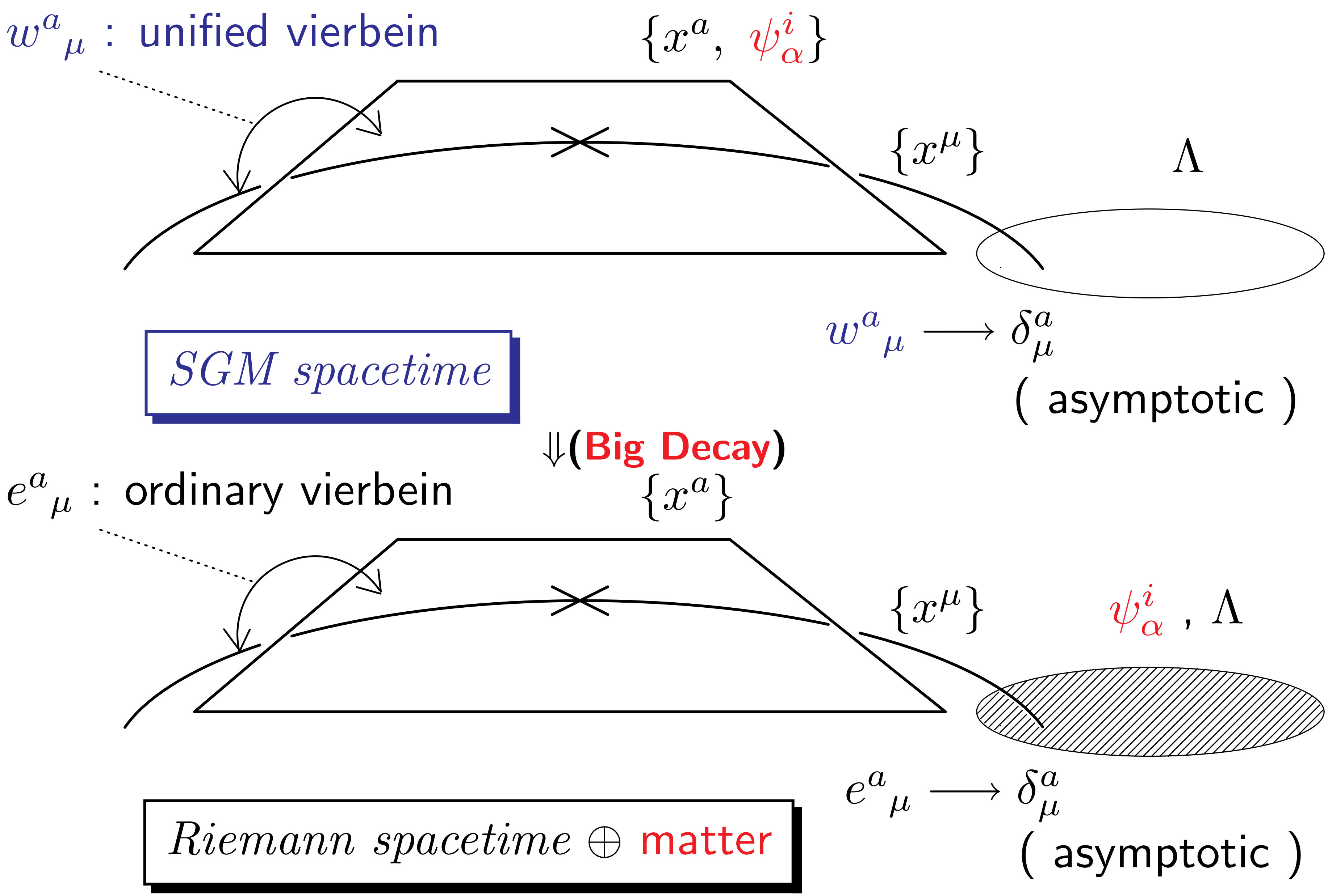
Big Decay of new space-time:

New space-time described by NLSUSY GR action (1) is unstable due to the global NLSUSY structure of tangent space-time and breakes down spontaneously to ordinary Riemann space-time(EH action) with superons(NG fermion) as follows (called Superon-Graviton Model) :

$$L(w) = L_{SGM}(e,\psi) = -\frac{c^4}{16\pi G}|e|\{R(e) + |w_{VA}|\Lambda + \tilde{T}(e,\psi)\}. \quad (12)$$

- $R(e)$: the scalar curvature of EH action
- Λ : the cosmological term
- $\tilde{T}(e,\psi)$: the gravitational interaction of superon.

- $L_{SGM}(e,\psi)$ produces N-extended NLSUSY action with $\kappa^2 = (\frac{c^4\Lambda}{8\pi G})^{-1}$ in asymptotic Riemann-flat($e^a{}_\mu(x) \to \delta^a{}_\mu$) space-time.



${w^a}_\mu$: unified vierbein
$\{x^a,\ \psi^i_\alpha\}$
$\{x^\mu\}$
Λ
${w^a}_\mu \longrightarrow \delta^a_\mu$
(asymptotic)
SGM spacetime
$\Downarrow$(Big Decay)
${e^a}_\mu$: ordinary vierbein
$\{x^a\}$
$\{x^\mu\}$
ψ^i_α , Λ
${e^a}_\mu \longrightarrow \delta^a_\mu$
(asymptotic)
Riemann spacetime $\oplus$ matter

3. Nonlinear-Linear SUSY Relation

Due to the high nonlinearity the physical consequences of $L_{SGM}(e, \psi)$ is unclear.

However,

- N-LSUSY theory *related(equivalent)* to N-NLSUSY theory can be constructed by persisting the SUSY algebra (in flat spacetime, at moment).

$\Longleftrightarrow$ NL/L SUSY relations

- The systematics for establishing NL/L SUSY relation are well understood and carried out for N=1,2,3 SUSY in flat space-time.

- NL/L SUSY relation solves the vacuum structure of SGM action.

Extracting low energy particle physics of SGM action for N=2:

- N=2 SUSY gives the minimal and physical (realistic) SUSY model in SGM scenario.

Because $J^P = 1^-$ U(1) geuge field appears in $N \geq 2$ SUSY.

$\Longrightarrow$ MSSM in SGM scenario is $N = 2$ LSUSY model.

- N=2 SGM in asymptotic Riemann-flat $({e^a}_\mu(x) \to {\delta^a}_\mu)$ space-time, where

$$L_{N=2SGM}(e,\psi) \to L_{N=2NLSUSY}(\psi) :$$ cosmological term of SGM.

The arguments are in two dimensional space-time for simplicity:

- N=2, d=2 NLSUSY model is given by

$$L_{\rm VA} = -\frac{1}{2\kappa^2}|w_{VA}| = -\frac{1}{2\kappa^2}\left[1 + t^a{}_a + \frac{1}{2}(t^a{}_a t^b{}_b - t^a{}_b t^b{}_a) + \cdots\right], \quad (13)$$

where,

$|w_{VA}| = \det w^a{}_b = \det(\delta^a_b + t^a{}_b)$,

$t^a{}_b = -i\kappa^2(\bar{\psi}_j \gamma^a \partial_b \psi^j - \bar{\psi}_j \gamma^a \partial_b \psi^j)$, $(j = 1, 2)$,

which is invariant under N=2 NLSUSY transformation,

$$\delta_\zeta \psi^j = \frac{1}{\kappa}\zeta^j - i\kappa(\bar{\zeta}_k \gamma^a \psi^k - \bar{\zeta}_k \gamma^a \psi^k)\partial_a \psi^j, (j = 1, 2).$$

N=2 LSUSY Theory:

- Helicity states of N=2 vector supermultiplet:

$$\begin{pmatrix} +1 \\ +\frac{1}{2}, \ +\frac{1}{2} \\ 0 \end{pmatrix} + [\mathrm{CPTconjugate}]$$

corresponds to N=2 LSUSY off-shell vector supermultiplet: $(v^a, \lambda^i, A, \phi, D;$i=1,2). in WZ.

- Helicity states of N=2 scalar supermultiplet:

$$\begin{pmatrix} +\frac{1}{2} \\ 0, \ 0 \\ -\frac{1}{2} \end{pmatrix} + [\mathrm{CPTconjugate}]$$

corresponds to N=2 LSUSY two scalar supermultiplets: $(\chi,\ B^i,\ \nu,\ F^i;\ i=1,2)$.

• **The most genaral $N=2$ LSUSYQED action** :

$$L_{N=2LSUSYQED} = L_{V0} + L'_{\Phi 0} + L_e + L_{Vf} + L_{Vm}, \tag{14}$$

$$\begin{aligned}
L_{V0} &= -\frac{1}{4}(F_{ab})^2 + \frac{i}{2}\bar{\lambda}^i \partial\!\!\!/ \lambda^i + \frac{1}{2}(\partial_a A)^2 + \frac{1}{2}(\partial_a \phi)^2 + \frac{1}{2}D^2 - \frac{\xi}{\kappa}D, \\
L'_{\Phi 0} &= \frac{i}{2}\bar{\chi}\partial\!\!\!/ \chi + \frac{1}{2}(\partial_a B^i)^2 + \frac{i}{2}\bar{\nu}\partial\!\!\!/ \nu + \frac{1}{2}(F^i)^2, \\
L_e &= e\Big\{ iv_a\bar{\chi}\gamma^a\nu - \epsilon^{ij}v^a B^i \partial_a B^j + \frac{1}{2}A(\bar{\chi}\chi + \bar{\nu}\nu) - \phi\bar{\chi}\gamma_5\nu \\
&\quad + B^i(\bar{\lambda}^i\chi - \epsilon^{ij}\bar{\lambda}^j\nu) - \frac{1}{2}(B^i)^2 D \Big\} + \frac{1}{2}e^2({v_a}^2 - A^2 - \phi^2)(B^i)^2, \\
L_{Vf} &= f\{A\bar{\lambda}^i\lambda^i + \epsilon^{ij}\phi\bar{\lambda}^i\gamma_5\lambda^j + (A^2 - \phi^2)D - \epsilon^{ab}A\phi F_{ab}\}, \\
L_{Vm} &= -\frac{1}{2}m\ (\bar{\lambda}^i\lambda^i - 2AD + \epsilon^{ab}\phi F_{ab}).
\end{aligned} \tag{15}$$

To see explicitly the local gauge invariance of the action, we define a complex (Dirac) spinor field χ_D and complex scalar fields (B^i, F^i) by

$$\chi_D = \frac{1}{\sqrt{2}}(\chi + i\nu), \quad B = \frac{1}{\sqrt{2}}(B^1 + iB^2), \quad F = \frac{1}{\sqrt{2}}(F^1 - iF^2), \tag{16}$$

and substitute them into $S'_{\Phi 0} + S_e$ in the action we obtain

$$\begin{aligned} S'_{\Phi 0} + S_e = & \int d^2x \{ i\bar{\chi}_D \not{\mathcal{D}} \chi_D + |\mathcal{D}_a B|^2 + |F|^2 \\ & + e(\bar{\chi}_D \lambda B + \bar{\lambda} \chi_D B^* - D|B|^2 + \bar{\chi}_D \chi_D A + i\bar{\chi}_D \gamma_5 \chi_D \phi) \\ & - e^2(A^2 + \phi^2)|B|^2 \} + [\text{ surface term }], \end{aligned} \tag{17}$$

with the covariant derivative $\mathcal{D}_a = \partial_a - iev_a$ and $\lambda = \frac{1}{\sqrt{2}}(\lambda^1 - i\lambda^2)$.

We can see the action is invariant under the ordinary local $U(1)$ gauge transformations,

$$\begin{aligned}(\chi_D, B, F) \quad &\to \quad (\chi'_D, B', F')(x) = e^{i\theta(x)}(\chi_D, B, F)(x), \\ v_a \quad &\to \quad v'_a(x) = v_a(x) + \frac{1}{e}\partial_a\theta(x).\end{aligned} \tag{18}$$

The commutor algebra for the fields (16) is also computed as

$$[\delta_{Q1}, \delta_{Q2}] = \delta_g(\mathcal{D}), \tag{19}$$

where $\delta_g(\mathcal{D})$ means a gauge covariant transformation according to $\mathcal{D} = \Xi^a\partial_a + ie\theta$.

$L_{\mathrm{N=2LSUSYQED}}$ is invariant under $N=2$ LSUSY parametrized by ζ^i.

- For the vector off-shell supermultiplet:

$$\begin{aligned}
\delta_\zeta v^a &= -i\epsilon^{ij}\bar{\zeta}^i\gamma^a\lambda^j, \\
\delta_\zeta \lambda^i &= (D - i\partial\!\!\!/ A)\zeta^i + \frac{1}{2}\epsilon^{ab}\epsilon^{ij}F_{ab}\gamma_5\zeta^j - i\epsilon^{ij}\gamma_5\partial\!\!\!/\phi\zeta^j, \\
\delta_\zeta A &= \bar{\zeta}^i\lambda^i, \\
\delta_\zeta \phi &= -\epsilon^{ij}\bar{\zeta}^i\gamma_5\lambda^j, \\
\delta_\zeta D &= -i\bar{\zeta}^i\partial\!\!\!/\lambda^i.
\end{aligned} \tag{20}$$

$$[\delta_{Q1}, \delta_{Q2}] = \delta_P(\Xi^a) + \delta_g(\theta), \tag{21}$$

where $\delta_g(\theta)$ is the $U(1)$ gauge transformation only for v^a with $\theta = -2(i\bar{\zeta}_1^i\gamma^a\zeta_2^i\, v_a - \epsilon^{ij}\bar{\zeta}_1^i\zeta_2^j A - \bar{\zeta}_1^i\gamma_5\zeta_2^i\phi)$.

- For the two scalar off-shell supermultiplets:

$$
\begin{aligned}
\delta_\zeta \chi &= (F^i - i\partial\!\!\!/ B^i)\zeta^i - e\epsilon^{ij} V^i B^j, \\
\delta_\zeta B^i &= \bar{\zeta}^i \chi - \epsilon^{ij} \bar{\zeta}^j \nu, \\
\delta_\zeta \nu &= \epsilon^{ij}(F^i + i\partial\!\!\!/ B^i)\zeta^j + eV^i B^i, \\
\delta_\zeta F^i &= -i\bar{\zeta}^i \partial\!\!\!/ \chi - i\epsilon^{ij}\bar{\zeta}^j \partial\!\!\!/ \nu \\
&\qquad -e\{\epsilon^{ij}\bar{V}^j \chi - \bar{V}^i \nu + (\bar{\zeta}^i \lambda^j + \bar{\zeta}^j \lambda^i)B^j - \bar{\zeta}^j \lambda^j B^i\}, \\
&\qquad [\delta_{\zeta_1}, \delta_{\zeta_2}]\chi = \Xi^a \partial_a \chi - e\theta\nu, \\
&\qquad [\delta_{\zeta_1}, \delta_{\zeta_2}]B^i = \Xi^a \partial_a B^i - e\epsilon^{ij}\theta B^j, \\
&\qquad [\delta_{\zeta_1}, \delta_{\zeta_2}]\nu = \Xi^a \partial_a \nu + e\theta\chi, \\
&\qquad [\delta_{\zeta_1}, \delta_{\zeta_2}]F^i = \Xi^a \partial_a F^i + e\epsilon^{ij}\theta F^j,
\end{aligned} \tag{22}
$$

with $V^i = iv_a\gamma^a\zeta^i - \epsilon^{ij}A\zeta^j - \phi\gamma_5\zeta^i$ and the U(1) gauge parameter θ.

The NL/L SUSY relation:

$$L_{\rm N=2LSUSYQED} = L_{\rm N=2NLSUSY} + [\text{surface terms}], \tag{23}$$

is established by SUSY invariant relations.

• SUSY invariant relations express uniquely all component fields of LSUSY supermultiplet as the composites of superons ψ_j of NLSUSY:

$$\sim \kappa^{n-1}(\psi^i)^n|w| + \cdots . \tag{24}$$

• Taking the NLSUSY transformations of the constituent superons ψ^j in SUSY invariant relations reproduce the familiar LSUSY transformations among the component fields of the supermultiplet.

• Substituting SUSY invariant relations into $L_{\rm N=2LSUSYQED}$, the NL/L SUSY relation is established.

- SUSY invariant relationsns for the vector off-shell supermultiplet:

$$\begin{aligned} v^a &= -\frac{i}{2}\xi\kappa\epsilon^{ij}\bar{\psi}^i\gamma^a\psi^j|w|, \\ \lambda^i &= \xi\psi^i|w|, \\ A &= \frac{1}{2}\xi\kappa\bar{\psi}^i\psi^i|w|, \\ \phi &= -\frac{1}{2}\xi\kappa\epsilon^{ij}\bar{\psi}^i\gamma_5\psi^j|w|, \\ D &= \frac{\xi}{\kappa}|w|. \end{aligned} \tag{25}$$

- Note that the global SU(2) emerges for N=2, d=4 SGM.

- SUSY invariant relations for scalar off-shell supermultiplets:

$$\chi = \xi^i \left[\psi^i |w| + \frac{i}{2}\kappa^2 \partial_a \{ \gamma^a \psi^i \bar{\psi}^j \psi^j |w| \} \right]$$

$$B^i = -\kappa \left(\frac{1}{2} \xi^i \bar{\psi}^j \psi^j - \xi^j \bar{\psi}^i \psi^j \right) |w|,$$

$$\nu = \xi^i \epsilon^{ij} \left[\psi^j |w| + \frac{i}{2}\kappa^2 \partial_a \{ \gamma^a \psi^j \bar{\psi}^k \psi^k |w| \} \right],$$

$$\tilde{F}^i = \frac{1}{\kappa} \xi^i \left\{ |w| + \frac{1}{8}\kappa^3 \partial_a \partial^a (\bar{\psi}^j \psi^j \bar{\psi}^k \psi^k |w|) \right\} - i\kappa \xi^j \partial_a (\bar{\psi}^i \gamma^a \psi^j |w|)$$
$$- \underline{\frac{1}{4} e \kappa^2 \xi \xi^i \bar{\psi}^j \psi^j \bar{\psi}^k \psi^k |w|}. \qquad (26)$$

The quartic fermion self-interaction term in $\tilde{F}^i$ is the origin of the local $U(1)$ gauge symmetry of LSUSY.

SUSY invariant relations produce a new off-shell commutator algebra which closes on only a translation:

$$[\delta_Q(\zeta_1), \delta_Q(\zeta_2)] = \delta_P(v), \tag{27}$$

where $\delta_P(v)$ is a translation with a parameter

$$v^a = 2i(\bar{\zeta}_{1L}\gamma^a\zeta_{2L} - \bar{\zeta}_{1R}\gamma^a\zeta_{2R}) \tag{28}$$

• Note that the commutator does not induce the U(1) gauge transformation, which is different from the ordinary LSUSY.

• Substituting these SUSY invariant relations into $L_{N=2LSUSYQED}$, we find NL/L SUSY relations:

$$L_{N=2LSUSYQED} = f(\xi,\xi^i)L_{\rm N=2NLSUSY} + [\text{suface terms}], \qquad (29)$$

$$f(\xi,\xi^i) = \xi^2 - (\xi^i)^2 = 1. \qquad (30)$$

$\Rightarrow$ composite eigenstates of global space-time (bulk) symmetry !?

• NL/L SUSY relation connects the cosmology

and the low energy particle physics in NLSUSY GR (in Sec. 4).

• The direct linearization of highly nonlinear SGM action (12), i.e. the construction of an equivalent and renormalizable broken LSUSY field theory of the LSUSY supermultiplet, remains to be carried out.

Broken $\mathcal{N}$-LSUSY(SUSYQCD) theory
emerges as massless composites states
from the $\mathcal{N}$-NLSUSY
of the cosmological constant of SGM.

♣ Systematics of NL/L SUSY relation and $N=2$ SUSY QED

The SUSY invariant relations

$\Longrightarrow$ are systematically obtained in the superfield formulation.

Linearization of NLSUSY in the $d=2$ superfield formulation

- General superfields are given for the $N=2$ vector supermultiplet by

$$\begin{aligned}\mathcal{V}(x,\theta^i) &= C(x)+\bar\theta^i\Lambda^i(x)+\frac{1}{2}\bar\theta^i\theta^jM^{ij}(x)-\frac{1}{2}\bar\theta^i\theta^iM^{jj}(x)+\frac{1}{4}\epsilon^{ij}\bar\theta^i\gamma_5\theta^j\phi(x)\\ &\quad -\frac{i}{4}\epsilon^{ij}\bar\theta^i\gamma_a\theta^jv^a(x)-\frac{1}{2}\bar\theta^i\theta^i\bar\theta^j\lambda^j(x)-\frac{1}{8}\bar\theta^i\theta^i\bar\theta^j\theta^jD(x), \qquad (31)\end{aligned}$$

and for the $N=2$ scalar supermultiplet by

$$\begin{aligned}\Phi^i(x,\theta^i) &= B^i(x)+\bar\theta^i\chi(x)-\epsilon^{ij}\bar\theta^j\nu(x)-\frac{1}{2}\bar\theta^j\theta^jF^i(x)+\bar\theta^i\theta^jF^j(x)-i\bar\theta^i\partial\!\!\!/ B^j(x)\theta^j\\ &\quad +\frac{i}{2}\bar\theta^j\theta^j(\bar\theta^i\partial\!\!\!/\chi(x)-\epsilon^{ik}\bar\theta^k\partial\!\!\!/\nu(x))+\frac{1}{8}\bar\theta^j\theta^j\bar\theta^k\theta^k\partial_a\partial^aB^i(x). \qquad (32)\end{aligned}$$

• Consider the general superfields on the following ψ^i-dependent specific supertranslations, $\leftarrow$ ordinary LSUSY with $-\kappa\psi(x)$,

$$x'^a = x^a + i\kappa\bar{\theta}^i\gamma^a\psi^i, \quad \theta'^i = \theta^i - \kappa\psi^i, \tag{33}$$

and we denote the general superfields on (x'^a, θ'^i) by

$$\tilde{\mathcal{V}}(x^a, \theta^i; \psi^i(x)) = \mathcal{V}(x'^a, \theta'^i), \quad \tilde{\Phi}(x^a, \theta^i; \psi^i(x)) = \Phi(x'^a, \theta'^i). \tag{34}$$

Under the the translation on (x'^a, θ'^i), i.e.
hybrid global SUSY transformation, $\delta^h = \delta^L(x.\theta) + \delta^{NL}(\psi)$:

$$\delta^h\tilde{\mathcal{V}}(x^a, \theta^i; \psi^i(x)) = \xi_\mu\partial^\mu\tilde{\mathcal{V}}(x^a, \theta^i; \psi^i(x)), \delta^h\tilde{\Phi}(x^a, \theta^i; \psi^i(x)) = \xi_\mu\partial^\mu\tilde{\Phi}(x^a, \theta^i; \psi^i(x)), \tag{35}$$

Therefore, the following conditions, i.e. SUSY invariant constraints available for eliminatingthe other d.o.f. than $\varphi^I_{\mathcal{V}}(x)$, $\varphi^I_{\Phi}(x)$ and ψ^i, can be imposed,

$$\tilde{\varphi}^I_{\mathcal{V}}(x) = \text{constant}, \quad \tilde{\varphi}^I_{\Phi}(x) = \text{constant}, \tag{36}$$

which are invariant (conserved quantities) under hybrid supertrasformations.

• Putting constants as follows:

$$\tilde{C} = \xi_c, \quad \tilde{\Lambda}^i = \xi^i_\Lambda, \quad \tilde{M}^{ij} = \xi^{ij}_M, \quad \tilde{\phi} = \xi_\phi, \quad \tilde{v}^a = \xi^a_v, \quad \tilde{\lambda}^i = \xi^i_\lambda, \quad \tilde{D} = \frac{\xi}{\kappa}, \tag{37}$$

$$\tilde{B}^i = \xi^i_B, \quad \tilde{\chi} = \xi_\chi, \quad \tilde{\nu} = \xi_\nu, \quad \tilde{F}^i = \frac{\xi^i}{\kappa}, \tag{38}$$

where the mass dimensions of constants (or constant spinors) in $d = 2$ are defined by $(-1, \frac{1}{2}, 0, 0, 0, -\frac{1}{2})$ for $(\xi_c, \xi^i_\Lambda, \xi^{ij}_M, \xi_\phi, \xi^a_v, \xi^i_\lambda)$, $(0, -\frac{1}{2}, -\frac{1}{2})$ for $(\xi^i_B, \xi_\chi, \xi_\nu)$ and 0 for ξ^i for convenience.

• SUSY invariant relations $\varphi^I_\mathcal{V} = \varphi^I_\mathcal{V}(\psi)$ are calculated systematically and straightforwardly as

$$\begin{aligned} C &= \xi_c + \kappa\bar{\psi}^i\xi^i_\Lambda + \frac{1}{2}\kappa^2(\xi^{ij}_M\bar{\psi}^i\psi^j - \xi^{ii}_M\bar{\psi}^j\psi^j) + \frac{1}{4}\xi_\phi\kappa^2\epsilon^{ij}\bar{\psi}^i\gamma_5\psi^j - \frac{i}{4}\xi^a_v\kappa^2\epsilon^{ij}\bar{\psi}^i\gamma_a\psi^j \\ &\quad -\frac{1}{2}\kappa^3\bar{\psi}^i\psi^i\bar{\psi}^j\xi^j_\lambda - \frac{1}{8}\xi\kappa^3\bar{\psi}^i\psi^i\bar{\psi}^j\psi^j, \\ \Lambda^i &= \xi^i_\Lambda + \kappa(\xi^{ij}_M\psi^j - \xi^{jj}_M\psi^i) + \frac{1}{2}\xi_\phi\kappa\epsilon^{ij}\gamma_5\psi^j - \frac{i}{2}\xi^a_v\kappa\epsilon^{ij}\gamma_a\psi^j \end{aligned}$$

$$
\begin{aligned}
& -\frac{1}{2}\xi_\lambda^i \kappa^2 \bar{\psi}^j \psi^j + \frac{1}{2}\kappa^2 (\psi^j \bar{\psi}^i \xi_\lambda^j - \gamma_5 \psi^j \bar{\psi}^i \gamma_5 \xi_\lambda^j - \gamma_a \psi^j \bar{\psi}^i \gamma^a \xi_\lambda^j) \\
& -\frac{1}{2}\xi \kappa^2 \psi^i \bar{\psi}^j \psi^j - i\kappa \partial\!\!\!/ C(\psi) \psi^i, \\
M^{ij} &= \xi_M^{ij} + \kappa \bar{\psi}^{(i} \xi_\lambda^{j)} + \frac{1}{2}\xi \kappa \bar{\psi}^i \psi^j + i\kappa \epsilon^{(i|k|} \epsilon^{j)l} \bar{\psi}^k \partial\!\!\!/ \Lambda^l(\psi) - \frac{1}{2}\kappa^2 \epsilon^{ik} \epsilon^{jl} \bar{\psi}^k \psi^l \partial^2 C(\psi), \\
\phi &= \xi_\phi - \kappa \epsilon^{ij} \bar{\psi}^i \gamma_5 \xi_\lambda^j - \frac{1}{2}\xi \kappa \epsilon^{ij} \bar{\psi}^i \gamma_5 \psi^j - i\kappa \epsilon^{ij} \bar{\psi}^i \gamma_5 \partial\!\!\!/ \Lambda^j(\psi) + \frac{1}{2}\kappa^2 \epsilon^{ij} \bar{\psi}^i \gamma_5 \psi^j \partial^2 C(\psi), \\
v^a &= \xi_v^a - i\kappa \epsilon^{ij} \bar{\psi}^i \gamma^a \xi_\lambda^j - \frac{i}{2}\xi \kappa \epsilon^{ij} \bar{\psi}^i \gamma^a \psi^j - \kappa \epsilon^{ij} \bar{\psi}^i \partial\!\!\!/ \gamma^a \Lambda^j(\psi) + \frac{i}{2}\kappa^2 \epsilon^{ij} \bar{\psi}^i \gamma^a \psi^j \partial^2 C(\psi) \\
& -i\kappa^2 \epsilon^{ij} \bar{\psi}^i \gamma^b \psi^j \partial^a \partial_b C(\psi), \\
\lambda^i &= \xi_\lambda^i + \xi \psi^i - i\kappa \partial\!\!\!/ M^{ij}(\psi) \psi^j + \frac{i}{2}\kappa \epsilon^{ab} \epsilon^{ij} \gamma_a \psi^j \partial_b \phi(\psi) \\
& -\frac{1}{2}\kappa \epsilon^{ij} \left\{ \psi^j \partial_a v^a(\psi) - \frac{1}{2}\epsilon^{ab} \gamma_5 \psi^j F_{ab}(\psi) \right\} \\
& -\frac{1}{2}\kappa^2 \{ \partial^2 \Lambda^i(\psi) \bar{\psi}^j \psi^j - \partial^2 \Lambda^j(\psi) \bar{\psi}^i \psi^j - \gamma_5 \partial^2 \Lambda^j(\psi) \bar{\psi}^i \gamma_5 \psi^j
\end{aligned}
$$

$$-\gamma_a \partial^2 \Lambda^j(\psi) \bar{\psi}^i \gamma^a \psi^j + 2\partial\!\!\!/ \partial_a \Lambda^j(\psi) \bar{\psi}^i \gamma^a \psi^j\} - \frac{i}{2}\kappa^3 \partial\!\!\!/ \partial^2 C(\psi) \psi^i \bar{\psi}^j \psi^j,$$

$$\begin{aligned} D \;=\; & \frac{\xi}{\kappa} - i\kappa \bar{\psi}^i \partial\!\!\!/ \lambda^i(\psi) \\ & + \frac{1}{2}\kappa^2 \left\{ \bar{\psi}^i \psi^j \partial^2 M^{ij}(\psi) - \frac{1}{2}\epsilon^{ij} \bar{\psi}^i \gamma_5 \psi^j \partial^2 \phi(\psi) \right. \\ & \left. + \frac{i}{2}\epsilon^{ij} \bar{\psi}^i \gamma_a \psi^j \partial^2 v^a(\psi) - i\epsilon^{ij} \bar{\psi}^i \gamma_a \psi^j \partial_a \partial_b v^b(\psi) \right\} \\ & - \frac{i}{2}\kappa^3 \bar{\psi}^i \psi^i \bar{\psi}^j \partial\!\!\!/ \partial^2 \Lambda^j(\psi) + \frac{1}{8}\kappa^4 \bar{\psi}^i \psi^i \bar{\psi}^j \psi^j \partial^4 C(\psi), \end{aligned} \tag{39}$$

while the SUSY invariant relations $\varphi^I_\Phi = \varphi^I_\Phi(\psi)$ are

$$\begin{aligned} B^i \;=\; & \xi^i_B + \kappa(\bar{\psi}^i \xi_\chi - \epsilon^{ij} \bar{\psi}^j \xi_\nu) - \frac{1}{2}\kappa^2 \{\bar{\psi}^j \psi^j F^i(\psi) - 2\bar{\psi}^i \psi^j F^j(\psi) + 2i\bar{\psi}^i \partial\!\!\!/ B^j(\psi) \psi^j\} \\ & - i\kappa^3 \bar{\psi}^j \psi^j \{\bar{\psi}^i \partial\!\!\!/ \chi(\psi) - \epsilon^{ik} \bar{\psi}^k \partial\!\!\!/ \nu(\psi)\} + \frac{3}{8}\kappa^4 \bar{\psi}^j \psi^j \bar{\psi}^k \psi^k \partial^2 B^i(\psi), \\ \chi \;=\; & \xi_\chi + \kappa\{\psi^i F^i(\psi) - i\partial\!\!\!/ B^i(\psi) \psi^i\} \end{aligned}$$

$$
\begin{aligned}
& -\frac{i}{2}\kappa^2[\partial\!\!\!/\chi(\psi)\bar{\psi}^i\psi^i - \epsilon^{ij}\{\psi^i\bar{\psi}^j\partial\!\!\!/\nu(\psi) - \gamma^a\psi^i\bar{\psi}^j\partial_a\nu(\psi)\}] \\
& +\frac{1}{2}\kappa^3\psi^i\bar{\psi}^j\psi^j\partial^2 B^i(\psi) + \frac{i}{2}\kappa^3\partial\!\!\!/ F^i(\psi)\psi^i\bar{\psi}^j\psi^j + \frac{1}{8}\kappa^4\partial^2\chi(\psi)\bar{\psi}^i\psi^i\bar{\psi}^j\psi^j, \\
\nu \ =\ & \xi_\nu - \kappa\epsilon^{ij}\{\psi^i F^j(\psi) - i\partial\!\!\!/ B^i(\psi)\psi^j\} \\
& -\frac{i}{2}\kappa^2[\partial\!\!\!/\nu(\psi)\bar{\psi}^i\psi^i + \epsilon^{ij}\{\psi^i\bar{\psi}^j\partial\!\!\!/\chi(\psi) - \gamma^a\psi^i\bar{\psi}^j\partial_a\chi(\psi)\}] \\
& +\frac{1}{2}\kappa^3\epsilon^{ij}\psi^i\bar{\psi}^k\psi^k\partial^2 B^j(\psi) + \frac{i}{2}\kappa^3\epsilon^{ij}\partial\!\!\!/ F^i(\psi)\psi^j\bar{\psi}^k\psi^k + \frac{1}{8}\kappa^4\partial^2\nu(\psi)\bar{\psi}^i\psi^i\bar{\psi}^j\psi^j, \\
F^i \ =\ & \frac{\xi^i}{\kappa} - i\kappa\{\bar{\psi}^i\partial\!\!\!/\chi(\psi) + \epsilon^{ij}\bar{\psi}^j\partial\!\!\!/\nu(\psi)\} \\
& -\frac{1}{2}\kappa^2\bar{\psi}^j\psi^j\partial^2 B^i(\psi) + \kappa^2\bar{\psi}^i\psi^j\partial^2 B^j(\psi) + i\kappa^2\bar{\psi}^i\partial\!\!\!/ F^j(\psi)\psi^j \\
& +\frac{1}{2}\kappa^3\bar{\psi}^j\psi^j\{\bar{\psi}^i\partial^2\chi(\psi) + \epsilon^{ik}\bar{\psi}^k\partial^2\nu(\psi)\} - \frac{1}{8}\kappa^4\bar{\psi}^j\psi^j\bar{\psi}^k\psi^k\partial^2 F^i(\psi).
\end{aligned}
\tag{40}
$$

• Simple SUSY invariant constraints of the component fields in $\tilde{\mathcal{V}}$ and $\tilde{\Phi}$,

$$\tilde{C} = \tilde{\Lambda}^i = \tilde{M}^{ij} = \tilde{\phi} = \tilde{v}^a = \tilde{\lambda}^i = 0, \tilde{D} = \frac{\xi}{\kappa}, \tilde{B}^i = \tilde{\chi} = \tilde{\nu} = 0, \quad \tilde{F}^i = \frac{\xi^i}{\kappa}, \tag{41}$$

give abovementioned simple SUSY invariant relations.

Actions in the $d = 2$, $N = 2$ NL/L SUSY relation

By changing the integration variables $(x^a, \theta^i) \longrightarrow (x'^a, \theta'^i)$, we can confirm systematically that LSUSY actions reduce to NLSUSY representation.

(a) The kinetic (free) action with the Fayet-Iliopoulos (FI) D term for the $N = 2$ vector supermultiplet $\mathcal{V}$ reduces to $S_{N=2\text{NLSUSY}}$;

$$S_{\mathcal{V}\text{free}} = \int d^2x \left\{ \int d^2\theta^i \, \frac{1}{32} (\overline{D^i \mathcal{W}^{jk}} D^i \mathcal{W}^{jk} + \overline{D^i \mathcal{W}_5^{jk}} D^i \mathcal{W}_5^{jk}) + \int d^4\theta^i \, \frac{\xi}{2\kappa} \mathcal{V} \right\}_{\theta^i = 0}$$
$$= \xi^2 S_{N=2\text{NLSUSY}}, \tag{42}$$

where

$$\mathcal{W}^{ij} = \bar{D}^i D^j \mathcal{V}, \quad \mathcal{W}_5^{ij} = \bar{D}^i \gamma_5 D^j \mathcal{V}. \tag{43}$$

(Note) The FI D term gives the correct sign of the NLSUSY action.

(b) Yukawa interaction terms for $\mathcal{V}$ vanish,
i.e.

$$
\begin{aligned}
S_{\mathcal{V}f} &= \frac{1}{8}\int d^2x\ f\left[\int d^2\theta^i\ \mathcal{W}^{jk}(\mathcal{W}^{jl}\mathcal{W}^{kl}+\mathcal{W}_5^{jl}\mathcal{W}_5^{kl})\right.\\
&\qquad \left.+\int d\bar{\theta}^i d\theta^j\ 2\{\mathcal{W}^{ij}(\mathcal{W}^{kl}\mathcal{W}^{kl}+\mathcal{W}_5^{kl}\mathcal{W}_5^{kl})+\mathcal{W}^{ik}(\mathcal{W}^{jl}\mathcal{W}^{kl}+\mathcal{W}_5^{jl}\mathcal{W}_5^{kl})\}\right]_{\theta^i=0}\\
&= 0, \qquad (44)
\end{aligned}
$$

by means of cancellations among four NG-fermion self-interaction terms.

(c) The *most general* gauge invariant action for $\mathcal{V}$ coupled with Φ^i reduces to $S_{N=2\mathrm{NLSUSY}}$;

$$S_{\mathrm{gauge}} = -\frac{1}{16}\int d^2x \int d^4\theta^i e^{-4e\mathcal{V}}(\Phi^j)^2 = -(\xi^i)^2 S_{N=2\mathrm{NLSUSY}}. \quad (45)$$

- Here $U(1)$ gauge interaction terms with the gauge coupling constant e produce four NG-fermion self-interaction terms as

$$S_e(\text{for the minimal off shell multiplet}) = \int d^2x \left\{\frac{1}{4}e\kappa\xi(\xi^i)^2\bar{\psi}^j\psi^j\bar{\psi}^k\psi^k\right\}, \quad (46)$$

which are absorbed in the SUSY invariant relation of the auxiliary field $F^i = F^i(\psi)$ by adding four NG-fermion self-interaction terms as (26):

$$\tilde{F}^i(\psi) = F^i(\psi) - \frac{1}{4}e\kappa^2\xi\xi^i\bar{\psi}^j\psi^j\bar{\psi}^k\psi^k|w_{VA}|. \quad (47)$$

Therefore,

under the SUSY invariant relations, which are obtained systematically, the $N = 2$ NLSUSY action $S_{N=2\mathrm{NLSUSY}}$ is related to $N = 2$ SUSY QED action by

$$f(\xi, \xi^i) S_{N=2\mathrm{NLSUSY}} = S_{N=2\mathrm{SQED}} \equiv S_{\mathcal{V}\mathrm{free}} + S_{\mathcal{V}f} + S_{\mathrm{gauge}} \qquad (48)$$

when $f(\xi, \xi^i) = \xi^2 - (\xi^i)^2 = 1$.

$\Longrightarrow$ This NL/L SUSY relation connecte the cosmology and the low energy particle physics in NLSUSY GR (in Sec. 4).

• The magnitude of the bare gauge coupling constant is predicted by taking the more general SUSY invariant constraints,i.e. vevs of auxiliary fields:

$$\tilde{C} = \xi_c, \tilde{\Lambda}^i = \tilde{M}^{ij} = \tilde{\phi} = \tilde{v}^a = \tilde{\lambda}^i = 0, \tilde{D} = \frac{\xi}{\kappa}, \tilde{B}^i = \tilde{\chi} = \tilde{\nu} = 0, \quad \tilde{F}^i = \frac{\xi^i}{\kappa}. \tag{49}$$

The bare gauge coupling constant (i.e. the fine structure constant $\alpha = \frac{e^2}{4\pi}$) is expressed (determined) in terms of *constant values of auxiliary-fields* :

$$f(\xi, \xi^i, \xi_c) = \xi^2 - (\xi^i)^2 e^{-4e\xi_c} = 1, \quad i.e., \ e = \frac{\ln(\frac{\xi^{i2}}{\xi^2 - 1})}{4\xi_c}, \tag{50}$$

where e is the bare gauge coupling constant.
This mechanism is natural and very favourable for SGM scenario as a theory for everything.

Broken N-LSUSY(SUSYQCD) theory emerges
as composites states in the true vacuum of N-NLSUSY.

4. Cosmology and Low Energy Physics in NLSUSY GR

The variation of SGM action $L_{N=2SGM}(e,\psi)$ with respect to $e^a{}_\mu$ yields the equation of motion for $e^a{}_\mu$ in Riemann space-time:

$$R_{\mu\nu}(e) - \frac{1}{2}g_{\mu\nu}R(e) = -\frac{8\pi G}{c^4}\{\tilde{T}_{\mu\nu}(e,\psi) - g_{\mu\nu}\frac{c^4\Lambda}{16\pi G}\}, \qquad (51)$$

where $\tilde{T}_{\mu\nu}(e,\psi)$ abbreviates the stress-energy-momentum of superon(NG fermion) including the gravitational interaction.

Note that $-\frac{c^4\Lambda}{16\pi G}$ can be interpreted as the negative energy density of empty space-time, i.e. the dark energy density ρ_D.
(The negative sign is unique.)

While, we have seen in the preceding section that

$N=2$ SGM is essentially N=2 NLSUSY action in asymptotic Riemann-flat (tangent) space-time.

- The low energy theorem for NLSUSY gives the following superon(massless NG fermion matter)-vacuum coupling

$$< \psi^{j}{}_{\alpha}(q)|J^{k\mu}{}_{\beta}|0 >= i\sqrt{\frac{c^4\Lambda}{16\pi G}}(\gamma^{\mu})_{\alpha\beta}\delta^{jk}, \qquad (52)$$

where $J^{k\mu} = i\sqrt{\frac{c^4\Lambda}{16\pi G}}\gamma^{\mu}\psi^{k} + \cdots$ is the conserved supercurrent.

$\sqrt{\frac{c^4\Lambda}{16\pi G}}$ is the coupling constant (g_{sv}) of superon with the vacuum.

For extracting the low energy particle physics contents of $N = 2$ SGM (NLSUSY GR) we consider in Riemann-flat asymptotic space-time, where NL/L SUSY relation in flat space-time gives:

$$L_{N=2\mathrm{SGM}} \longrightarrow L_{N=2\mathrm{NLSUSY}} + [\text{suface terms}] = L_{N=2\mathrm{SUSYQED}}. \quad (53)$$

- Now we study the vacuum structure of $N = 2$ LSUSY QED action in stead of $N = 2$ SGM.

The vacuum is determined by the minimum of the potential $V(A, \phi, B^i, D)$ of $L_{N=2LSUSYQED}$,

$$V(A, \phi, B^i, D) = -\frac{1}{2}D^2 + \left\{ \frac{\xi}{\kappa} - f(A^2 - \phi^2) + \frac{1}{2}e(B^i)^2 \right\} D. \quad (54)$$

Substituting the solution of the equation of motion for the auxiliary field D we obtain

$$V(A,\phi,B^i) = \frac{1}{2}f^2\left\{A^2-\phi^2-\frac{e}{2f}(B^i)^2-\frac{\xi}{f\kappa}\right\}^2+\frac{1}{2}e^2(A^2+\phi^2)(B^i)^2 \geq 0. \tag{55}$$

The configurations of the fields corresponding to the vacua in (A,ϕ,B^i)-space are classified according to the signatures of the parameters e, f, ξ, κ as follows:

(I) For $ef > 0$, $\frac{\xi}{f\kappa} > 0$ case,

$$A^2-\phi^2-(\tilde{B}^i)^2 = k^2. \quad \left(\tilde{B}^i = \sqrt{\frac{e}{2f}}B^i, \quad k^2 = \frac{\xi}{f\kappa}\right) \tag{56}$$

(II) For $ef < 0, \quad \frac{\xi}{f\kappa} > 0$ case,

$$A^2 - \phi^2 + (\tilde{B}^i)^2 = k^2. \quad \left(\tilde{B}^i = \sqrt{-\frac{e}{2f}} B^i, \quad k^2 = \frac{\xi}{f\kappa} \right) \tag{57}$$

(III) For $ef > 0, \quad \frac{\xi}{f\kappa} < 0$ case,

$$-A^2 + \phi^2 + (\tilde{B}^i)^2 = k^2. \quad \left(\tilde{B}^i = \sqrt{\frac{e}{2f}} B^i, \quad k^2 = -\frac{\xi}{f\kappa} \right) \tag{58}$$

(IV) For $ef < 0, \quad \frac{\xi}{f\kappa} < 0$ case,

$$-A^2 + \phi^2 - (\tilde{B}^i)^2 = k^2. \quad \left(\tilde{B}^i = \sqrt{-\frac{e}{2f}} B^i, \quad k^2 = -\frac{\xi}{f\kappa} \right) \tag{59}$$

We find that the vacua (I) and (IV) are unphysical, for they produce pathological wrong sign kinetic terms for the fields expanded around the vacuum.

As for the vacua (II) and (III) we perform similar arguments as shown below and find that two different physical vacua appear.

The physical particle spectrum is obtained by expanding the fields (A, ϕ, B^i) around the vacuum.

• Expressions for the case (II):

Case (IIa)

$$\begin{aligned}
A &= (k+\rho)\sin\theta\cosh\omega, \\
\phi &= (k+\rho)\sinh\omega, \\
\tilde{B}^1 &= (k+\rho)\cos\theta\cos\varphi\cosh\omega, \\
\tilde{B}^2 &= (k+\rho)\cos\theta\sin\varphi\cosh\omega
\end{aligned}$$

Case (IIb)

$$\begin{aligned}
A &= -(k+\rho)\cos\theta\cos\varphi\cosh\omega, \\
\phi &= (k+\rho)\sinh\omega, \\
\tilde{B}^1 &= (k+\rho)\sin\theta\cosh\omega, \\
\tilde{B}^2 &= (k+\rho)\cos\theta\sin\varphi\cosh\omega.
\end{aligned}$$

• For the case (III) the arguments hold by exchanging A and ϕ, called (IIIa) and (IIIb).

Substituting these expressions into $V(A,\phi,B^i)$ and expanding around the vacuum configuration we obtain the physical particle contents.

- For the cases (IIa) and (IIIa) we obtain

$$
\begin{aligned}
L_{N=2\mathrm{SUSYQED}} = & \frac{1}{2}\{(\partial_a \rho)^2 - 2(-ef)k^2\rho^2\} \\
& + \frac{1}{2}\{(\partial_a \theta)^2 + (\partial_a \omega)^2 - 2(-ef)k^2(\theta^2 + \omega^2)\} \\
& + \frac{1}{2}(\partial_a \varphi)^2 \\
& - \frac{1}{4}(F_{ab})^2 + (-ef)k^2 v_a^2 \\
& + \frac{i}{2}\bar{\lambda}^i \partial\!\!\!/ \lambda^i + \frac{i}{2}\bar{\chi}\partial\!\!\!/ \chi + \frac{i}{2}\bar{\nu}\partial\!\!\!/ \nu + \sqrt{-2ef}(\bar{\lambda}^1 \chi - \bar{\lambda}^2 \nu) + \cdots ,
\end{aligned}
\tag{60}
$$

and the following mass spectra

$$m_\rho^2 = m_\theta^2 = m_\omega^2 = m_{v_a}^2 = 2(-ef)k^2 = -\frac{2\xi e}{\kappa},$$
$$m_{\lambda^i} = m_\chi = m_\nu = m_\varphi = 0. \tag{61}$$

The vacuum breaks both SUSY and the local $U(1)$ spontaneously.

(φ is the NG boson for the spontaneous breaking of $U(1)$ symmetry, i.e. the $U(1)$ phase of B, and totally gauged away by the Higgs-Kibble mechanism with $\Omega(x) = \sqrt{e\kappa/2}\varphi(x)$ for the $U(1)$ gauge (28).)

All bosons have the same mass, and remarkably all fermions remain massless.

The off-diagonal mass terms $\sqrt{-2ef}(\bar\lambda^1\chi - \bar\lambda^2\nu) = \sqrt{-2ef}(\bar\chi_{\rm D}\lambda + \bar\lambda\chi_{\rm D})$ would induce mixings of fermions. $\Rightarrow$ pathological?

- For (IIb) and (IIIb) we obtain

$$
\begin{aligned}
L_{N=2\mathrm{SUSYQED}} = & \frac{1}{2}\{(\partial_a \rho)^2 - 4f^2k^2\rho^2\} \\
& + \frac{1}{2}\{(\partial_a \theta)^2 + (\partial_a \varphi)^2 - e^2k^2(\theta^2 + \varphi^2)\} \\
& + \frac{1}{2}(\partial_a \omega)^2 \\
& - \frac{1}{4}(F_{ab})^2 \\
& + \frac{1}{2}(i\bar{\lambda}^i \partial\!\!\!/ \lambda^i - 2fk\bar{\lambda}^i\lambda^i) \\
& + \frac{1}{2}\{i(\bar{\chi}\partial\!\!\!/ \chi + \bar{\nu}\partial\!\!\!/ \nu) - ek(\bar{\chi}\chi + \bar{\nu}\nu)\} + \cdots . \qquad (62)
\end{aligned}
$$

and the following mass spectra:

$$m_\rho^2 = m_{\lambda^i}^2 = 4f^2k^2 = \frac{4\xi f}{\kappa},$$
$$m_\theta^2 = m_\varphi^2 = m_\chi^2 = m_\nu^2 = e^2k^2 = \frac{\xi e^2}{\kappa f},$$
$$m_{v_a} = m_\omega = 0, \tag{63}$$

which can produce mass hierarchy by the factor $\frac{e}{f}$.

SUSY is broken spontaneously alone.

The massless scalar ω is a NG boson for the degeneracy of the vacuum in $(A, \tilde{B}_2)$-space, which is gauged away provided the gauge symmetry between the vector and the scalar multiplet is introduced.

• We have shown explicitly that N=2 LSUSY QED, i.e. the matter sector (in asymptotic flat-space) of $N=2$ SGM, possesses a unique true vacuum type (b) with $V=0$.

The resulting model describes:

one massive charged Dirac fermion (${\psi_D}^c \sim \chi + i\nu$),
one massive neutral Dirac fermion (${\lambda_D}^0 \sim \lambda^1 - i\lambda^2$),
one massless vector (a photon) (v_a),
one charged scalar ($\phi^c \sim \theta + i\varphi$),
one neutral complex scalar ($\phi^0 \sim \rho(+i\omega)$),

which are the composites of superons.

c.f. N=1MSSM

• As for cosmological meanings of $N = 2$ LSUSY QED in the SGM scenario, the unique vacuum type (b) may simply explain the observed mysterious (numerical) relations and give a new insight into the origin of mass:

$$\text{(dark) energy density of the universe} \sim {m_\nu}^4 \sim (10^{-12} GeV)^4 \sim {g_{sv}}^2,$$

provided λ^i is identified with neutrino. [$D = 4$ as well]

• While the vacua of (IIa) and (IIIa) inducing the fermion mixing, unphysical so far, may give new features characteristic of $N = 2$.

They may be generic for $N > 2$ and deserve further investigations.

5. Nonlinear vector-spinor SUSY GR

• New SUSY algebra containing spinor-vector generators Q^{μ}_{α}:

$$\{Q^{\mu}_{\alpha}, Q^{\nu}_{\beta}\} = \varepsilon^{\mu\nu\lambda\rho} P_{\lambda}(\gamma_{\rho}\gamma_5 C)_{\alpha\beta}, \tag{64}$$

$$[Q^{\mu}_{\alpha}, P^{\nu}] = 0, \tag{65}$$

$$[Q^{\mu}_{\alpha}, J^{\lambda\rho}] = \frac{1}{2}(\sigma^{\lambda\rho}Q^{\mu})_{\alpha} + i\eta^{\lambda\mu}Q^{\rho}_{\alpha} - i\eta^{\rho\mu}Q^{\lambda}_{\alpha}, \tag{66}$$

where Q^{μ}_{α} are vector-spinor generators satisfying Majorana condition $Q^{\mu}_{\alpha} = C_{\alpha\beta}\overline{Q}^{\mu}_{\alpha}$.
• Consider the following global (3/2 super)translations:

$$\psi^{a}_{\alpha} \longrightarrow \psi^{a}_{\alpha} + \zeta^{a}_{\alpha}. \tag{67}$$

$$x_a \longrightarrow x_a + i\kappa\varepsilon_{abcd}\bar{\psi}^{b}\gamma^{c}\gamma_5\zeta^{d}, \tag{68}$$

where ζ^{a}_{α} is a constant Majorana tensor-spinor parameter.
• The invariant differential forms become:

$$\omega_a = dx_a + i\kappa\varepsilon_{abcd}\bar{\psi}^{b}\gamma^{c}\gamma_5 d\psi^{d}. \tag{69}$$

- Invariant action of nonlinear representation of vector-spinor SUSY:

$$S = \frac{1}{\kappa}\int \omega_0 \wedge \omega_1 \wedge \omega_2 \wedge \omega_3 = \frac{1}{\kappa}\int \det w_{ab} d^4x, \tag{70}$$

$$w_{ab} = \delta_{ab} + t_{ab}, \quad t_{ab} = i\kappa\varepsilon_{acde}\bar{\psi}^c\gamma^d\gamma_5\partial_b\psi^e, \tag{71}$$

- Following the geometrical arguments of SGM we obtain vector-spinor NLSUSY GR:

$$L_{vsNLSUSYGR} = -\frac{c^3}{16\pi G}|w|\{\Omega(w^a{}_\mu) + \Lambda\}, \tag{72}$$

$$|w| = detw^a{}_\mu = det(e^a{}_\mu + t^a{}_\mu), \tag{73}$$

- Unified vierbein become:

$$w^a{}_\mu(x) = e^a{}_\mu(x) + t^a{}_\mu(x), \quad t^a{}_\mu(x) = i\kappa\varepsilon^{abcd}\bar{\psi}_b\gamma_c\gamma_5\partial_\mu\psi_d, \tag{74}$$

- $L_{vsNLSUSYGR}$ possesses similar symmetry properties as SGM.

4. Summary

NLSUSY GR(SGM) scenario:

Ultimate entity,
New unstable (empty) space-time :$[\mathbf{x^a}, {\psi_\alpha}^{\mathbf{N}}; \mathbf{x}^\mu]$
$[L = L_{NLSUSGR}(w)] \Longleftarrow$ NLSUSY GR for empty space-time with Λ
Mach principle is encoded geometrically

$\Longrightarrow$ **Big Decay (due to false vacuum $V = \Lambda > 0$)** $\Longrightarrow$

The creation of Riemann space-time $[\mathbf{x^a}; \mathbf{x}^\mu]$ and massless fermionic matter $[{\psi_\alpha}^{\mathbf{N}}]$
$[L = L_{EH}(e) - \Lambda + T(\psi.e)] \Longleftarrow$ Einstein GR with $V = \Lambda > 0$ and N superon(SGM)

$\Longrightarrow$ **Superfluidity of space-time and matter** $[{e^a}_\mu, {\psi_\alpha}^N]$ $\Longrightarrow$

Phase transition to true vacuum $V = \Lambda = 0$
achieved by composite (massless) eigenstates of LSUSY staffs,
Ignittion of Big Bang, Inflation

$\Longrightarrow$

In asymptotic flat space-time, broken N-LSUSY theory emerges from the N-NLSUSY cosmological term of SGM via NL/L SUSY relation. $\Longleftrightarrow$ GL and BCS

Predictions and Conjectures: [Qualitative, accessible ones]

@(Group theory SO(N) sP with $N = \underline{10} = \underline{5} + \underline{5}^*$ of superon-quintet(SQ) hypothesis) :

- Lepton-type spin 3/2 doublet
- Doubly charged spin 1/2 particles $E^{2+} \Longleftrightarrow$ **same sign lepton pair**
- Proton decay modes in GUTs are forbidden due to compositeness. $\Rightarrow$ stable proton

New wine(superons-quintet:SQ) in Old Bottle ($\underline{5}$ of SU(5) GUT)!

@(Field theory via Linearization):

- SGM scenario predicts **4 dimensional spacetime**.
- neutral scalar particle $\sim O(m_\nu) \Longleftrightarrow$ **dark matter**
- N-LSUSY from N-NLSUSY $\Longleftrightarrow$ **SQ hypothesis** for all particles except gravity
- Superfluidity of space-time $\Longleftrightarrow$ κ^{-2}**: chemical potential**

The cosmological constant is the constant for everything!

Many Open Questions ! e.g.,

- D=4 case is urgent,
- Large N case(especially N=5 and N=10), $\cdots$, partial N SUSY breaking?.
- Direct linearization of SGM action in curved space-time.
- What is the equivalent LSUSY theory?
- Complete Detour of No-Go Th.! (Massive high-spins in linearized theory)
- SGM scenario suggests $N \geq 2$ low energy MSSM, SUSY GUT.
- equivalence principle and NLSUSYGR.
- Physical Consequences of spin 3/2 NLSUSY GR.

$\vdots$