

# Viability of MSSM scenarios at very large $\tan \beta$

based on arXiv:1004.1993 [hep-ph]

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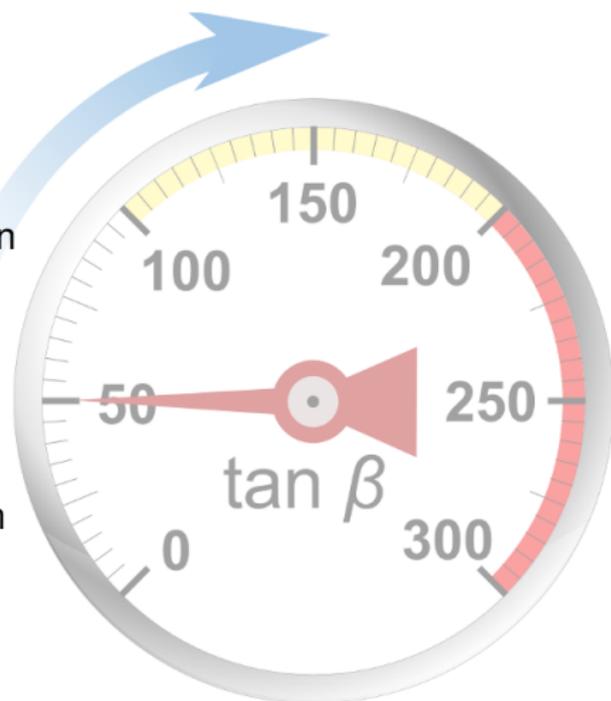
SUSY10  
Bonn  
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Technische Universität München



- 1 Why not very large  $\tan \beta$ ?  
Yukawa perturbativity  
FCNCs
- 2 Why very large  $\tan \beta$ ?  
The  $B_\mu$  problem of gauge mediation  
Uplifted SUSY
- 3 Numerical analysis  
Perturbativity of Yukawa couplings  
Low energy scan  
Very large  $\tan \beta$  in gauge mediation
- 4 Conclusions



# Demotivation 1: perturbativity of Yukawa couplings

3rd generation fermion masses at tree level:

$$m_t = v_u y_t, \quad m_b = v_d y_b, \quad m_\tau = v_d y_\tau \quad (1)$$

$$\tan \beta = \frac{v_u}{v_d}, \quad v_u^2 + v_d^2 = (174 \text{ GeV})^2$$

Large  $\tan \beta \Rightarrow$  small  $v_d \Rightarrow$  large down-type Yukawas

Using these tree-level relations,

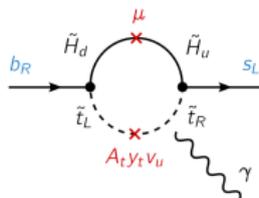
- $y_b$  and  $y_\tau$  would be non-perturbative for  $\tan \beta \gtrsim 250$
- $y_b$  and  $y_\tau$  would have a Landau pole below  $M_{\text{GUT}}$  for  $\tan \beta \gtrsim 75$

**Ways out:**

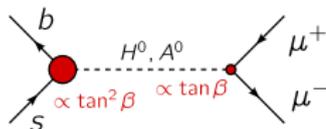
- $\tan \beta$  enhanced threshold corrections to (1)
- Perturbativity up to  $M_{\text{GUT}}$  not strictly necessary

## Demotivation 2: flavour violation

Large  $\tan \beta \Rightarrow$  enhanced contributions to  $B$  decays even for flavour-blind soft terms:



$$\frac{\text{BR}(B \rightarrow X_s \gamma)}{\text{BR}(B \rightarrow X_s \gamma)_{\text{SM}}} - 1 \propto -\frac{\mu A_t}{\tilde{m}_t^2} \tan \beta$$

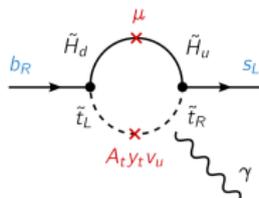


$$\frac{\text{BR}(B_s \rightarrow \mu^+ \mu^-)}{\text{BR}(B_s \rightarrow \mu^+ \mu^-)_{\text{SM}}} - 1 \propto \frac{|\mu A_t|^2}{M_A^4} (\tan \beta)^6$$

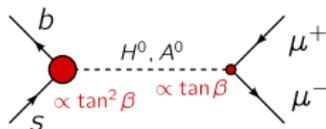
Under control if **trilinear terms are small** (as in gauge mediation)

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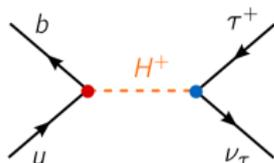


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Under control if **trilinear terms are small** (as in gauge mediation)



$$\frac{\text{BR}(B^+ \rightarrow \tau^+ \nu)}{\text{BR}(B^+ \rightarrow \tau^+ \nu)_{\text{SM}}} \simeq \left( 1 - \frac{m_{B^+}^2}{M_{H^+}^2} \frac{\tan^2 \beta}{(1 + \epsilon_0 t_\beta)(1 + \epsilon_\ell t_\beta)} \right)^2$$

Under control if  **$M_{H^+}$  is large**

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# Motivation 1: the $\mu/B\mu$ problem of gauge mediation

$$\mathcal{W} \supset \mu H_u H_d, \quad \mathcal{L}_{\text{soft}} \supset -B\mu H_u H_d + \text{h.c.}$$

- Difficult to generate  $B\mu$  of the right size in GMSB  
 $\Rightarrow B\mu(M) = 0$  natural possibility
- $\tan \beta = \frac{m_A^2}{B\mu}$  at tree level
- Non-zero  $B\mu$  generated by RG effects  $\Rightarrow$  large  $\tan \beta$   
[Dine, Nir, Shirman (1993)]

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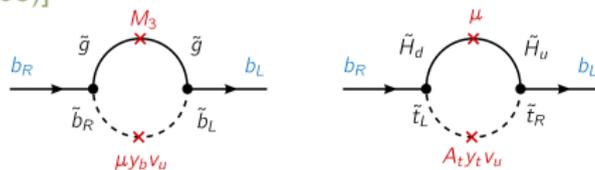
Additional nice features:

- Soft terms are flavour blind
- Trilinear couplings are small
- Mediation scale can be quite low (perturbativity!)

## Motivation 2: Uplifted SUSY

Tree-level relations between masses and Yukawas are modified by  $\tan \beta$  enhanced threshold corrections:

[Hall, Rattazzi, Sarid (1993)]



$$m_{b,\tau} = y_{b,\tau} v_d + \Delta m_{b,\tau} = y_{b,\tau} (v_d + \epsilon_{b,\tau} v_u) = y_{b,\tau} v_d (1 + \epsilon_{b,\tau} \tan \beta)$$

$$\epsilon_b = \epsilon_b^{\tilde{B}} + \epsilon_b^{\tilde{W}} + \epsilon_b^{\tilde{g}} + \epsilon_b^{\tilde{H}}$$

$$\epsilon_\tau = \epsilon_b^{\tilde{B}} + \epsilon_b^{\tilde{W}}$$

“Uplifted SUSY” [Dobrescu, Fox (2010)]

For  $\tan \beta \gg 50$ , one could have  $\Delta m_{b,\tau} \gtrsim y_{b,\tau} v_d$

- Loop contribution to  $b/\tau$  mass **comparable** to tree-level contribution
- Yukawas remain **perturbative** even for very large  $\tan \beta$
- Can be a consequence of  $B\mu = 0$  at some scale

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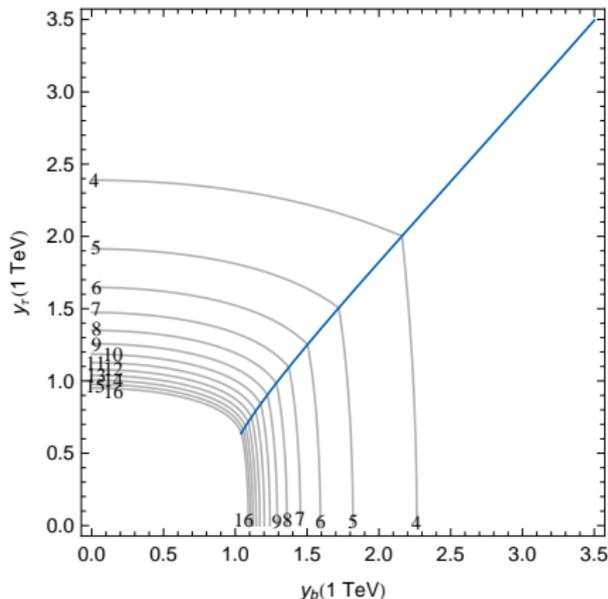
# RG evolution of Yukawa couplings

## Yukawa RGE:

$$\frac{dy_i}{dt} = \frac{y_i}{16\pi^2} \sum_{j,k} (a_j y_j^2 - b_k g_k^2)$$

## Gray contours:

The scale ( $10^X$  GeV) where  $y_b$  or  $y_\tau$  reach  $y^{\max} = \sqrt{4\pi}$ , depending on their value at the TeV scale.



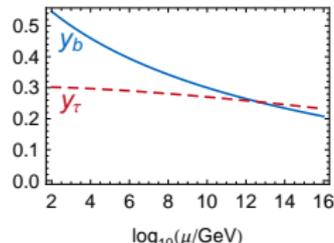
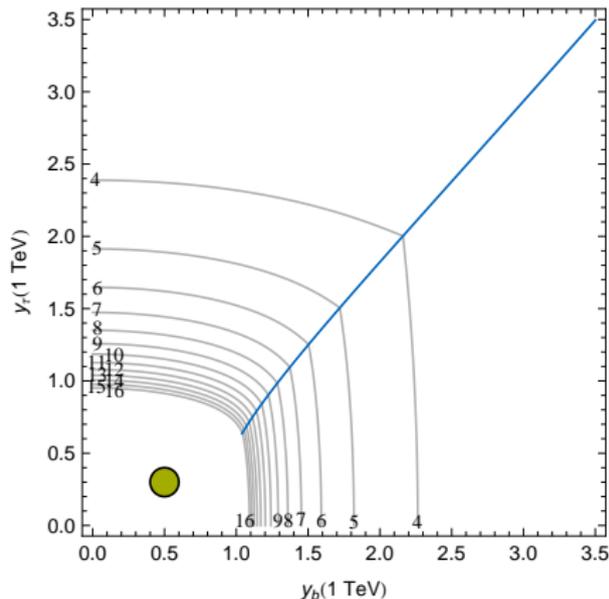
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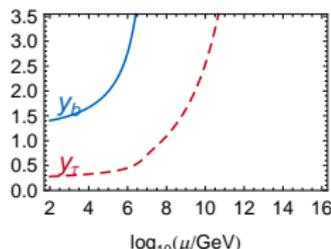
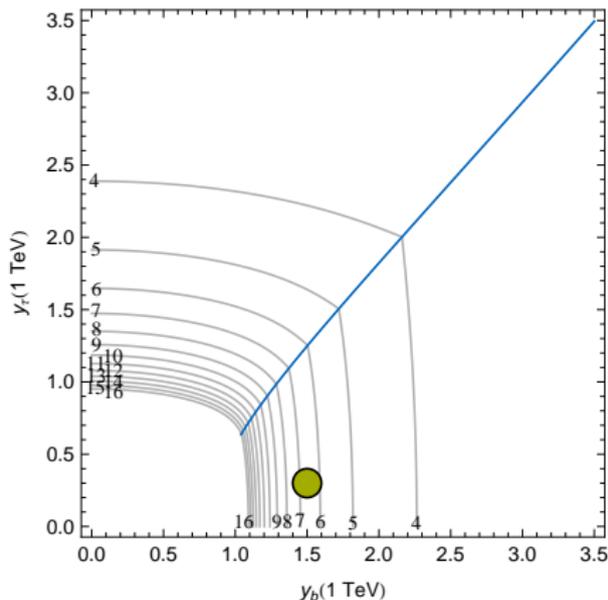
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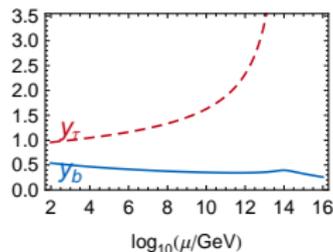
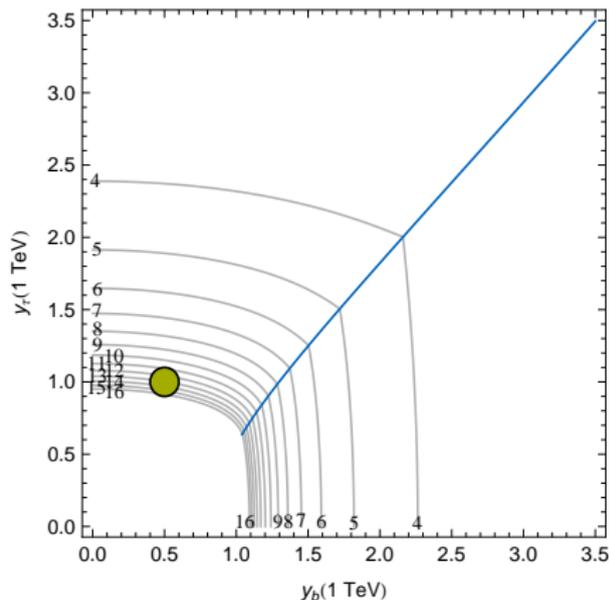
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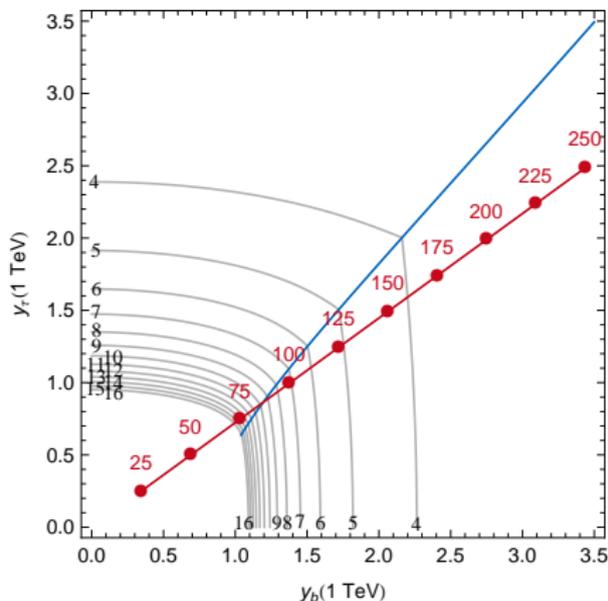
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## Red line:

Value of  $y_{b,\tau}$  at 1 TeV without threshold corrections



# RG evolution of Yukawa couplings

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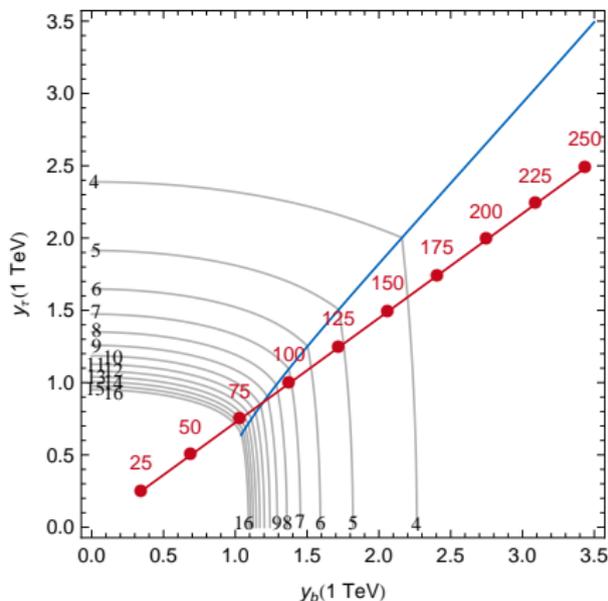
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At what scale do the Yukawas become non-perturbative  
 $\Leftrightarrow$  how far can the actual Yukawas deviate from the red line?

## Low-energy parameter scan

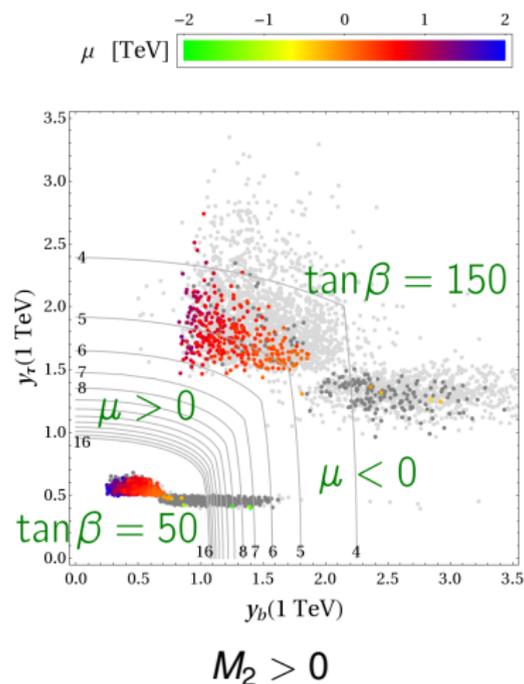
Vary the low-energy MSSM parameters for fixed values of  $\tan \beta$ , assuming

- no new sources of CP violation
- flavour-blind soft terms

Scan ranges:

$$\begin{aligned} m_{Q,U,D,L,E} &\in [0.1, 2] \text{ TeV}, & A_{u,d,l} &\in [-0.5, 0.5] \text{ TeV}, \\ \mu &\in [-2, 2] \text{ TeV}, & M_{A^0} &\in [0.1, 2] \text{ TeV}, \\ M_1 &\in [-1, 1] \text{ TeV}, & M_2 &\in [-2, 2] \text{ TeV}, & M_3 &\in [0, 6] \text{ TeV}, \end{aligned}$$

# Numerical results: Yukawas



Threshold correction to  $y_\tau$

$$-\Delta y_\tau^{\tilde{W}} \propto \tan\beta \times \text{sgn}(\mu M_2)$$

Threshold correction to  $y_b$

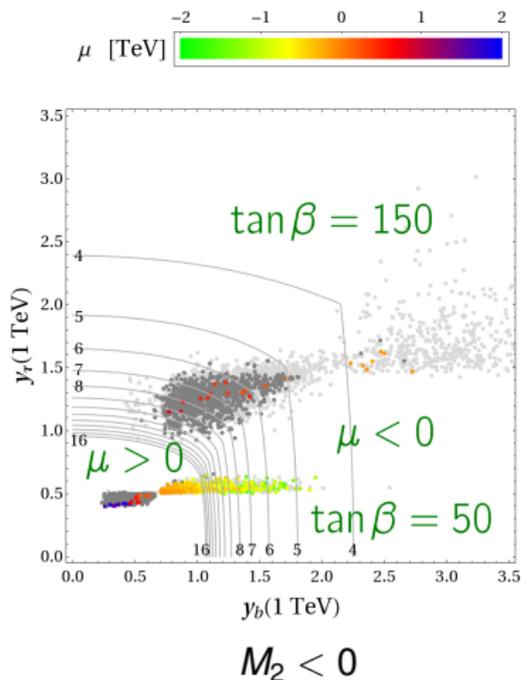
$$\Delta y_\tau^{\tilde{g}} \propto \tan\beta \times \text{sgn}(\mu M_3)$$

The muon  $g - 2$

$$\Delta a_\mu^{\text{exp}} = (25.5 \pm 8.0) \times 10^{-10}$$

$$\Delta a_\mu^{\text{SUSY}} \propto \tan\beta \times \text{sgn}(\mu M_2)$$

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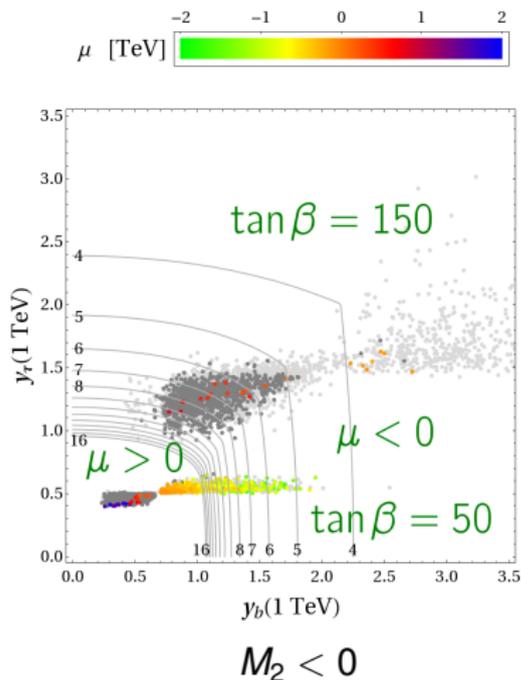
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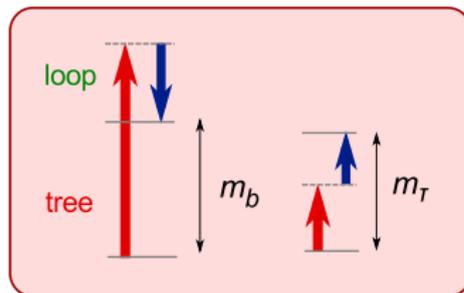
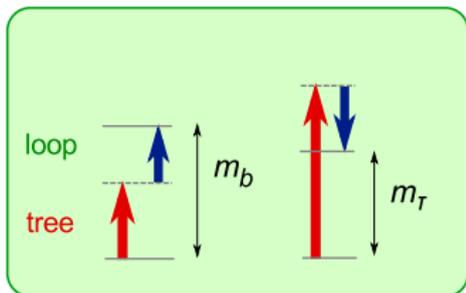
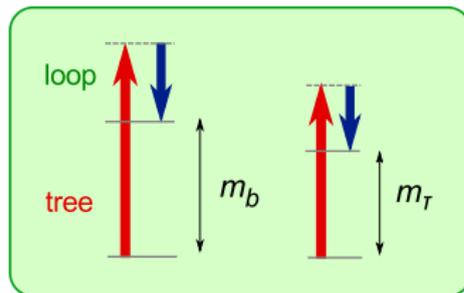
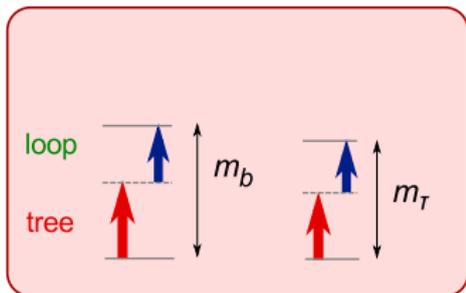
$$\Delta a_\mu^{\text{exp}} = (25.5 \pm 8.0) \times 10^{-10}$$

$$\Delta a_\mu^{\text{SUSY}} \propto \tan\beta \times \text{sgn}(\mu M_2)$$

Simultaneous  $\Delta y_\tau > 0$  and  $\Delta y_b > 0$  strongly disfavoured by  $(g - 2)_\mu$

# Threshold corrections vs. $g - 2$

ruled out by  $g-2$



ruled out by  $g-2$

# Large $\tan \beta$ in gauge mediation

## Gauge mediation

- Can naturally lead to  $B\mu(M) = 0$  ( $\Rightarrow$  large  $\tan \beta$ )
- **Small**  $A$  terms suppress  $\tan \beta$  enhanced FCNCs
- **Low** mediation scale welcome due to Yukawa Landau poles

## Setup: General gauge mediation (GGM) [Meade, Seiberg, Shih (2008)]

Large class of GM models described while the soft terms are parametrized in terms of a small number of parameters

$$M_k = g_k^2 M B_k, \quad m_f^2 = g_1^2 Y_f \zeta + \sum_k g_k^4 C_2(f, k) A_k, \quad A_{u,d,l}^{IJ} = 0,$$

## GGM parameter scan

- Mediation scale fixed to 100 TeV
- Gaugino masses assumed to be positive

# GGM: Numerical results – Yukawa couplings

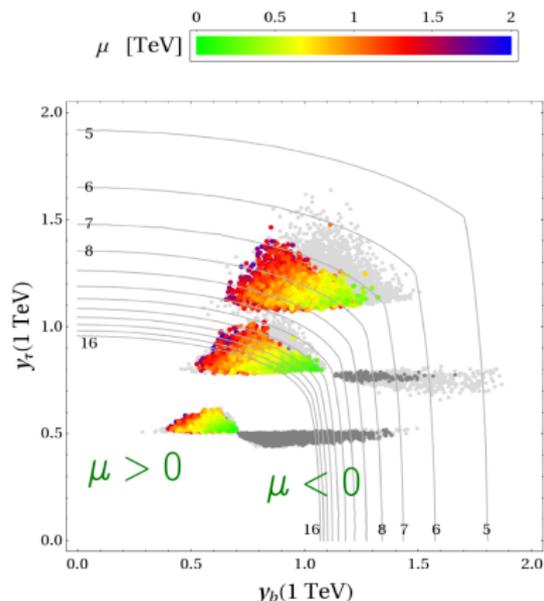
$$M = 10^5 \text{ GeV}, M_2 > 0$$

$$\tan \beta =$$

100

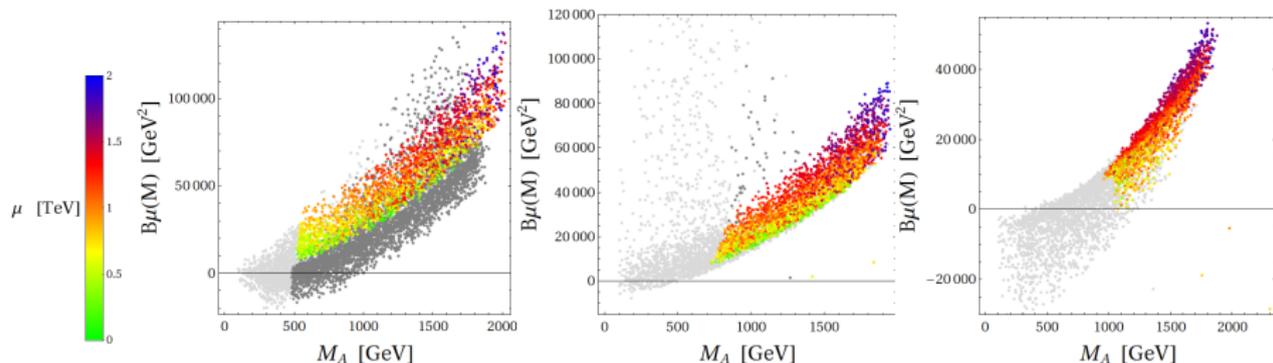
75

50



No convergence for  $\tan \beta \gg 100$ : Yukawas non-perturbative at mediation scale  $M$

# GGM: Numerical results – $B_\mu$ term



$\tan \beta =$

50

75

100

Points with  $B_\mu(M) = 0$  ruled out by  $\text{BR}(B \rightarrow \tau \nu)$  and/or  $(g - 2)_\mu$

## Conclusion

- $B_\mu = 0$  at  $M = 100$  TeV not compatible with  $\tan \beta \gg 50$
- $B_\mu(M) = 0$  possible for  $M > 100$  TeV, but then  $\tan \beta < 100$

## Low-energy analysis of the MSSM with very large $\tan\beta$

- At  $\tan\beta = 100$  (150), Yukawas are at most perturbative up to  $10^{12}$  ( $10^6$ ) GeV.
- $(g - 2)_\mu$  rules out  $\Delta y_b > 0$  and  $\Delta y_\tau > 0$  at  $\tan\beta \gg 50$

## General gauge mediation with very large $\tan\beta$

With  $M = 100$  TeV,

- $\tan\beta \sim 100$  allowed for  $\mu > 0$
- $\tan\beta \sim 150$  ruled out by perturbativity
- $B\mu(M) = 0$  ruled out by  $(g - 2)_\mu$  and  $\text{BR}(B \rightarrow \tau\nu)$