

# Bound-state effects on gluino-pair production

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# Introduction :

- Exploring physics beyond the SM at the LHC :

New particles at **TeV scale** are expected

**Colored particles** have large cross-section at Hadron Colliders : **squarks and gluino**

- Sparticle production at Hadron Colliders

## NLO SUSY-QCD

Beenakker, Hopker, Spira, Zerwas ('96)

Beenakker, Kramer, Plehn, Spira, Zerwas ('98)

## One-Loop EW

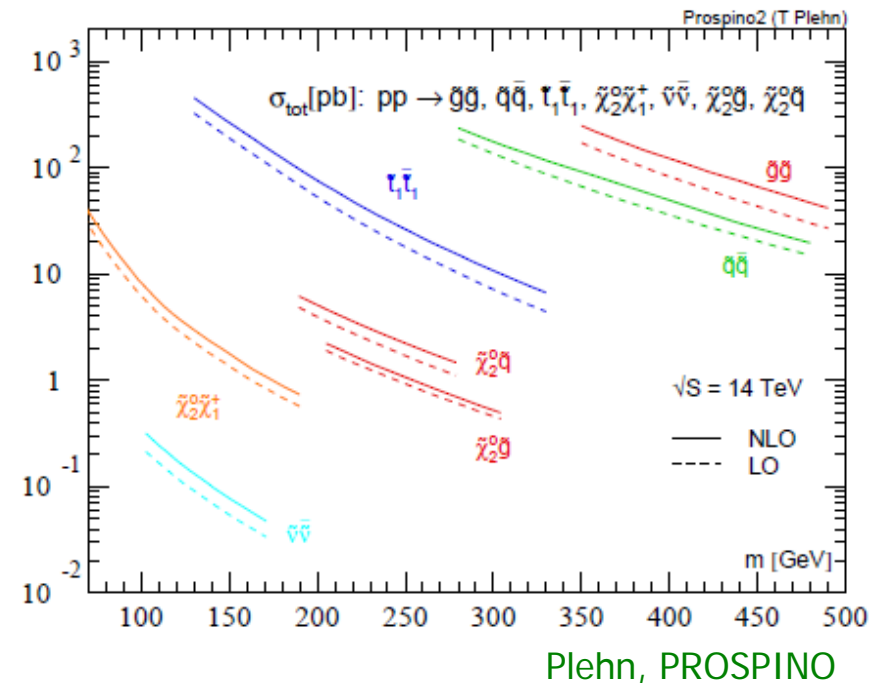
Hollik, Kollar, Trenkel ('07)

Hollik, Mirabella ('08)

Hollik, Mirabella, Trenkel ('08)

Mirabella ('08)

Germer, Hollik, Mirabella, Trenkel ('09)





# Introduction :

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- Beyond the NLO corrections :
  - dominant NNLO contribution for squark-pair prod. [Langenfeld,Moch\('09\)](#)
- Resummation technique :
  - Threshold resummation : [Kulesza,Motyka\('08,'09\)](#),  
[Beenakker,Brening,Kramer,Kulesza,Laenern,Niessen\('09\)](#)
  - Coulomb summation : Sommerfeld factor; [Kulesza,Motyka\('09\)](#)  
(unstable) bound-state effects for gluino-pair prod.;  
(stable) Gluinonia; (NNLO pot., NLO corr.) [Hagiwara,HY\('09\)](#)  
[Kauth,Kuhn,Marquard,Steinhauser\('09\)](#)
  - Factorization of soft and Coulomb functions : [Beneke,Falgari,Schwinn\('09,'10\)](#)  
+ numerical study for squark-antisquark prod.

# Glauino-pair production

- NLO correction near the partonic threshold :  $\beta = \sqrt{1 - 4m^2/\hat{s}} \rightarrow 0$

$$\hat{\sigma}_i^{(c),\text{NLO}} \sim \hat{\sigma}_i^{(c),\text{LO}} \left[ 1 + \frac{\alpha_s}{\pi} \left\{ \underbrace{A_i \ln^2(8\beta^2)}_{\text{red}} + \underbrace{B_i^{(c)} \ln(8\beta^2)}_{\text{green}} + \underbrace{C_i^{(c)} \frac{\pi^2}{\beta}}_{\text{green}} + \underbrace{D_i^{(c)}}_{\text{blue}} + \mathcal{O}(\beta) \right\} \right]$$

**Threshold logs:** emission of soft and/or collinear gluon in initial-state and final-state

→ **Threshold resummation**  
Kulesza, Motyka('08, '09)

**Coulomb singularity:**  
Coulomb gluon exchange between final-state

→ **Coulomb summation,** Kulesza, Motyka('09)  
**Bound-state formation** Hagiwara, HY('09)

**Hard correction:**  
process dependent

- Factorization of soft and Coulomb functions : Beneke, Falgari, Schwinn('09, '10)

+ numerical study for squark-antisquark prod.

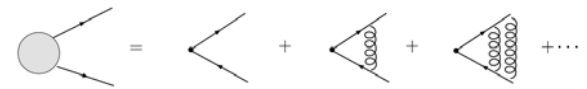
# Coulomb summation

- Coulomb summation to all-order

- One-loop level  $\mathcal{O}(\alpha_s) \propto C^{(c)} \frac{\alpha_s}{\beta}$

- Summation of ladder diagrams = Sommerfeld factor Sommerfeld; Sakharov,

$$S(z) = \frac{z}{1 - \exp[z]} \quad \text{with } z = C^{(c)} \pi \alpha_s / \beta$$



- Green's function formalism (NRQCD) Fadin, Khoze('87)

Schrodinger's Equation :

$$\left[ (E + i\Gamma_{\tilde{g}}) - \left\{ -\frac{\nabla^2}{m_{\tilde{g}}} + V_{QCD}^{(c)}(r) \right\} \right] G^{(c)}(\vec{x}, E + i\Gamma_{\tilde{g}}) = \delta^3(\vec{x})$$



finite width effects by complex energy

$$G(\vec{x}, E) \simeq \sum_n \frac{\Psi_n(\vec{x}) \Psi_n(0)^*}{E - E_n + i\Gamma_n/2} + \text{continuum}$$

# Threshold Logs vs. Coulomb Singularities

NLL threshold resummation and  
Coulomb summation by Sommerfeld factor

Kulesza, Motyka ('09)

- Large corrections in gluino production
- Coulomb summation can overtake the threshold resummation

gluino-pair

squark-pair

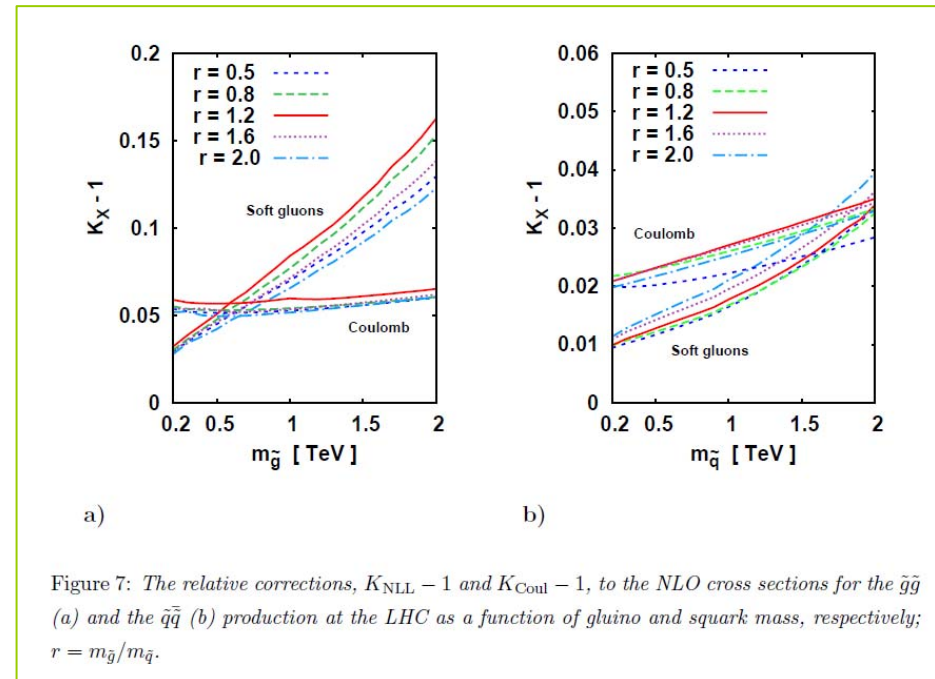


Figure 7: The relative corrections,  $K_{\text{NLL}} - 1$  and  $K_{\text{Coul}} - 1$ , to the NLO cross sections for the  $\tilde{g}\tilde{g}$  (a) and the  $\tilde{q}\tilde{q}$  (b) production at the LHC as a function of gluino and squark mass, respectively;  $r = m_{\tilde{g}}/m_{\tilde{q}}$ .

Kulesza, Motyka ('09)

# Color structure of Gluino-Pair Production

- Color decomposition :

$$8 \otimes 8 = 1 \oplus 8_S \oplus 8_A \oplus 10 \oplus \overline{10} \oplus 27$$

Color-factor in QCD Potential :  $V^{(i)}(r) = C_i \frac{\alpha_s}{r}$ , with  $C_i = \left\{ -C_A, -\frac{C_A}{2}, -\frac{C_A}{2}, 0, 1 \right\}$

- Gluinonia Classification :

Kuhn, Ono ('84), Goldman, Haber ('85),  
Kauth, Kuhn, Marquard, Steinhauser ('09)

color	symmetric (1, 8 <sub>S</sub> , 27)	anti-symmetric (8 <sub>A</sub> )
$\tilde{g}\tilde{g}$	$^1S_0, ^3P_{0,1,2}, ^1D_2, \dots$	$^3S_1, ^1P_1, ^3D_{1,2,3}, \dots$
$i = gg$	$^1S_0, ^3P_{0, 2}, ^1D_2, \dots$	$^1P_1, ^3D_{1, 3}, \dots$
$i = q\bar{q}$	$^3P_{1,2}, \dots$	$^3S_1, ^3D_{1,2,3}, \dots$

only even S+L

only odd S+L

due to the Majorana nature  
(MSSM)

# Coulomb summation

- Green's function formalism (NRQCD)

$$G(\vec{x}, E) \simeq \sum_n \frac{\Psi_n(\vec{x})\Psi_n(0)^*}{E - E_n + i\Gamma_n/2} + \text{continuum}$$

- Binding energy :  $E_B \simeq C^2 m_{\tilde{g}} \alpha_s^2 / 4$

$E_B > \Gamma_B \simeq 2\Gamma_{\tilde{g}}$  should be satisfied to form bound-states

- Bohr radius :  $\mu_B \simeq C m_{\tilde{g}} \alpha_s / 2$

typical momentum of Coulomb gluon  
 $\alpha_s = \alpha_s(\mu_B)$

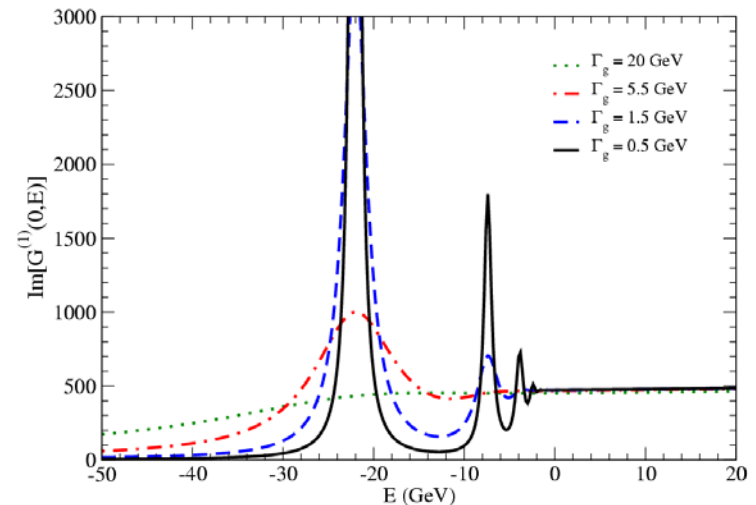
- Green's function for color-singlet :

for  $m_{\tilde{g}} = 608 \text{ GeV}$

and  $\Gamma_{\tilde{g}} = 0.5, 1.5, 5.5, 20 \text{ GeV}$

(SPS1a:  $m_{\tilde{g}} = 608 \text{ GeV}$ ,  $\Gamma_{\tilde{g}} = 5.5 \text{ GeV}$ )

$$E = m_{\tilde{g}\tilde{g}} - 2m_{\tilde{g}}$$





# Glauino decay-width

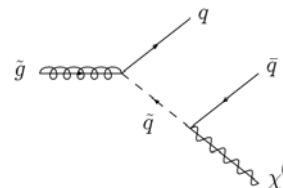
- BS Physics crucially depends on the decay-width

Estimate the gluino decay-width by

$$\tilde{g} \rightarrow q\bar{q}\tilde{B}, \quad \tilde{g} \rightarrow q\bar{q}'\tilde{W} \quad \text{Barnett, Gunion, Haber ('88), ...}$$

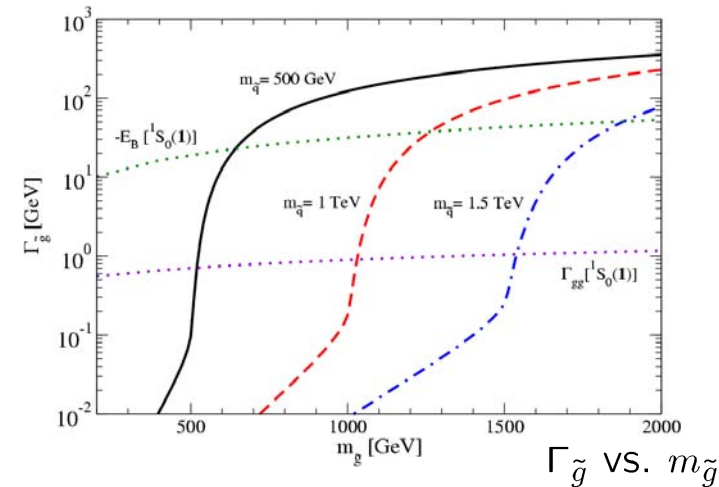
$$m_{\tilde{g}} > m_{\tilde{q}} \quad \Gamma_{\tilde{g}} \simeq \mathcal{O}(10^{0-1}) \text{ GeV}$$

$$m_{\tilde{g}} < m_{\tilde{q}} \quad \Gamma_{\tilde{g}} < \mathcal{O}(10^{-1}) \text{ GeV}$$



Assuming 5-flavor massless quarks and common squark mass, and gaugino mass relation;  $m_{\tilde{g}} : m_{\tilde{W}} : m_{\tilde{B}} = 7 : 2 : 1$ .

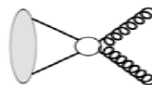
Decay into top and stop are neglected.



and the relation with the other two scales :

- Binding energy :  $|E_B| \simeq C_A^2 m_{\tilde{g}} \alpha_s^2 / 4$

- Annihilation decay-width :  $\Gamma_{gg} \propto C_A^2 \alpha_s^2 |\psi(0)|^2 / m_{\tilde{g}}^2$  with  $|\psi(0)|^2 \propto \alpha_s^3 m_{\tilde{g}}^3$



hidden gluino  $\rightarrow$  no cascade decay, but di-jet, di-photon signal

# Glauino decay-width

- Four scenarios :

$A : \Gamma_{\tilde{g}} \gtrsim |E_B|$  : gluinos decay before they form a bound-state.

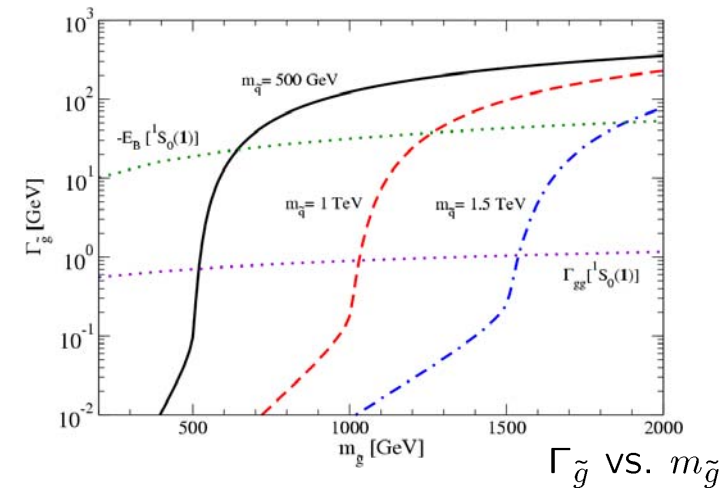
$B : |E_B| \gtrsim \Gamma_{\tilde{g}} \gg \Gamma_{gg}$  : a broad resonance enhancement, similar to the top-quark case.

$C : |E_B| \gg \Gamma_{\tilde{g}} \gtrsim \Gamma_{gg}$  : few narrow resonances can be formed, while the decay is dominated by the constituent gluino's decay.

$D : \Gamma_{\tilde{g}} < \Gamma_{gg}$  : dominantly decays into jets, but not in cascade. If  $\Gamma_{\tilde{g}} < \Lambda_{QCD}$ , hadronization (R-hadron).

Assuming 5-flavor massless quarks and common squark mass, and gaugino mass relation;  $m_{\tilde{g}} : m_{\tilde{W}} : m_{\tilde{B}} = 7 : 2 : 1$ .

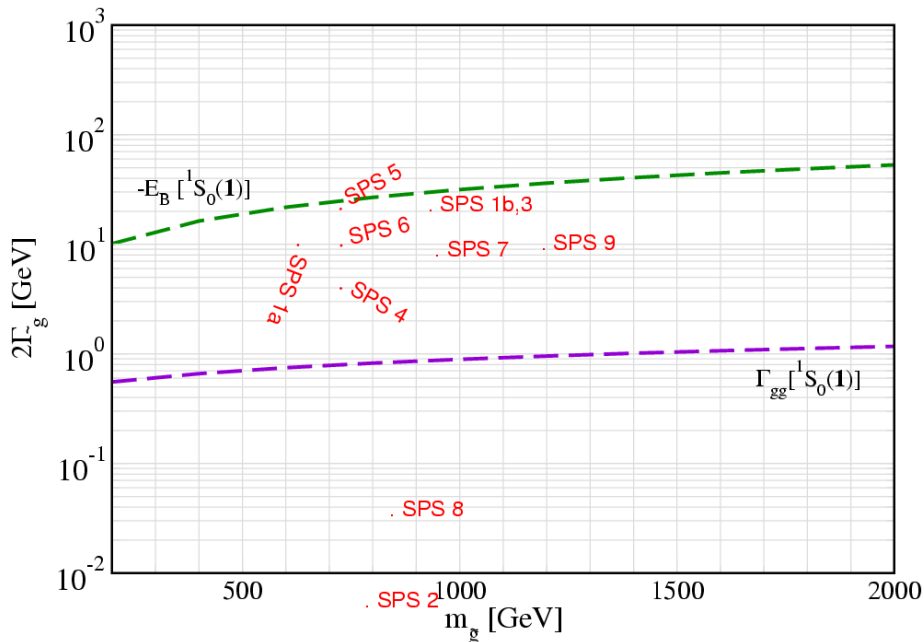
Decay into top and stop are neglected.



# Glauino decay-width

- Various MSSM points :

Model	$m_{\tilde{g}}$ GeV	$m_{\tilde{t}}$ GeV	$E_b^1$ GeV	$E_b^8$ GeV	$2\Gamma_{\tilde{g}}$ GeV	$\Gamma_{1g}^{\text{ann}}$ MeV	$\Gamma_{8g}^{\text{ann}}$ MeV	$(\Gamma_{8A}^{\text{ann}})$ MeV	$\Gamma_{8A}^{\text{ann}}$ MeV	$\sigma^{\text{cont}}$ pb	$\sigma^{\text{bound}}$ fb	$\sigma_{jj}^{\text{ann}}$ fb	$\sigma_{t\bar{t}}^{\text{ann}}$ fb
SPS 1a	607	400	18	5.3	11	460	18	25	0.5	6.3	240	7.2	0.14
SPS 1b	938	660	25	7.4	20	530	21	29	0.6	0.30	13	0.3	0.00
SPS 2	782	950	22	6.5	0.0052	500	20	27	8.2	1.7	51	46	0.41
SPS 3	935	648	25	7.4	23	530	21	29	0.6	0.34	13	0.2	0.00
SPS 4	733	545	21	6.1	4.2	490	19	26	0.3	1.8	70	5.5	0.06
SPS 5	722	262	20	6.1	23	490	19	26	1.6	2.4	77	1.2	0.02
SPS 6	720	503	20	6.1	11	490	19	26	0.5	2.0	79	2.5	0.03
SPS 7	950	807	25	7.5	8.4	530	21	29	0.2	0.30	12	0.5	0.00
SPS 8	839	978	23	6.8	0.034	510	20	27	1.7	0.81	28	22	0.10
SPS 9	1182	930	30	8.8	9.1	570	23	31	0.3	0.027	2.2	0.1	0.00



Kats, Schwartz('09)

$$A : \Gamma_{\tilde{g}} \gtrsim |E_B| \quad \text{SPS 5}$$

$$B : |E_B| > \Gamma_{\tilde{g}} \gg \Gamma_{gg} \quad \text{SPS 1a, 1b, 3, 6, 7, 9}$$

$$C : |E_B| \gg \Gamma_{\tilde{g}} > \Gamma_{gg} \quad \text{SPS 4}$$

$$D : \Gamma_{\tilde{g}} < \Gamma_{gg} \quad \text{SPS 2, 8}$$

# Initial-state/Final-state radiation

- Gluino-Pair Invariant-Mass Distribution :

$$\frac{d\sigma}{dM}(s, M^2) = \hat{\sigma}_{B,i}^{(c)}(M^2) \cdot K_i^{(c)} \int_{\tau_0}^1 \frac{dz}{z} F_i^{(c)}(z) \frac{d\mathcal{L}_i}{d\tau}(\tau_0/z)$$

$$\tau_0 = m_{tt}^2/s$$

$$\sigma_{B,i}^{(c)} = \sigma_{0,i}^{(c)} \cdot \text{Im}[G^{(c)}(\vec{0}, E)]$$

$\mathcal{L}$  : partonic luminosity

- Initial-state/Final-state radiation :

$$F_i^{(c)}(z) = \delta(1-z) + \frac{\alpha_s}{\pi} \left[ A_i \left\{ \left( \frac{\ln(1-z)}{1-z} \right)_+ - \left( \frac{1}{1-z} \right)_+ \ln \left( \frac{\mu_F}{2m_t} \right) \right\} \right. \\ \left. + D_{tt}^{(c)} \left( \frac{1}{1-z} \right)_+ + k_i^{(c)} \delta(1-z) \right]$$

resum log? → Kiyoy et al (tt) ('09)  
Kulesza, Motyka ('09)  
see previous talk.

- Hard-gluon correction :

$$K_i^{(c)} = 1 + \frac{\alpha_s}{\pi} h_{i,1}^{(c)}$$

matching coefficient at NLO

# Glino-pair invariant-mass distribution

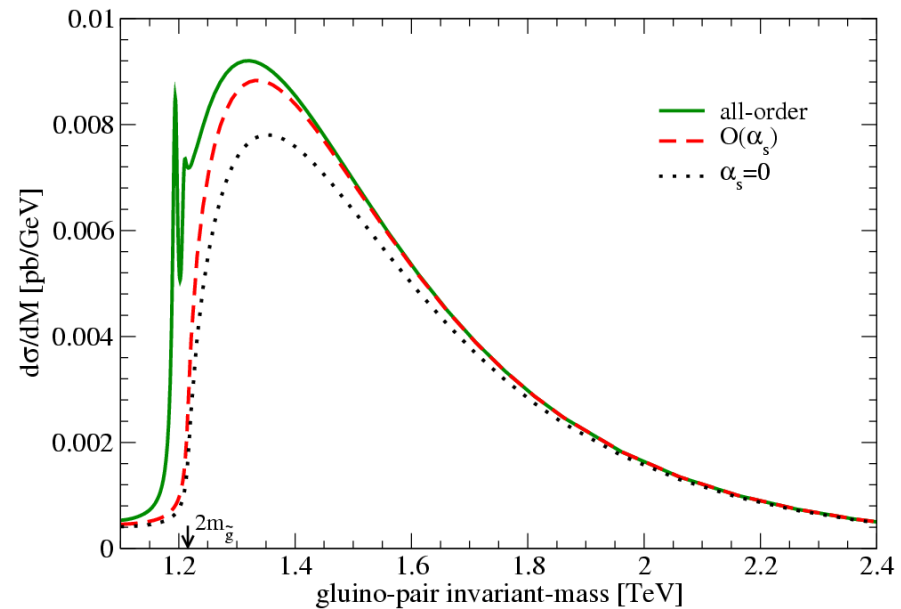
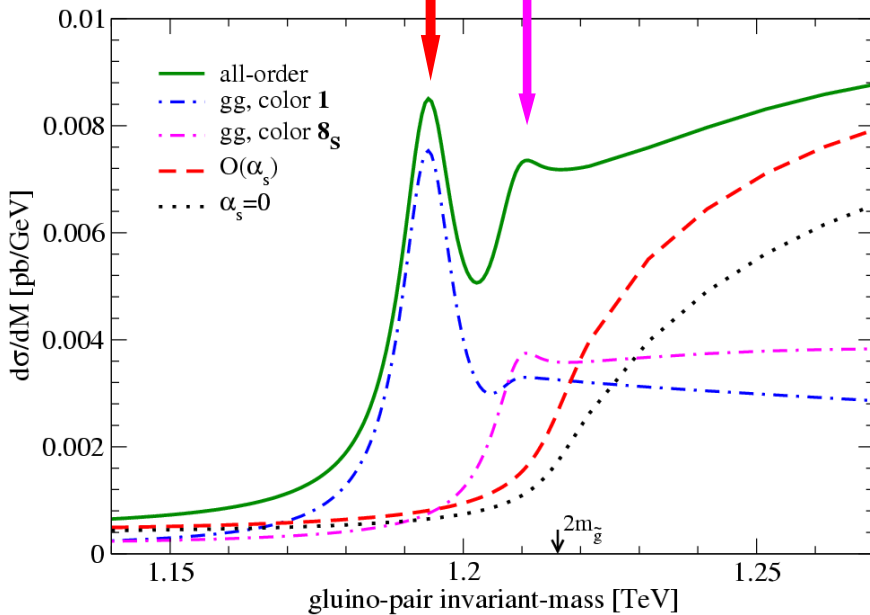
Example : SPS1a  $m_{\tilde{g}} = 608$  GeV and  $\Gamma_{\tilde{g}} = 5.5$  GeV ( $m_{\tilde{q}} \simeq 547$  GeV)

(Scenario B)

Glino-pair inv.-mass dist. in threshold region

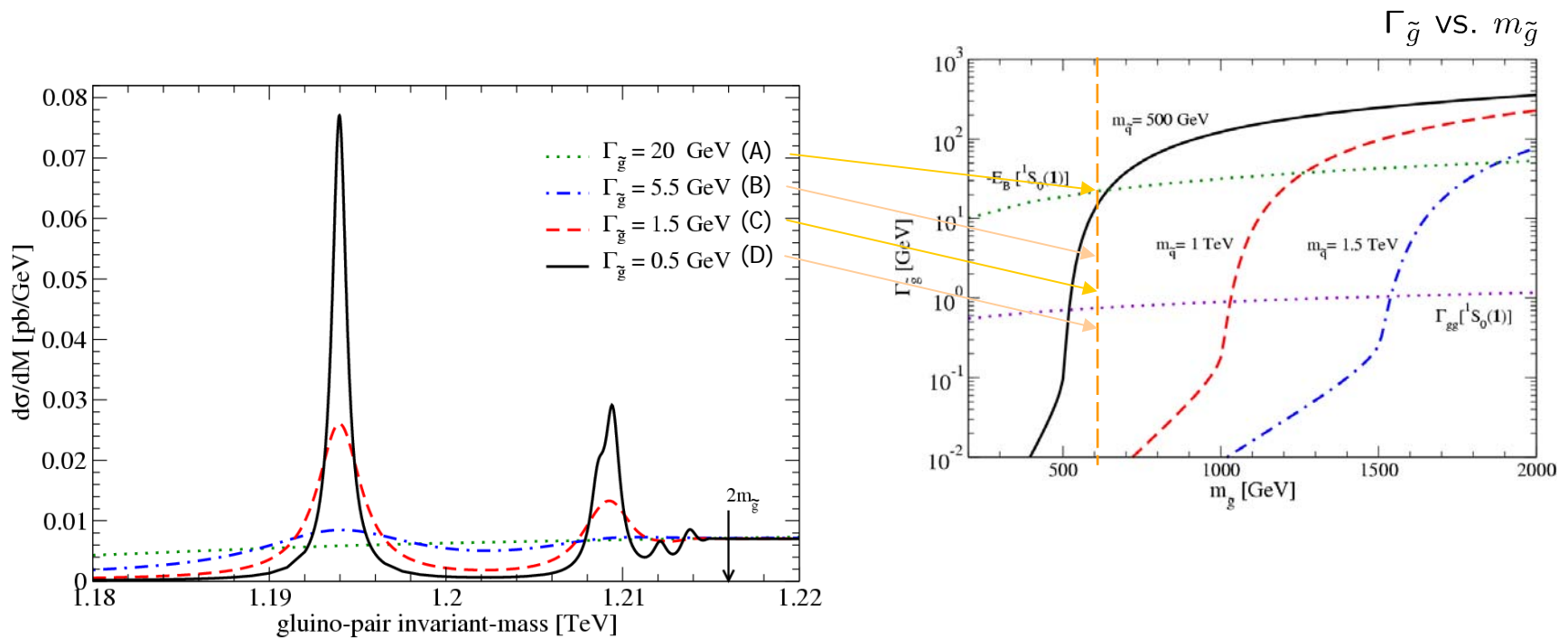
- Enhancement only near the threshold
- Above the threshold is described by Sommerfeld correction

$E_{1S}[1] \sim -22\text{GeV}$   
 $E_{1S}[8_s] \sim E_{2S}[1] \sim -7\text{GeV}$



# Glino-pair invariant-mass distribution

- Varying gluino decay-width : ( $m_{\tilde{g}} = 608 \text{ GeV}$ )



- The narrower  $\Gamma_{\tilde{g}}$ , the sharper resonances.
- For (C)&(D), non-negligible branching ratio to the annihilation decay;

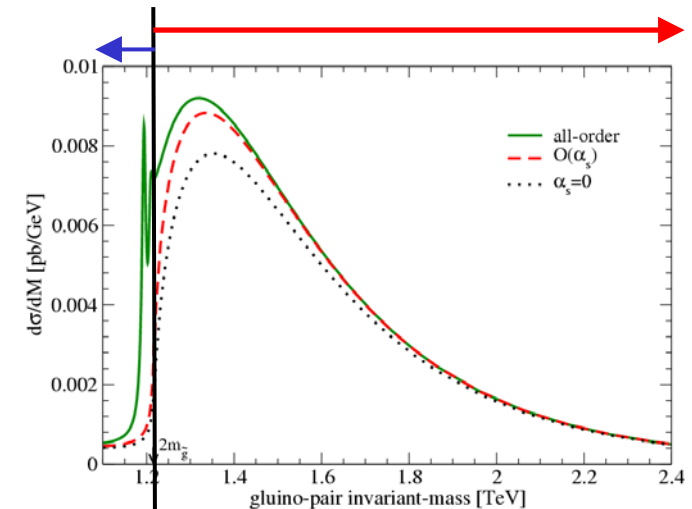
$$B((\tilde{g}\tilde{g}) \rightarrow gg) \simeq \Gamma_{gg}/(2\Gamma_{\tilde{g}} + \Gamma_{gg})$$

di-jets & di-photon signal Kats, Schwartz('09)

# Effect to the Total Cross-section

**Above threshold** : described by Sommerfeld correction.  
independent of  $\Gamma_{\tilde{g}}$  ~ 5% from NLO

**Below threshold** : Resonances + smearing.  
one of gluinos is off-shell



- The proportion of the contribution below the threshold to the total cross-section?

$m_{\tilde{g}}$	A: $\Gamma_{\tilde{g}} = E_B$	B: $\Gamma_{\tilde{g}} = E_B/2$	C: $\Gamma_{\tilde{g}} = 2\Gamma_{gg}$	D: $\Gamma_{\tilde{g}} = \Gamma_{gg}/2$
200 [GeV]	7.5 [4.5]	5.0 [1.8]	4.0 [0.3]	3.9 [0.1]
400 [GeV]	7.1 [4.2]	4.8 [1.7]	3.8 [0.2]	3.8 [0.1]
600 [GeV]	7.2 [4.2]	5.0 [1.7]	3.9 [0.2]	4.2 [0.0]
1 [TeV]	7.9 [4.6]	5.5 [1.8]	4.3 [0.2]	4.4 [0.0]
1.5 [TeV]	9.2 [5.3]	6.3 [2.1]	5.0 [0.2]	5.1 [0.0]
2 [TeV]	10.7 [6.3]	7.4 [2.5]	5.9 [0.2]	5.9 [0.0]

4~6%



## Summary :

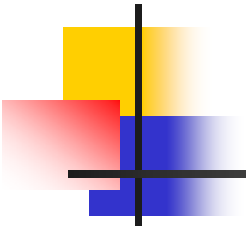
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- Sparticles production at the LHC → heavy colored ones (squark, gluino)  
large QCD corrections beyond NLO
- Summations of **Soft/Collinear gluon** & **Coulomb gluon** effects :  
Coulomb-gluon summation results **Bound-State formation (Gluinonia)**  
deform the pair invariant-mass distribution :  
resonances structure (smeared by finite decay-width),  
enhancement only near the threshold  
correction to the total cross-sections :
  - **4~6%** for gluino pair
  - **~2%** for squark pair Beneke, Falgari, Schwinn('10)
  - ( **~1%** for top-quark pair) Hagiwara, Sumino, HY('08),  
Kiyono et al ('09)

if gluino is stable, clean signal from annihilation decay?

- Gluinonia Physics crucially depend on the gluino decay-width, ie  $m_{\tilde{g}}$  vs.  $m_{\tilde{q}}$





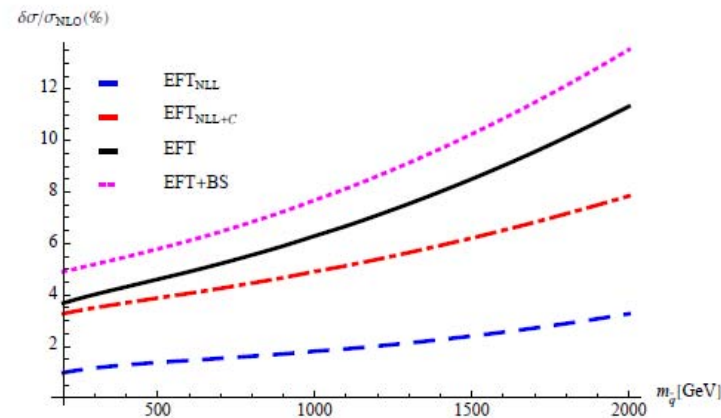
## Squark-antisquark resummed cross section

Beneke, PF, Schwinn, PRELIMINARY

- **EFT<sub>NLL</sub>**: NLL soft resummation, no Coulomb resummation
- **EFT<sub>NLL+C</sub>**: NLL soft resummation **AND** Coulomb resummation (above threshold).  
No soft/Coulomb interference
- **EFT**: NLL soft resummation + Coulomb resummation (above threshold)  
+ soft/1st Coulomb interference
- **EFT + BS**: **EFT** + Bound-state effects

## Setup:

- PP@ 14 TeV
- MSTW2008 PDFs
- equal squark masses
- no stops
- $m_{\tilde{g}} = 1.25m_{\tilde{q}}$
- $\mu_f = m_{\tilde{q}}$



BS effect 1~2%

interference between  
soft/1<sup>st</sup> CoulombEFT<sub>NLL</sub> result agrees well with Kulesza, Motyka '09

for stable gluinos;  $\Gamma_{\tilde{g}} \rightarrow 0$  cf. Split-SUSY

- Gluinonia using NNLO potential

pole mass scheme with  $\rightarrow$

$$\mu_S = m_{\tilde{g}} C_A \alpha_s(\mu_S)$$

also corrections to the wave-function at the origin

- Production and decay at NLO
- Decay ratio to  $t\bar{t}$ bar, two photons  $\sim$  few percent,  $10^{-5}$
- Signal-to-Background ratio in dijet decay  $\sim 0.5 \%$

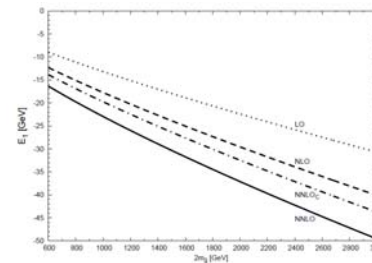


Figure 3: Ground state energy  $E_1$  in the pole mass scheme as function of twice the constituent mass.

### Potential Subtracted scheme

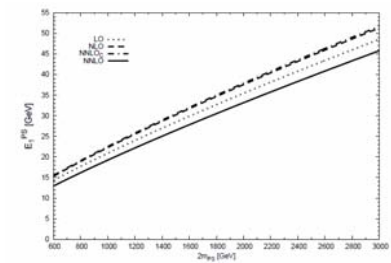


Figure 4: Ground state energy  $E_1$  in the potential subtracted mass scheme as function of twice the potential subtracted mass. The curves have been obtained using the pole mass as input and evaluating both the potential subtracted mass (using  $E_1$  (7)) and  $E_1^{PS}$  to a given order in  $\alpha_s$ . Note that the dash-dotted and dashed curves are almost on top of each other.

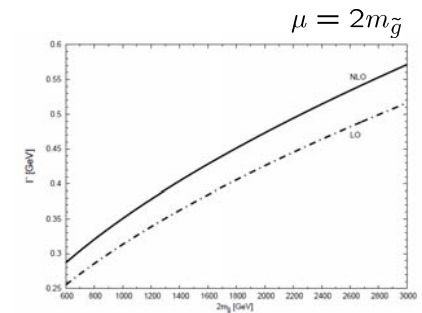


Figure 9: Decay rate of the 1S state into gluons.