Stability of Scalar Fields in Warped Extra Dimensions

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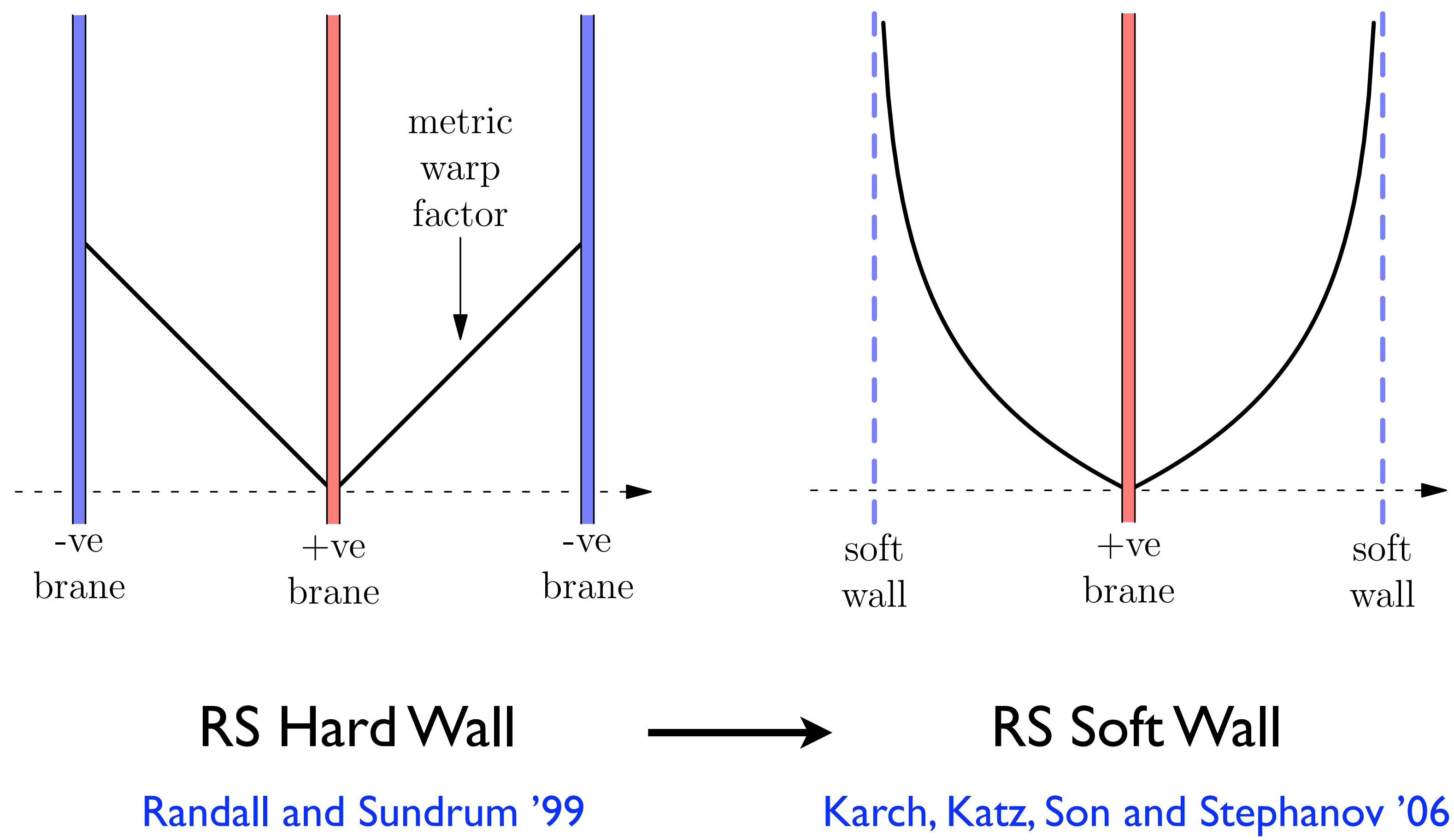
Compact extra dimensions

Models with compact extra dimensions offer new ways of understanding the stability of electroweak scale against ultraviolet physics.



- Original soft-wall motivation: AdS/QCD and linear Regge trajectories
- Now exist early BSM models

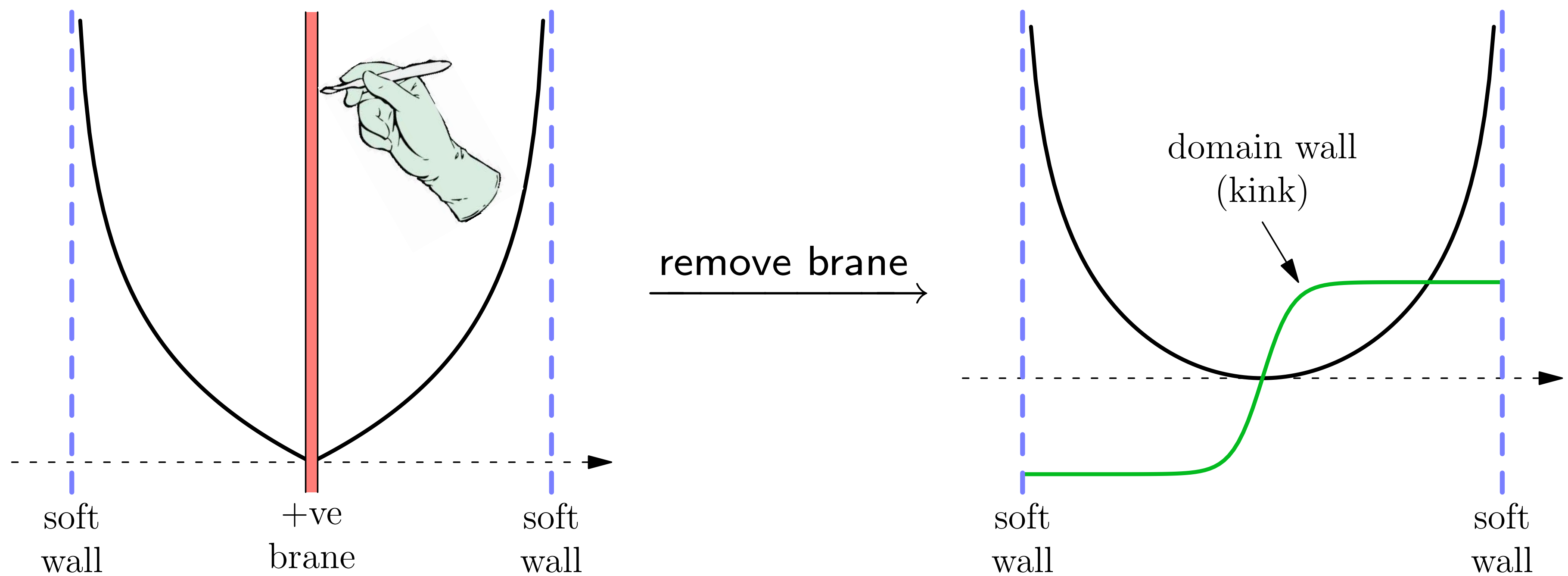
Batell and Gherghetta '08, Falkowski and Perez-Victoria '08, Batell, Gherghetta and Sword '08
Cabrer, von Gersdorff and Quiros '09, MA and Santiago '09

Branes: you don't need them!

"Can't you give me brains?" asked the Scarecrow.

"You don't need them!" replied the Wizard.

Our aim: replace the brane with a domain wall.
$\longrightarrow$ *Must ensure stability*



Why interesting ? $\longrightarrow$ Purely field theoretical construction and requires no appeal to string theory.

General set-up; background configuration

General framework: N scalar fields coupled to gravity;

$$\mathcal{S} = \int d^4x\, dy \left[\sqrt{-g}\left(M^3 R + \mathcal{L}_{\mathrm{matter}}\right)\right]$$

with $$\mathcal{L}_{\mathrm{matter}} = -\frac{1}{2}\sum_i g^{MN}\partial_M\Phi_i\partial_N\Phi_i - V\left(\{\Phi_i\}\right)$$

Background ansatz: $ds^2 = e^{-2\sigma(y)}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2$, $\Phi_i(x^\mu, y) = \phi_i(y)$.

Background fields σ and $\phi_i(y)$ are solutions of Einstein's and Euler-Lagrange equations.

We specialize to potentials generated using the superpotential approach!

Superpotential approach

Construct a superpotential *W* such that the original potential is given by

$$V\left(\{\Phi_i\}\right) = \sum_i \frac{1}{2}\left(\frac{\partial W(\{\Phi_i\})}{\partial \Phi_i}\right)^2 - \frac{1}{3M^3}\left(W(\{\Phi_i\})\right)^2$$

DeWolfe, Freedman, Gubser and Karch '00
Freedman, Nunez, Schnabl and Skenderis '04

Restricts potentials since not all potentials can be generated this way!

For a generic *W* solutions to Einstein's and Euler-Lagrange equations are given by

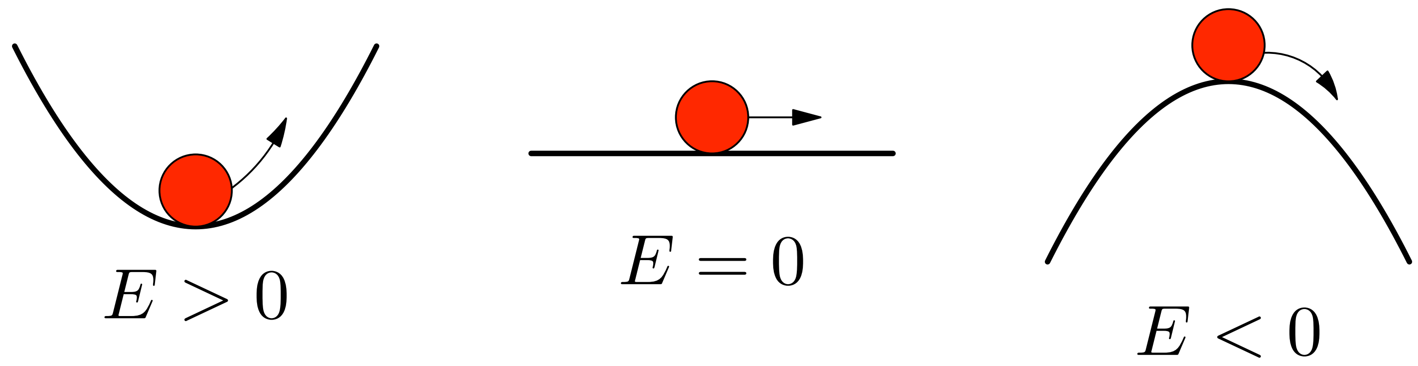
$$\begin{aligned} \sigma' &= \frac{1}{3M^3} W(\{\phi_i\}) \\ \phi_i' &= \frac{\partial W(\{\phi_i\})}{\partial \phi_i} \end{aligned}$$

- Set of first order differential equations
- Can take $\sigma(y_0) = 0$ without loss of generality
- Have *N* integration constants : set of values $\{\phi_i(y_0)\}$

$\{W, \phi_i(y_0)\}$ uniquely define a configuration. Is it stable ??

Perturbative stability

Perturb the background configuration $\rightarrow$ eigenvalue problem.



Spin-0 and spin-2 perturbations described by:

$$ds^2 = e^{-2\sigma}\left[(1 - 2F(x^\mu, y)\eta_{\mu\nu} + h_{\mu\nu}(x^\mu, y)\right] dx^\mu dx^\nu + \left[1 + 4F(x^\mu, y)\right] dy^2,$$
$$\Phi_i(x^\mu, y) = \phi_i(y) + \varphi_i(x^\mu, y)\ .$$

Spin-2: $h_{\mu\nu}$ decouples from F and φ_i. Is non-tachyonic. Has a zero mode (4D graviton). Known RS2 result.

Spin-0: Non-trivial. Physical modes are mixtures of F and φ_i.

Systems with one scalar field were shown to be stable without any tachyonic or zero modes!

DeWolfe, Freedman, Gubser and Karch '00

We want: generalization to N>1 scalar fields.

Scalar perturbations for arbitrary N

For spin-0 the equations to solve are

$$
\begin{aligned}
6M^3(F' - 2\sigma' F) &= \phi_i' \varphi_i \\
6M^3(-e^{2\sigma}\Box F - 2\sigma' F' + F'') &= 2\phi_i \varphi_i' \\
e^{2\sigma}\Box\varphi_i + \varphi_i'' - 4\sigma'\varphi_i' - 6\phi_i' F' - 4V_i F - V_{ij}\varphi_j &= 0
\end{aligned}
$$

We can do it! Go to conformal coordinates ($dy = e^{-\sigma} dz$), rescale fields:

$$F(y) = \frac{1}{\sqrt{12}} e^{3\sigma/2} \chi(z(y)) \,, \qquad \varphi_i(y) = M^{3/2} e^{3\sigma/2} \psi_i(z(y)) \,.$$

For a generic potential *V* we then can write the above equations as

$$-\begin{pmatrix} \chi \\ \psi_i \end{pmatrix}'' + \begin{pmatrix} \mathcal{V}_{00} & \mathcal{V}_{0j} \\ \mathcal{V}_{0i} & \mathcal{V}_{ij} \end{pmatrix} \begin{pmatrix} \chi \\ \psi_j \end{pmatrix} = \Box \begin{pmatrix} \chi \\ \psi_i \end{pmatrix}$$

Superpotential approach is powerful !

Using superpotential approach we find that $\mathcal{V} = \mathcal{S}^2 + \mathcal{S}'$

$$\mathcal{S} = e^{-\sigma} \left(\begin{array}{cc} \frac{1}{12M^3} W & \frac{1}{\sqrt{3M^3}} \frac{\partial W}{\partial \Phi_j} \\ \frac{1}{\sqrt{3M^3}} \frac{\partial W}{\partial \Phi_i} & -\frac{1}{4M^3} \delta_{ij} W + \frac{\partial W}{\partial \Phi_i \partial \Phi_j} \end{array} \right) \Bigg|_{\mathrm{bg}}$$

Write $\Psi = (\chi, \psi_i)^T$. Perturbations obey $(\partial_z + \mathcal{S})(-\partial_z + \mathcal{S})\Psi = \Box\Psi$. Fourier transform on x^μ, multiply by $\Psi^\dagger$ on left and integrate:

$$\int |(-\partial_z + \mathcal{S})\Psi|^2 dz + (\text{boundary terms}) = E \int |\Psi|^2 dz$$

Boundary terms vanish for warped metric. $\boxed{\text{We find } E \geq 0.}$

What about $E = 0$? Such modes can correspond to changes in the size of the compact extra dimension.

N>1 and the zero-mode criterion

System with N=1 is well understood and there does not exist a zero-mode in the spin-0 sector. For N>1 there may or may not be a zero mode!

DeWolfe, Freedman, Gubser and Karch '00

Criteria: *For a system of definite parity with N scalar fields that couple to gravity, the number of independent normalisable zero modes is at most equal to the number of fields whose background solutions are even.*

Proof: If a zero mode exists, adding it to the background takes you to another background, generated using the same superpotential but with different integration constants.

Zero modes $\longleftrightarrow$ available integration constants
No integration constants $\longrightarrow$ no zero modes

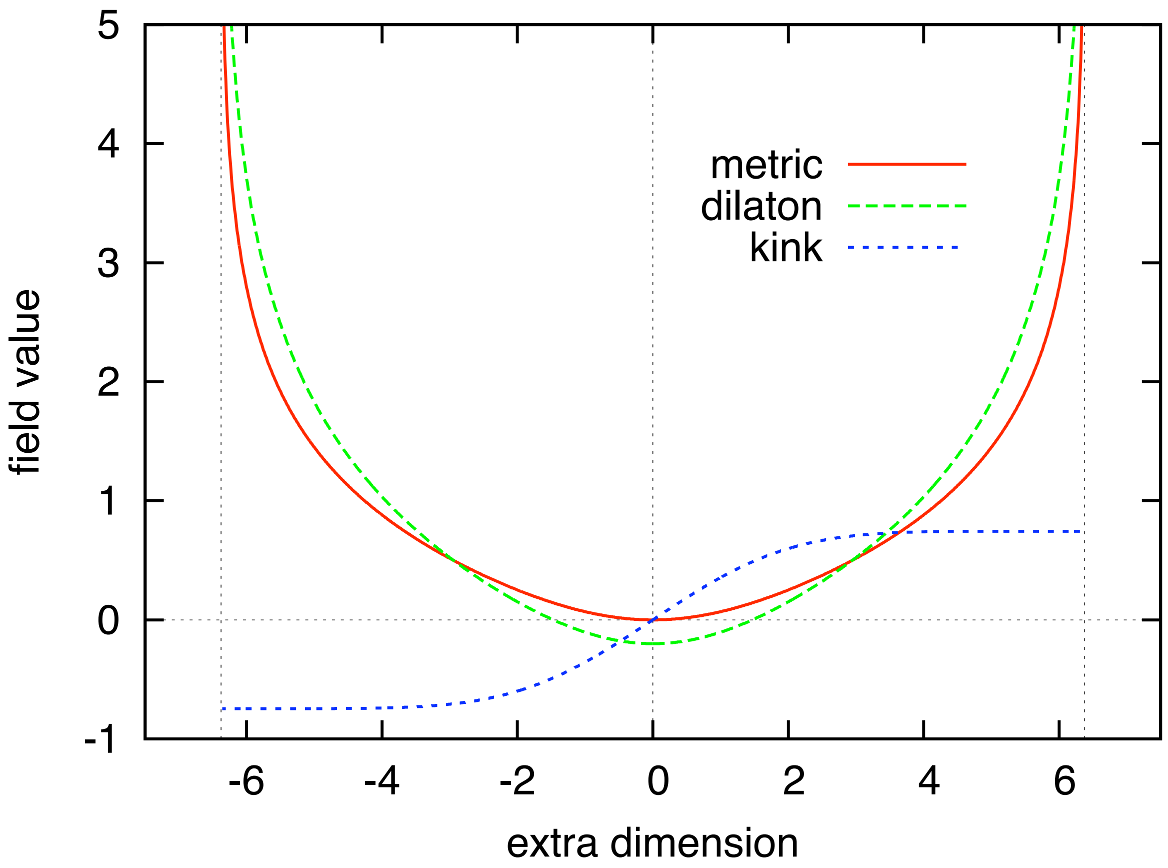
Look at some examples with *N=2 scalar fields*.

Example: N=2 unstable system

Φ_1 even, Φ_2 odd, σ even

Superpotential: $W(\Phi_1, \Phi_2) = e^{\nu \Phi_1} \left(a\Phi_2 - b\Phi_2^3 \right)$

- choice of integration constant
- zero mode ⟷ integration constant
- size of extra dimension is *not* fixed



$(\nu = 1.4,\ a = 0.5,\ b = 0.3)$

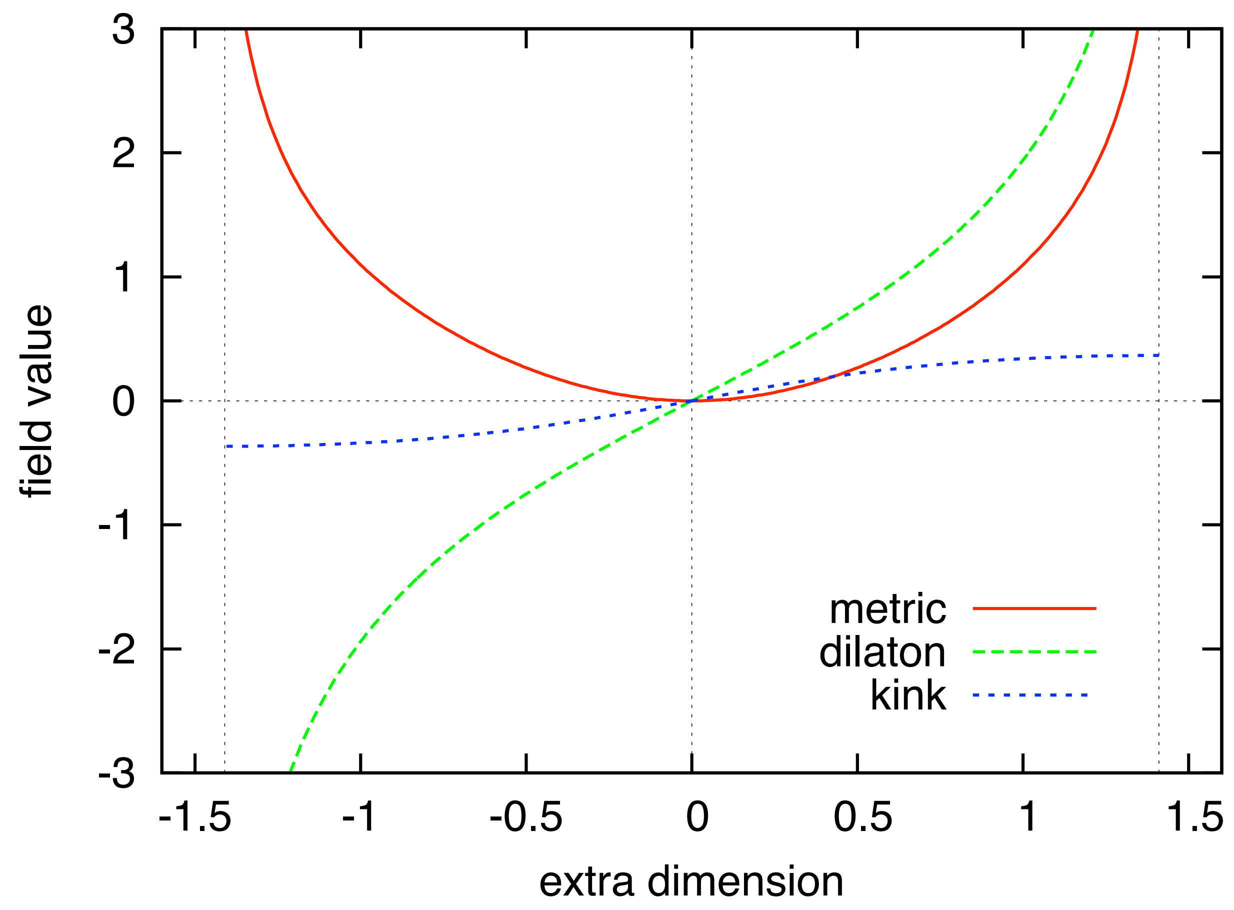
For this case, we have found the explicit solution for the zero mode.

Example: N=2 stable system

$$\Phi_1 \text{ odd}, \;\; \Phi_2 \text{ odd}, \;\; \sigma \text{ even}$$

$$\text{Superpotential}: \;\; W\left(\Phi_1, \Phi_2\right) = \alpha \sinh(\nu \Phi_1) + \left(a\Phi_2 - b\Phi_2^3\right)$$

- no integration constants to choose
- background solution is unique
- no normalizable zero modes
- size of extra dimension is fixed by parameters in *W*



$(\alpha = 1, \; \nu = 1.4, \; a = 0.5, \; b = 0.3)$

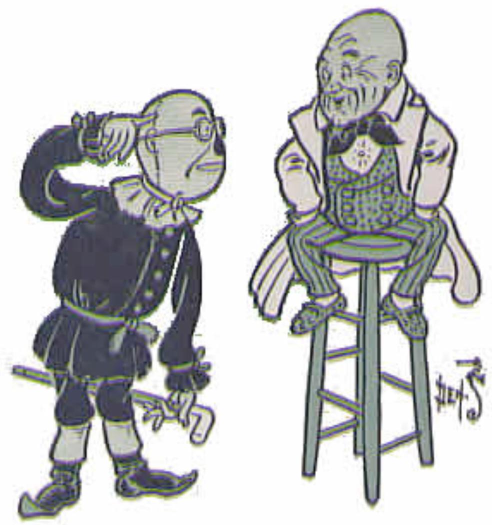
An example of a domain-wall model with a soft-wall that stabilizes the size of the extra dimension.

Summary and outlook

Dynamic branes not necessary!

We can have a stable compact extra dimension

- Soft wall at the edge of the space; replaces negative tension brane
- Domain wall at the origin; replaces positive tension brane
- Additional scalar (dilaton) cuts off space



Using superpotential approach, need definite parity and all scalars must be odd to eliminate zero modes!

Technical questions:

- No superpotential: Can we have even fields?
- No definite parity: Are there zero modes?

Other interesting questions:

- Can we solve the hierarchy problem?
- Can we construct a realistic BSM model?