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Supersymmetric Yukawaon Model

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Contents

1 What is the yukawaon model?

My home town, Kanazawa city

Standard Model

The origin of the mass spectra lies in the Yukawa coupling constants Y_f which are fundamental constants in the theory. I cannot believe that the nature needs such so many fundamental constants.

Yukawaon Model

The mass spectra are understood as dynamical quantities, i.e. VEV matrices of new gauge singlet scalars Y_f :

$$Y_f^{eff} = \frac{y_f}{\Lambda}\langle Y_f \rangle \quad (1.1)$$

We refer to these scalars as “yukawaons” .

Those VEVs can, in principle, be calculated dynamically.

1.3 Basic assumptions

(i) Would-be yukawa interactions are given by

$$H_Y = \sum_{i,j} \frac{y_u}{\Lambda} u_i^c (Y_u)_{ij} q_j H_u + \sum_{i,j} \frac{y_d}{\Lambda} d_i^c (Y_d)_{ij} q_j H_d$$

$$+ \sum_{i,j} \frac{y_\nu}{\Lambda} \ell_i (Y_\nu)_{ij} \nu_j^c H_u + \sum_{i,j} \frac{y_e}{\Lambda} \ell_i (Y_e)_{ij} e_j^c H_d + h.c. + \sum_{i,j} y_R \nu_i^c (Y_R)_{ij} \nu_j^c \quad (1.2)$$

(ii) We assume a family symmetry, e.g. U(3) or O(3).
For the time being, we will assume O(3) symmetry:

All of Y_f belong to $(\mathbf{3} \times \mathbf{3})_S = \mathbf{1} + \mathbf{5}$ of O(3).

(iii) In order to distinguish each Y_f from others,

we have assumed a $U(1)_X$ symmetry:

$Q_X(f^c) = -x_f$, $Q_X(Y_f) = +x_f$ and $Q_X(Y_R) = 2x_\nu$

The $SU(2)_L$ doublet fields are assigned as $Q_X = 0$

1.4 How to obtain VEV relations

(i) We give an O(3) and $U(1)_X$ invariant superpotential for yukawaons Y_f

(ii) We solve SUSY vacuum conditions $\partial W/\partial Y_f = 0$ **.**

(iii) Then, we obtain relations among $\langle Y_f \rangle$ **.**

Example:

$$W_e = \lambda_e \mathrm{Tr}[\Phi_e \Phi_e \Theta_e] + \mu_e \mathrm{Tr}[Y_e \Theta_e] + W_\Phi \qquad (1.3)$$

$$\frac{\partial W}{\partial \Theta_e} = \lambda_e \Phi_e \Phi_e + \mu_e Y_e = 0 \quad \Rightarrow \quad \langle Y_e \rangle = -\frac{\lambda_e}{\mu_e} \langle \Phi_e \rangle \langle \Phi_e \rangle \qquad (1.4)$$

$$\frac{\partial W}{\partial Y_e} = \mu_e \Theta_e = 0 \quad \Rightarrow \quad \langle \Theta_e \rangle = 0 \qquad (1.5)$$

2 An attempt of unified description of quark and lepton mass matrices

Night in Osaka

2.1 Overview

(i) VEV relations among yukawaons Y_f are obtained from SUSY vacuum conditions.

(ii) The VEV relations are valid only a specific flavor basis "*e*-basis" in which the charged lepton mass matrix $\langle Y_e \rangle$ is diagonal. Hereafter, we denote a VEV matrix $\langle A \rangle$ in the "e-basis" as $\langle A \rangle_e$.)

(iii) VEVs of all yukawaons $\langle Y_f \rangle$ are described in terms of a VEV of a fundamental yukawaon ("ur-yukawaon")

$$\langle \Phi_e \rangle_e \propto \mathrm{diag}(\sqrt{m_e}, \sqrt{m_\mu}, \sqrt{m_\tau}) \quad (2.1)$$

For a superpotential form which leads to Eq.(2.1), for example, see Y.K., PRD79, 033009 (2009); PLB687, 219 (2010).

All of quark and lepton mass matrices are describe in terms of the VEV of the ur-yukawaon $\langle \Phi_e \rangle$

$$\langle Y_e \rangle_e \propto \langle \Phi_e \rangle_e \langle \Phi_e \rangle_e \qquad (2.2)$$

$$\langle Y_u \rangle_e \propto \langle \Phi_u \rangle_e \langle \Phi_u \rangle_e \qquad (2.3)$$

$$\langle \Phi_u \rangle_e \propto \langle \Phi_e \rangle_e \langle S_u \rangle_e \langle \Phi_e \rangle_e \qquad (2.4)$$

$$\langle Y_d \rangle_e \propto \langle \Phi_e \rangle_e \langle S_d \rangle_e \langle \Phi_e \rangle_e \qquad (2.5)$$

$$\langle S_q \rangle_e \propto X + a_q \mathbf{1} \equiv \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + a_q \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad (2.6)$$

For an explicit form of the superpotential which leads to these VEV relations, see Y.K., PLB680, 76 (2009).

Note that the quark mass matrices are described only by parameters a_u and a_d.

(iv) For the neutrino sector, we adopt the conventional seesaw type mass matrix model:

$$M_\nu \propto \langle Y_\nu \rangle \langle Y_R \rangle^{-1} \langle Y_\nu \rangle \quad (2.7)$$

However, in this model, the Dirac and Majorana mass matrices are given by $\langle Y_\nu \rangle \propto \langle Y_e \rangle$ **(2.8)**

$$\langle Y_R \rangle_e \propto \langle Y_e \rangle_e \langle P_u \rangle_e \langle \Phi_u \rangle_e + \langle \Phi_u \rangle_e \langle P_u \rangle_e \langle Y_e \rangle_e$$
$$+\xi(\langle P_u \rangle_e \langle Y_e \rangle_e \langle \Phi_u \rangle_e + \langle \Phi_u \rangle_e \langle Y_e \rangle_e \langle P_u \rangle_e) \quad (2.9)$$

$$\langle \Phi_u \rangle_u \propto \mathrm{diag}(\sqrt{m_u}, \sqrt{m_c}, \sqrt{m_t}) \quad (2.10)$$

$$\langle P_u \rangle_u = v_P \mathrm{diag}(+1, -1, +1) \quad (2.11)$$

At present, the model for quarks and leptons is a phenomenological one, because we have assigned the $U(1)_X$ charges so that the above relations can be derived.

Y.K. PLB 680, 76 (2009) [arXiv:0904.1644]

In spite of few parameters, the model can give good fitting

Sector	Parameters	Predictions					
		$\sin^2 2\theta_{atm}$	$\tan^2\theta_{solar}$	$\lvert U_{13}\rvert$			
M_ν	$\xi = +0.0005$	0.982	0.449	0.012			
	$\xi = -0.0012$	0.990	0.441	0.014			
$M_u^{1/2}$	$a_u = -0.56$	$\sqrt{\frac{m_u}{m_c}} = 0.0425$	$\sqrt{\frac{m_c}{m_t}} = 0.0570$				
	two parameters	5 observables: fitted excellently					
M_d	$a_d e^{i\alpha_d}$	$\sqrt{\frac{m_d}{m_s}}$,	$\sqrt{\frac{m_s}{m_b}}$,	$\lvert V_{us}\rvert$,	$\lvert V_{cb}\rvert$,	$\lvert V_{ub}\rvert$,	$\lvert V_{td}\rvert$
	two parameters	6 observables: not always excellent					

Note that we have obtained a nearly tribimaximal mixing without assuming any discrete symmetry for neutrino sector.

However, the model for down-quark sector still need an improvement.


3 New aspect opened
by the Sumino model
Festival in Osaka

3.1 What did he take seriously?

We know the empirical relation

$$K \equiv \frac{m_e + m_\mu + m_\tau}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2} = \frac{2}{3}. \quad (3.1)$$

[One of the purposes of the yukawaon model was to derive Eq.(3.1)]

The relation (3.1) is satisfied with an accuracy of 10^{-5}

$$K^{pole} = \frac{2}{3} \times (0.999989 \pm 0.000014) \quad (3.2)$$

for the observed charged lepton masses (pole masses).

However, in conventional mass matrix models,

"mass" means not "pole mass", but "running mass".

The formula is only valid with an accuracy of 10^{-3} for $m(\mu)_{ei}$

$$K^{run}_{\mu=M_Z} = \frac{2}{3} \times (1.00189 \pm 0.00002) \quad (3.3)$$

Why is the mass formula (3.1) so remarkably satisfied with the pole masses?

This has been a mysterious problem for long years.

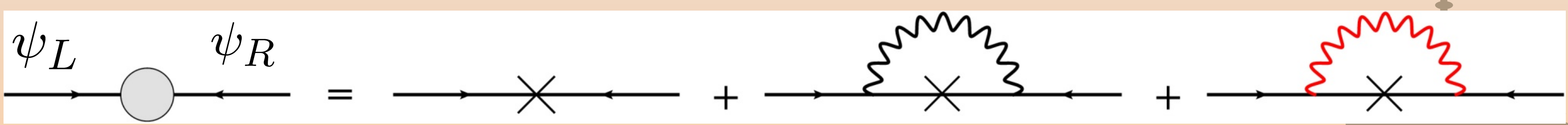
Sumino mechanism

Recently, a possible solution of this problem has been proposed by Sumino [Y.S., PLB 671, 477 (2009); JHEP 0905, 075 (2009)].

The deviation of $K^{run}(\mu)$ **from** K^{pole} **is caused by a term** $m_i \log(\mu/m_i)$ **in the running mass terms.**

He considers that a family symmetry is gauged, and the logarithmic term in the radiative correction to $K^{run}(\mu)$ **due to photon is exactly canceled by that due to family gauge bosons.**

photon **new family gauge bosons**



e e g $-g$

$m_i(\mu)$ m_i^{pole}

$(\psi_L, \psi_R) = (\mathbf{3}, \mathbf{3}^*)$ of U(3)

The family gauge bosons A_i^j **acquire their masses**

$$m_{fij} \equiv m(A_i^j) \propto \sqrt{m_{ei} + m_{ej}} \qquad \textbf{(3.4)}$$

through the VEV $\langle(\Phi_e)_{ii}\rangle_e \propto \sqrt{m_{ei}}$.

Thus, the Sumino model ensures that the charged lepton mass formula (3.1) can reasonably give $K(\mu) = K^{pole}$. Besides, it provides characteristic new effects in TeV region physics, e.g. $e^- + e^- \to \mu^- + \mu^-$ and so on.

For phenomenological aspects of the Sumino model, see our recent paper

Y.K, Sumino & Yamanaka, arXive:1007.4739 [hep-ph]

I think that it is worthwhile noticing the Sumino model, so that my yukawaon model may be modified a little bit.

4 Concluding remarks

Summary

(i) I have considered that the Yukawa coupling constants are not fundamental constants in the theory, but those can be dynamically calculated, i.e. Y_f^{eff} are given by $\langle Y_f \rangle / \Lambda$

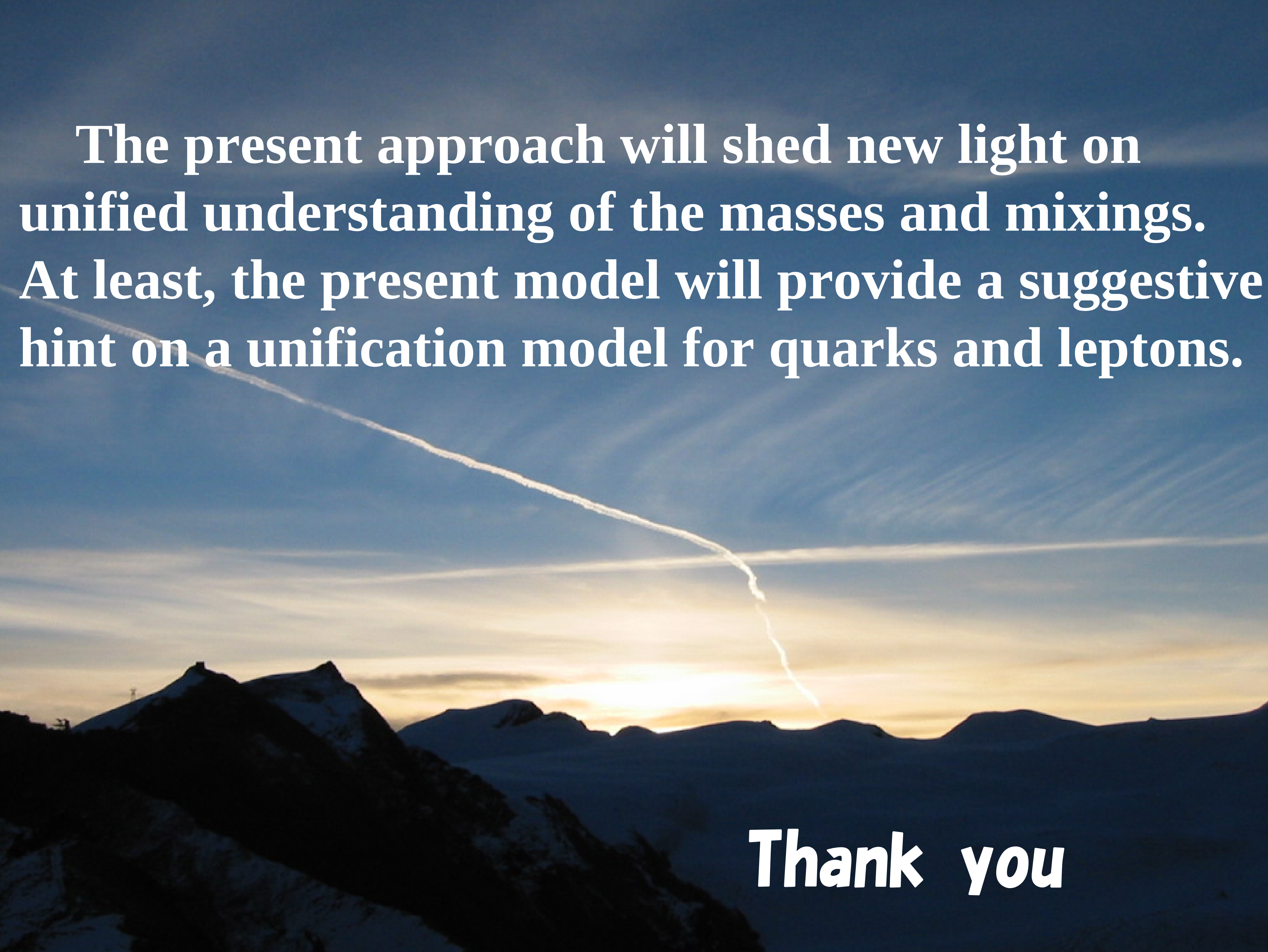
(ii) VEVs of all yukawaons are related to among them by SUSY vacuum conditions.

(iii) There is a possibility that we can describe the quark and lepton mass matrices in terms of a more fundamental VEV matrix $\langle \Phi_e \rangle_e \propto \mathrm{diag}(\sqrt{m_e}, \sqrt{m_\mu}, \sqrt{m_\tau})$

(iv) The "*e*-basis", in which the charged lepton mass matrix is diagonal, has a special meaning in this unified description.

Many problems are still remained

(i) The present model is based on an effective theory.
How can we build a model without Λ ?
(ii) There are many fields in this model.
How can we economize those fields?
(If we want a model without Λ , this will be possible, but we will need many new fields furthermore.)
(iii) In this model, the spectrum of $\langle \Phi_e \rangle$ is essential.
How can we give this spectrum $\langle \Phi_e \rangle \propto \mathrm{diag}(\sqrt{m_e}, \sqrt{m_\mu}, \sqrt{m_\tau})$ more naturally?
(iv) We have used SUSY vacuum conditions in order to obtain yukawaon relations. On the other hand, we know that SUSY is broken. How about the effects on our yukawaon relations?
(v) How to incorporate Sumino's idea into the yukawaon model?


The present approach will shed new light on unified understanding of the masses and mixings. At least, the present model will provide a suggestive hint on a unification model for quarks and leptons.
Thank you