SOMMERFELD AND SUDAKOV CORRECTIONS IN HEAVY NEUTRALINO ANNIHILATION

Guillaume CHALONS

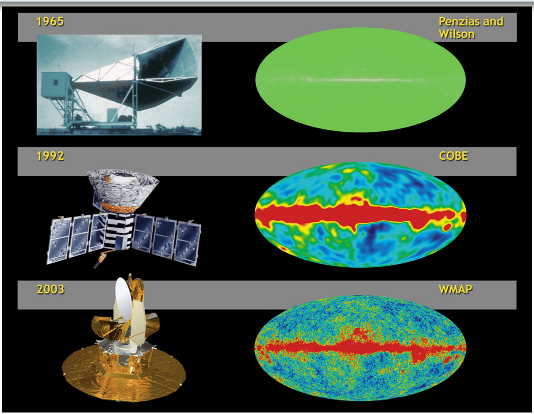
in collaboration with N. Baro, F. Boudjema, Sun Hao

August, 26^{th}, 2010

ERA OF PRECISION MEASUREMENTS

RELIC DENSITY OF DARK MATTER

- WMAP : $0.0997 < \Omega_{DM}h^2 < 0.1221$ (10% precision)
- PLANCK : 2% precision



PRECISION MEASUREMENTS

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$$\Omega_{DM} h^2 \simeq \frac{3 \times 10^{-27} cm^3 s^{-1}}{\langle \sigma(\chi\chi \to SM) v \rangle}$$

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PRECISION

- Need for precise theoretical predictions w.r.t experimental measurements.
- Precision needed at the level of $\sigma \Rightarrow$ One-loop calculations (at least).
- If SUSY found $\Rightarrow$ Reconstruction of fundamental underlying parameters.
- Radiative corrections must be under control to be able to constrain the cosmological underlying scenario.

SOME PREVIOUS WORK AT 1-L IN SUSY

EW + QCD corrections

- $\tilde{\chi}_1^0 \tilde{\chi}_1^0 \to \gamma\gamma, Z\gamma, gg$: Boudjema,Semenov,Temes, *Phys. Rev.* **D72**, 055024 (2005)
- $\tilde{\chi}_1^0 \tilde{\chi}_1^0 \to ZZ, W^+W^-$: Baro,Boudjema,Semenov, *Phys. Lett.* **B660** (2008) 550
  Baro,Boudjema, G.C, Sun Hao, *Phys. Rev* **D81** (2008) 015005
- $\tilde{\chi}_1^0 \tilde{\chi}_1^0 \to \tau^+\tau^-, b\bar{b}$: Baro,Boudjema,Semenov, *Phys. Lett* **B660** (2008) 550
- Co-annihilation with $\tilde{\tau}$: Baro,Boudjema,Semenov, *Phys. Lett* **B660** (2008) 550,

QCD corrections

- Co-annihilation with $\tilde{t}$ Freitas *Phys. Lett.* **B652** (2007) 280
- Annihilation into massive quarks Hermann,Klasen,Kovarik *Phys. Rev.* **D79** (2009)
  Herrmann, Klasen, *Phys. Rev.* **D76** (2007) 117704
  Herrmann, Klasen and Kovarik, *Phys. Rev.* **D80** (2009) 085025

- At tree-level we have for $\tilde{\chi}_1^0\tilde{\chi}_1^0 \to WW$ 7 diagrams.
- Relic density predictions involve many annihilation (and coannihilation) channels.

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Some efficient tree-level codes already exist for relic density calculations :

- DarkSUSY [Bergström *et al.* (2004)]
- micrOMEGAs [Bélanger, Boudjema, Pukhov, Semenov (2002)]
- Mainly $2 \to 2$ processes are taken into account in the computation.

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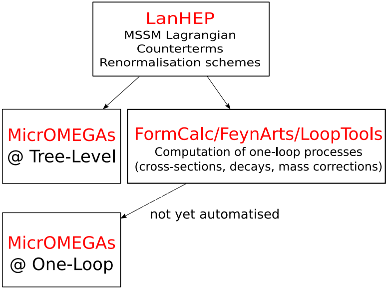
$\longrightarrow$ To deal with IR and collinear divergencies$\longrightarrow$ include bremsstrahlung.

$\longrightarrow$ To evaluate many processes entering $\langle \sigma v \rangle$.



- Evaluation of one-loop diagrams including a complete and coherent renormalisation of each sector of the MSSM with an OS scheme.
- Modularity between different renormalisation schemes.
- Non-linear gauge fixing.
- Handles a large number of Feynman diagrams.
- Checks : results UV,IR finite and gauge independent.

http://code.sloops.free.fr/

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Input parameters as in the Standard Model $\boxed{m_f, \alpha(0), M_W, M_Z}$

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NEUTRALINOS/CHARGINOS SECTOR

Input parameters : 2 charginos $m_{\tilde{\chi}_1^\pm}, m_{\tilde{\chi}_2^\pm}$ and 1 neutralino $\tilde{\chi}_1^0$

RELIC DENSITY OF A HEAVY-WINO

COSMOLOGY (MicrOmegas)

THERMAL RELIC DENSITY

- Solve the Boltzmann equation

$$dn/dt = -3Hn - \langle \sigma v \rangle (n^2 - n_{\text{eq}}^2)$$

Processes with gauge boson production are the most difficult because gauge invariance plays a dominant role.

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THERMAL RELIC DENSITY

- Solve the Boltzmann equation
- Thermal average (MB approx.)

$$\langle \sigma v \rangle \propto \int_0^\infty (\sigma v) v^2 e^{-xv^2/4}\, dv$$

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GAUGE BOSON PRODUCTION

- $SU(2)_L$ type couplings
- Channels contributions $>$ 5% to Ωh^2 at TL corrected at one-loop
- Coannihilation channels
- Large number of diagrams to compute

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$\tilde{\chi}_1^0 \tilde{\chi}_1^\pm W^\pm$ and $\tilde{\chi}_1^\pm \tilde{\chi}_1^\pm Z^0$ vertices

$\alpha(0) \to \alpha(M_Z^2)$: 13% corrections absorbed.

Also $q\bar{q}'$ production $\to$ QCD corrections

Processes with gauge boson production are the most difficult because gauge invariance plays a dominant role.

- Singularities arise in scalar triangle C_0 and box D_0 loop integrals when $\beta \longrightarrow 0$.

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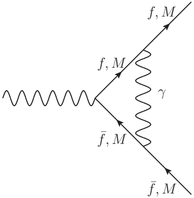

$$C_0 \xrightarrow{\beta \to 0} -\frac{\pi^2}{Q^2 \beta}$$

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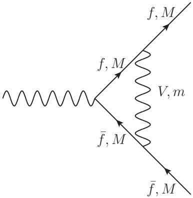

$$C_0 \xrightarrow{\beta \to 0} -\frac{\pi}{m \times M}$$

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- If two heavy masses M and one internal mass very small $m \ll M$
- See also study of these integrals in the non-relativistic limit with application to the relic density [Drees, Kim, Nagao *Phys. Rev.* **D81** (2010) 105004] and Kim's talk

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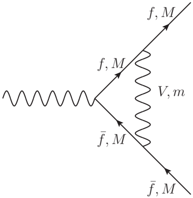

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Results in numerical instabilities (vanishing Gram determinant).
Avoided using Segmentation of the loop integrals. [Boudjema-Semenov-Temes (2005)].
Idea : split 4 pt function $\longrightarrow$ 3 pt function when $\beta \longrightarrow 0$.

Parameter	M_1	M_2	μ	t_β	M_3	$M_{\tilde{L},\tilde{Q}}$	A_i	M_{A^0}
Value(GeV)	3500	1800	4500	15	5000	5000	0	5000

$$\tilde{\chi}_1^0 = 0.000\tilde{B} - 0.999\tilde{W} + 0.004\tilde{H}_1^0 + 0.032\tilde{H}_2^0$$

		Tree
$\tilde{\chi}_1^0\tilde{\chi}_1^0 \to W^+W^-$ [10%]	a	$+2.43$
	b	$+0.52$
$\tilde{\chi}_1^+\tilde{\chi}_1^+ \to W^+W^+$ [10%]	a	$+1.22$
	b	$+0.26$
$\tilde{\chi}_1^0\tilde{\chi}_1^+ \to Z^0W^+$ [9%]	a	$+0.51$
	b	$+0.12$
$\tilde{\chi}_1^0\tilde{\chi}_1^+ \to t\bar{b}$ [9%]	a	$+0.54$
	b	-0.23
$\tilde{\chi}_1^0\tilde{\chi}_1^+ \to u\bar{d}$ [9%]	a	$+0.54$
	b	-0.23
$\tilde{\chi}_1^+\tilde{\chi}_1^- \to Z^0Z^0$ [6%]	a	$+0.73$
	b	$+0.16$
$\tilde{\chi}_1^+\tilde{\chi}_1^- \to W^+W^-$ [6%]	a	$+0.65$
	b	$+0.17$
$\Omega_\chi h^2$		0.0997

- $m_{\tilde{\chi}_1^0} = 1799.1$ GeV
- $\delta(m_{\tilde{\chi}_1^+} - m_{\tilde{\chi}_1^0}) = 0.0003$ GeV

- $\sigma_0 v = a + bv^2$
- $m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_1^\pm}$ almost degenerate
- Coannihilation very important
- Degeneracy between processes $\tilde{\chi}_1^0\tilde{\chi}_1^0 \to W^+W^-$ and $\tilde{\chi}_1^+\tilde{\chi}_1^+ \to W^+W^+$
- A lot of processes contribute

HEAVY-WINO NEUTRALINO

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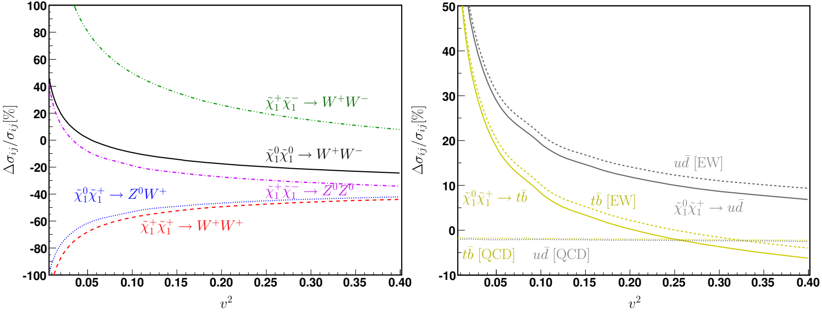

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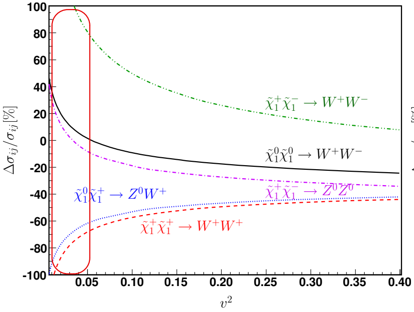
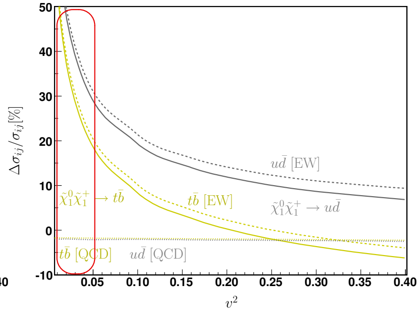
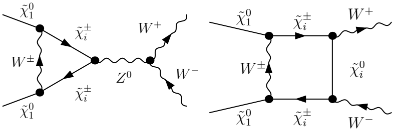
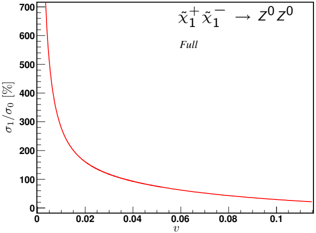




- $M_W/m_{\tilde{\chi}_1^0} = 0.045 \Rightarrow W^\pm, Z^0$ bosons almost considered as massless.
- $v \longrightarrow 0$: Large Sommerfeld (QED+EW) enhancement.



- The EW Sommerfeld effect is expected to be cut-off, as opposed to the QED one.
- To extract it, remove the QED Coulomb effect first.

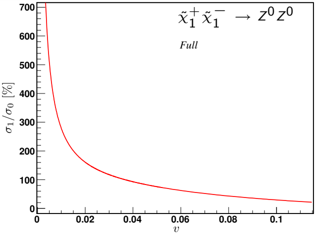


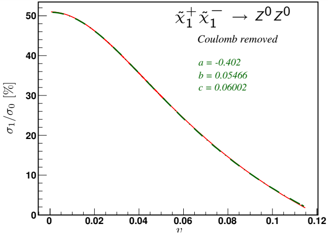
- $S_{nr} = X_{nr}/(1 - e^{-X_{nr}})$ $\quad X_{nr} = 2\pi\alpha Q_i Q_j / v$
- $S_{1L} = \frac{\pi\alpha}{v} \times \sigma_0 Q_i Q_j$

- The EW Sommerfeld effect is expected to be cut-off, as opposed to the QED one.
- To extract it, remove the QED Coulomb effect first.
- Then, as behavior expected to be cut-off, fit with,

$$\sigma_1/\sigma_0 = a + \frac{b}{\sqrt{v^2 + c^2}}$$

where c is supposed to be the cut-off, of order $M_W/m_{\tilde{\chi}_1^0}$.





- Large corrections but $<$ QED Sommerfeld for $v \to 0$.

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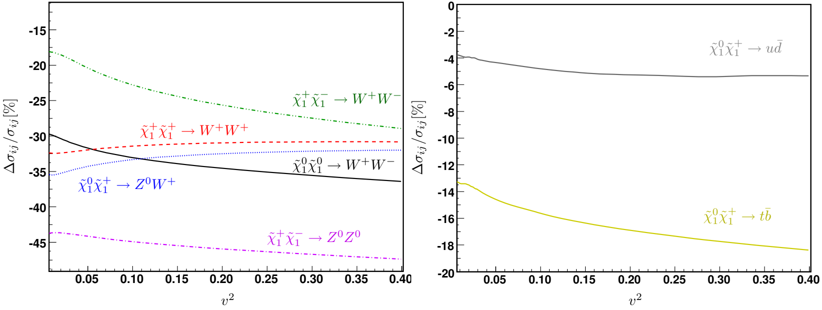

EXTRACTING THE ONE-LOOP SOMMERFELD EFFECT

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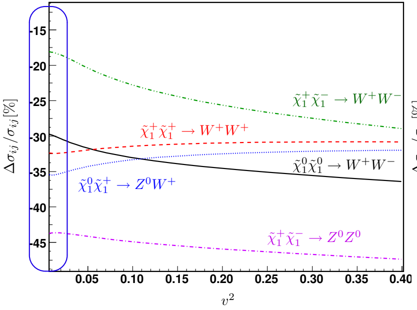
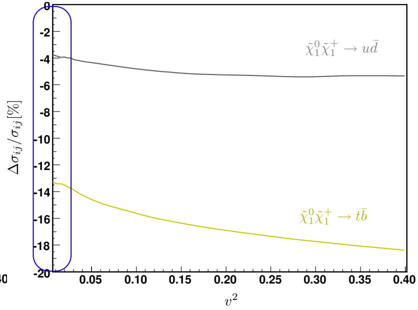
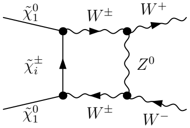


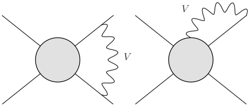


- $v \to 1$: Large negative corrections of Sudakov type.



- Originate from vertex and box diagrams involving virtual bosons.
- General form of one-loop Sudakov corrections

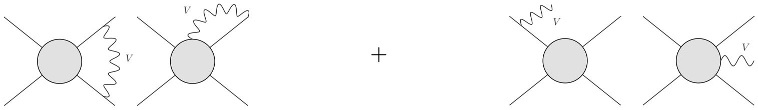
$$\alpha\left[C_2 \underbrace{\ln^2\left(\frac{s}{M_V^2}\right)}_{\text{LL}} + C_1 \underbrace{\ln^1\left(\frac{s}{M_V^2}\right)}_{\text{NLL}} + C_0\right] + \mathcal{O}\left(\frac{M_V^2}{s}\right) \quad V = \gamma, W^{\pm}, Z^0$$



- The $\ln(s/M_V^2)$ represents mass singularities and originate from soft and collinear regions.
- For QED corrections always present ($M_\gamma \to 0$), for EW ones when $s \gg M_{W,Z}^2$.

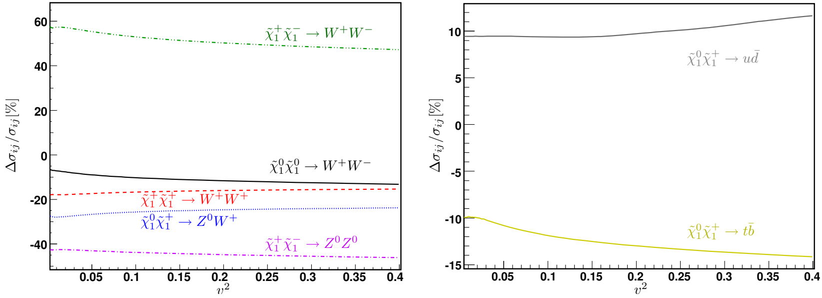
- Originate from vertex and box diagrams involving virtual bosons.
- General form of one-loop Sudakov corrections

$$\alpha\left[C_2 \underbrace{\ln^2\left(\frac{s}{M_V^2}\right)}_{\text{LL}} + C_1 \underbrace{\ln^1\left(\frac{s}{M_V^2}\right)}_{\text{NLL}} + C_0\right] + \mathcal{O}\left(\frac{M_V^2}{s}\right) \quad V = \gamma, W^\pm, Z^0$$



- The $\ln(s/M_V^2)$ represents mass singularities and originate from soft and collinear regions.
- For QED corrections always present ($M_\gamma \to 0$), for EW ones when $s \gg M_{W,Z}^2$.
- Dependency on M_γ unphysical $\Rightarrow$ removed by adding real emission as stated by the Bloch-Nordsieck theorem [Bloch,Nordsieck(1937)].
- For EW corrections, $M_{W,Z}$ physical and retained in the calculation.
- Adding real emission can counterbalance virtual corrections.

ADDING REAL EMISSION AND SUBSTRACTING SOMMERFELD



- Still large correction for some processes (-45% corrections for $\tilde{\chi}_1^+\tilde{\chi}_1^- \to Z^0 Z^0$).

RELIC DENSITY

	Tree	$\mathrm{A}_{\tau\tau}$	$+ Z^0$ brem	+ Coul resum	EW Som removed
$\Omega_\chi h^2$	0.0993	0.104	0.0934	0.0934	0.103
$\frac{\delta\Omega_\chi h^2}{\Omega_\chi h^2}$		$+4.7\%$	-5.9%	-5.9%	$+3.7\%$

- Taking into account Z^0 real emission or not has an important effect.
- The QED Coulomb one-loop effect is enough, and doesn't change the result.
- One-loop EW Sommerfeld still relevant for relic density calculation.
- Large corrections for individual processes but final result not so affected, large compensation between processes.

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ORIGIN OF LARGE REMAINING CORRECTIONS

- Non-compensation between real and virtual corrections.
- Due to Bloch-Norsieck violations [Ciafaloni, Ciafaloni, Comelli (2000)] ?
- Non cancellation between real and virtual contributions due to W emission.
- One-loop expansion not enough $\Rightarrow$ Two-loop effects important ?

- Importance of radiative corrections in the relic density calculations.
- Need to control them to be able to extract informations from it and to constrain the underlying cosmological scenario.
- For a heavy neutralino scenarios taking into account $2 \longrightarrow 3$ processes is necessary.
- Large corrections due to soft/collinear logs and Sommerfeld enhancement.
- Study of the dependency of the results on the chargino/neutralino renormalisation scheme.
- Improve the interface with `micrOMEGAs` .

BACKUP

A WORD ABOUT LOOP INTEGRALS

- Loop tensor integrals reduced to a basis of scalar integrals [Passarino-Veltman (1979)]
- Reduction method rely on a kinematical ingredient : The Gram Determinant.
- For $2 \longrightarrow 2$ processes, Gram determinant vanishes when relative velocity $v \longrightarrow 0$
- In this case reduction method inefficient $\Rightarrow$ different approach

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- In this case reduction method inefficient $\Rightarrow$ different approach

- Segmentation has been used to study the analytical and numerical behaviour for $v \longrightarrow 0$ [Boudjema-Semenov-Temes (2005)].

$$\begin{aligned}
\frac{1}{D_0 D_1 D_2 D_3} &= \left(\frac{1}{D_0 D_1 D_2} - \alpha \frac{1}{D_0 D_2 D_3} - \beta \frac{1}{D_0 D_1 D_3} + (\alpha + \beta - 1) \frac{1}{D_1 D_2 D_3} \right) \times \\
& \quad \frac{1}{A + 2\ell \cdot (s_3 - \alpha s_1 - \beta s_2)} \\
A &= (s_3^2 - M_3^2) - \alpha(s_1^2 - M_1^2) - \beta(s_2^2 - M_2^2) - (\alpha + \beta - 1) M_0^2 . \\
D_i &= (\ell + s_i)^2 - M_i^2 , \; s_i = \sum_{j=1}^{i} p_j
\end{aligned}$$

Relevant mostly for indirect detection : $\chi\chi \rightarrow W^+ W^-, \gamma\gamma \cdots$ in our galaxy ($v \simeq 10^{-3} c$).

SEGMENTATION

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$$
\begin{aligned}
\frac{1}{D_0 D_1 D_2 D_3} &= \left(\frac{1}{D_0 D_1 D_2} - \alpha \frac{1}{D_0 D_2 D_3} - \beta \frac{1}{D_0 D_1 D_3} + (\alpha+\beta-1) \frac{1}{D_1 D_2 D_3} \right) \times \\
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\end{aligned}
$$

- For any graph if $\det G(s_1, s_2, s_3) \simeq 0$, construct all 3 sub-determinants $\det G(s_1, s_2)$ and take the couple s_i, s_j (as independant basis) that corresponds to $\mathrm{Max}|\det G(s_i, s_j)|$.Then

$$
s_3 = \alpha s_1 + \beta s_2 + \varepsilon_T \quad \text{with } s_1 \cdot \varepsilon_T = s_2 \cdot \varepsilon_T = 0
$$

$$
\alpha = \frac{s_2^2 (s_3 \cdot s_1) - (s_1 \cdot s_2)(s_2 \cdot s_3)}{\det G(s_1, s_2)}, \quad \beta = \alpha (s_1 \leftrightarrow s_2)
$$

$$
\det G(s_1, s_2, s_3) = \varepsilon_T^2 \det G(s_1, s_2)
$$

GAUGE FIXING

Linear gauge fixing

$$\mathcal{L}_{GF} = -\frac{1}{\xi_W}|\partial_\mu W^{\mu +} + i\xi_W \frac{g}{2} v G^+ |^2 - \frac{1}{2\xi_Z}(\partial_\mu Z^\mu + \xi_Z \frac{g}{2c_w} v G^0)^2 - \frac{1}{2\xi_A}(\partial_\mu A^\mu)^2$$

$$\Gamma^{VV} = \frac{-i}{q^2 - M_V^2 + i\epsilon}\left[g_{\mu\nu} + (\xi_V - 1)\frac{q_\mu q_\nu}{q^2 - \xi_V M_V^2}\right]$$

Linear gauge fixing

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$\xi_{W,Z,A} = 1$ (Feynman gauge)

Non-Linear gauge fixing

$$
\begin{aligned}
\mathcal{L}_{GF} \quad = \quad & -\frac{1}{\xi_W}|(\partial_\mu - ie\tilde{\alpha}A_\mu - igc_w\tilde{\beta}Z_\mu)W^{\mu\,+} \\
& +i\xi_W\frac{g}{2}(v+\tilde{\delta}h^0+\tilde{\omega}H^0+i\tilde{\kappa}G^0+i\tilde{\rho}A^0)G^+\,|^2 \\
& -\frac{1}{2\xi_Z}(\partial_\mu Z^\mu+\xi_Z\frac{g}{2c_w}(v+\tilde{\epsilon}h^0+\tilde{\gamma}^0_H)G^0)^2 \\
& -\frac{1}{2\xi_A}(\partial_\mu A^\mu)^2
\end{aligned}
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GAUGE FIXING

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& -\frac{1}{2\xi_A}(\partial_\mu A^\mu)^2
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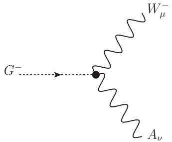

$eM_W(1 + \tilde{\alpha}g_{\mu\nu})$

$\xi_{W,Z,A} = 1$ (Feynman gauge)

→ Gauge parameter dependence in gauge/Goldstone/ghost vertices.
→ No "unphysical" threshold, no higher rank tensor.

SUDAKOV VIRTUAL+REAL CORRECTIONS : ABELIAN EXAMPLE

- However adding real emission of EW gauge boson can counterbalance virtual effects.
- Abelian $Z' \to \bar{\nu}\nu + Z^0$ (of mass $\sqrt{s}$) as an example (in the limit $s \gg M_Z^2$) :

$$\Gamma^{V}{}_{\nu\bar{\nu}} = - \Gamma^0_{\nu\bar{\nu}} \frac{\alpha_Z}{4\pi}\left[2\left(\ln^2\left(\frac{m_Z^2}{s}\right) + 3\ln\left(\frac{m_Z^2}{s}\right)\right) - \frac{2\pi^2}{3} + 7\right]$$

$$\Gamma^{R}{}_{\nu\bar{\nu}} = + \Gamma^0_{\nu\nu} \frac{\alpha_Z}{4\pi}\left[2\left(\ln^2\left(\frac{m_Z^2}{s}\right) + 3\ln\left(\frac{m_Z^2}{s}\right)\right) - \frac{2\pi^2}{3} + 10\right]$$

- Complete compensation between virtual and real logarithmic corrections.

- Factorisation and universality : $\mathcal{M}_1^{Sud} = \mathcal{M}_0 \times$ Sudakov form factor
- For our heavy-wino case Sudakov corrections important ($M_W^2/m^2_{\tilde{\chi}_1^0} = 2.10^{-3}$).
- $2 \to 3$ to be taken into account for relic density.
- Real emission of a Z^0 boson added.