Dirac-Born-Infeld actions and the dilaton in N=2 Supersymmetry

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General setting

- Type II string compactifications on Calabi-Yau to $D = 4 \Rightarrow N = 2$ supersymmetry in the bulk
- D-branes break half of supersymmetry which is then nonlinearly realised on their worldvolume
- Gauge field on D-brane described by Dirac-Born-Infeld (DBI) action
- Complete effective action includes a topological term with coupling to RR fields
- Dilaton belongs to Universal Hypermultiplet, which has Heisenberg symmetry preserved in string perturbation theory
- Shift symmetries of the Universal Hypermultiplet allow dualisation of one or two scalars into two-forms $\Rightarrow$ single-tensor or double-tensor descriptions

Description

- Global $N = 2$ supersymmetry realised on $N = 1$ superfields
- Hypermultiplet dualised into single-tensor to allow off-shell supersymmetric description
- DBI and nonlinear supersymmetry introduced by constrained superfields
- Lagrangian given by supersymmetrisation of $b \wedge F$ interaction with one linear and one nonlinear supersymmetry
- Nonlinear $N = 2$ vector multiplet coupled to full single-tensor: no orientifold truncation
- Bosonic fields are:

  In abelian vector: A_μ ($F_{\mu\nu}$) and the auxiliary d_2.
  In single-tensor: $b_{\mu\nu}$, C (dilaton), Φ (complex RR scalar), 4-form $C_{\mu\nu\rho\sigma}$ (see later).

Dilaton-Brane coupling and DBI

After integration of auxiliary field d_2 the bosonic Lagrangian is

$$\begin{aligned}\mathcal{L}_{DBI,bos.} &= \frac{\mathrm{Re}\,\phi}{4\kappa}\left[1 - \sqrt{1 + \frac{C^2}{2(\mathrm{Re}\,\phi)^2}}\sqrt{-\det(\eta_{\mu\nu} + 2\sqrt{2}\kappa F_{\mu\nu})}\right] \\ &\quad + \epsilon^{\mu\nu\rho\sigma}\left(\frac{\kappa}{4}\mathrm{Im}\,\phi\, F_{\mu\nu}F_{\rho\sigma} - \frac{1}{4}b_{\mu\nu}F_{\rho\sigma} + \frac{1}{24\kappa}C_{\mu\nu\rho\sigma}\right)\end{aligned}$$

κ (mass^{-2}) is the nonlinear scale.
We recovered the DBI action with

- field dependent coefficient in front of the square root generated by integration of auxiliary field
- Full topological term $\sim \sum_k C_k \wedge e^F$
- Invariance under non-linearly realised 2nd SUSY
- C arises as the $M_P \to \infty$ limit of the dilaton for a quaternionic-Kähler manifold with Heisenberg symmetry (in string perturbation theory) [AADT, 1005.0323]

Analysis of the vacuum: super-Higgs without gravity

There is a scalar potential

$$V(C, \mathrm{Re}\,\phi) = \frac{\mathrm{Re}\,\phi}{4\kappa}\left[\sqrt{1 + \frac{C^2}{2(\mathrm{Re}\,\phi)^2}} - 1\right]$$

- $\mathrm{Im}\,\phi$ is a flat direction.
- SUSY vacuum at $\langle C \rangle = 0$, for any value of $\mathrm{Re}\,\phi$.

Properties of SUSY vacuum:

- ϕ is massless
- C is massive $m_C^2 = \frac{1}{4\kappa\langle \mathrm{Re}\,\phi \rangle}$, same mass as the vector
- Partial breaking of supersymmetry $N = 2$ to $N = 1$
- Gaugino transforms as a goldstino, but is not massless
- The mixing term $\chi\lambda$ gave a mass to the gaugino, but 2nd SUSY preserved since its variation is canceled by the variation of the 4-form.

Realisation of $N = 2$ on $N = 1$ superfields

Off-shell realisation with two real $N = 1$ superfields V_1, V_2 $(16_B + 16_F)$:

$$\delta^* V_1 = -\frac{i}{\sqrt{2}}(\eta D - \overline{\eta D})V_2 \,, \quad \delta^* V_2 = \sqrt{2}i(\eta D - \overline{\eta D})V_1$$

Works for two off-shell $N = 2$ multiplets (not for hypermultiplets):

1. Impose supersymmetric constraint on V_1 and V_2:
   $V_1 = L$ linear $(\overline{DD}L = 0)$, $V_2 = \Phi + \overline{\Phi}$, Φ chiral.
   $\Rightarrow$ Single-tensor multiplet (L, Φ), $8_B + 8_F$ off-shell.
   Bosons: real C, complex Φ, $b_{\mu\nu}$ with $\delta b_{\mu\nu} = 2\partial_{[\mu}\Lambda_{\nu]}$, auxiliary f_Φ.
2. Impose gauge invariance on V_1 and V_2
   Use the single-tensor $(\Lambda_\ell, \Lambda_c)$ as gauge parameters:

$$\delta_{gauge} V_1 = \Lambda_\ell \,, \qquad \delta_{gauge} V_2 = \Lambda_c + \overline{\Lambda}_c$$

$\Rightarrow$ $N = 2$ abelian vector multiplet, $8_B + 8_F$ off-shell.

The constrained vector multiplet

Gauge invariant vector multiplet:

$$X = \frac{1}{2}\overline{DD}V_1 \,, \qquad W_\alpha = -\frac{1}{4}\overline{DD}D_\alpha V_2 \,.$$

Can be embedded into usual $N = 2$ chiral superfield

$$\mathcal{W} = X + \sqrt{2}i\widetilde{\theta}^\alpha W_\alpha - \frac{1}{4}\widetilde{\theta}\widetilde{\theta}\overline{DDX}$$

Nonlinear supersymmetry can be obtained imposing a quadratic constraint

$$\left(\mathcal{W} - \frac{1}{2\kappa}\widetilde{\theta}\widetilde{\theta}\right)^2 = \mathcal{W}^2 - \frac{1}{\kappa}\widetilde{\theta}\widetilde{\theta}\mathcal{W} = 0 \Longleftrightarrow WW - \frac{1}{2}X\overline{DDX} = \frac{1}{\kappa}X$$

- Need to deform the 2nd SUSY transformation of W_α
- Gaugino transforms as a goldstino.
- Solution of the constraint $X(WW)$ contains the Born-Infeld

Single-tensor version II

The $N = 1$ superfields (L, Φ) supersymmetrise $H_{\mu\nu\rho} = 3\partial_{[\mu} B_{\nu\rho]}$ with $8_B + 8_F$ off-shell.

To supersymmetrise $b_{\mu\nu}$ one needs $16_B + 16_F$ with gauge invariance: (Y, χ_α, Φ) with $L = D^\alpha \chi_\alpha - \overline{D}_{\dot\alpha} \overline{\chi}^{\dot\alpha}$ and Y a chiral superfield containing the four-form $C_{\mu\nu\rho\sigma}$

We can construct a chiral $N = 2$ superfield

$$\mathcal{Y} = Y + \sqrt{2}\widetilde{\theta}^\alpha \chi_\alpha - \widetilde{\theta}\widetilde{\theta} \left[\frac{i}{2}\Phi + \frac{1}{4}\overline{DDY} \right]$$

Bosonic fields:

$N = 1$ superfield	Field	Gauge invariance	Number of fields
χ_α	$b_{\mu\nu}$	$\delta b_{\mu\nu} = 2\,\partial_{[\mu}\Lambda_{\nu]}$	$6_B - 3_B = 3_B$
	C		1_B
Φ	Φ		2_B
	f_Φ		2_B (auxiliary)
Y	$C_{\mu\nu\rho\sigma}$	$\delta\, C_{\mu\nu\rho\sigma} = 4\,\partial_{[\mu}\Lambda_{\nu\rho\sigma]}$	$1_B - 1_B = 0_B$

Supersymmetric coupling

- D-brane gauge field is described by constrained vector multiplet $\mathcal{W}_{nl} = \mathcal{W} - \frac{1}{2\kappa}\widetilde{\theta}\widetilde{\theta}$, $\mathcal{W}_{nl}^2 = 0$
- Dilaton multiplet is described by single-tensor $\mathcal{Y}$
- $N = 2$ Supersymmetric coupling with one linear and one nonlinear supersymmetry is given by

$$\mathcal{L} = \int d^2\theta d^2\widetilde{\theta}\, i\mathcal{Y}\mathcal{W}_{nl} + \mathrm{c.c.} + \text{single-tensor kinetic terms}$$

which is the supersymmetrisation of $b \wedge F$ (Chern-Simons).

Conclusions

- Global $N = 2$ supersymmetry and Heisenberg symmetry uniquely determine the form of the theory
- Generalized the derivation of the DBI from constrained superfields to the coupling to the dilaton (single-tensor) with one linear and one nonlinear supersymmetry
- Nonlinear $N = 2$ vector multiplet coupled to full single-tensor: no orientifold truncation needed
- New super-Higgs without gravity
- Low energy effective theory description of string theory compactified on rigid Calabi-Yau including perturbative corrections

Web of supersymmetric dualities