Monte Carlo simulation of Wilson loop operator in N=4 SYM

Goro Ishiki (CQUeST, Korea)

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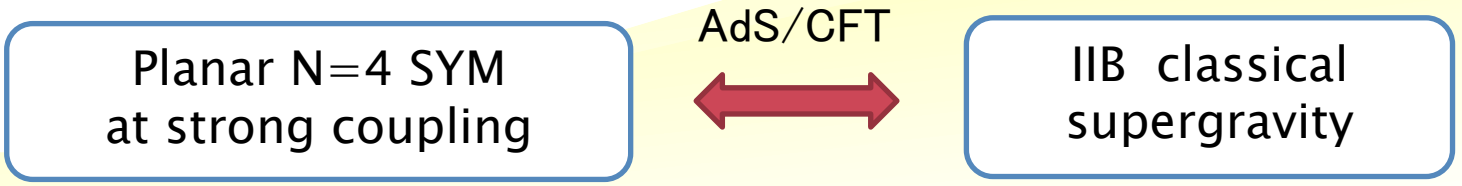
In collaboration with
M. Honda (KEK), J. Nishimura (KEK), A. Tsuchiya (Shizuoka U.)

AdS/CFT correspondence



- We need to study strongly coupled planar N=4 SYM to show AdS/CFT.
  - BPS region, BMN limit, Integrability
  - Numerical approach, Lattice theory ?

No lattice theory for N=4 SYM (SUSY is difficult to keep on lattice.)

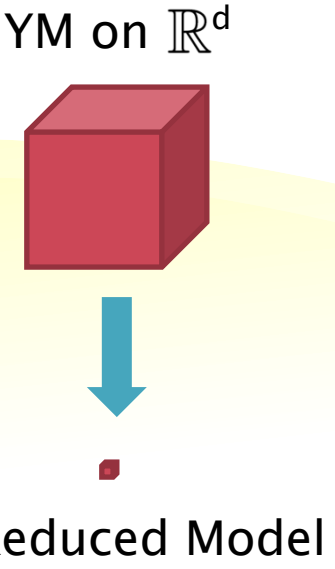
→ We can use the plane wave (BMN) matrix model instead of lattice to analyze planar N=4 SYM on $R \times S^3$ non-perturbatively !!!

- One can preserve gauge sym. and SUSY naturally in the matrix reg.
  - PWMM has gauge sym. and SU(2|4) sym (16 SUSY)
  - They are manifestly preserved in this method.

Large N reduction [Eguchi-Kawai]

"volume independence" at large N.

YM theory on $\mathbb{R}^d$ is equivalent to its "dimensionally reduced matrix model" in the large N limit.



◆ Matrix models give non-perturbative definition of large N theories.

Matrix regularization

N=4 SYM on $R \times S^3$

N→∞ (large N reduction)

Plane wave matrix model

Cf.) Lattice regularization

QCD

Continuum limit

Lattice QCD

One can simulate using computer !!!

Aim of this project

- Non-perturbative analysis of N=4 SYM using PWMM.
- Applying this method to AdS/CFT.

Our Results

- We give a nonperturbative formulation of planar N=4 SYM on $R \times S^3$ in terms of the plane wave matrix model (PWMM).
- We reproduced free energy in SYM at weak coupling from PWMM.
- We compute circular BPS Wilson loop operator.

Plan of Talk

1. Introduction

2. Non-perturbative reguralization of N=4 SYM on $R \times S^3$

3. Free energy at week coupling from PWMM

4. Circular Wilson loop from PWMM

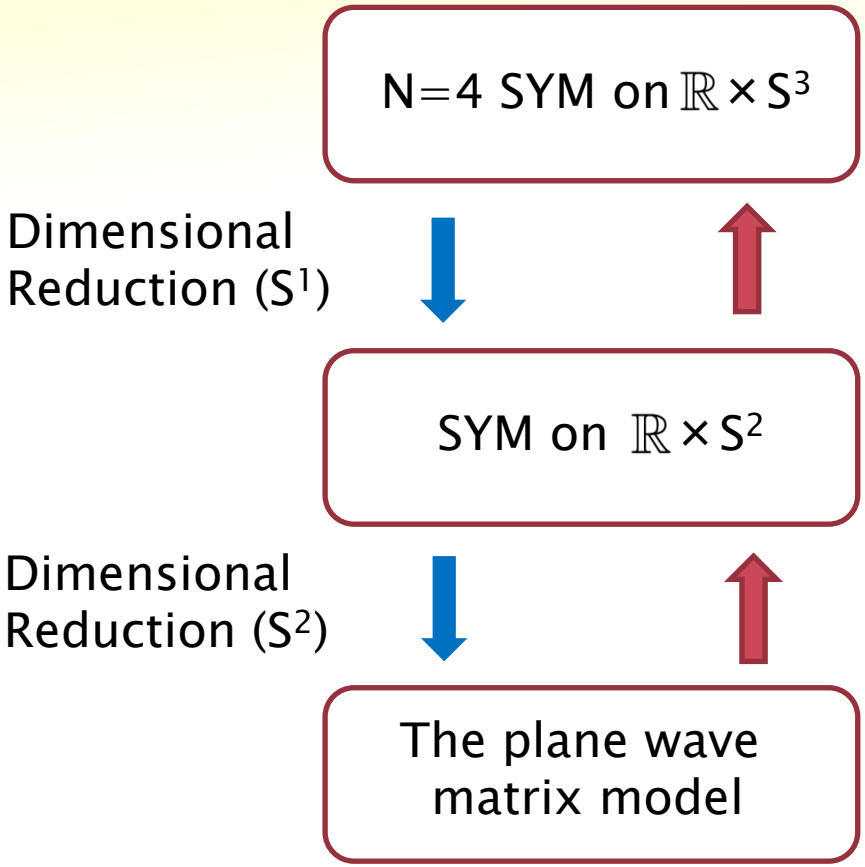
5. Summary & Outlook

2. Non-perturbative reguralization of planar N=4 SYM on $R \times S^3$

Construction of S^3 from reduced model

[Ishii-Ishiki-Simasaki-Tsuchiya, Ishiki-Shimasaki-Takayama-Tsuchiya]

N=4 SYM on $\mathbb{R} \times S^3$ is realized from the reduced model as follows,



Locally, $S^3 \sim S^1 \times S^2$

(1) Large N reduction
(Realization of S^1)

(2) Commutative limit of fuzzy sphere
(Realization of S^2)

By combining (1) & (2),
one can realize a gauge theory on S^3
from its reduced matrix model.

Dimensional reduction

- N=4 Super Yang-Mills on $R \times S^3$

$$S_{R\times S^3/Z_k} = \frac{1}{g^2_{YM3}} \int dt d\Omega_3 \mathrm{Tr} \left(-\frac{1}{4} F_{ij} F_{ij} + (\dot{A}_i, A_0, X^m, \lambda) \right)$$

Dropping the derivative for S^3

- Plane wave matrix model [BMN]

$$S_{PW} = \frac{1}{g^2_{PW}} \int dt \ \mathrm{Tr} \left(-\frac{1}{2} (Y_i - \frac{i}{2} \epsilon_{ijk} [Y_j, Y_k])^2 + \cdots \right)$$

[Ishii-GI-Shimasaki-Tsuchiya]

Planar N=4 SYM on $\mathbb{R} \times S^3$ is realized from PWMM as follows.

$$S_{PWMM} = \frac{1}{g^2} \int dt \, \mathrm{tr} \left\{ -\frac{1}{2} \left(X_i + \frac{i}{2} \epsilon_{ijk} [X_j, X_k] \right)^2 + \cdots \right\} \qquad i = 1, 2, 3$$

We expand the matrices in PWMM around the following vacuum,

$$X_i = \begin{pmatrix} \ddots & & & \\ & L_i^{[j_s-1]} & & \\ & & L_i^{[j_s]} & \\ & & & L_i^{[j_s+1]} \\ & & & & \ddots \end{pmatrix} \otimes \mathbf{1}_{k \times k}$$

$$2j_s + 1 = N_0 + s \qquad s = -\frac{\Lambda}{2}, -\frac{\Lambda}{2} + 1, \cdots, \frac{\Lambda}{2}$$

continuum limit

$N_0 \to \infty$ (S^2)

$\Lambda \to \infty$ (S^1)

$k \to \infty$

Then, PWMM around this vacuum $\equiv$ planar N=4 SYM on $R \times S^3$

↑

Fuzzy sphere + Large N reduction

3. Free energy at weak couopling from PWMM

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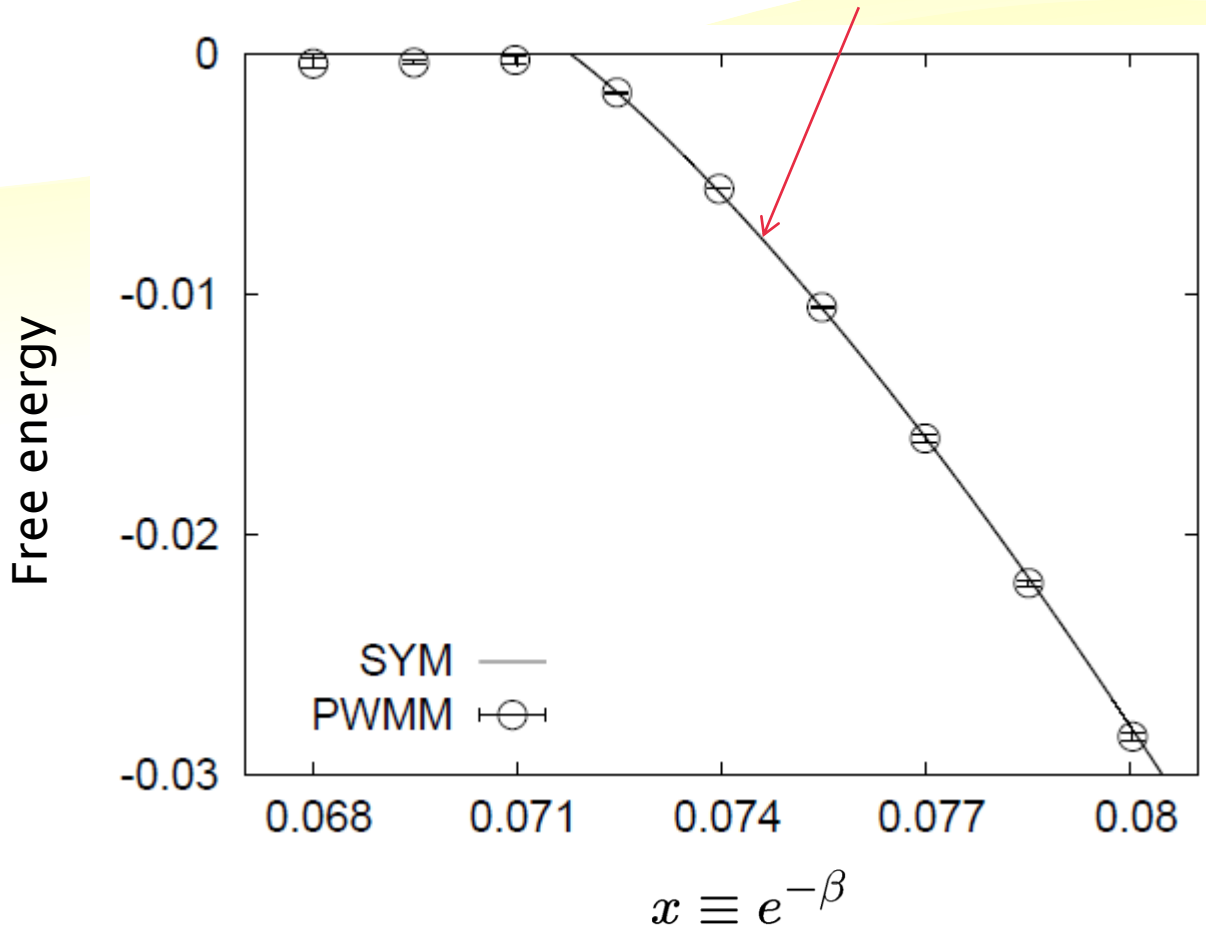
[GI-Kim-Nishimura-Tsuchiya]

Free Energy

1-loop Free energy in N=4 SYM on $R \times S^3$
[Aharony-Marsano-Minwalla-Papadodimas-Raamsdonk]



- Free energy at 1-loop level is reproduced from PWMM.
- Behavior at high temp. ~ T^4 is also reproduced. [Kitazawa-Matsumoto]
- One can derive the correct critical temperature from PWMM analytically.

4. Circular BPS Wilson loop from PWMM

Wilson Loop operator in PWMM

[Ishii-GI-Shimasaki-Ohta-Tsuchiya]

Wilson loop operator on noncommutative plane

$$W(C) = \mathrm{Tr} P \exp\left(\int_C \hat{A}_\mu dx^\mu\right)$$ [Ishibashi-Iso-Kawai-Kitazawa]

One can also construct an operator in PWMM which is reduced to the Wilson loop operator in SYM on $\mathbb{R} \times S^3$ in the continuum limit.

$$W(C) = \mathrm{Tr} P \exp\left(\int_C X_A \, e_M^A(z) dz^M\right) \to \mathrm{Tr} P \exp\left(\int_C A_M(z) dz^M\right)$$

Continuum limit

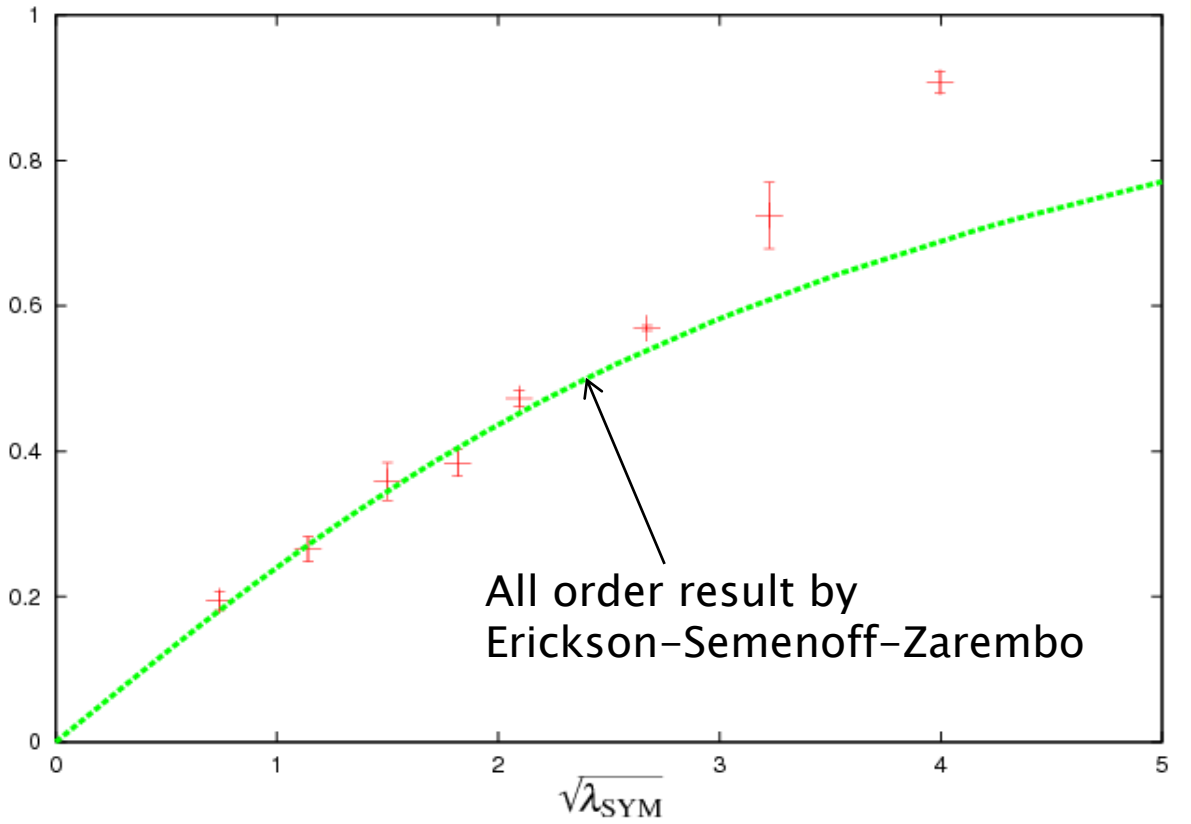
$e_M^A(z)dz^M$: Right-invariant 1-form on $S^3 \sim SU(2)$

$X_A \ \ (A = 1, 2, 3)$: SO(3) scalars in PWMM.

Circular BPS Wilson loop

[in progress with Honda, Nishimura, Tsuchiya]

$$\frac{1}{\sqrt{\lambda_{\rm SYM}}} \log W(\text{circle})$$



First result for non-perturbative simulation of N=4 SYM !!!

Summary

- Planar N=4 SYM on $R \times S^3$ is regularized by PWMM nonperturbatively. Gauge sym. and SU(2|4) sym (16 susy) are preserved.

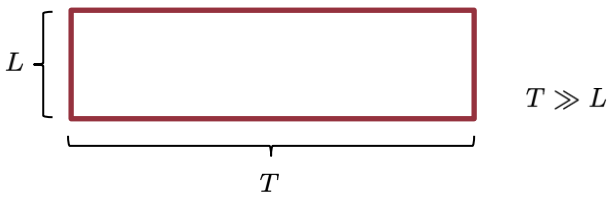
- Free energy at weakly coupling and circular BPS Wilson loop are reproduced from PWMM.

Future work

- Simulation of strongly coupled N=4 SYM at finite temperature.
  - Hawking-Page transition from N=4 SYM ?

- Computation of rectangular Wilson loop at strong coupling.
  - Gravity side predicts the result at strong coupling. [Alday-Maldacena]

$$\log\langle W(\text{rectangle})\rangle = \sqrt{\lambda}\frac{4\pi^2}{\Gamma(1/4)^4}\frac{T}{L}$$



[in progress with Honda, Nishimura, Tsuchiya]