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On the origin of neutrino flavour symmetry

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Tri-bimaximal lepton mixing

$$\text{PMNS} \;\approx\; U_{TB} \;\equiv\; \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$\Rightarrow$

PMNS-angles	tri-bimax.	1σ exp.
$\sin^2\theta_{12}$:	$\frac{1}{3}$	$0.288 - 0.326$
$\sin^2\theta_{23}$:	$\frac{1}{2}$	$0.44 - 0.57$
$\sin^2\theta_{13}$:	0	≤ 0.026

Schwetz, Tórtola, Valle (2008)

Non-Abelian family symmetries

The symmetries of the neutrino mass matrix

neutrino flavour basis

$$M_\ell = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} \qquad M_\nu \sim U_{TB}\, M_\nu^{\mathrm{diag}}\, U_{TB}^T$$

$$\sim \begin{pmatrix} \alpha & \beta & \beta \\ \beta & \gamma & \alpha+\beta-\gamma \\ \beta & \alpha+\beta-\gamma & \gamma \end{pmatrix}$$

$$\left[\alpha = \frac{1}{3}(2m_{\nu_1} + m_{\nu_2}) \quad \beta = \frac{1}{3}(-m_{\nu_1} + m_{\nu_2}) \quad \gamma = \frac{1}{6}(m_{\nu_1} + 2m_{\nu_2} + 3m_{\nu_3})\right]$$

How can we obtain such relations between elements of a mass matrix?

invoke a symmetry $\mathcal{G} \ni S, T, U, \dots$ Lam (2008)

$$M_\nu \;=\; S^T M_\nu S \;=\; U^T M_\nu U$$

$$\Rightarrow \quad S \;=\; \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix} \qquad U \;=\; -\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\bar{M}_\ell \;=\; T^T \bar{M}_\ell T^* \qquad (\bar{M}_\ell \;=\; M_\ell M_\ell^\dagger)$$

$$\Rightarrow \quad T \;=\; \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^{-1} & 0 \\ 0 & 0 & \omega \end{pmatrix} \qquad \text{e.g.} \quad \omega = e^{2\pi i/3}$$

$S, T, U =$ generators of the triplet representation of the permutation group S_4
(symmetry group of the octahedron and the cube)

S_4 symmetry

presentation

$$S^2 = T^3 = U^2 = \mathbb{1}$$
$$(ST)^3 = (SU)^2 = (TU)^2 = (STU)^4 = \mathbb{1}$$

irreducible representations

	S	T	U
$\mathbf{1}$	1	1	1
$\mathbf{1}'$	1	1	-1
$\mathbf{2}$	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} \omega & 0 \\ 0 & \omega^{-1} \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
$\mathbf{3}$	$\frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^{-1} & 0 \\ 0 & 0 & \omega \end{pmatrix}$	$-\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$
$\mathbf{3}'$	$\frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^{-1} & 0 \\ 0 & 0 & \omega \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

A_4 symmetry

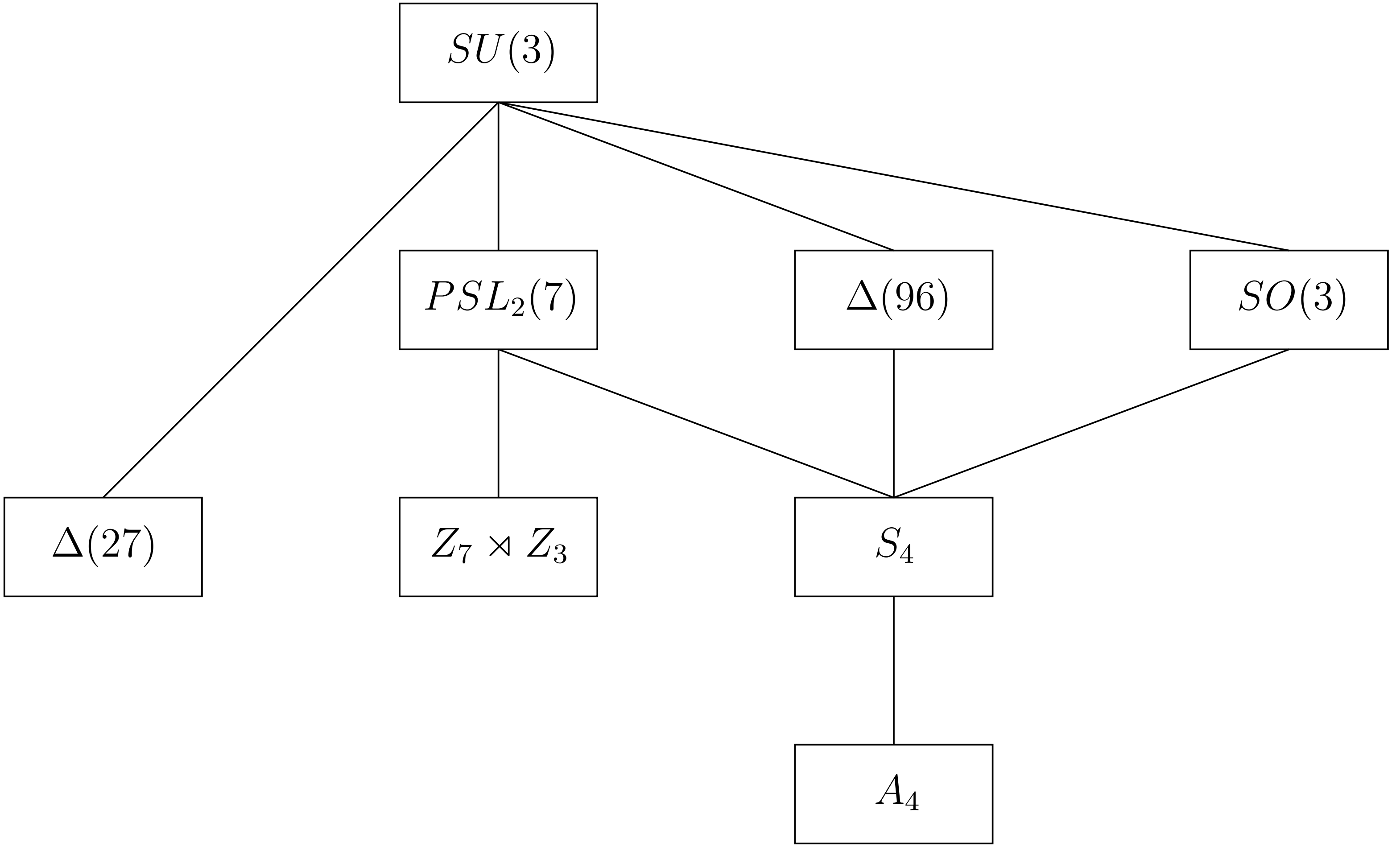
presentation

$$S^2 = T^3 = \mathbb{1}$$
$$(ST)^3 = \mathbb{1}$$

irreducible representations

	S	T	
$\mathbf{1}$	1	1	
$\begin{pmatrix} \mathbf{1}' \\ \mathbf{1}'' \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} \omega & 0 \\ 0 & \omega^{-1} \end{pmatrix}$	
$\mathbf{3}$	$\frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^{-1} & 0 \\ 0 & 0 & \omega \end{pmatrix}$	

Other candidates for $\mathcal{G}$



Direct and indirect models

S, U generate the Klein four-group $\mathcal{K} = Z_2 \times Z_2$

	direct models	indirect models
family symmetry $\mathcal{G}$	$\mathcal{K} \subset \mathcal{G}$	$\mathcal{K}$ not necessarily $\subset \mathcal{G}$
flavon VEVs $\langle \Phi^\nu \rangle$	do not break $\mathcal{K}$	break $\mathcal{K}$, however...
	$S \cdot \langle \Phi^\nu \rangle \; = \; \langle \Phi^\nu \rangle$	$S \cdot \langle \Phi^\nu \rangle \; = \pm \langle \Phi^\nu \rangle$
	$U \cdot \langle \Phi^\nu \rangle \; = \; \langle \Phi^\nu \rangle$	$U \cdot \langle \Phi^\nu \rangle \; = \pm \langle \Phi^\nu \rangle$
effective Lagrangian	linear in Φ^ν	quadratic in Φ^ν
	$\mathcal{L}_\nu \sim \sum_a \nu \, \Phi^\nu_a \, \nu \; H \, H$	$\mathcal{L}_\nu \sim \sum_a \nu^T \Phi^\nu_a \Phi^{\nu\,T}_a \nu \; H \, H$

Alignment in direct models – $S_4\,,\ PSL_2(7)\,,\ \Delta(96)$

effective Lagrangian $\sum_a \nu\,\Phi^\nu_a\,\nu\ H\,H$ respects $\mathcal{K}$

in S_4: multiplication rule $\mathbf{3}\otimes\mathbf{3} = \underbrace{(\mathbf{1}+\mathbf{2}+\mathbf{3}')_s}_{\text{flavon representations}} + \mathbf{3}_a$

S_4 irrep	S	U	alignment of Φ^ν_a
$\mathbf{1}$	1	1	1
$\mathbf{2}$	$\begin{pmatrix}1&0\\0&1\end{pmatrix}$	$\begin{pmatrix}0&1\\1&0\end{pmatrix}$	$\begin{pmatrix}1\\1\end{pmatrix}$
$\mathbf{3}'$	$\frac{1}{3}\begin{pmatrix}-1&2&2\\2&-1&2\\2&2&-1\end{pmatrix}$	$\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix}$	$\begin{pmatrix}1\\1\\1\end{pmatrix}$

Alignment in indirect models – $\Delta(27)\,,\ Z_7 \rtimes Z_3\,,\ A_4$

central terms in (scalar) flavon potential

$(i)\quad \kappa \sum_i \phi_i^\dagger \phi_i \phi_i^\dagger \phi_i$

$$\kappa > 0 \quad \rightarrow \quad \langle \phi_{123} \rangle \ \propto\ \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\kappa < 0 \quad \rightarrow \quad \langle \phi_1 \rangle \ \propto\ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$(ii)\quad \tilde{\kappa} \sum_{i,j} (\phi_i^\dagger \tilde{\phi}_i)(\tilde{\phi}_j^\dagger \phi_j)$

$\tilde{\kappa} > 0 \quad \rightarrow$ orthogonality condition $\langle \phi \rangle \perp \langle \tilde{\phi} \rangle$

e.g. $\langle \phi_{23} \rangle \ \propto\ \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$

or $\langle \phi_{\perp} \rangle \ \propto\ \frac{1}{\sqrt{6}} \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$

flavon VEVs break neutrino flavour symmetry $\mathcal{K}$ (generated by S, U)

$$\langle\phi_\perp\rangle \;\sim\; \frac{1}{\sqrt{6}}\begin{pmatrix}-2\\1\\1\end{pmatrix} \qquad \langle\phi_{123}\rangle \;\sim\; \frac{1}{\sqrt{3}}\begin{pmatrix}1\\1\\1\end{pmatrix} \qquad \langle\phi_{23}\rangle \;\sim\; \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\-1\end{pmatrix}$$

$$\begin{aligned} S\cdot\langle\phi_\perp\rangle &= -\langle\phi_\perp\rangle & \qquad U\cdot\langle\phi_\perp\rangle &= -\langle\phi_\perp\rangle \\ S\cdot\langle\phi_{123}\rangle &= +\langle\phi_{123}\rangle & \qquad U\cdot\langle\phi_{123}\rangle &= -\langle\phi_{123}\rangle \\ S\cdot\langle\phi_{23}\rangle &= -\langle\phi_{23}\rangle & \qquad U\cdot\langle\phi_{23}\rangle &= +\langle\phi_{23}\rangle \end{aligned}$$

effective neutrino Lagrangian respects $\mathcal{K}$

$$\mathcal{L}_\nu \;\sim\; \nu^T(m_{\nu_1}\,\phi_\perp\phi_\perp^T \;+\; m_{\nu_2}\,\phi_{123}\phi_{123}^T \;+\; m_{\nu_3}\,\phi_{23}\phi_{23}^T)\,\nu\;H\,H$$

A direct $SU(5) \times S_4$ model

Particle content

Hagedorn, King, Luhn (2010)

matter	T_3	T	F	N	H_5	$H_{\overline{5}}$	$H_{\overline{45}}$
$SU(5)$	$\mathbf{10}$	$\mathbf{10}$	$\overline{\mathbf{5}}$	$\mathbf{1}$	$\mathbf{5}$	$\overline{\mathbf{5}}$	$\overline{\mathbf{45}}$
S_4	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$U(1)$	0	5	4	-4	0	0	1

flavons	ζ_2^u	$\widetilde{\zeta}_2^u$	ϕ_2^d	$\widetilde{\phi}_{23}^d$	ζ_1^d	ξ^ν	ζ_{12}^ν	$\phi'^{\,\nu}_{123}$
$SU(5)$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
S_4	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{3'}$
$U(1)$	-10	0	-4	-11	1	8	8	8

notation for flavons

singlets $\rightarrow$ ξ

doublets $\rightarrow$ ζ

triplets $\rightarrow$ ϕ, ϕ'

$(\text{flavon field})^{\text{sector}}_{\text{alignment}}$

Neutrino sector

neutrino masses (type I seesaw)

$$F N H_{\mathbf{5}} \;\rightarrow\; M_D \qquad\qquad N N (\xi^\nu + \zeta^\nu_{12} + \phi'^{\,\nu}_{123}) \;\rightarrow\; M_R$$

$$\Longrightarrow \quad M_\nu \;=\; -\, M_D \, M_R^{-1} \, M_D^T$$

· three light neutrino masses $\leftrightarrow$ three input parameters

neutrino mixing

· M_D symmetric under S_4

· M_R symmetric under $\mathcal{K}$

· hence M_ν symmetric under S and U

· tri-bimaximal structure robust up to $\mathcal{O}(10^{-3})$

Charged fermions

up sector $T_3T_3H_5 + \frac{1}{M}TT\zeta_2^u H_5 + \frac{1}{M^2}TT\zeta_2^u\widetilde{\zeta}_2^u H_5$

$$M_u \sim \begin{pmatrix} \lambda^8 & 0 & 0 \\ 0 & \lambda^4 & 0 \\ 0 & 0 & 1 \end{pmatrix} v_u$$

- diagonal up quark matrix
- renormalisable top Yukawa

down sector $\frac{1}{M}FT_3\phi_2^d H_{\overline{5}} + \frac{1}{M^2}(F\widetilde{\phi}_{23}^d)_1(T\zeta_1^d)_1 H_{\overline{45}} + \frac{1}{M^3}(F\zeta_1^d\zeta_1^d)_3(T\widetilde{\phi}_{23}^d)_3 H_{\overline{5}}$

$$M_d \sim \begin{pmatrix} 0 & \lambda^5 & \lambda^5 \\ \lambda^5 & \lambda^4 & \lambda^4 \\ 0 & 0 & \lambda^2 \end{pmatrix} v_d \qquad M_\ell \sim \begin{pmatrix} 0 & \lambda^5 & 0 \\ \lambda^5 & 3\,\lambda^4 & 0 \\ \lambda^5 & 3\,\lambda^4 & \lambda^2 \end{pmatrix} v_d$$

- CKM mixing from down sector
- Georgi-Jarlskog relations $m_b \sim m_\tau$, $m_s \sim \frac{1}{3}m_\mu$, $m_d \sim 3m_e$
- Gatto-Sartori-Tonin relation $\theta_{12}^d \sim \sqrt{\frac{m_d}{m_s}}$

An indirect $SU(5) \times A_4$ model

Particle content

Cooper, King, Luhn (2010)

matter	T_3	T_2	T_1	F	$\psi_{\mathbf{24}}$	N	$H_{\mathbf{5}}$	$H_{\overline{\mathbf{5}}}$	$H_{\overline{\mathbf{45}}}$
$SU(5)$	**10**	**10**	**10**	$\overline{\mathbf{5}}$	**24**	**1**	**5**	$\overline{\mathbf{5}}$	$\overline{\mathbf{45}}$
A_4	**1**	**1**	**1**	**3**	**1**	**1**	**1**	**1**	**1**
$U(1)$	0	1	4	0	-1	2	0	0	2
$Z_2 \times \tilde{Z}_2$	$+-$	$++$	$++$	$++$	$-+$	$-+$	$++$	$-+$	$-+$

flavons	ϕ_{123}	ϕ_{23}	ξ	ϕ_3	$\widetilde{\xi}$	ϕ_1
$SU(5)$	**1**	**1**	**1**	**1**	**1**	**1**
A_4	**3**	**3**	**1**	**3**	**1**	**3**
$U(1)$	1	-2	-1	0	-4	q_1
$Z_2 \times \tilde{Z}_2$	$-+$	$-+$	$++$	$--$	$-+$	$++$

notation for flavons

singlets $\longrightarrow$ ξ

triplets $\longrightarrow$ ϕ

(flavon field)$_{\text{alignment}}$

Neutrino sector

Dirac $\frac{1}{M}\underbrace{\phi_{23}FN}_{\propto (F_2-F_3)N}H_{\mathbf{5}} \quad + \quad \frac{1}{M}\underbrace{\phi_{123}F\psi_{\mathbf{24}}}_{\propto (F_1+F_2+F_3)\psi_{\mathbf{24}}}H_{\mathbf{5}}$

Majorana $\left(\frac{\phi_{23}^2}{M}+\frac{\xi^4}{M^3}\right)NN \quad + \quad \frac{\phi_{123}^2}{M}\,\mathrm{Tr}\left({\psi_{\mathbf{24}}}^2\right)$

after integrating out seesaw particles N and $\rho_0\,,\ \rho_3 \subset \psi_{\mathbf{24}}$

$$\mathcal{L}_\nu \;\sim\; \frac{1}{M^2}\,F^T\left(\frac{\phi_{23}\phi_{23}^T}{m_N}+\frac{\phi_{123}\phi_{123}^T}{m_{\rho_0}}+\frac{\phi_{123}\phi_{123}^T}{m_{\rho_3}}\right)F\;H_{\mathbf{5}}\,H_{\mathbf{5}}$$

$$\Longrightarrow \quad M_\nu \;\sim\; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix}\frac{v_u^2}{M} + \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}\frac{v_u^2}{M}$$

· m_{ν_3} due to N & m_{ν_2} due to $\psi_{\mathbf{24}}$ & $m_{\nu_1}=0$

· tri-bimaximal mixing in an indirect model

Charged fermions

up sector $$T_3T_3H_{\mathbf{5}} + \frac{\xi^2}{M^2}T_2T_2H_{\mathbf{5}} + \frac{\tilde{\xi}^2}{M^2}T_1T_1H_{\mathbf{5}}$$

$$+ \left(\frac{\phi_{23}^2\xi}{M^3} + \frac{\xi^5}{M^5}\right)T_1T_2H_{\mathbf{5}} + \frac{\phi_{23}\phi_3\xi^2}{M^4}T_1T_3H_{\mathbf{5}} + \frac{\phi_{123}\phi_3\xi^2}{M^4}T_2T_3H_{\mathbf{5}}$$

$$M_u \sim \begin{pmatrix} \lambda^8 & \lambda^8 & \lambda^9 \\ \lambda^8 & \lambda^4 & \lambda^9 \\ \lambda^9 & \lambda^9 & 1 \end{pmatrix} v_u$$

- almost diagonal up quark matrix
- renormalisable top Yukawa

down sector $$\frac{\phi_3}{M}FT_3H_{\overline{\mathbf{5}}} + \frac{\xi\phi_{23}}{M^2}FT_2H_{\overline{\mathbf{45}}} + \frac{\xi^2\phi_{123}}{M^3}FT_2H_{\overline{\mathbf{5}}} + \frac{\xi^2\phi_{23}}{M^3}FT_1H_{\overline{\mathbf{5}}}$$

$$M_d \sim \begin{pmatrix} 0 & \lambda^4 & \lambda^4 \\ \lambda^4 & \lambda^3 & \lambda^3 \\ 0 & 0 & \lambda \end{pmatrix} v_d \qquad M_\ell \sim \begin{pmatrix} 0 & \lambda^4 & 0 \\ \lambda^4 & 3\,\lambda^3 & 0 \\ \lambda^4 & 3\,\lambda^3 & \lambda \end{pmatrix} v_d$$

- CKM mixing from down sector
- Georgi-Jarlskog relations $m_b \sim m_\tau \ , \ m_s \sim \frac{1}{3}m_\mu \ , \ m_d \sim 3m_e$

Quick summary

· tri-bimaximal lepton mixing more than a numerical accident

· non-Abelian family symmetry $\mathcal{G}$ at work

· direct vs. indirect nature of $\mathcal{G}$

· two examples

(*i*) $SU(5) \times S_4$ model (direct)

(*ii*) $SU(5) \times A_4$ model (indirect)

Thank you