Cascade Textures in SUSY GUT

SUSY10

August 23-28, 2010, Bonn, Germany

Ryo Takahashi (MPIK)

References

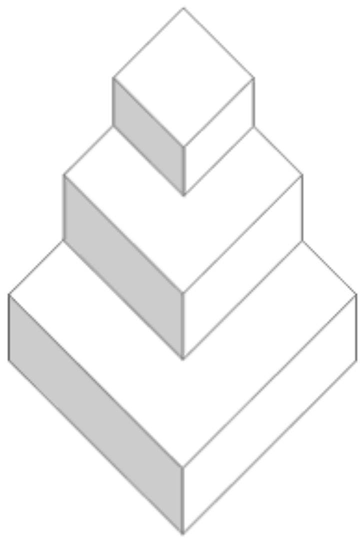
A. Adulpravitchai, K. Kojima and RT, [to appear]

K. Kojima, H. Sawanaka and RT, [to appear]

N. Haba, RT, M. Tanimoto and K. Yoshioka, PRD78 (2008) 113002

Contents

1. Introduction
2. Cascade Hierarchy
3. Cascade Textures in SUSY GUT
4. Summary



cascade

1. Introduction

Neutrino Oscillation Experiments

T. Schwetz *et al*, arXiv:0808.2016 [hep-ph] (ver.3, 3σ)

Best Fit

$\Downarrow$

$$\sin^2\theta_{12} = 0.27 \;-\; 0.32 \;-\; 0.38 \quad : \text{Large}$$
$$\sin^2\theta_{23} = 0.36 \;-\; 0.50 \;-\; 0.67 \quad : \text{Maximal}$$
$$\sin^2\theta_{13} \leq 0.053 \quad : \text{Small}$$

$\Downarrow$

$$\sin^2\theta_{12} \simeq \frac{1}{3}$$
$$\sin^2\theta_{23} \simeq \frac{1}{2}$$
$$\sin^2\theta_{13} \sim 0$$

Tri-bimaximal Generation Mixing

Harrison, Perkins, Scott (2002)

$$V_{\rm TB} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \Rightarrow \begin{array}{l} \sin^2\theta_{12} = \frac{1}{3} \\ \sin^2\theta_{23} = \frac{1}{2} \\ \sin^2\theta_{13} = 0 \end{array} \Leftrightarrow \begin{array}{l} \textbf{Experiments} \\ \sin^2\theta_{12} \simeq \frac{1}{3} \\ \sin^2\theta_{23} \simeq \frac{1}{2} \\ \sin^2\theta_{13} \leq 0.050 \end{array}$$

- ν_2 : Tri-maximal Mixture of $\nu_e, \ \nu_\mu, \ \nu_\tau$
- ν_3 : Bi -maximal Mixture of $\nu_\mu, \ \nu_\tau$

$$\boxed{V_{\rm TB} \simeq V_{\rm PMNS}^{\rm exp}}$$

$$\Downarrow$$

- Good theoretical motivation to look for flavor structure

  - Flavor symmetry

    e.g.) Discrete symmetry : S_3, A_4,,,

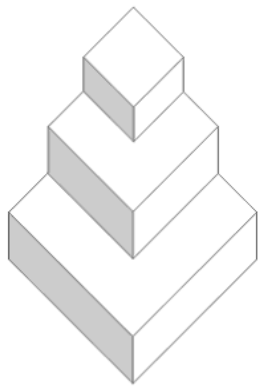
  - Texture analysis

    e.g.) “Cascade” matrix,,,

2. Cascade Hierarchy

Cascade (mass) matrix

$$M_{\text{cas}} = \begin{pmatrix} \delta & \delta & \delta \\ \delta & \lambda & \lambda \\ \delta & \lambda & 1 \end{pmatrix} v, \quad \delta \ll \lambda \ll 1$$



("cascade hierarchy" by Dorsner and Barr, (2001))

(mass) Eigenvalues : $m_1 : m_2 : m_3 \sim \delta : \lambda : 1$

Mixing angles :

$$\theta_{12} \sim \frac{\delta}{\lambda}, \quad \theta_{23} \sim \lambda, \quad \theta_{13} \sim \delta, \quad \left[\theta_{ij} \sim \frac{m_i}{m_j}\right]$$

- The cascade mass matrix leads to small mixing angles.
- It can lead to nearly tri-bimaximal generation mixing of the lepton sector in the seesaw mechanism if the neutrino Dirac mass matrix is a cascade form. Haba, RT, Tanimoto, Yoshioka, (2008)

2. Cascade Hierarchy

Neutrino Sector (Charged lepton sector: Diagonal)

- **What is the form of M_ν for the tri-bimaximal mixing?**

$$M_\nu = V_{\rm TB}^* \cdot {\rm Diag}\{m_1, m_2, m_3\} \cdot V_{\rm TB}^\dagger$$
$$= \frac{m_1}{6}\begin{pmatrix} 4 & -2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 1 \end{pmatrix} + \frac{m_2}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{m_3}{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix}$$

Cascade Matrix (See-saw machanism : $M_\nu \simeq M_{\nu D} M_R^{-1} M_{\nu D}^T$)

Dirac Mass Matrix

$$M_{\nu D} = \begin{pmatrix} \delta & \delta & \delta \\ \delta & \lambda & -\lambda \\ \delta & -\lambda & 1 \end{pmatrix} v, \quad (\delta \ll \lambda \ll 1)$$

Majorana Mass Matrix

$$M_R = \begin{pmatrix} M_1 & & \\ & M_2 & \\ & & M_3 \end{pmatrix}$$

$$M_\nu \simeq \frac{v^2}{M_3}\begin{pmatrix} \delta^2 & \delta\lambda & \delta \\ \delta\lambda & \lambda^2 & -\lambda \\ \delta & -\lambda & 1 \end{pmatrix} + \frac{v^2}{M_1}\delta^2 \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{v^2}{M_2}\lambda^2 \begin{pmatrix} \delta^2/\lambda^2 & \delta/\lambda & -\delta/\lambda \\ \delta/\lambda & 1 & -1 \\ -\delta/\lambda & -1 & 1 \end{pmatrix}$$

$\Rightarrow |m_1| \ll |m_{2,3}|$ (Normal Hierarchy), $|\delta m_3| \ll |\lambda m_2|$

Cascade matrix (seesaw mechanism)

Dirac mass matrix **Majorana mass matrix**

$$M_\nu^D = \begin{pmatrix} \delta & \delta & \delta \\ \delta & \lambda & -\lambda \\ \delta & -\lambda & 1 \end{pmatrix} v, \quad (\delta \ll \lambda \ll 1) \qquad M_R = \begin{pmatrix} M_1 & & \\ & M_2 & \\ & & M_3 \end{pmatrix}$$

Mass eigenvalues

$$(m_1,\ m_2,\ m_3) = \left(\frac{v^2}{6M_3},\ \frac{3\delta^2 v^2}{M_1} + \frac{v^2}{3M_3},\ \frac{2\lambda^2 v^2}{M_2} + \frac{v^2}{2M_3} \right)$$

Mixing angles

$$\sin^2\theta_{12} = \left| \frac{1}{\sqrt{3}} - \mathcal{O}\left(\frac{m_1}{m_2}\right) \right|^2$$

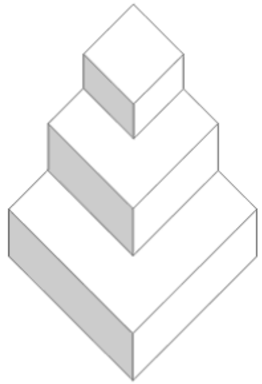
$$\sin^2\theta_{23} = \left| \frac{-1}{\sqrt{2}} + \mathcal{O}\left(\frac{m_1}{m_3}\right) + \mathcal{O}\left(\frac{\delta}{\lambda}\frac{m_2}{m_3}\right) \right|^2$$

$$\sin^2\theta_{13} = \left| \mathcal{O}\left(\frac{\delta}{\lambda}\right) + \mathcal{O}\left(\frac{m_1 m_2}{m_3^2}\right) \right|^2$$

3. Cascade Textures in SUSY GUT

Cascade (mass) matrix

$$M_{\rm cas} = \begin{pmatrix} \delta & \delta & \delta \\ \delta & \lambda & \lambda \\ \delta & \lambda & 1 \end{pmatrix} v, \quad \delta \ll \lambda \ll 1$$



("cascade hierarchy" by Dorsner and Barr, (2001))

(mass) Eigenvalues : $m_1 : m_2 : m_3 \sim \delta : \lambda : 1$

Mixing angles :

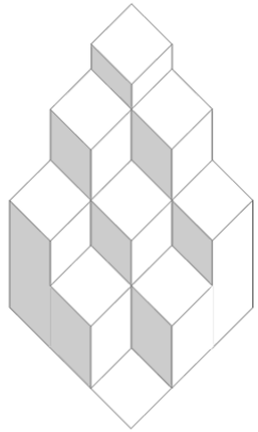
$$\theta_{12} \sim \frac{\delta}{\lambda}, \quad \theta_{23} \sim \lambda, \quad \theta_{13} \sim \delta, \quad \left[\theta_{ij} \sim \frac{m_i}{m_j}\right]$$

- The cascade mass matrix leads to small mixing angles.
- It can lead to nearly tri-bimaximal mixing of the lepton sector if the neutrino Dirac mass matrix is described by the cascade matrix with diagonal charged lepton and right-handed neutrino Majorana mass matrices.

3. Cascade Textures in SUSY GUT

Waterfall (mass) matrix

$$M_{\rm wat} = \begin{pmatrix} \delta^2 & \delta\lambda & \delta \\ \delta\lambda & \lambda^2 & \lambda \\ \delta & \lambda & 1 \end{pmatrix} v, \quad \delta \ll \lambda \ll 1$$



(mass) Eigenvalues : $m_1 : m_2 : m_3 \sim \delta^2 : \lambda^2 : 1$

Mixing angles :

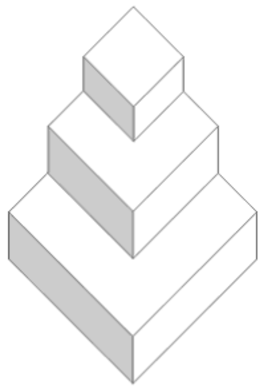
$$\theta_{12} \sim \frac{\delta}{\lambda}, \quad \theta_{23} \sim \lambda, \quad \theta_{13} \sim \delta, \qquad \left[\theta_{ij} \sim \sqrt{\frac{m_i}{m_j}} \right]$$

- This cannot lead to the LMA of lepton sector even in the seesaw.
- If $M_d \simeq M_{\rm wat}$, it cannot even reproduce the CKM mixing angles, θ^q_{ij} :
  $\theta^q_{12} \sim \sqrt{m_{d_1}/m_{d_2}}$, $\theta^q_{23} \sim m_{d_2}/m_{d_3}$, $\theta^q_{13} \sim \theta^q_{12}\theta^q_{23}$

3. Cascade Textures in SUSY GUT

Cascade (mass) matrix

$$M_{\rm cas} = \begin{pmatrix} \delta & \delta & \delta \\ \delta & \lambda & \lambda \\ \delta & \lambda & 1 \end{pmatrix} v, \quad \delta \ll \lambda \ll 1$$



("cascade hierarchy" by Dorsner and Barr, (2001))

(mass) Eigenvalues : $m_1 : m_2 : m_3 \sim \delta : \lambda : 1$

Mixing angles :

$$\theta_{12} \sim \frac{\delta}{\lambda}, \quad \theta_{23} \sim \lambda, \quad \theta_{13} \sim \delta, \qquad \left[\theta_{ij} \sim \frac{m_i}{m_j}\right]$$

Quark sector :

$$\theta_{12}^q \sim \sqrt{m_{d_1}/m_{d_2}}, \; \theta_{23}^q \sim m_{d_2}/m_{d_3}, \; \theta_{13}^q \sim \theta_{12}^q \theta_{23}^q$$

3. Cascade Textures in SUSY GUT

If M_d is taken as the cascade form, θ^q_{23} can be reproduced but θ^q_{12} and θ^q_{13} cannot be reproduced.

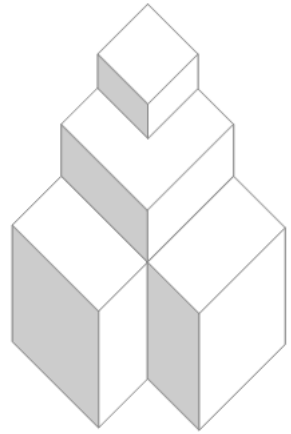
If M_d is taken as the waterfall form, θ^q_{12} can be reproduced but θ^q_{23} and θ^q_{13} cannot be reproduced.

Let us mix the cascade with the waterfall.

3. Cascade Textures in SUSY GUT

Hybrid Cascade (H.C.) matrix

$$M_{\text{hyb}} = \begin{pmatrix} \epsilon & \delta & \delta \\ \delta & \lambda & \lambda \\ \delta & \lambda & 1 \end{pmatrix} v, \quad \epsilon \ll \delta \ll \lambda \ll 1$$



(mass) Eigenvalues : $m_1 : m_2 : m_3 \sim \delta^2/\lambda : \lambda : 1$

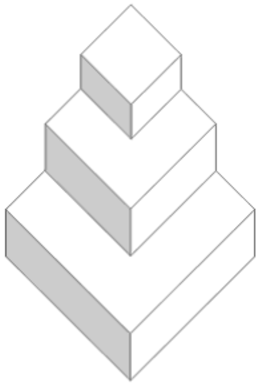
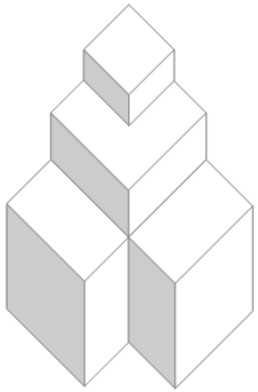
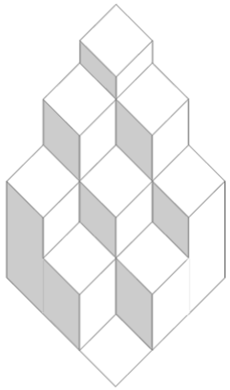
Mixing angles :

$$\theta_{12} \sim \frac{\delta}{\lambda} \sim \sqrt{\frac{m_1}{m_2}}, \quad \theta_{23} \sim \lambda \sim \frac{m_2}{m_3}, \quad \theta_{13} \sim \delta,$$

Quark sector :

$$\theta^q_{12} \sim \sqrt{\frac{m_{d_1}}{m_{d_2}}}, \quad \theta^q_{23} \sim \frac{m_{d_2}}{m_{d_3}}, \quad \theta^q_{13} \sim \theta^q_{12}\theta^q_{23}$$

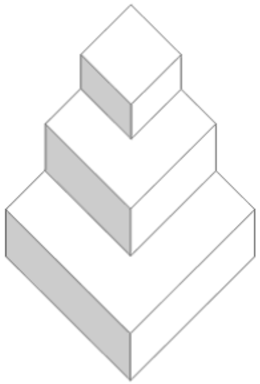
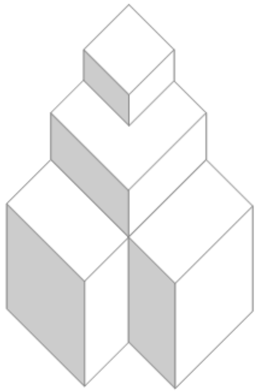
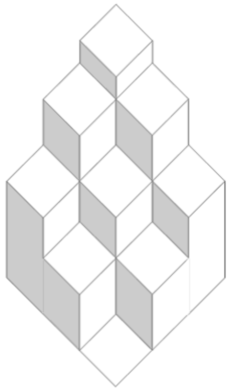
3. Cascade Textures in SUSY GUT

	Cas.	H.C.	Wat.	
				$\begin{pmatrix} * & 0 & 0 \\ 0 & * & 0 \\ 0 & 0 & * \end{pmatrix}$
$M_{\nu D}$	○	×	×	×
M_e				○
M_d	×	○	×	
M_R				○

- $M_{\nu D}$ =Cas. can lead to LMA in basis of diagonal M_e & M_R

Haba, RT, Tanimoto, Yoshioka, PRD 78 (2008) 113002

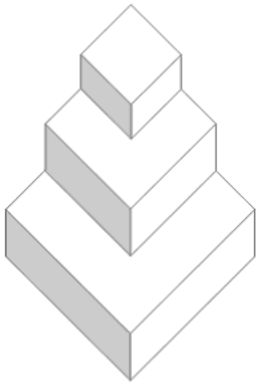
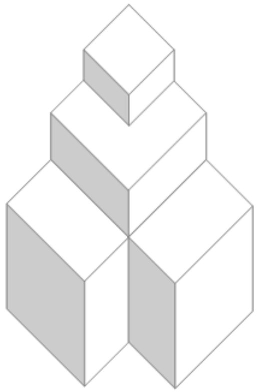
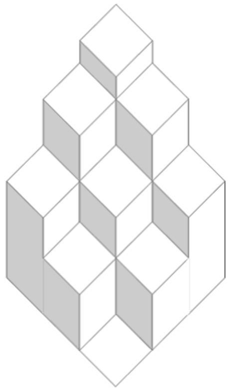
3. Cascade Textures in SUSY GUT

	Cas. 	H.C. 	Wat. 	$\begin{pmatrix} * & 0 & 0 \\ 0 & * & 0 \\ 0 & 0 & * \end{pmatrix}$
$M_{\nu D}$	○	×	×	×
M_e	○			○
M_d	×	○	×	
M_R	○			○

- $M_{\nu D}$ =Cas. is also OK even if M_e & M_R =Cas..

Haba, RT, Tanimoto, Yoshioka, PRD 78 (2008) 113002

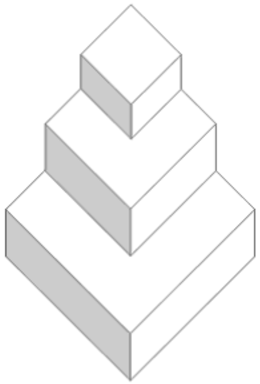
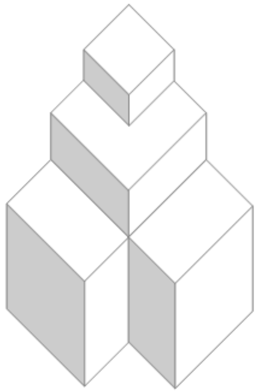
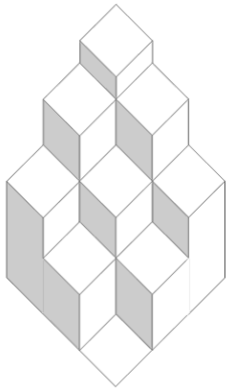
3. Cascade Textures in SUSY GUT

	Cas. 	H.C. 	Wat. 	Small Mixing or $\begin{pmatrix} * & 0 & 0 \\ 0 & * & 0 \\ 0 & 0 & * \end{pmatrix}$
$M_{\nu D}$	○	×	×	×
M_e	○	○	○	○
M_d	×	○	×	
M_R	○	○	○	○

- $M_{\nu D}$ =Cas. is OK as long as M_e & M_R lead to only small mixings.

Kojima, Sawanaka, RT, [to appear]

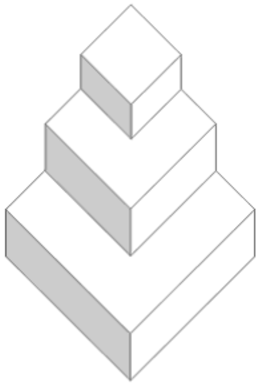
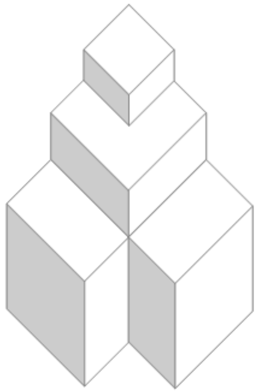
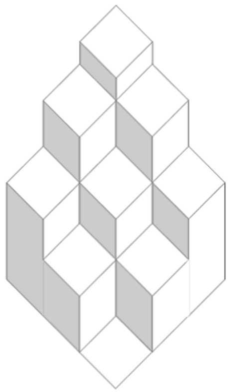
3. Cascade Textures in SUSY GUT

	Cas.	H.C.	Wat.	Small Mixing or $\begin{pmatrix} * & 0 & 0 \\ 0 & * & 0 \\ 0 & 0 & * \end{pmatrix}$
				
$M_{\nu D}$	○	×	×	×
M_e	○	○	○	○
M_d	×	○	×	×
M_u	○	○	○	○
M_R	○	○	○	○

- V_{CKM} is almost determined by M_d: M_u gives only corrections.

Kojima, Sawanaka, RT, [to appear]

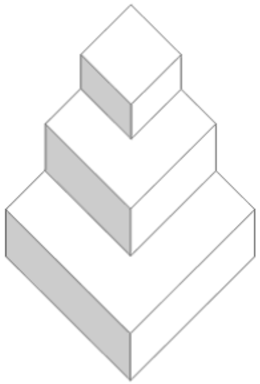
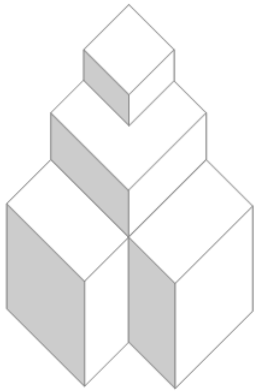
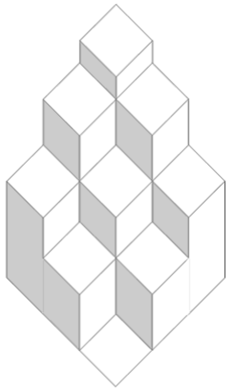
3. Cascade Textures in SUSY GUT

	Cas.	H.C.	Wat.	Small Mixing or
				$\begin{pmatrix} * & 0 & 0 \\ 0 & * & 0 \\ 0 & 0 & * \end{pmatrix}$
$M_{\nu D}$	○	×	×	×
M_e	○	○	○	○
M_d	×	○	×	×
M_u	○	○	○	○
M_R	○	○	○	○

- It would seem that we can explain the masses and mixings of the quark and lepton sectors in terms of only Cas. & H.C. hierarchies$\cdots$.

Kojima, Sawanaka, RT, [to appear]

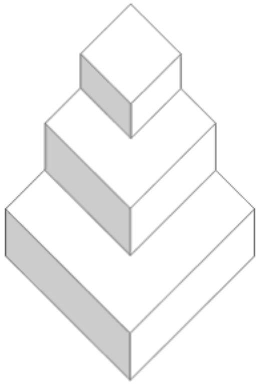
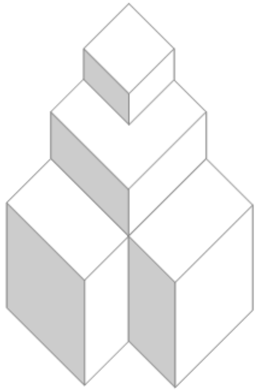
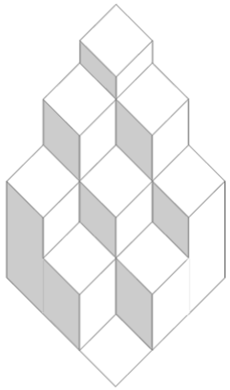
3. Cascade Textures in SUSY GUT

	Cas. 	H.C. 	Wat. 	Small Mixing or $\begin{pmatrix} * & 0 & 0 \\ 0 & * & 0 \\ 0 & 0 & * \end{pmatrix}$
$M_{\nu D}$	○	×	×	×
M_e	○	○	○	○
M_d	×	○	×	×
M_u	○	○	○	○
M_R	○	○	○	○

• **SUSY-GUT ?**

Kojima, Sawanaka, RT, [to appear]

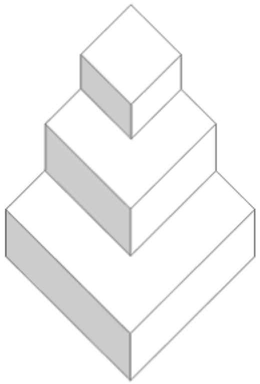
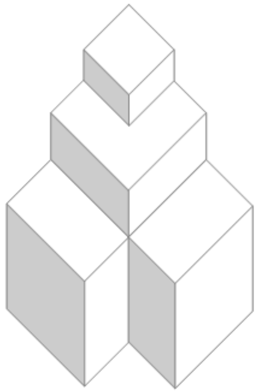
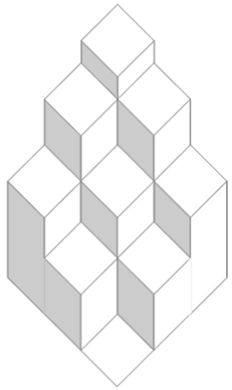
3. Cascade Textures in SUSY GUT

	Cas. 	H.C. 	Wat. 	Small Mixing or $\begin{pmatrix} * & 0 & 0 \\ 0 & * & 0 \\ 0 & 0 & * \end{pmatrix}$
$M_{\nu D}$	○	×	×	×
M_e	×	○	×	×
M_d	×	○	×	×
M_u	○	○	○	○
M_R	○	○	○	○

- **For example, $M_e \simeq M_d^T$ in $SU(5)$ GUT.**

Kojima, Sawanaka, RT, [to appear]

3. Cascade Textures in SUSY GUT

	Cas.	H.C.	Wat.	Small Mixing or
				$\begin{pmatrix} * & 0 & 0 \\ 0 & * & 0 \\ 0 & 0 & * \end{pmatrix}$
$M_{\nu D}$	○	×	×	×
M_e	×	○	×	×
M_d	×	○	×	×
M_u	○	×	×	×
M_R	○	○	○	○

- **For example, $M_e \simeq M_d^T$ and $M_{\nu D} \simeq M_u$ in $SO(10)$ GUT.**

Adulpravitchai, Kojima, RT, [to appear]

3. Cascade Textures in SUSY GUT

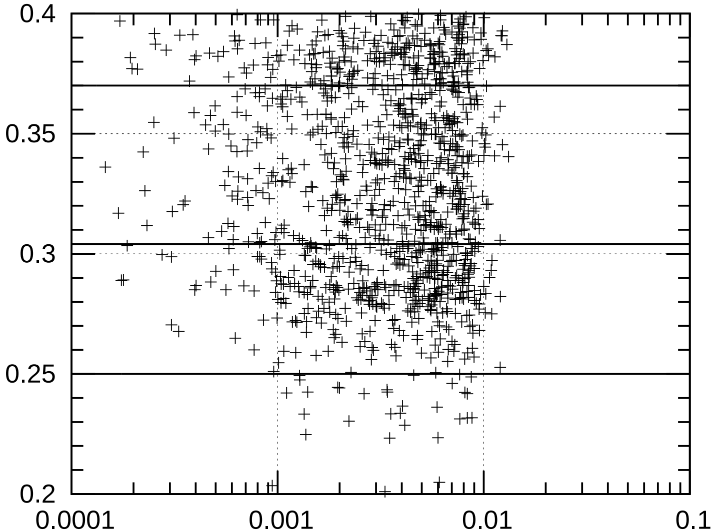
Numerical analyses of PMNS mixing angles in SU(5) case

**Input: 3 CKM mixing angles and 1 phase,
quark and charged lepton masses, neutrino mass ratio**

$$M_{\nu D} \simeq \begin{pmatrix} \lambda^3 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda & -\lambda \\ \lambda^3 & -\lambda & 1 \end{pmatrix} v_u, \qquad M_e \simeq \begin{pmatrix} 0 & \lambda^3 & \lambda^3 \\ \lambda^3 & -3\lambda^2 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \lambda v_d,$$

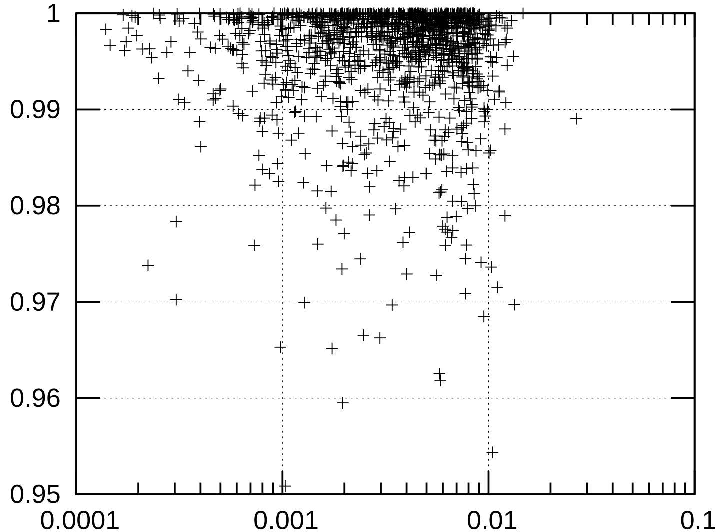
$$M_u \simeq \begin{pmatrix} 0 & \lambda^8 & \lambda^8 \\ \lambda^8 & \lambda^4 & \lambda^4 \\ \lambda^8 & \lambda^4 & 1 \end{pmatrix} v_u, \qquad M_d \simeq \begin{pmatrix} 0 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \lambda v_d.$$

$\sin^2\theta_{12}$



$\sin^2\theta_{13}$

$\sin^2 2\theta_{23}$



$\sin^2\theta_{13}$

3. Cascade Textures in SUSY GUT

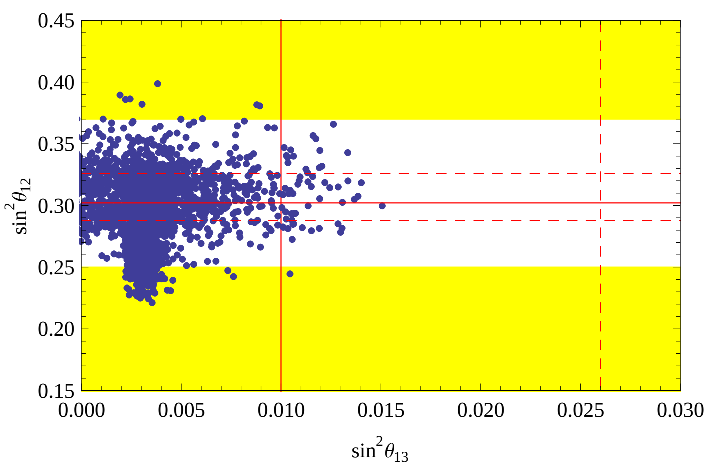
Numerical analyses of PMNS mixing angles in SO(10) case

Input: 3 CKM mixing angles and 1 phase,
quark and charged lepton masses, neutrino mass ratio

$$M_{\nu D} \simeq \begin{pmatrix} \lambda^8 & \lambda^8 & \lambda^8 \\ \lambda^8 & \lambda^4 & -\lambda^4 \\ \lambda^8 & -\lambda^4 & 1 \end{pmatrix} v_u, \qquad M_e \simeq \begin{pmatrix} 0 & \lambda^3 & \lambda^3 \\ \lambda^3 & -3\lambda^2 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \lambda v_d,$$

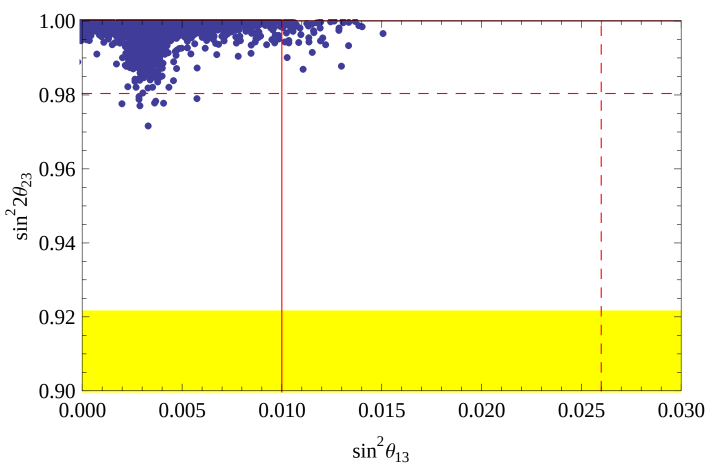
$$M_u \simeq \begin{pmatrix} 0 & \lambda^8 & \lambda^8 \\ \lambda^8 & \lambda^4 & \lambda^4 \\ \lambda^8 & \lambda^4 & 1 \end{pmatrix} v_u, \qquad M_d \simeq \begin{pmatrix} 0 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \lambda v_d.$$

$\sin^2\theta_{12}$



$\sin^2\theta_{13}$

$\sin^2 2\theta_{23}$

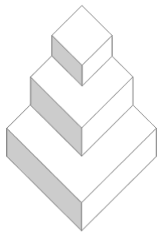
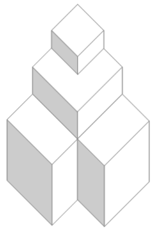


$\sin^2\theta_{13}$

4. Summary

Texture Analyses of Cascading Matrices

$\Uparrow$

PMNS $\Leftarrow$  **and**  $\Rightarrow$ **CKM**

$\Downarrow$

$\sim$ Cascading Matrices & SUSY-GUT $\sim$

Comprehensive understanding of mixings/masses for quark/lepton sectors.

For example, in $SU(5)$ and $SO(10)$ SUSY GUT models.

$\Downarrow$

θ_{13}, LFV, and Leptogenesis

Appendix

Realization of cascade matrix from $U(1)$ flavor theory

	L_1	L_2	L_3	R_1	R_2	R_3	ϕ_1	ϕ_2	ϕ_3
$U(1)_f$	$2m+1$	1	0	$2m+1$	1	0	$-2m-3$	-2	-1

(m : Positive Integer)

$$M_D = \begin{pmatrix} \frac{\phi_1\phi_2^{m-1}\phi_3}{\Lambda^{m+1}} & \frac{\phi_2^{m+1}}{\Lambda^{m+1}} & \frac{\phi_2^{m}\phi_3}{\Lambda^{m+1}} \\ \frac{\phi_2^{m+1}}{\Lambda^{m+1}} & \frac{\phi_2}{\Lambda} & \frac{\phi_3}{\Lambda} \\ \frac{\phi_2^{m}\phi_3}{\Lambda^{m+1}} & \frac{\phi_3}{\Lambda} & 1 \end{pmatrix} v$$

$$\Downarrow$$

$$\Downarrow \ \langle\phi_1\rangle \simeq \langle\phi_2\rangle \simeq \langle\phi_3\rangle \equiv \lambda\Lambda$$

$$\Downarrow$$

$$M_D \simeq \begin{pmatrix} \lambda^{m+1} & \lambda^{m+1} & \lambda^{m+1} \\ \lambda^{m+1} & \lambda & \lambda \\ \lambda^{m+1} & \lambda & 1 \end{pmatrix} v, \qquad \delta = \lambda^{m+1}$$

Appendix

$SU(5)$ ($SO(10)$) model

$$
\begin{aligned}
M_u &= Y^{10}\langle 5(10)\rangle + Y^{120}\langle 45(120)\rangle + Y^{1\bar{2}6}\langle 5(1\bar{2}6)\rangle \\
&\equiv Y_u v_u, \\
M_d &= Y^{10}\langle \bar{5}(10)\rangle + Y^{120}[\langle \bar{5}(120)\rangle + \langle 4\bar{5}(120)\rangle] \\
&\quad + Y^{1\bar{2}6}\langle 4\bar{5}(1\bar{2}6)\rangle \\
&\equiv Y_d' v_d, \\
M_{\nu D} &= Y^{10}\langle 5(10)\rangle + Y^{120}\langle 5(120)\rangle - 3Y^{1\bar{2}6}\langle 5(1\bar{2}6)\rangle \equiv Y_\nu' v_u, \\
M_e &= Y^{10}\langle \bar{5}(10)\rangle + Y^{120}[\langle \bar{5}(120)\rangle - 3\langle 4\bar{5}(120)\rangle] \\
&\quad -3Y^{1\bar{2}6}\langle 5(1\bar{2}6)\rangle \\
&\equiv Y_e' v_e, \\
M_R &= Y^{1\bar{2}6}\langle 1(1\bar{2}6)\rangle, \\
M_L &= Y^{1\bar{2}6}\langle 15(1\bar{2}6)\rangle,
\end{aligned}
$$

Appendix

Input for numerical analyses

$$\frac{m_u}{m_c} = 0.0026(6), \quad \frac{m_c}{m_t} = 0.0023(2), \quad y_t = 0.51(2),$$

$$\frac{m_d}{m_s} = 0.051(7), \quad \frac{m_s}{m_b} = 0.018(2), \quad y_b = 0.34(3),$$

$$A = 0.73(3), \quad \lambda_c = 0.227(1), \quad \bar{\rho} = 0.22(6), \quad \bar{\eta} = 0.33(4),$$

$$\tan\beta = 38, \quad \gamma_b = -0.22, \quad \gamma_d = -0.21, \quad \gamma_t = 0,$$

$$\gamma_t \sim y_t^2 \frac{\tan\beta}{32\pi^2}\frac{\mu A_t}{m_{\tilde{t}}^2}, \quad \gamma_u \sim 0, \quad \gamma_b \sim \frac{4}{3} g_3^2 \frac{\tan\beta}{16\pi^2}\frac{\mu \bar{M}_3}{m_{\tilde{b}}^2},$$

$$\gamma_d \sim \frac{4}{3} g_3^2 \frac{\tan\beta}{16\pi^2}\frac{\mu \bar{M}_3}{m_{\tilde{d}}^2},$$

Appendix

Input for numerical analyses

SU(5) case

$$M_R = \begin{pmatrix} f_R\lambda^7 & e_R\lambda^9 & d_R\lambda^5 \\ e_R\lambda^9 & c_R\lambda^4 & b_R\lambda^3 \\ d_R\lambda^5 & b_R\lambda^3 & a_R \end{pmatrix} M,$$

SO(10) case

$$M_R = \begin{pmatrix} f_R\lambda^{17} & e_R\lambda^{17} & d_R\lambda^{10} \\ e_R\lambda^{17} & c_R\lambda^{10} & b_R\lambda^7 \\ d_R\lambda^{10} & b_R\lambda^7 & a_R \end{pmatrix} M,$$