

The Flavor structure of UED with KK-parity

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Outline

Goal:

A model of UED with KK-parity with

- 1 anarchic Yukawas
- 2 mass hierarchies from localization

Outline:

- Reminder: How does RS-GIM work?
- Mass Hierarchy in UED with KK-parity
or: KK-parity and the problem of light modes
- Bounds from FCNC in UED with KK-parity

Motivation: Hierarchy Problem vs. Flavor Physics

- “tension” between EWSB & flavor

Unitarity

$$m_{Higgs} \lesssim 1 \text{ TeV}$$

Flavor

$$\Lambda_{new} \gtrsim 10^4 - 10^5 \text{ TeV}$$

⇒ We need **new physics** $\lesssim 1 \text{ TeV}$ to ensure that Higgs is light, but it **must not violate flavor**.

- Does a model of EWSB naturally respect flavor?

⇒ In general: No. In some cases: Yes.

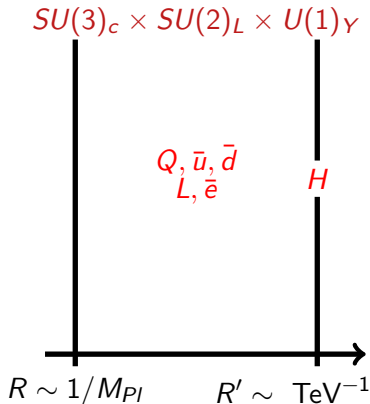
Reminder: Flavor in RS

Randall-Sundrum

- warped metric

$$ds^2 = \left(\frac{R}{z}\right)^2 (dx^2 - dz^2)$$

- Higgs on the IR brane.
 - SM fields in the bulk.
-
- Higgs mass is suppressed
by $\frac{R'}{R} \sim 10^{16}$



Flavor in RS: The Yukawa couplings

- Fermion bulk mass: $\sim \frac{c^i}{z} \bar{\Psi}_i \Psi_i$

Normalized zero mode solutions

$$g_0 \sim z^{2-c} f(c) \quad \text{and} \quad f_0 \sim z^{2+c} f(-c)$$

$$\text{with } f(c) = \sqrt{\frac{1-2c}{1-(R'/R)^{2c-1}}}$$

- $f(c)$ strongly hierarchical for SM fermions.
- Yukawa couplings:

$$\mathcal{L}_Y = -\frac{v}{\sqrt{2}} \frac{R^4}{R'^3} \left[\bar{\Psi}_q \tilde{Y}_u \Psi_u + \bar{\Psi}_q \tilde{Y}_d \Psi_d + h.c. \right] \Big|_{z=R'}$$

\tilde{Y}_u, \tilde{Y}_d are both $\mathcal{O}(1)$, anarchic flavor matrices.

Flavor in RS: The SM masses

⇒ Mass matrices are given by

$$\begin{aligned} m_u &= \frac{v}{\sqrt{2}} f_q \tilde{Y}_u f_u \\ m_d &= \frac{v}{\sqrt{2}} f_q \tilde{Y}_d f_d \end{aligned} \quad \text{with} \quad f_c = \text{diag}[\{f(c_i)\}]$$

- Now usual SM prescription applies (here: $U_{ij} \sim f_i/f_j$)

$$m^{SM} = U_L m U_R^\dagger \quad \text{and} \quad V_{CKM} = U_{Lu}^\dagger U_{Ld}$$

$$\Rightarrow \left(m_{u,d}^{SM}\right)_{ii} \sim \frac{v}{\sqrt{2}} Y_* f_{q_i} f_{u_i,d_i}$$

⇒ We got flavor hierarchy, but what about new FCNC contributions?

Flavor in RS: Hierarchy of f_i 's

- First let us check the order the f_i 's:

Left-handed f_i 's are determined from $V_{CKM} \sim f_{qi}/f_{qj}$

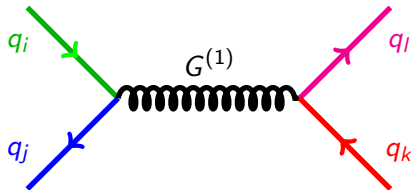
$$\frac{f_{q2}}{f_{q3}} \sim \lambda^2, \quad \frac{f_{q1}}{f_{q3}} \sim \lambda^3 \quad \text{with } \lambda \sim \sin \theta_c \sim 0.2$$

Right-handed f_{u_i, d_i} 's fixed by fermion masses hierarchy

$$\begin{aligned} \frac{f_{u1}}{f_{u3}} &\sim \frac{m_u}{m_t} \frac{1}{\lambda^3}, & \frac{f_{u2}}{f_{u3}} &\sim \frac{m_c}{m_t} \frac{1}{\lambda^2}, \\ \frac{f_{d1}}{f_{u3}} &\sim \frac{m_d}{m_t} \frac{1}{\lambda^3}, & \frac{f_{d2}}{f_{u3}} &\sim \frac{m_s}{m_t} \frac{1}{\lambda^2}, & \frac{f_{d3}}{f_{u3}} &\sim \frac{m_b}{m_t} \end{aligned}$$

Flavor in RS: New Contributions to FCNC

- Strongest bound: FCNC due to KK-gluon exchange:



$$\sim g_{L,u}^{ij} \bar{u}_L^i \gamma^\mu G_\mu^{(1)} u_L^j + g_{L,d}^{kl} \bar{d}_L^k \gamma^\mu G_\mu^{(1)} d_L^l + (L \rightarrow R)$$

- Plug in the wave function: In original (gauge) basis = diagonal

$$g_x \approx g_{s*} \left[\underbrace{-\frac{1}{\log R'/R}}_{\text{universal}} + \underbrace{\frac{f_x^2 \gamma(c_x)}{\gamma \sim 1}}_{\text{non-universal: } \gamma \sim 1} \right]$$

⇒ Universal part does not contribute: $U^\dagger \mathbb{1} U = \mathbb{1}$.

⇒ Non-universal part: **New source of FCNCs.**

Flavor in RS: RS-GIM

- How big is the size of flavor violation?
- Rotate with $U \sim f_i/f_j$
⇒ Off-diagonal KK-gluon couplings

$$g^{ij} \sim g_{s*} f_i f_j \gamma \quad (\text{for } q, u, d)$$

RS - GIM $\hat{=}$ g^{ij} is automatically suppressed

- for L: by ratios of CKM-elements.
- for R: by mass hierarchy.

Flavor in RS: How strong is RS-GIM?

- How strong is this suppression for $\Delta F = 2$ operators?

Effective Hamiltonian

$$\mathcal{H} = C^1(\bar{q}_L^i q_L^j)(\bar{q}_L^k q_L^l) + C^4(\bar{q}_R^i q_L^k)(\bar{q}_L^l q_R^j) + C^5(\bar{q}_R^i q_L^l)(\bar{q}_L^k q_R^j)$$

- Strongest bound comes from the Kaon system:
 $|C_K^4|$ suppressed by $10^4 - 10^5$ TeV.

⇒ in RS:

$$C_K^4 \sim \frac{g_{s^*}^2}{M^2} f_{q1} f_{q2} f_{d1} f_{d2} \sim \frac{g_{s^*}^2}{M^2} \frac{m_d m_s}{v^2}$$

$$\Rightarrow \boxed{M \sim 20 \text{ TeV}}$$

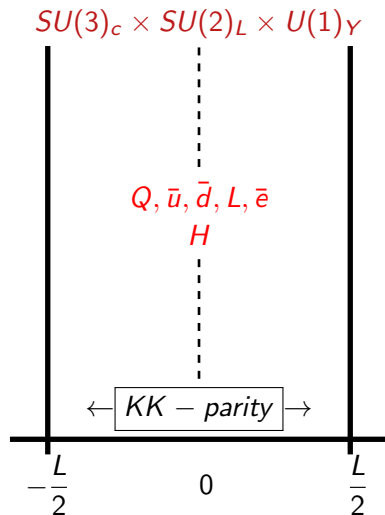
Does a similar mechanism exist in UED?

UED with KK-parity

UED

- Flat metric
 $ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu - dy^2$
- All SM fields in the bulk.

- SM fields are flat.
- Add **KK-parity**: $y \rightarrow -y$
 - + Improves EWPC.
 - + DM candidate.



UED with KK-parity: Fermion zero mode

- Let's add a **bulk mass for the fermions** like we did in RS:

$$S = \int d^4x \int dy \left[\frac{i}{2} (\bar{\Psi} \Gamma^M \overleftrightarrow{\partial}_M \Psi) - m \bar{\Psi} \Psi \right]$$

- Equations of motion

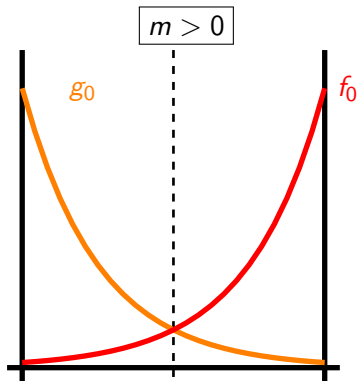
$$\frac{dg_n}{dy} + m g_n - m_n f_n = 0$$

$$\frac{df_n}{dy} - m f_n - m_n g_n = 0$$

- Zero mode solutions

$$\text{left-handed: } g_0 \sim e^{-my}$$

$$\text{right-handed: } f_0 \sim e^{+my}$$



⇒ not KK-Parity invariant

Split-UED

- To maintain KK-parity and also allow bulk masses choose

$$m = m(y) = \begin{cases} \mu & , y < 0 \\ -\mu & , y > 0 \end{cases}$$

- For $y \neq 0$, we still have

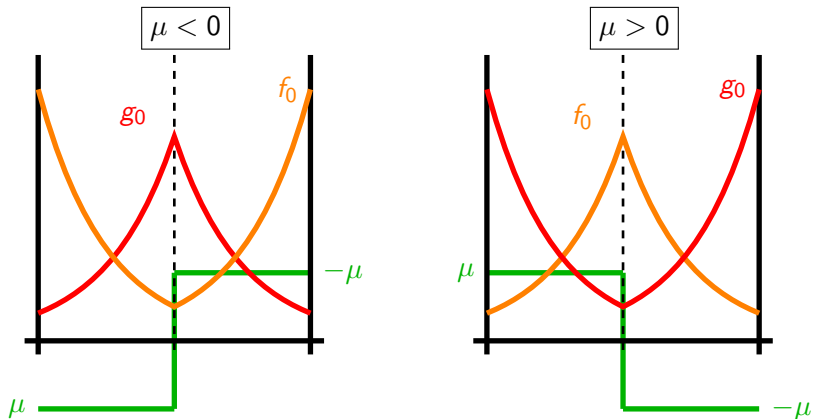
$$g'_n + m(y) g_n - m_n f_n = 0$$

$$f'_n - m(y) f_n - m_n g_n = 0$$

\Rightarrow Invariant under KK-parity: $m(y) \rightarrow -m(y)$

Split-UED: Fermion zero mode

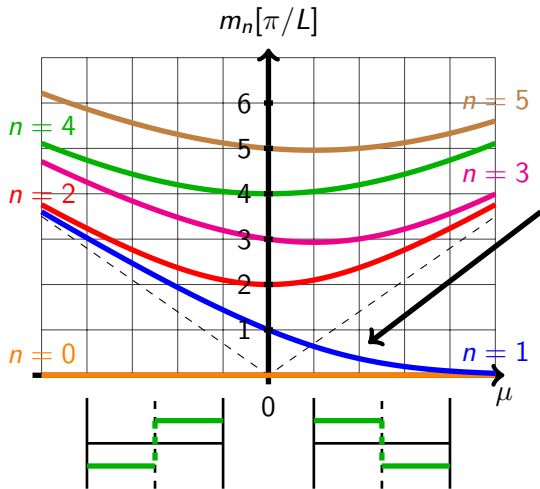
Different localization, depending on sign of μ .



\Rightarrow Could give mass hierarchy due to **small overlap**.

Fermion spectrum: For a LH zero mode

- Let's examine the complete fermion spectrum:
Lefthanded zero mode.



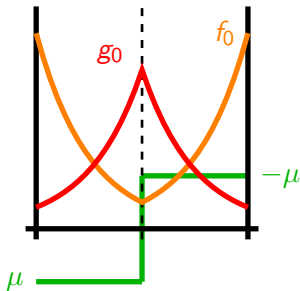
$$m_1 \approx 2\mu e^{-\mu L/2}$$

\Rightarrow too light
(we need $\mu L \approx 10$)

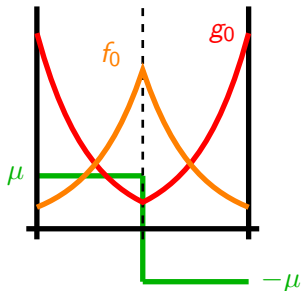
Fermion spectrum: Origin of the extra light mode

- Where does this light mode come from?

Solve:
$$\begin{aligned} g'_n + mg_n - m_n f_n &= 0 \\ f'_n - mf_n - m_n g_n &= 0 \end{aligned}$$
 with BC: $f_n(\pm L/2) = 0$



BC: $f_0 \neq 0 \Rightarrow$ gets heavy.

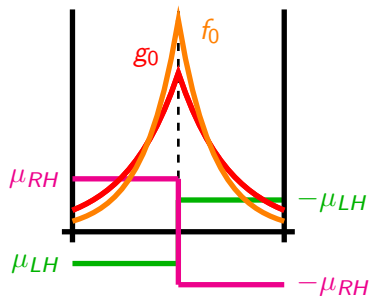


BC: $f_0 \approx 0 \Rightarrow f_0$ remains light
[Equivalently, take the limit $L \rightarrow \infty$.]

Fermion spectrum: Consequences

- What does this mean for our model?

- Need to choose $\mu_{LH} < 0$ to localize g_0 in the middle
 - For same reason we need f_0 in middle ($\mu_{RH} > 0$)
- ⇒ Need all SM localized at $y = 0$ to avoid light modes.



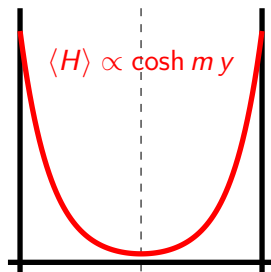
⇒ **No small overlap:** How will we get the hierarchy now?

Localizing the Higgs

- To obtain hierarchy:
Need to exponentially **localize the Higgs at the boundaries**
(or put it directly on the boundary)

Localizing the Higgs

- Add bulk potential
 $\mathcal{V} = m^2 |H|^2$
- Add boundary potentials
 $V \propto \lambda (|H|^2 - v^2)^2$



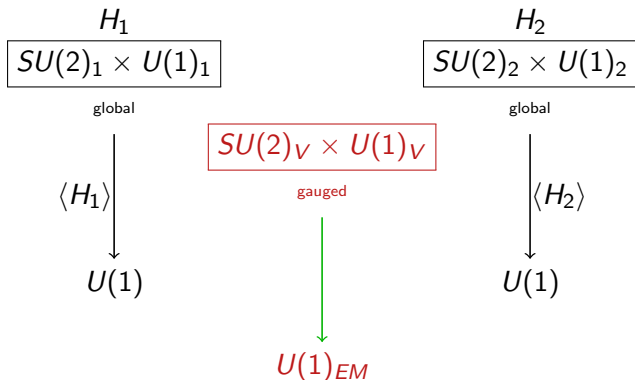
⇒ Gives hierarchy:

$$\mathcal{L}_Y \approx -\frac{v}{\sqrt{2}} \left[\bar{\Psi}_q \tilde{Y}_u \Psi_u + \bar{\Psi}_q \tilde{Y}_d \Psi_d + h.c. \right] \Big|_{y=\pm \frac{L}{2}}$$

- BUT, also gives **very light, KK-odd mode**.

Localizing the Higgs: Two site model

- Where are these (Pseudo)Goldstones from?
- For simplicity, consider a 2 site model: $H \rightarrow H_1$ & H_2



- Two independent H_i + global symmetries \doteq 6 Goldstones
 - 3 KK-even: $\pi_1 + \pi_2 \rightarrow$ get eaten
 - 3 KK-odd: $\pi_1 - \pi_2 \rightarrow$ remain in spectrum!

Mass of Pseudo-Goldstones

Global $[SU(1)_1 \times U(1)_1] \times [SU(2)_2 \times U(1)_2]$ is explicitly broken by

- Having localized Higgs, not 2-site model (small correction)

$$\langle H \rangle \propto \cosh(my) \quad \Rightarrow \quad m_0 \propto m e^{-mL/2}$$

- Gauging $SU(2)_V \times U(1)_V$
- Introducing Yukawa couplings:

$$\mathcal{L}_Y \sim \bar{\Psi}_q H_1 \Psi_u + \bar{\Psi}_q H_2 \Psi_u + (\text{down})$$

Coleman-Weinberg potential

\Rightarrow All the KK-odd Goldstones get mass from fermion (and gauge) loops.

[If you don't like loops: Modify setup slightly (e.g. extra $U(1)$ in bulk,...)]

Flavor in UED with KK-parity

- Now, we can proceed analogous to the RS case:

Normalized zero mode solutions

$$g_0, f_0 \sim f(c) \exp \left[-c \left(\frac{|y|}{L} - \frac{1}{2} \right) \right] \quad \text{with } c = \mu L$$

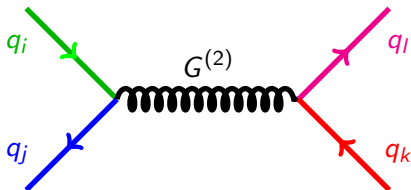
$$\text{with } f(c) = \sqrt{\frac{c}{e^c - 1}} = \frac{1}{\log R'/R} f^{RS}(c_{RS})$$

- Yukawa couplings are given by

$$\mathcal{L}_Y = -\frac{v}{\sqrt{2}} \left[\bar{\Psi}_q \tilde{Y}_u \Psi_u + \bar{\Psi}_q \tilde{Y}_d \Psi_d + h.c. \right] \Big|_{y=\pm L/2}$$

- \tilde{Y}_u, \tilde{Y}_d are both $\mathcal{O}(1)$, anarchic flavor matrices.
- ⇒ This is completely analogous to the RS case ...

... until



- Plug in the wave function: In original basis = diagonal

$$g_x \approx g^{4D} \sqrt{2} \left[\underbrace{1}_{\text{universal}} - \underbrace{f_x^2 \gamma(c_x)}_{\text{non-universal}} \right]$$

- Here $\gamma \propto \frac{e^c}{c^3}$, but to obtain mass hierarchy we need $c \sim 0 \dots 15$.

Unlike RS: $\gamma \approx 1 \Rightarrow$ NO protection from FCNCs.

[There is a way of fixing this, but that leads back to RS.]

Conclusion

- UED with KK-parity can localize fermions (but only in the middle).
 - Localizing the Higgs on the boundaries gives Pseudo-Goldstones: but they get loop masses \rightarrow no problem.
- \Rightarrow Get flavor hierarchy, but no protection from FCNC.

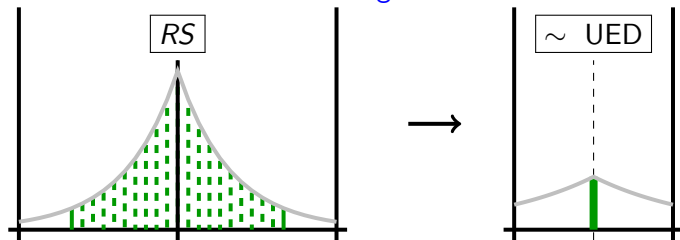
Anarchic flavor in UED is difficult.

The End

Thank you.

RS vs. UED with KK-parity

- So why does this work in RS, but not in UED with KK-parity?
- Can't we think of UED as integrated out RS?



⇒ Effect of integrating out: Add “boundary” kinetic term

$$S_{\text{fermion}} = \int d^5x \left\{ \frac{i}{2} \bar{\Psi} \Gamma^\mu \overleftrightarrow{\partial}_\mu \Psi \right\} \kappa L \delta(y)$$

RS vs. UED with KK-parity

- Only changes the function $f(c, \kappa) = \sqrt{\frac{c}{(1 + c\kappa)e^c - 1}}$.
- Form of γ_x is not changed.

Can suppress $f \sim 1/\kappa$, while keeping $\gamma \sim \frac{e^c}{c} \sim 1$.

- However, **this is basically RS** and not UED!