Stochastic supersymmetry

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Plan of the talk:

- Stochastic superspace
- MSSM with stochastic supersymmetry
- Models incorporating neutrino masses:
  - R-parity violating SSM
  - SSM with type – I see-saw
- Conclusion & outlook

Supersymmetry & superspace

Relativistic invariance

- The concept of space-time:

$$x^\mu = (t, x, y, z)$$

- 4 dimensions, coordinates are c-numbers,

$$x^\mu x^\nu - x^\nu x^\mu = 0$$

- 10 parameter Poncare group:

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu$$

$$\eta_{\mu\nu} = (1, -1, -1 - 1)$$

- Quantum field F(x)
  Particle – representation of the Poincare group

Supersymmetry

- The concept of superspace:

$$X^M = \left(x^\mu, \theta^\alpha, \bar{\theta}^{\dot{\alpha}}\right)$$

- 8 dimensions (N=1 case), θ's are the Grassmann-numbers,

$$\theta^\alpha \theta^\beta + \theta^\beta \theta^\alpha = 0, \quad e.g., \theta^\alpha \theta^\gamma \theta^\delta = 0$$

- 14 parameter super-Poincare group:

$$dS^2 = G_{MN} dX^M dX^N$$

$$G_{MN} = (\eta_{\mu\nu}, \epsilon^{\alpha\beta}, \epsilon_{\dot{\alpha}\dot{\beta}})$$

- Superfield $S(x, \theta, \bar{\theta})$ – describes particles with different spins which form reps of super-Poincare group

Supersymmetry & superspace

- Taylor expansion: $S(x,\theta,\bar{\theta}) = \phi(x) + \theta\psi_1(x) + \bar{\theta}\bar{\psi}_2(x) + \theta^2 M(x) + \bar{\theta}^2 N(x)$
  $\theta\sigma^\mu\bar{\theta}A_\mu(x) + \theta^2\bar{\theta}\bar{\lambda}_1(x) + \bar{\theta}^2\theta\lambda_2(x) + \theta^2\bar{\theta}^2 D(x)$

- Basic features: Number of fermionic and bosonic degrees of freedom are equal; Fermions and bosons in the same superfield are degenerate in mass.

- Good thing: Improved high energy behavior – supersymmetric QFT are logarithmically divergent at most – might play the role in stabilizing the electroweak scale against quantum corrections.

- Bad things: We do not see such spectrum of elementary particles – supersymmetry must be softly broken – more than 100 extra soft breaking parameters, potentially large contributions to FCNC processes, CP violation etc.

- Accepted picture: Supersymmetry is spontaneously broken in some hidden sector and it is transmitted to the visible sector through some interactions – gravity, extra gauge interactions, superconformal anomaly.

Stochastic superspace

- Stochastic superspace is simply understood as a superspace where the anti-commuting Grassmannian coordinates are stochastic variables
- Generic distribution describing stochasticity of Grassmannian coordinates is:

$$\mathcal{P}(\theta,\bar{\theta}) = A + \theta^{\alpha}\Psi_{\alpha} + \bar{\theta}_{\dot{\alpha}}\bar{\Xi}^{\dot{\alpha}} + \theta^{\alpha}\theta_{\alpha}B + \bar{\theta}_{\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}C$$
$$+\theta^{\alpha}\sigma^{\mu}{}_{\alpha\dot{\beta}}\bar{\theta}^{\dot{\beta}}V_{\mu} + \theta^{\alpha}\theta_{\alpha}\bar{\theta}_{\dot{\alpha}}\bar{\Lambda}^{\dot{\alpha}} + \bar{\theta}_{\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\theta^{\alpha}\Sigma_{\alpha} + \theta^{\alpha}\theta_{\alpha}\bar{\theta}^{\dot{\alpha}}\bar{\theta}_{\dot{\alpha}}D$$

- Relativistic invariance requires that all Lorentz non-invariant moments must vanish:

$$\langle\theta\rangle = \langle\bar{\theta}\rangle = \langle\theta\bar{\theta}\rangle = \langle\theta^2\bar{\theta}\rangle = \langle\theta\bar{\theta}^2\rangle = 0, \quad \Longrightarrow \quad \Psi = \bar{\Xi} = V_{\mu} = \bar{\Lambda} = \Sigma = 0$$

$$\langle\ldots\rangle \overset{\text{def.}}{\equiv} \int d^4\theta\mathcal{P}\left[\ldots\right]$$

Stochastic superspace

- Lorentz invariant moments are:

$$\langle\theta^\alpha\theta^\beta\rangle = \frac{1}{2\xi}\epsilon^{\alpha\beta},\ \langle\bar\theta_{\dot\alpha}\bar\theta_{\dot\beta}\rangle = \frac{1}{2\xi^*}\epsilon_{\dot\alpha\dot\beta},\ \langle\theta^\alpha\theta^\beta\bar\theta_{\dot\alpha}\bar\theta_{\dot\beta}\rangle = \langle\theta^\alpha\theta^\beta\rangle\langle\bar\theta_{\dot\alpha}\bar\theta_{\dot\beta}\rangle = \frac{1}{4|\xi|^2}\epsilon^{\alpha\beta}\epsilon_{\dot\alpha\dot\beta}$$

$$\Downarrow$$

$$B = C^* = \frac{1}{\xi},\ A = \frac{1}{|\xi|^2}$$

where ξ is the stochasticity parameter with dimension of mass

- Normalization: $$\int d^2\theta d^2\bar\theta\mathcal{P}(\theta,\bar\theta) = 1,\ \Longrightarrow\ D = 1$$

- Probability distribution is:

$$\mathcal{P}(\theta,\bar\theta)|\xi|^2 \equiv \tilde{\mathcal{P}}(\theta,\bar\theta) = 1 + \xi^*(\theta\theta) + \xi(\bar\theta\bar\theta) + |\xi|^2(\theta\theta)(\bar\theta\bar\theta)$$

$$P = 1 + \xi^*\theta^2,\ \bar P = 1 + \xi\bar\theta^2\ ,$$

$$\tilde{\mathcal{P}} = P\bar P,\quad \int d^2\theta\frac{1}{\xi^*}P = \int d^2\bar\theta\frac{1}{\xi}\bar P = 1$$

Wess-Zumino model in stochastic superspace

- The probaility distribution is a spurion field with non-zero F and D terms:

$$[P]_{\mathrm{F}} = \xi^* \neq 0,\ \ [\tilde{\mathcal{P}}]_{\mathrm{D}} = |\xi|^2 \neq 0$$

- Consider a chiral superfield:

$$\Phi(x,\theta,\bar{\theta}) = \phi(x) + \sqrt{2}\theta\psi(x) + \theta^2 F(x)$$
$$+i\theta\sigma^\mu\bar{\theta}\partial_\mu\phi(x) + \frac{i}{\sqrt{2}}\theta^2\partial_\mu\psi(x)\sigma^\mu\bar{\theta} - \frac{1}{4}\theta^2\bar{\theta}^2\partial_\mu\partial^\mu\phi(x)$$

- Super-Lagrangian density of the Wess-Zumino model:

$$\mathcal{L}_{\mathrm{WZ}} = \mathcal{L}_{\mathrm{kin}} + [W + \mathrm{h.c.}]$$

$$\mathcal{L}_{\mathrm{kin}} = \Phi^+\Phi \quad (\mathrm{D-density})$$

$$W = \frac{m}{2}\Phi^2 + \frac{h}{3}\Phi^3 \quad (\mathrm{F-density})$$

Wess-Zumino model in stochastic superspace

- The Lagrangian density of WZ model in stochastic superspace is the averaged WZ sper-Lagrangian density:

$$L = \langle \mathcal{L}_{\mathrm{WZ}} \rangle$$

- Explicitly, we have

$$L_{\mathrm{kin}} = \int d^2\theta d^2\bar{\theta} P\bar{P}\Phi^+\Phi = L_{\mathrm{kin-SUSY}} + |\xi|^2\phi^*(x)\phi(x) + \xi^*\phi(x)F^*(x) + \xi\phi^*(x)F(x)\ ,$$

$$V = \int d^2\theta PW + \mathrm{h.c.} = V_{\mathrm{SUSY}} + \left(\frac{\xi^* m}{2}\phi^2 + \frac{\xi^* h}{3}\phi^3 + \mathrm{h.c.}\right)$$

- Solve for the auxiliary field: $F = -\xi^*\phi + m^*\phi^* + h^*\phi^{*2}$

- We obtain: $L = L_{\mathrm{on-shell-SUSY}} - \left(\frac{\xi^* m}{2}\phi^2 + \frac{2\xi^* h}{3}\phi^3 + \mathrm{h.c.}\right)$

Mass spectrum: $m_{\phi_1} = \sqrt{|m|^2 - |\xi^* m|}\ ,\quad m_{\phi_2} = \sqrt{|m|^2 + |\xi^* m|}\ ,\quad m_\psi = |m|$

$$\mathrm{note:}\quad STrM^2 = 0\ .$$

Gauge theory in stochastic superspace

- Super-Yang-Mills theory is described by a real superfield (D-density, written in the WZ gauge)

$$V(x,\theta,\bar\theta) = -\theta\sigma^\mu\bar\theta A_\mu(x) + i\theta^\alpha\theta_\alpha\bar\theta_{\dot\alpha}\bar\lambda^{\dot\alpha}(x) - i\bar\theta_{\dot\alpha}\bar\theta^{\dot\alpha}\theta^\alpha\lambda_\alpha(x) + \frac{1}{2}\theta^2\bar\theta^2 D(x) \ ,$$

- The chiral field-strength superfield (F-density):

$$W^\alpha = -i\lambda^\alpha(y) + \theta^\alpha D(y) + \theta^\alpha\sigma^{\mu\nu}_\alpha{}^\beta F_{\mu\nu}(y) - \theta^2\bar\sigma^{\mu\ \dot\beta\alpha} D_\mu\bar\lambda_{\dot\beta}(y)$$

- The Lagrangian density for the stochastic super-Yang-Mills:

$$L_{\rm gauge} = \langle\frac{1}{2}{\rm Tr}W^\alpha W_\alpha\rangle + {\rm h.c.} = \left[\frac{1}{2}{\rm Tr}W^\alpha W_\alpha\right]_F - \frac{\xi^*}{2}{\rm Tr}\lambda\lambda + {\rm h.c.}$$

Gaugino mass: $m_{1/2} = 0.5|\xi|$

- $L_{\rm kin-gauge} = \langle{\rm Tr}\Phi^+ e^{2gV}\Phi\rangle$ no breaking terms are produced

MSSM with stochastic SUSY

- Matter fields (quarks and leptons) are residing in chiral superfields, while SU(3) × SU(2) × U(1) gauge fields are placed in real superfields.
- MSSM superpotential

$$W_{\rm MSSM} = \mu H_u H_d + \hat{y}^{\rm up} Q U^c H_u + \hat{y}^{\rm down} Q D^c H_d + \hat{y}^{\rm lept} L E^c H_d$$

$$L_{\rm soft-scalar} = -\xi^* \mu H_u H_d - 2\xi^* \left[\hat{y}^{\rm up} \tilde{Q} \tilde{U}^c H_u + \hat{y}^{\rm down} \tilde{Q} \tilde{D}^c H_d + \hat{y}^{\rm lept} \tilde{L} \tilde{E}^c H_d\right] + {\rm h.c.}$$

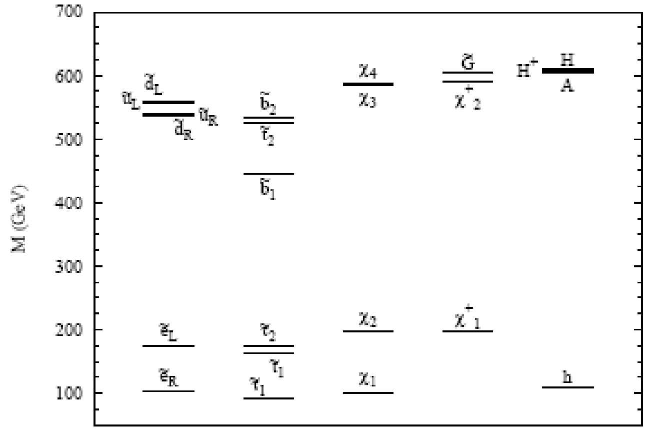
- Soft breaking terms in stochastic MSSM:
  - The bilinear Higgs soft term with $B_\mu = \xi^*$.
  - The trilinear scalar soft terms proportional to the Yukawa couplings, with the universal constant $A_0 = 2\xi^*$.
  - The universal gaugino masses, $m_{1/2} = \frac{1}{2}|\xi|$.
  - The scalar soft masses are absent, $m_0^2 = 0$.

MSSM with stochastic SUSY

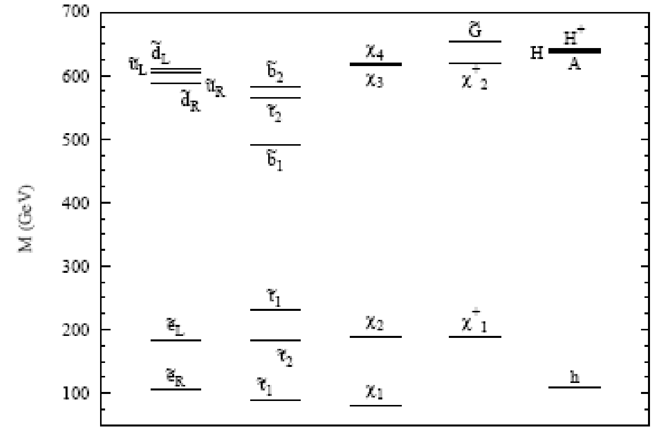
- Soft terms are defined at some high energy scale Λ. Soft parameters at low energies ~M_Z are defined as through the solutions to the RG equations.
- Predictions:
  - $\tan\beta \approx 4 - 5$ (very mild dependence on ξ and Λ)
  - $m_h \approx 112 - 115$ GeV (enhanced due to the near "maximal mixing")
  - Light stop, $m_{\tilde{t}_1} \approx 150 - 250$ GeV, for $\xi < -500$ GeV (favourable for baryogenesis). $\theta_t \approx 0.3 - 0.4$ (determine from backward-forward asymmetry of leptonic and hadronic decays of polarizied top from $\tilde{t}_1 \to t\tilde{\chi}_1^0$).
  - stau– co-annihilation region: $m_{\tilde{\tau}_1} \approx 5\text{–}10 \cdot m_{\chi 1}$, for $\Lambda > 6 \cdot 10^{17}$ GeV (dark matter abundance, $(g-2)_\mu$)

For $\Lambda \lesssim 6 \cdot 10^{17}$ GeV, $m_{\tilde{\tau}_1} < 250$ GeV is the LSP (excluded by the CDF CHAMP search)

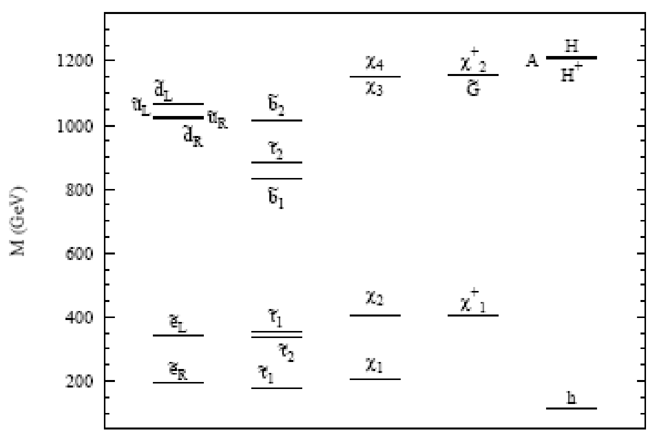
MSSM with stochastic SUSY



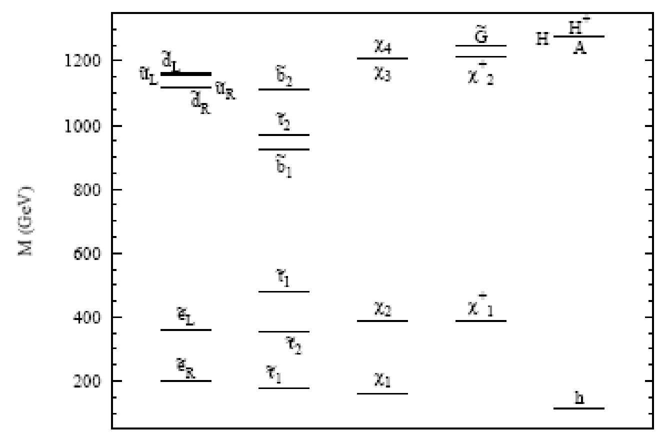
(a)$\Lambda = M_{\rm GUT}, \xi = -500$ GeV.



(b)$\Lambda = M_{\rm Pl}, \xi = -500$ GeV.



(e)$\Lambda = M_{\rm GUT}, \xi = -1000$ GeV.



(f)$\Lambda = M_{\rm Pl}, \xi = -1000$ GeV.

FIG. 1: Sparticle spectra computed using SOFTSUSY3.0, for various values of the parameters Λ and ξ.

RPV SSM with stochastic supersymmetry

$$W_{\not{R}_p} = \epsilon_{mn}\left[\mu_i L_i^m H_2^n + \frac{1}{2}\lambda_{ijk}L_i^m L_j^n E_k^c + \lambda'_{ijk}L_i^m Q_j^n D_k^c\right] + \frac{1}{2}\epsilon^{\alpha\beta\gamma}\lambda''_{ijk}U_{i\alpha}^c D_{j\beta}^c D_{k\gamma}^c$$

- R-parity violation – neutrino mass generation

$$[m_\nu]_{ij} = [m_{\text{tree-level}}]_{ij}^{(\mu\mu)} + [m_{\text{loop}}]_{ij}^{(\lambda\lambda)} + [m_{\text{loop}}]_{ij}^{(\lambda'\lambda')}$$

$$[m_{\text{tree-level}}]_{ij}^{(\mu\mu)} \approx \frac{\cos^2\beta}{M_{\text{SUSY}}}\mu_i\mu_j,$$

$$[m_{\text{loop}}]_{ij}^{(\lambda\lambda)} \approx \sum_{k,l}\frac{1}{8\pi^2}\lambda_{ikl}\lambda_{jlk}\frac{m_{l_k}m_{l_l}}{M_{\text{SUSY}}},$$

$$[m_{\text{loop}}]_{ij}^{(\lambda'\lambda')} \approx \sum_{k,l}\frac{3}{8\pi^2}\lambda'_{ikl}\lambda'_{jlk}\frac{m_{d_k}m_{d_l}}{M_{\text{SUSY}}},$$

- Simplified assumption: the dominant set of couplings

$$\{\lambda_{133}, \lambda_{233}, \lambda'_{133}, \lambda'_{233}, \lambda'_{333}\}$$

RPV SSM with stochastic supersymmetry

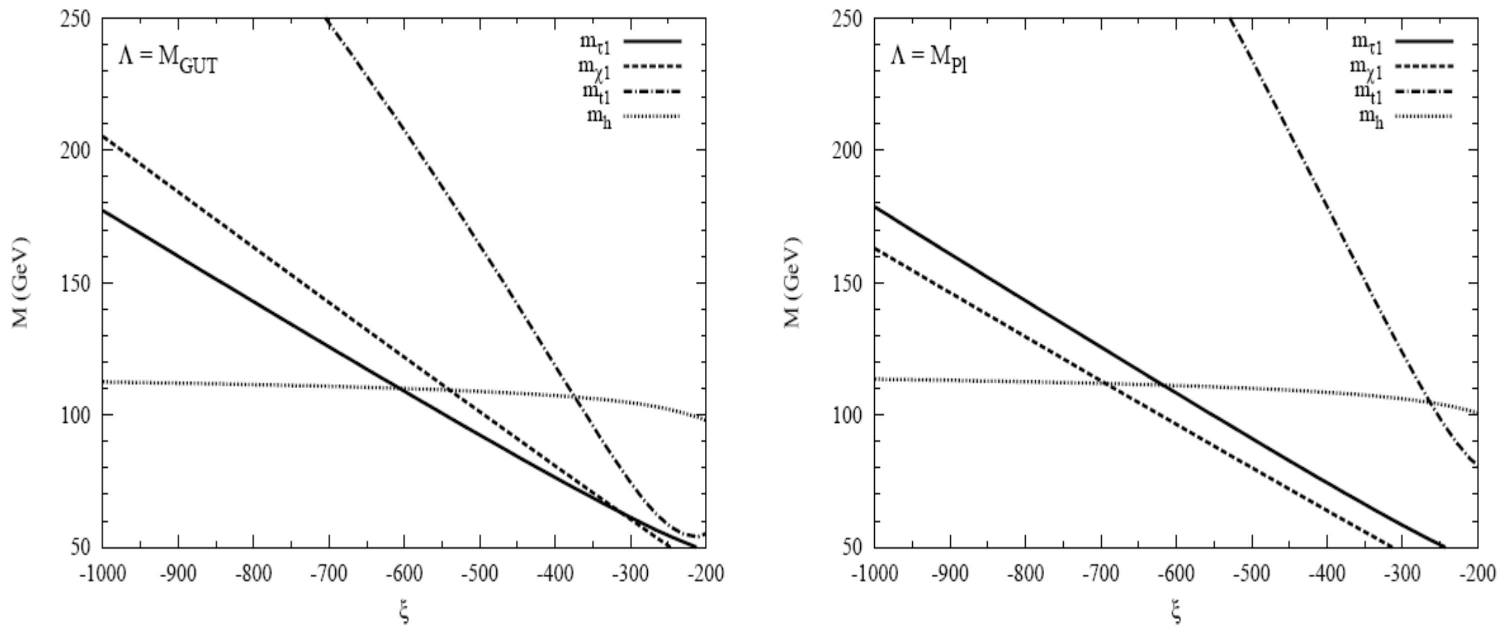
- Tribimaximal mixing: $$U = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix},$$

$$\begin{aligned} m_\nu &= U\,\mathrm{diag}(m_1, m_2, m_3) U^T \\ &= \begin{pmatrix} \frac{1}{3}(2m_1 + m_2) & \frac{1}{3}(-m_1 + m_2) & \frac{1}{3}(-m_1 + m_2) \\ \frac{1}{3}(-m_1 + m_2) & \frac{1}{6}(m_1 + 2m_2 + 3m_3) & \frac{1}{6}(m_1 + 2m_2 - 3m_3) \\ \frac{1}{3}(-m_1 + m_2) & \frac{1}{6}(m_1 + 2m_2 - 3m_3) & \frac{1}{6}(m_1 + 2m_2 + 3m_3) \end{pmatrix} \end{aligned}$$

Parameter	Normal Hierarchy		Inverted Hierarchy	
λ_{133}	8.058×10^{-5}	$- + - +$	3.503×10^{-4}	$- + - +$
λ_{233}	1.612×10^{-4}	$- + - +$	0	
λ'_{133}	6.802×10^{-6}	$- - + +$	6.339×10^{-7}	$- - + +$
λ'_{233}	5.035×10^{-5}	$+ + - -$	6.077×10^{-5}	$- - + +$
λ'_{333}	6.395×10^{-5}	$- - + +$	6.077×10^{-5}	$- - + +$

TABLE I: R-parity violating parameter sets (at weak scale) that achieve tribimaximal mixing in the normal hierarchy ($m_1 = 0$, $m_2 = 8.75 \times 10^{-12}$ GeV, $m_3 = 4.90 \times 10^{-11}$ GeV) and the inverted hierarchy ($m_1 = 4.90 \times 10^{-11}$ GeV, $m_2 = 4.98 \times 10^{-11}$ GeV, $m_3 = 0$). The columns of '+' and '−' represent the eight possible cases, indicating the sign of the corresponding parameter whose magnitude does not change for a given choice of mass hierarchy.

RPV SSM with stochastic supersymmetry



(a)$\Lambda = M_{\rm GUT}$. For $\xi \lesssim -300$ GeV, the stau is the LSP.

(b)$\Lambda = M_{\rm Pl}$. The neutralino is the LSP.

FIG. 2: These plots display the $\xi-$dependence of the masses of the lightest stau ($m_{\tau 1}$), lightest neutralino ($m_{\chi 1}$), lightest stop (m_{t1}) and lightest CP-even Higgs for two representative choices of the cut-off scale, Λ.

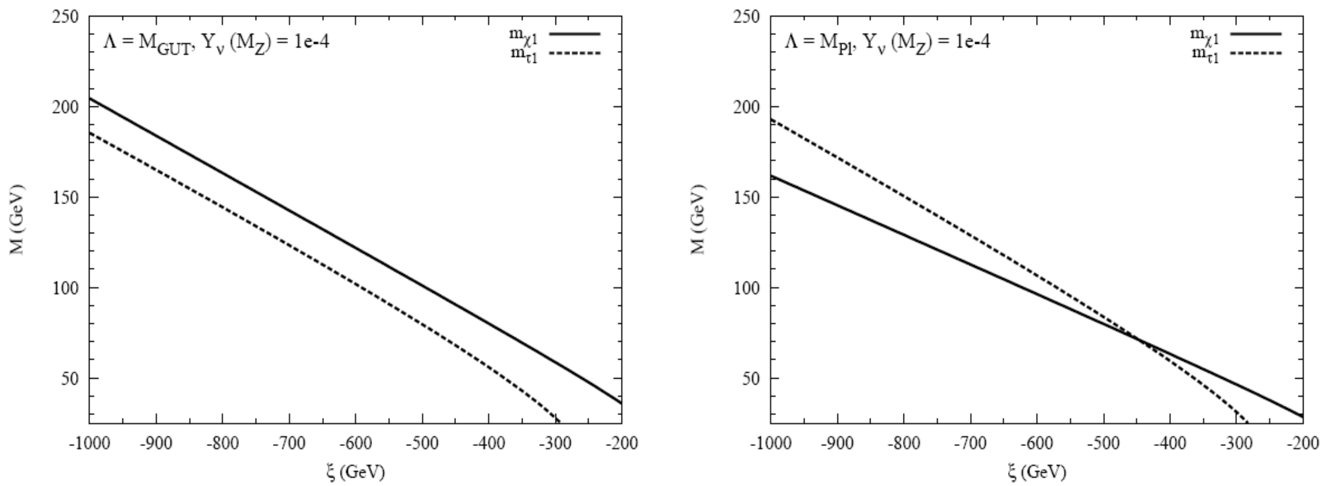
See-saw SSM with stochastic supersymmetry

- Type I see-saw model:

$$W_{\text{seesaw}} = \mu H_u H_d + \hat{y}^{\text{up}} Q U^c H_u + \hat{y}^{\text{down}} Q D^c H_d + \hat{y}^{\text{lept}} L E^c H_d + \hat{y}^{\text{neut}} L N^c H_u + \frac{1}{2} M_R N^c N^c,$$

$$M_R \sim Y_N \times 10^{17}\text{GeV}, \qquad Y_N \equiv \frac{(\hat{y}^{\text{neut}})^2}{16\pi^2}$$

- Type I see-saw works for $Y_N < 0.05 Y_t.$



(a)$\Lambda = M_{\text{GUT}}$. (b)$\Lambda = M_{\text{Pl}}$.

FIG. 3: LSP and NLSP masses as a function of ξ for type-1 seesaw mechanism in stochastic superspace. These figures demonstrate the increased mass gap between the lightest stau and neutralino compared to the minimal stochastic superspace model.

Conclusions

- Field theories on stochastic superspace are equivalent to softly broken supersymmetric theories with a specific and rather constrained pattern of soft breaking terms.

- With this pattern of soft breaking parameters no large contributions to FCNC processes are generated.

- Specific mass spectrum and collider signatures are predicted for MSSM, RPV and type I see-saw models

- Other promising extensions of the minimal model are currently under the investigation, e.g. stochastic supergravity.

- The models with stochastic supersymmetry are generally very predictive, and can be falsified in upcoming experiments at LHC.