Higgs inflation in supergravity

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Outline

1. Inflation and SM Higgs
2. Unitarity bound and naturalness
3. Higgs inflation in supergravity

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Standard Model Higgs inflation

[Bezrukov, Shaposhnikov (2007); De Simone et al (2008); Bezrukov et al (2008)]

- Higgs boson is believed to be present in the SM, triggering electroweak symmetry breaking and generating all the fermion masses.
- If Higgs boson is the inflaton, the required self-coupling ($\lambda \sim 10^{-13}$) would not be compatible with the experimental value, $0.11 < \lambda \lesssim 0.27$, from $114.4\mathrm{GeV} < m_h \lesssim 182\mathrm{GeV}$ (LEP direct bound + precision electroweak data at 95% CL).
- The Higgs inflation with non-minimal coupling may be a new interesting possibility. The non-minimal coupling to gravity is always present in a renormalized scalar theory with self interaction in curved spacetime.

- The Lagrangian proposed for the SM Higgs inflation is in Jordan frame

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{1}{2}M_P^2\mathcal{R} + \xi|\mathcal{H}|^2\mathcal{R} - |D_\mu\mathcal{H}|^2 - \lambda\Big(|\mathcal{H}|^2 - \frac{v^2}{2}\Big)^2.$$

- After Weyl-scaling to Einstein frame, there appears a plateau in the Higgs potential at $h \gg M_P/\sqrt{\xi}$ (with $\mathcal{H} = \frac{1}{\sqrt{2}}h$):

$$V_E = \frac{\frac{1}{4}\lambda(h^2 - v^2)^2}{(1+\xi h^2/M_P^2)^2} \simeq \frac{M_P^4}{4\xi}\Big(1 - 2e^{-2\phi/(\sqrt{6}M_P)}\Big)$$

with ϕ being the canonically normalized Higgs.

- For $N_{\rm efold} \simeq 60$, spectral index is $n_s \simeq 0.968$ while tensor-to-scalar ratio is $r \simeq 3 \times 10^{-3}$, being consistent with observations.
- But, the COBE normalization gives rise to $\xi \simeq 5 \times 10^4\sqrt{\lambda}$ so one needs a large non-minimal coupling ξ for the phenomenologically favored Higgs mass range.

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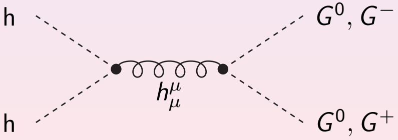
Unitarity bound

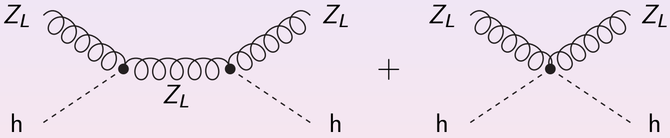
[Burgess, HML, Trott (2009,2010); Barbon, Espinosa (2009); Hertzberg (2010)]

- For a large non-minimal coupling ξ, perturbativity is in question. We focus on the expansion of the Higgs field around the SM vacuum: $\mathcal{H}^T = (G^+, v + h + iG^0)/\sqrt{2}$.
- In Jordan frame, unitarity is violated at $E \sim \Lambda$ via the effective dimension-5 operator, $\mathcal{L}_J = \frac{1}{\Lambda}(h^2 + (G^0)^2 + G^+G^-)\Box h^\mu_\mu$ with $\Lambda \equiv \frac{M_P}{\xi}$:



Power counting shows that the scattering amplitude is $A_4(E) \simeq \left(\frac{E}{\Lambda}\right)^2 \left(\frac{E}{4\pi\Lambda}\right)^{2L}$ at L-loop order.

- In Einstein frame, via the modified gauge-Higgs couplings, $\mathcal{L}_E = -\frac{M_Z^2}{2v^2} Z_\mu Z^\mu (2avh + bh^2)$ with $v = 246\mathrm{GeV}$, $a = 1 - \frac{3v^2}{\Lambda^2}$, $b = 1 - \frac{12v^2}{\Lambda^2}$, the tree-level unitarity is explicitly shown to be violated at $E \simeq \Lambda$:



- Power counting in both frames shows that the effective field theory with a large non-minimal coupling breaks down at $\Lambda = \frac{M_P}{\xi}$ so new physics enters at that scale.
- See the discussion on the validity of perturbative expansion during inflation.

[Lerner, McDonald(2009); Ferrara, Kallosh, Linde, Marrani, Van Proeyen (2010, paper2)]

Naturalness of Higgs inflation

[Burgess, HML, Trott (2009); Barbon, Espinosa (2009)]

- The effective theory of inflation as a derivative expansion breaks down unless the Hubble scale satisfies $H \ll \Lambda$ where Λ is unitarity bound. The Hubble scale in Higgs inflation is $H \simeq \sqrt{\lambda}\,\Lambda$, which is close to unitarity bound.
- The effective Higgs potential and the non-minimal coupling below Λ are expanded as $V_J = \sum_n a_n \frac{h^{4+2n}}{\Lambda^{2n}}$ and $f = \sum_n b_n \frac{h^{2+2n}}{\Lambda^{2n}}$, respectively. But the Higgs inflation takes place for a large Higgs vev, $h \gg \frac{M_P}{\sqrt{\xi}} \sim \sqrt{\xi}\Lambda$. Thus, one needs to arrange the higher order terms for maintaining the plateau of the Higgs potential.
- We consider the embedding of the Higgs inflation in supergravity and discuss the naturalness issues.

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Inflation in Jordan frame supergravity

[Einhorn, Jones (2009); Ferrara, Kallosh, Linde, Marrani, Van Proeyen (2010); HML (2010); Kallosh, Linde (2010)]

- In SUSY, the scalar potential is given by the Kähler potential K and the superpotential W.
- The 4D supergravity action in Jordan frame is

$$\frac{\mathcal{L}_J}{\sqrt{-g_J}} = -\frac{1}{6}\Omega\mathcal{R} - \Omega_{i\bar{j}} D_\mu\phi^i D^\mu\bar{\phi}^{\bar{j}} + \Omega b_\mu^2 - V_J$$

where $\Omega = -3e^{-K/3}$, $b_\mu = -\frac{i}{2\Omega}(D_\mu\phi^i\partial_i\Omega - D_\mu\bar{\phi}^{\bar{i}}\partial_{\bar{i}}\Omega)$ and V_J is related to the standard Einstein-frame potential as $V_J = \frac{\Omega^2}{9}V_E$ with $V_E = e^K(K^{i\bar{j}}(D_iW)(D_{\bar{j}}W^\dagger) - 3|W|^2) + V_D$.

- For the canonical scalar kinetic terms, the frame function is

$$\Omega = -3 + \delta_{i\bar{j}}\phi^i\bar{\phi}^{\bar{j}} - \frac{3}{2}(F(\phi) + \mathrm{h.c.})$$

Then, a non-minimal coupling can be introduced for an appropriate holomorphic function F.

- Consider a toy inflation model with two singlets S and X with the following frame function and superpotential:

$$\begin{aligned}\Omega &= -3 + S^\dagger S + X^\dagger X - \frac{3}{2}(\chi S^2 + \mathrm{h.c.}),\\ W &= \frac{1}{2}\lambda X S^2\end{aligned}$$

where a global $U(1)$ with charges $q_X = -2$ and $q_S = +1$ is broken only by the non-minimal coupling.

- For $|X|, |S| \ll 1$ and $\chi |S|^2 \gg 1$, the Jordan-frame scalar potential is

$$V_J \simeq \frac{1}{4}|\lambda|^2 |S|^4 - \frac{|\lambda|^2}{6\chi}|X|^2 (S^2 + S^{\dagger 2}) + \mathcal{O}\Big(\frac{|\lambda|^2}{\chi^2}|X|^4\Big).$$

So, X becomes tachyonic along the direction of $\mathrm{Re}(S)$.

- After stabilizing the angular mode of S, the Lagrangian density in Einstein frame is

$$\mathcal{L}_E \simeq -\frac{1}{2}(\partial_\mu\varphi)^2 - e^{-2\varphi/(\sqrt{6}M_P)}|\partial_\mu X|^2 - \frac{|\lambda|^2 M_P^4}{4\chi^2}\Big[1 - 2e^{-2\varphi/(\sqrt{6}M_P)} - \frac{2}{3}e^{-2\varphi/(\sqrt{6}M_P)}\frac{|X|^2}{M_P^2}\Big].$$

- The X singlet has a tachyonic effective mass of order the Hubble scale as $m_X^2 \simeq -2H^2$, spoiling the slow-roll inflation along the S singlet. This problem is generic for the canonical scalar kinetic terms in Jordan frame.

[Ferrara, Kallosh, Linde, Marrani, Van Proeyen (2010)]

A solution to tachyonic mass problem

- Introduce a higher order correction $\Delta\Omega = -\frac{\gamma}{M_P^2}(X^\dagger X)^2$ in the frame function to cure the tachyonic mass problem. Then, for $e^{-2\varphi/(\sqrt{6}M_P)} \sim 0.02$ and $\gamma > 0.003$, the singlet mass is
$$m_X^2 \simeq (12\gamma e^{2\varphi/(\sqrt{6}M_P)} - 2)H^2 > 0.$$
- Take two heavy chiral superfields $\Phi_{1,2}$ with $U(1)$ charges $q_1 = +1$ and $q_2 = -1$, and $Z_2 : \Phi_{1,2} \to -\Phi_{1,2}$ and $S, X \to S, X$. Then, the additional superpotential is $W' = \frac{1}{2}\kappa X \Phi_1^2 + M\Phi_1\Phi_2$ and the frame function is canonical. The one-loop correction is then
$$\Delta\Omega = -\frac{1}{32\pi^2}\left[2M^2 \ln\left(\frac{M^2}{\mu^2}\right) + \left\{\ln\left(\frac{M^2}{\mu^2}\right) + 2\right\}|\kappa X|^2 + \frac{|\kappa X|^4}{6M^2}\right].$$
- Then, we get $\gamma = \frac{|\kappa|^4}{192\pi^2}\frac{M_P^2}{M^2}$: for $M \sim \frac{M_P}{\xi}$, we need $|\kappa| > 0.97/\sqrt{\chi}$.

Naturalness revisited

- Unitarity bound is satisfied for $H \simeq \frac{|\lambda| M_P}{\chi} \ll \Lambda \simeq \frac{M_P}{\chi}$, i.e. a small singlet coupling $|\lambda| \ll 1$. Thus, from the constraint coming from COBE, $\frac{\chi}{|\lambda|} \simeq 5 \times 10^4$, the non-minimal coupling does not have to be very large.
- Higher order holomorphic corrections for S in frame function and the superpotential are not generated by non-renormalization theorem.
- But, higher order non-holomorphic corrections in frame function must be suppressed at cutoff scale and be not generated by heavy thresholds as in the previous example. For instance, for $\Delta\Omega = \frac{c}{\Lambda^2}(S^\dagger S)^2$, we need $|c| \ll 1$ for the non-minimal coupling to be dominant.

Higgs inflation in NMSSM

- The Higgs inflation in MSSM does not work because the D-term leads to a fast rolling along the $\tan\beta$ direction.
- In NMSSM, the singlet coupling to the Higgs doublets leads to an additional Higgs potential. The NMSSM Higgs inflation is described by [Einhorn, Jones (2009); Ferrara et al (2010)]

$$\begin{aligned}\Omega &= -3 + H_u^\dagger H_u + H_d^\dagger H_d + X^\dagger X + \frac{3}{2}(\chi H_u H_d + \mathrm{h.c.}), \\ W &= \frac{1}{2}\lambda X H_u H_d + \frac{1}{3}\rho X^3.\end{aligned}$$

- Setting the D-term to zero, our toy model analysis applies. Higgs inflation is compatible with unitarity bound for a small singlet coupling $|\lambda| \ll 1$ and is tachyon-free with a higher order correction for X in Jordan frame function.

- Due to the nontrivial Higgs dependence of the frame function or the Kähler potential, the effective μ term has an additional contribution,

$$\mu = \frac{3}{2}\chi m_{3/2} + \frac{1}{2}\lambda\langle X\rangle.$$

- For $\lambda \sim 0.01$ and $\chi \sim 10^2$, the soft mass of order 100 GeV leads to the gravitino mass $m_{3/2} \sim 1\mathrm{GeV}$ and $\langle X\rangle \leq 10\mathrm{TeV}$. When R-parity is conserved, the gravitino is LSP and can be either a non-thermal dark matter with neutralino NLSP or a thermal dark matter for the reheating temperature $T_R \sim 10^8\mathrm{GeV}$.
- For a small λ, the tree-level singlet contribution to the Higgs mass is suppressed and the mixing between the singlet and the neutral component of the MSSM Higgs doublets is small. So, the lighetest neutral Higgs is of the MSSM type.

Conclusion

- Higgs inflation can be embedded in NMSSM in which the extra Higgs potential drives the inflation without unitarity problem.
- An introduction of higher order terms is necessary for curing the tachyonic mass problem in supersymmetric Higgs inflation. On the other hand, one must suppress the accompanying dangerous higher order term for the inflaton.
- From a simple toy model for heavy physics, we showed that the tachyonic mass problem can be resolved without spoiling the slow-roll conditions by a Z_2 symmetry.
- In the NMSSM Higgs inflation, a large non-minimal coupling generates the μ term larger than gravitino mass, so gravitino can be LSP and a dark matter candidate.