Measuring Unification

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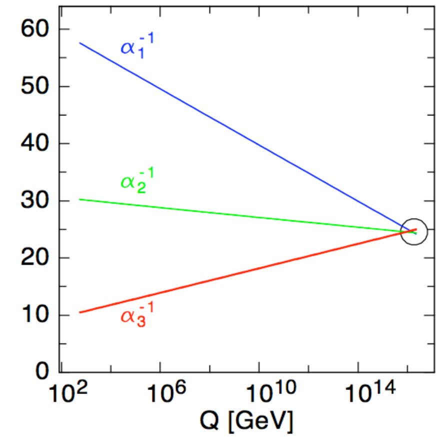
What does « measuring » stand for ?

It's been known since a long time that while the 3 gauge couplings α_i do not unify in the Standard Model, including supersymmetry can naturally lead to such a unification.

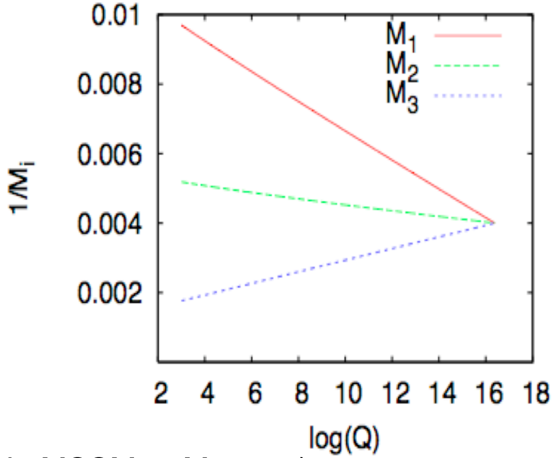


Our question is thus :

Given the expected precision on the MSSM parameters extracted from LHC measurements, what are the chances to observe the "expected" unification of gauginos ?

What about sfermions ?

How well can we measure the underlying m_0 and $m_{1/2}$?



In earlier publications, the SFitter team has worked on the extraction of the MSSM parameters from the set of expected measurements at LHC. Tools like SUSPECT allow to predict the evolution of the model parameters as a function of Q^2 :

"bottom up" (if they are defined at the EW scale - like in the MSSM)
or *"top down"* (if they are defined at the GUT scale - like in the High Scale MSSM or Msugra)

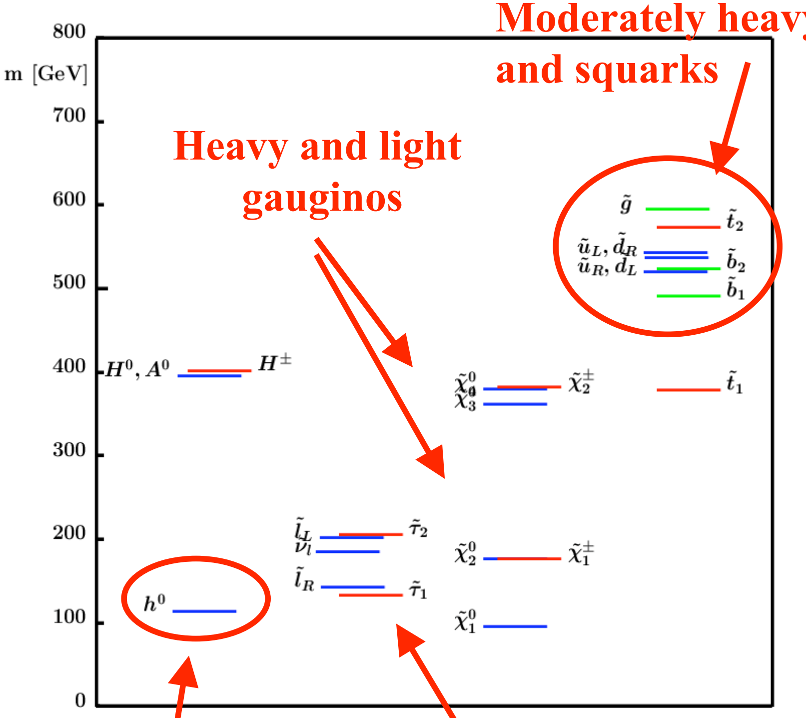
Building blocks (1) : the model

SPS1a : favorable for LHC and ILC, extensively studied (hep-ph/0410364)
very close to latest EW fits (hep-ph/0908.4300)

$m_0 = 100\text{GeV}$; $m_{1/2} = 250\text{GeV}$; $A_0 = -100\text{GeV}$; $\tan\beta = 10$; $\text{sign}(\mu) = +$

Moderately heavy gluinos and squarks

Heavy and light gauginos



Higgs at the limit of LEP reach

light sleptons

For a given model (MSSM), the mass spectrum at the EW scale is calculated at the 2 loops level by :
Suspect (hep-ph/0211331)
or SoftSusy (hep-ph/0104145)

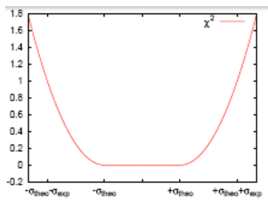
We now forget about the source and fit the MSSM parameters

$\tan\beta$
M_1
M_2
M_3
$M_{\tilde{\tau}_L}$
$M_{\tilde{\tau}_R}$
$M_{\tilde{\mu}_L}$
$M_{\tilde{\mu}_R}$
$M_{\tilde{e}_L}$
$M_{\tilde{e}_R}$
$M_{\tilde{q}3_L}$
$M_{\tilde{t}_R}$
$M_{\tilde{b}_R}$
$M_{\tilde{q}_L}$
$M_{\tilde{q}_R}$
A_τ
A_t
A_b
m_A
μ

Building blocks (2) : measurements @LHC

Set of expected results used as an input : “Physics Interplay of the LHC and ILC” hep-ph/0410364

- Some variables cannot be measured (in particular χ^0_3, charginos, …), some are not well constrained (e.g. 1 measurement for 2 stau masses).
- Experimental errors are statistical (Gaussian, uncorrelated) or systematical (Gaussian, fully correlated)
- Theoretical errors have a flat distribution within the allowed range :
  They are included using the Rfit scheme



$$\chi^2 = \sum_{\text{measurements}} \begin{cases} 0 & \text{for } |x_{\text{data}} - x_{\text{pred}}| < \sigma_{\text{theo}} \\ \left(\dfrac{|x_{\text{data}} - x_{\text{pred}}| - \sigma_{\text{theo}}}{\sigma_{\text{exp}}}\right)^2 & \text{for } |x_{\text{data}} - x_{\text{pred}}| \geq \sigma_{\text{theo}} \end{cases}$$

Theoretical errors used for the MSSM fit :

0.5 % for the masses of colorless particles (neutralinos, charginos, sleptons)

1 % for the masses of gluinos and squarks

300 fb^{-1}

	type of measurement	nominal value	stat.	LES error	JES error	theo.
m_h		108.7	0.01	0.25		2.0
m_t		171.20	0.01		1.0	
$m_{\tilde{l}_L} - m_{\chi_1^0}$		102.38	2.3	0.1		1.1
$m_{\tilde{g}} - m_{\chi_1^0}$		511.38	2.3		6.0	6.1
$m_{\tilde{q}_R} - m_{\chi_1^0}$		446.39	10.0		4.3	5.5
$m_{\tilde{g}} - m_{\tilde{b}_1}$		89.01	1.5		1.0	8.0
$m_{\tilde{g}} - m_{\tilde{b}_2}$		62.93	2.5		0.7	8.2
$m_{ll}^{\max}$:	three-particle edge(χ_2^0,$\tilde{l}_R$,χ_1^0)	80.852	0.042	0.08		1.2
$m_{llq}^{\max}$:	three-particle edge($\tilde{q}_L$,χ_2^0,χ_1^0)	449.08	1.4		4.3	5.1
m_{lq}^{low}:	three-particle edge($\tilde{q}_L$,χ_2^0,$\tilde{l}_R$)	326.32	1.3		3.0	5.2
$m_{ll}^{\max}(\chi_4^0)$:	three-particle edge(χ_4^0,$\tilde{l}_L$,χ_1^0)	277.36	3.3	0.3		2.0
$m_{\tau\tau}^{\max}$:	three-particle edge(χ_2^0,$\tilde{\tau}_1$,χ_1^0)	83.21	5.0		0.8	1.0
m_{lq}^{high}:	four-particle edge($\tilde{q}_L$,χ_2^0,$\tilde{l}_R$,χ_1^0)	390.18	1.4		3.8	5.0
m_{llq}^{thres}:	threshold($\tilde{q}_L$,χ_2^0,$\tilde{l}_R$,χ_1^0)	216.00	2.3		2.0	3.3
m_{llb}^{thres}:	threshold($\tilde{b}_1$,χ_2^0,$\tilde{l}_R$,χ_1^0)	198.41	5.1		1.8	3.1

Overview of the parameter space

Previous SFitter publication (arXiv.0709.3985 [hep-ph]) :
« For a « typical » point (SPS1a) and two physics models (MSUGRA and MSSM), it was shown that a Likelihood map could be built, maxima identified, and that the parameters could be extracted with some error »

LHC provides 23 measurements for 20 parameters => 3 DOF

For LHC+ILC : 41 measurements, 22 parameters => 18 DOF

- The structure of the RGE shows that the gaugino parameters are decoupled from the scalar sector (Yukawa couplings enter only at two-loop level)
- Some sectors are not fully constrained (StauR, StauL, StopR, m_A) : We chose to let them free in the fit (the errors will be very big)
- LHC has no sensitivity to the trilinear couplings : We chose to fix Atau and Ab to the “central value” their range : = 0 And estimated the bias introduced

To first approximation, the neutralino masses are :

$$m_{\chi_i^0}(i=1,..,4) = \mathrm{M}_1, \mathrm{M}_2, |\mu|, |\mu|$$

The mass predictions are insensitive to a swap :

One can expect 6 solutions x 2 because the sign of μ is not measured @LHC

$\tan\beta$

$\boxed{\mathrm{M}_1 \quad \mathrm{M}_2 \quad \mathrm{M}_3}$

$\mathrm{M}_{\tilde{\tau}_L}$

$\mathrm{M}_{\tilde{\tau}_R}$

$\mathrm{M}_{\tilde{\mu}_L}$

$\mathrm{M}_{\tilde{\mu}_R}$

$\mathrm{M}_{\tilde{e}_L}$

$\mathrm{M}_{\tilde{e}_R}$

$\mathrm{M}_{\tilde{q}3_L}$

$\mathrm{M}_{\tilde{t}_R}$

$\mathrm{M}_{\tilde{b}_R}$

$\mathrm{M}_{\tilde{q}_L}$

$\mathrm{M}_{\tilde{q}_R}$

$\boxed{\mathrm{A}_\tau \quad \mathrm{A}_t \quad \mathrm{A}_b}$

m_A

μ

Fit results : 8 solutions

Fit result (all parameters fitted, only 4 shown in the table)

closest to SPS1a

	DS1	DS2	DS3	DS4	DS7	DS8	DS9	DS10
$\tan\beta$	12.3±5.6	12.4±5.0	14.9±9.8	8.9±5.9	13.8±7.5	12.6±7.9	19.2±14.3	23.0±15.6
M_1	102.7±7.1	189.5±6.2	107.2±9.2	383.2±9.1	105.0±6.9	191.7±6.6	116.3±7.5	380.9±9.3
M_2	185.5±7.0	96.±6.4	356.9±8.7	114.2±10.7	194.7±7.3	105.5±7.3	354.0±8.2	137.2±9.1
μ	-362.7±7.8	-364.7±6.8	-186.0±8.5	-167.0±9.6	353.0±7.7	357.1±8.3	188.9±7.1	172.8±8.7

Sign of μ

Swaps of M1,M2, μ
errors ~ 20%
Fit results consistent with theoretical expectations
All other parameters are only slightly shifted
(see paper for details)

Prediction from theory

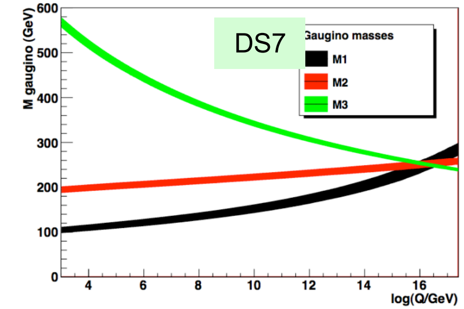
solution	M_1	M_2	μ	$m_{\chi^0_{pred}}$
DS1	97.66	187.35	-358.43	367.7
DS2	182.44	98.54	-361.64	371.8
DS3	102.35	354.88	-184.61	195.5
DS4	368.7	120.16	-165.49	197.0
~~DS5~~	~~168.0~~	~~357.44~~	~~-115.27~~	~~127.3~~
~~DS6~~	~~369.45~~	~~144.05~~	~~-77.94~~	~~55.2~~
DS7	100.41	196.68	349.34	355.6
DS8	184.73	106.47	354.69	361.5
DS9	109.26	350.25	185.84	193.2
DS10	367.13	140.59	170.86	215.9
~~DS11~~	~~163.59~~	~~354.52~~	~~126.55~~	~~134.6~~
~~DS12~~	~~368.88~~	~~136.13~~	~~83.44~~	~~46.7~~

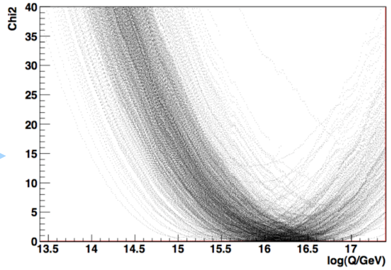
Why 2x4 instead of 2x6 DS ?
Because 4 out of 12 are not compatible
with experimental constraints @ LHC
(the unmeasured χ^0 would be the LSP)

Evolution versus Q2 ? Gauginos

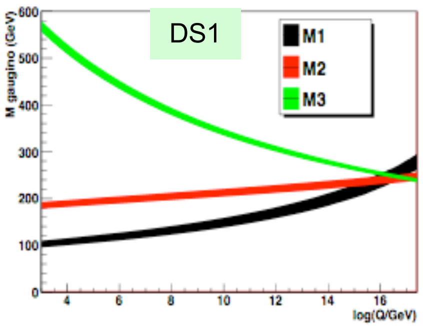
The analytical propagation of errors as function of the scale is quite tedious and not always possible → toys

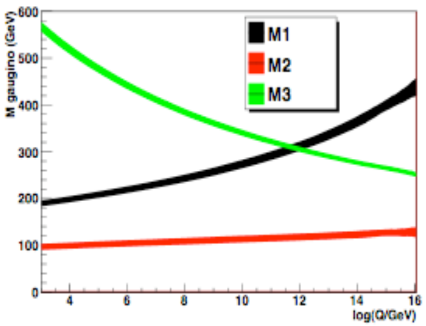
- ~ 5000 « data sets » are generated, where the expected LHC measurements are smeared according to their errors
- The determination of the parameters is performed
- Suspect or SoftSusy provide the RGE running of the fitted parameters between EW and GUT scales
- At any given scale, the width of the parameter distribution is the error on the parameter (width on the plots = RMS).

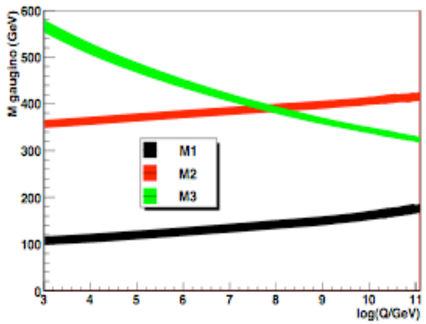


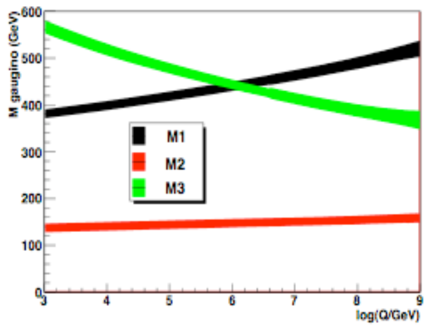


Build a χ^2 which is used to :
- count the fraction of toys « unifying »
- measure the unification mass $m_{1/2}$ and scale
(see paper for details on the χ^2)









M_i RGE are (almost) linear :
"swaps" do not unify !

Out of the 2x4 DS, only 2 "unify" : they differ by the sign of μ
95% of the toys « unify » for DS7 (~ SPS1a)
37% of the toys « unify » for DS1 (wrong μ sign)

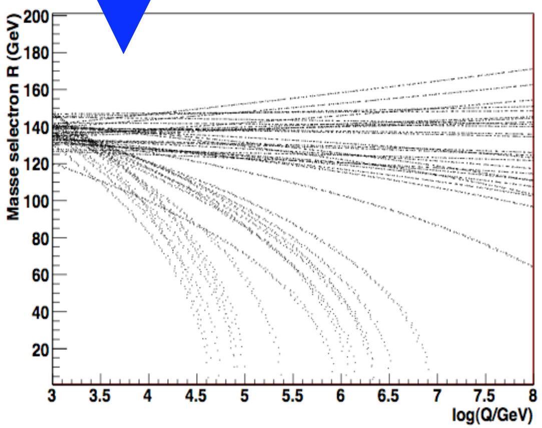
Effect of "unmeasured" parameters @LHC : the "tachyonic" sets

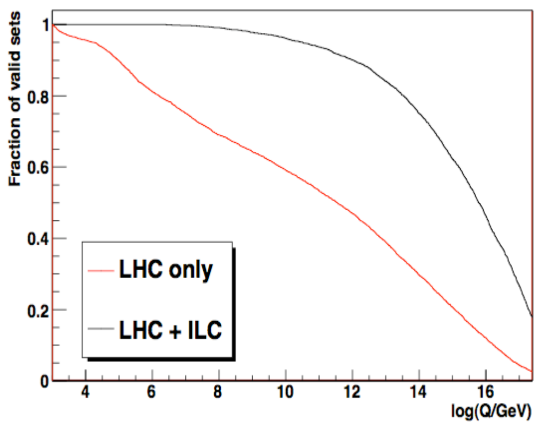
$$\frac{d}{dt}\mathrm{M}^2_{\tilde{q}_L} = \frac{1}{2\pi}\left(\frac{\alpha_1}{10}\mathrm{Tr}[Ym^2] - \frac{\alpha_1}{15}M_1^2 - 3\alpha_2 M_2^2 - \frac{16}{3}\alpha_3 M_3^2\right)$$

$$\begin{aligned}\mathrm{Tr}[Ym^2] = {}& \mathrm{M}^2_{\mathrm{H}_2} - \mathrm{M}^2_{\mathrm{H}_1} \\ & + \mathrm{M}^2_{\tilde{q}1_L} - \mathrm{M}^2_{\tilde{e}_L} - 2\mathrm{M}^2_{\tilde{u}_R} + \mathrm{M}^2_{\tilde{d}_R} + \mathrm{M}^2_{\tilde{e}_R} \\ & + \mathrm{M}^2_{\tilde{q}2_L} - \mathrm{M}^2_{\tilde{\mu}_L} - 2\mathrm{M}^2_{\tilde{c}_R} + \mathrm{M}^2_{\tilde{s}_R} + \mathrm{M}^2_{\tilde{\mu}_R} \\ & + \mathrm{M}^2_{\tilde{q}3_L} - \mathrm{M}^2_{\tilde{\tau}_L} - 2\mathrm{M}^2_{\tilde{t}_R} + \mathrm{M}^2_{\tilde{b}_R} + \mathrm{M}^2_{\tilde{\tau}_R}\end{aligned}$$

$$\frac{d}{dt}\mathrm{M}^2_{\tilde{e}_R} = \frac{1}{2\pi}\frac{3}{5}\alpha_1\left(\mathrm{Tr}[Ym^2] - 4M_1^2\right)$$

Tr[Ym^2] vanishes at the tree level… but contains squared masses :
the "unmeasured" ones can "spoil" the evolution of others … which can even become tachyonic :-(





→ Remove all sets where 1 mass becomes tachyonic

→ For LHC : 7% make it up to GUT

Adding measurements (ILC here) obviously helps !

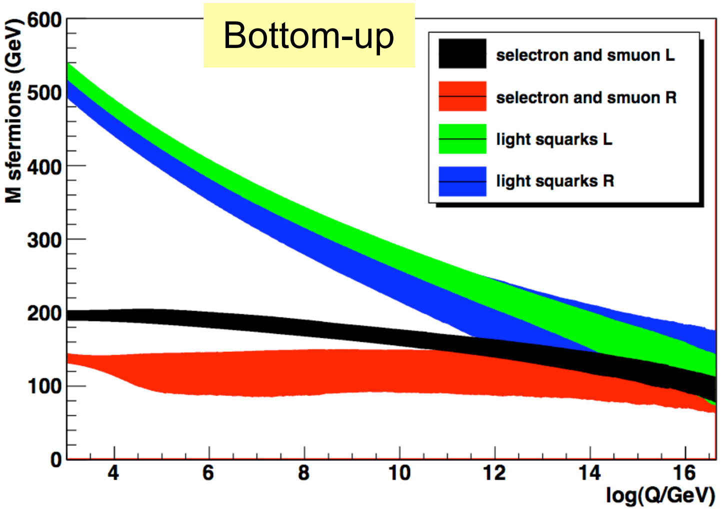
Claire Adam-Bourdarios

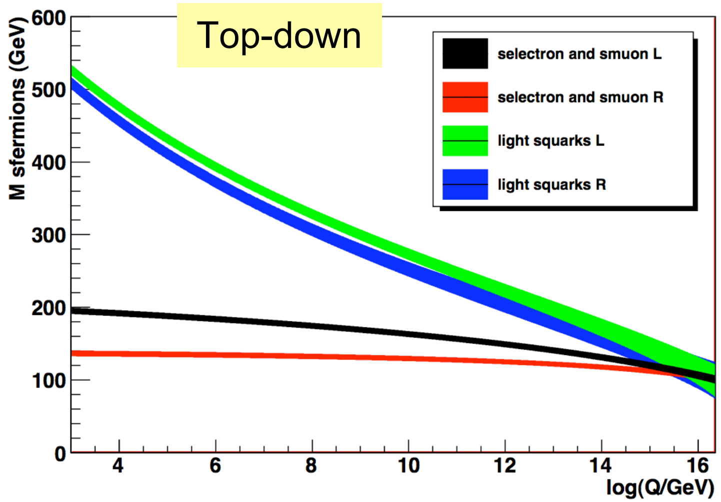
« bottom up » ≠ « top down » ?
extrapolation of sfermions masses

MSSM parameters can be defined at any scale. The 2 following approaches should give identical results :

- At EW scale : parameters are defined and fitted at the EW scale. RGE only enter afterwards, for the extrapolation to GUT. The broadening is a natural effect of error propagation : e.g. 1% on M3 @EW scale gives 20% to 100% @GUT scale

- High-Scale MSSM : parameters are defined at GUT scale, extrapolated « top-down », and fitted => this eliminates solutions where the « non measured parameters » are too far off !
  e.g : moving M stauL by 200 GeV changes the selectron and smuon masses by 5 GeV





Sfermions
1rst / 2nd
generation

The parameter space is obviously not constrained in the same way : we need a real bottom-up approach !

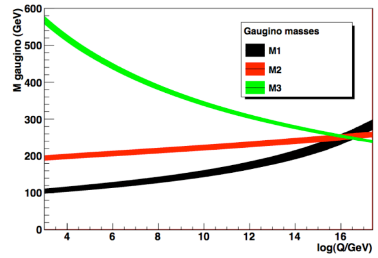
Results for LHC

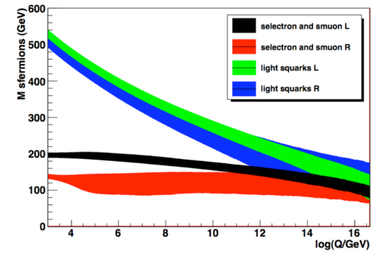
- Life will be difficult :
  - 8 solutions : we will not “prove” unification,
  - We can use it to select the 2 which “unify” (and differ by the sign of μ)
  - Only 7 % of the toys make it up to the GUT scale : shall we have to use toys for data ?

- Using the toy experiment technique the unified parameters (and scale) are determined:

$m_{1/2}$ measured to 2%
m_0 measured to 10%
Fixing the scale divides the error by 2

Name	Unified Parameter	Unification Scale $[\log(Q/\mathrm{GeV})]$	Parameter at $1.7 \cdot 10^{16}$ GeV
$m_{1/2}$	251.9 ± 5.9	16.23 ± 0.29	252.3 ± 3.2
m_0	98.5 ± 10.5	16.5 ± 0.6	100.8 ± 4.9





- Systematics related to the extrapolation ?
  - Threshold effects : used 3% on M3
  - results do not depend on spectrum calculator and RGE extrapolator: same results obtained with SUSPECT and SOFTSUSY.

SFitter

Adding the ILC

Fit results

	DS1	DS2	DS3	DS4	DS7	DS8	DS9	DS10
$\tan\beta$	12.3±5.6	12.4±5.0	14.9±9.8	8.9±5.9	13.8±7.5	12.6±7.9	19.2±14.3	23.0±15.6
M_1	102.7±7.1	189.5±6.2	107.2±9.2	383.2±9.1	105.0±6.9	191.7±6.6	116.3±7.5	380.9±9.3
M_2	185.5±7.0	96.±6.4	356.9±8.7	114.2±10.7	194.7±7.3	105.5±7.3	354.0±8.2	137.2±9.1
μ	-362.7±7.8	-364.7±6.8	-186.0±8.5	-167.0±9.6	353.0±7.7	357.1±8.3	188.9±7.1	172.8±8.7
$\Delta\chi^2_{\mathrm{ILC}}$	73	22000	1700	25000	0.4	22000	2000	24000
ILC	$\tilde{\tau}_1$	$\chi_1^\pm$	χ_3^0	$\chi_1^\pm$	$\tilde{\tau}_1$	$\chi_1^\pm$	χ_3^0	$\chi_1^\pm$
Ωh^2	0.17 ± 0.07	$(4\pm2)\cdot10^{-4}$	0.14 ± 0.08	$(8\pm4)\cdot10^{-4}$	0.16 ± 0.07	$(4\pm3)\cdot10^{-4}$	0.11 ± 0.06	$(9\pm4)\cdot10^{-4}$

Thanks to the additional measurements :

- only 1 solution remains
- all masses are measured
- Trilinear couplings : only sensitive to Atop

The relic density is calculated with the MicrOMEGAS program (arXiv:0803.2360 [hep-ph]) :

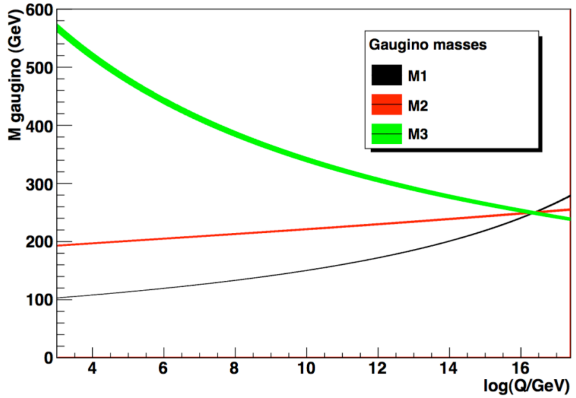
it is sensitive to swaps between M1 and M2

→ allows to select 4 of the 8 LHC solutions…

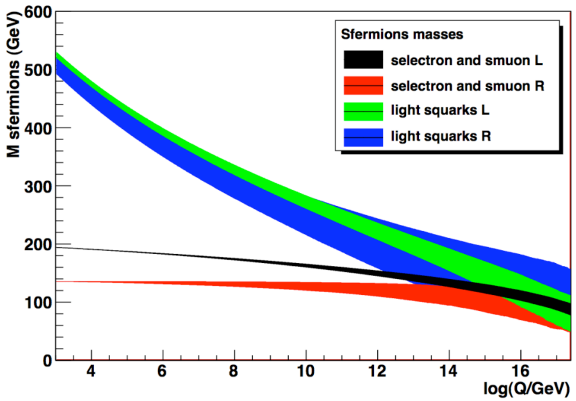
Fit results

	LHC	LHC+ILC	SPS1a
$\tan\beta$	13.8± 7.4	10.7± 3.1	10.0
M_1	105.0± 6.9	103.1± 0.7	103.1
M_2	194.7± 7.3	193.0± 1.6	192.9
M_3	568.3± 11.6	568.5± 7.8	567.7
$M_{\tilde{\tau}_L}$	321.4± 248	192.4± 4.7	193.5
$M_{\tilde{\tau}_R}$	164.3± 120	134.9± 5.7	133.4
$M_{\tilde{\mu}_L}$	196.3± 7.6	194.4± 1.2	194.3
$M_{\tilde{\mu}_R}$	138.0± 7.0	135.8± 0.6	135.8
$M_{\tilde{e}_L}$	196.4± 7.5	194.3± 0.8	194.3
$M_{\tilde{e}_R}$	137.9± 7.1	135.8± 0.6	135.8
$M_{\tilde{q}3_L}$	491.4± 16.2	486.2± 11.1	481.1
$M_{\tilde{t}_R}$	483.4± 232	409.6± 17.1	409.4
$M_{\tilde{b}_R}$	502.6± 15.3	499.1± 13.1	502.7
$M_{\tilde{q}_L}$	529.6± 12.1	526.4± 5.3	526.4
$M_{\tilde{q}_R}$	508.9± 16.4	507.8± 14.4	506.8
A_τ	fixed 0	-102.9± 681	-249.3
A_t	-394.4± 353	-497.3± 74	-496.8
A_b	fixed 0	-274.2±1830	-764.0
m_A	558.2±271.2	394.9± 1.5	394.9
μ	353.1± 7.7	350.8± 2.5	351.0

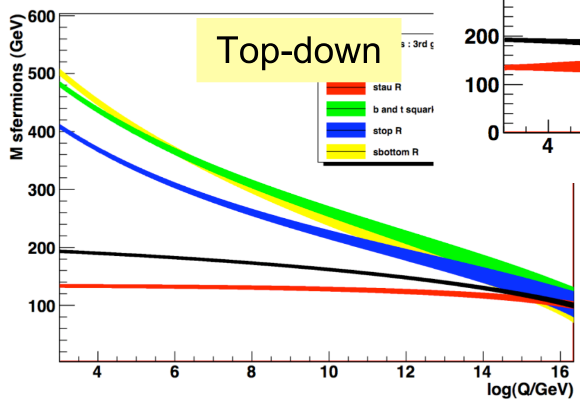
Results for LHC+ILC

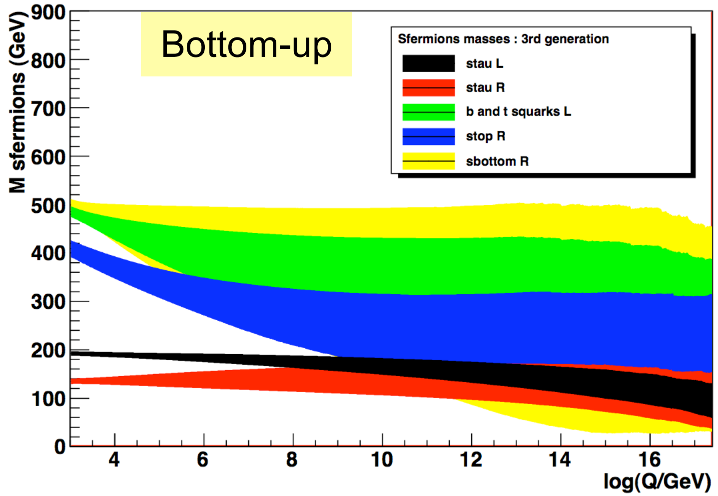


Name	Unified Parameter	Unification Scale $[\log(Q/\mathrm{GeV})]$	Parameter at $2.33 \cdot 10^{16}$ GeV
$m_{1/2}$	249.5±1.8	16.37±0.05	249.6±1.5
$m_0^{1/2\mathrm{Gen}}$	98.2±10.7	16.5±0.7	100.4±2.5
$m_0^{3\mathrm{Gen}}$	117.1±27	15.4±1.3	103.1±25
m_0	105.3±9.1	15.9±0.6	99.4±2.0
A_0	-164±182	14.8±4.5	-133.8±207



Sfermions of the 3rd generation





Conclusion

In regions similar to SPS1a :

MSSM parameters can be determined @LHC up to an 8 fold ambiguity in the gaugino sector

- Additional studies of the dark matter relic density will help, but..
- Removing all ambiguities will require the full mass spectrum (I.e. the ILC)

Meanwhile, one will at least be able to classify solutions : 2 of them should be compatible with unification, with opposite μ sign.

The unified gaugino mass parameter can be measured bottom-up to about 2%, and the logarithm of the unification scale to 1.7%

The robustness of our results has been checked using two tools (Suspect and SoftSusy)

And, clearly … " bottom up " # " top down "