
QCD corrections to dark matter annihilation: Recent developments

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1. Introduction and Motivation
2. Radiative corrections to neutralino pair annihilation
3. Few numerical examples
4. Conclusion and Outlook

Introduction

New physics provides interesting candidates for cold dark matter

Consider Minimal Supersymmetric Standard Model (MSSM) with R-parity conservation

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$$\frac{dn}{dt} = -3Hn - \langle \sigma_{\text{ann}} v \rangle (n^2 - n_{\text{eq}}^2) \longrightarrow \Omega_{\text{CDM}} h^2 \propto n_0 \propto \frac{1}{\langle \sigma_{\text{ann}} v \rangle}$$

Cosmology allows to identify (dis)favoured regions of parameter space

$$0.1053 < \Omega_{\text{CDM}} h^2 < 0.1193$$

[Komatsu et al. (WMAP) 2010]

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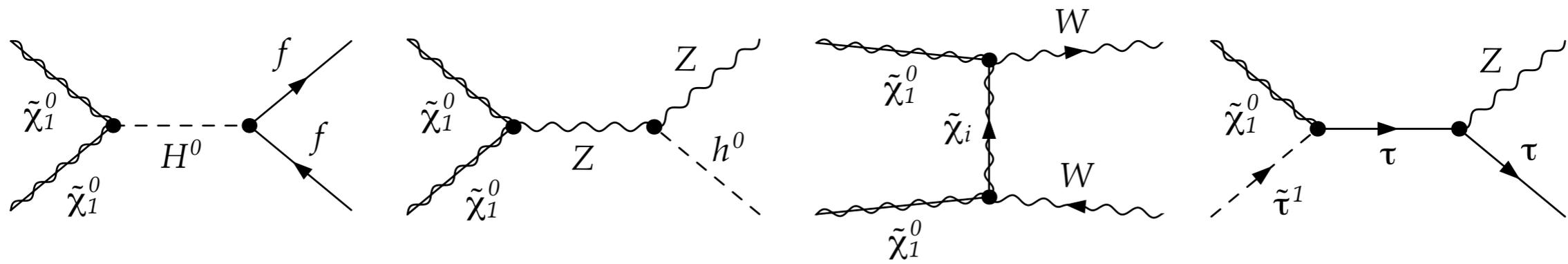
Public program packages

DarkSUSY [Gondolo et al. 2000-2010], micrOMEGAs [Bélanger et al. 2003-2010],

SuperIso Relic [Arbey and Mahmoudi 2009], ...

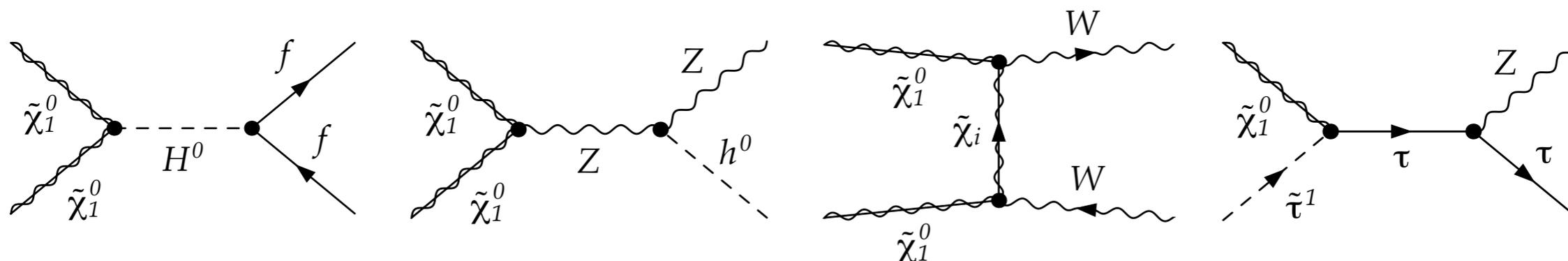
Motivation: Why radiative corrections...?

All (co)annihilation processes are implemented in public codes at leading order



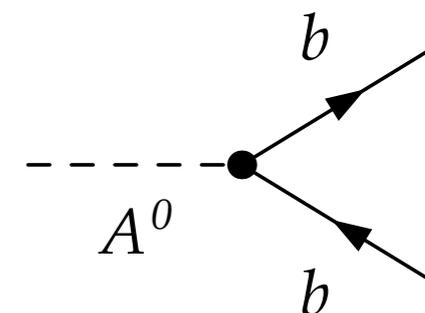
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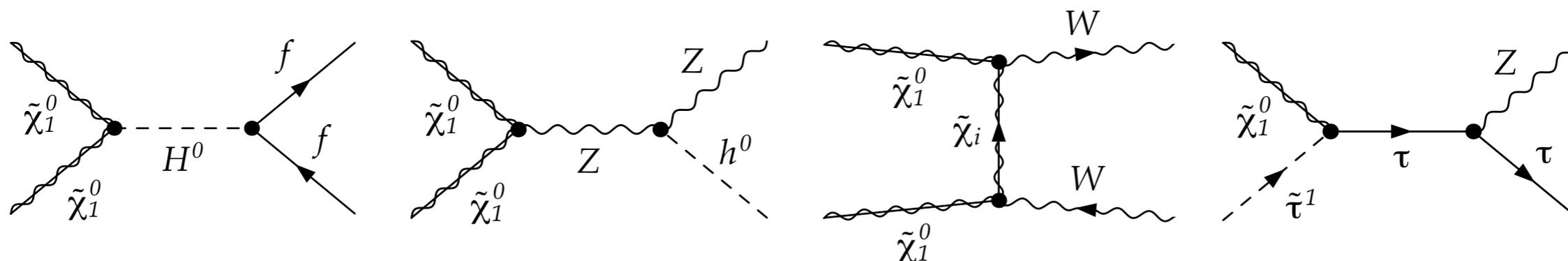
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e.g. bottom Yukawa coupling

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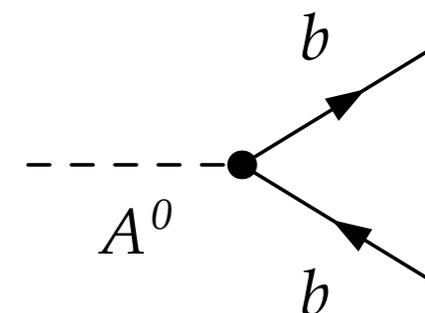
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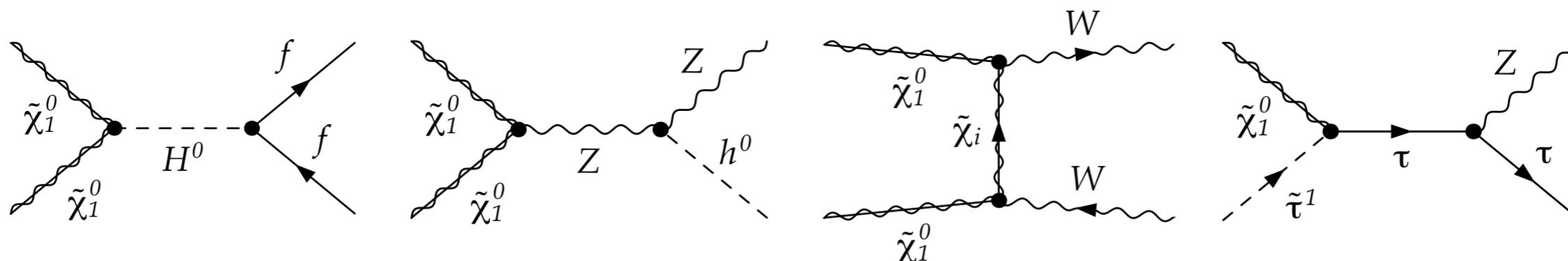
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Planck satellite delivers more precise cosmological data since may 2009

Better theoretical precision needed to keep up with experimental improvements

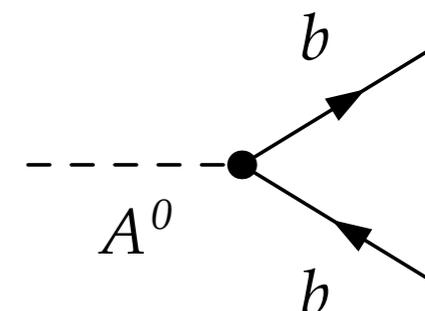
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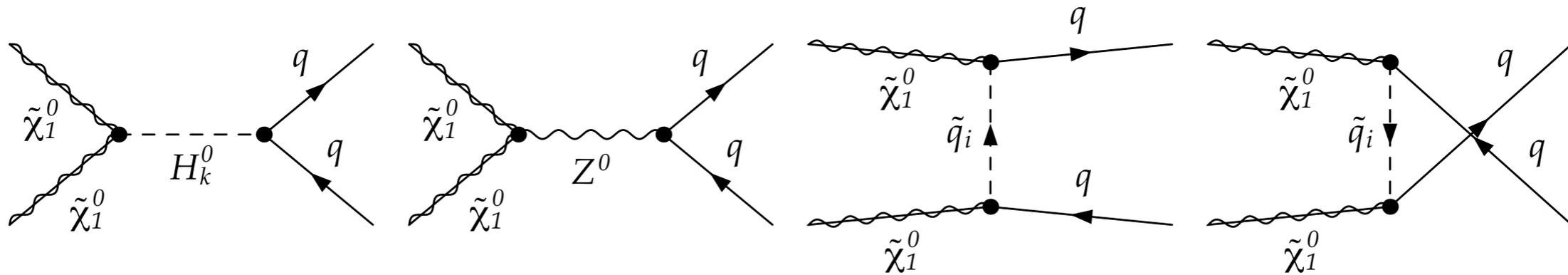
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QCD corrections numerically most important due to strong coupling constant, but also electroweak corrections can have a sizeable impact

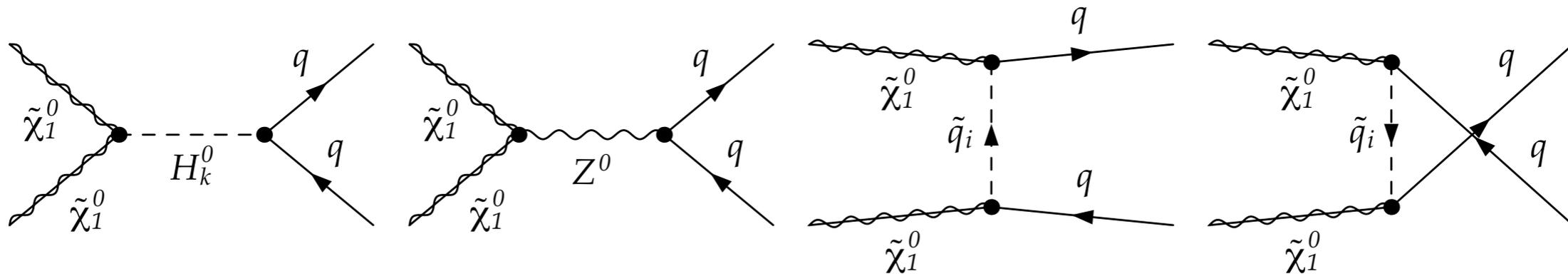
Neutralino annihilation into (heavy) quarks

In “mSUGRA-like” scenarios, annihilation into heavy quarks is dominant



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Higgs-exchange dominant in mSUGRA

- ▶ Low $m_{1/2}$ (if not excluded by LEP)

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- ▶ A-Funnel at high $\tan\beta$

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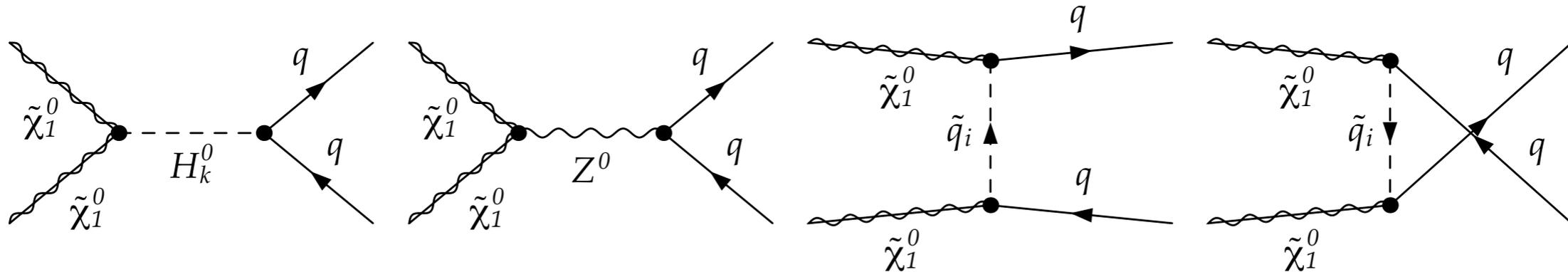
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[B. Herrmann and M. Klasen, PRD 76 (2007)]

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Relax scalar or gaugino mass unification

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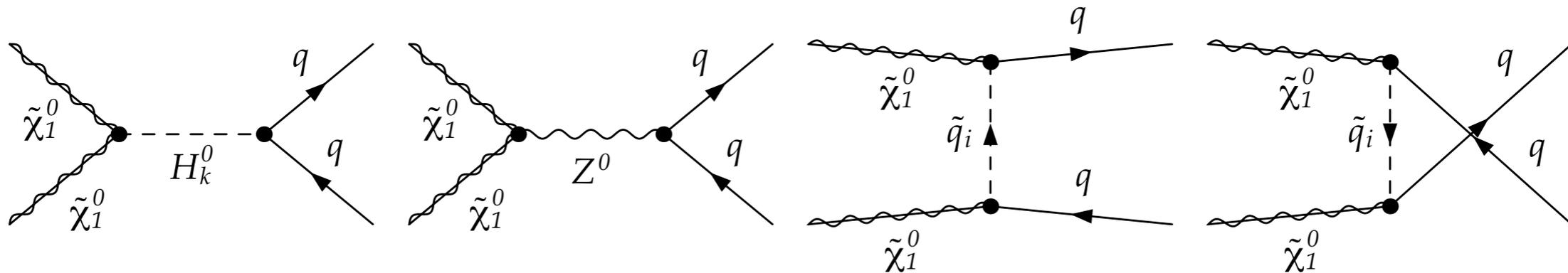
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Large mass splitting favours squark exchange

- ▶ Large trilinear coupling A_0

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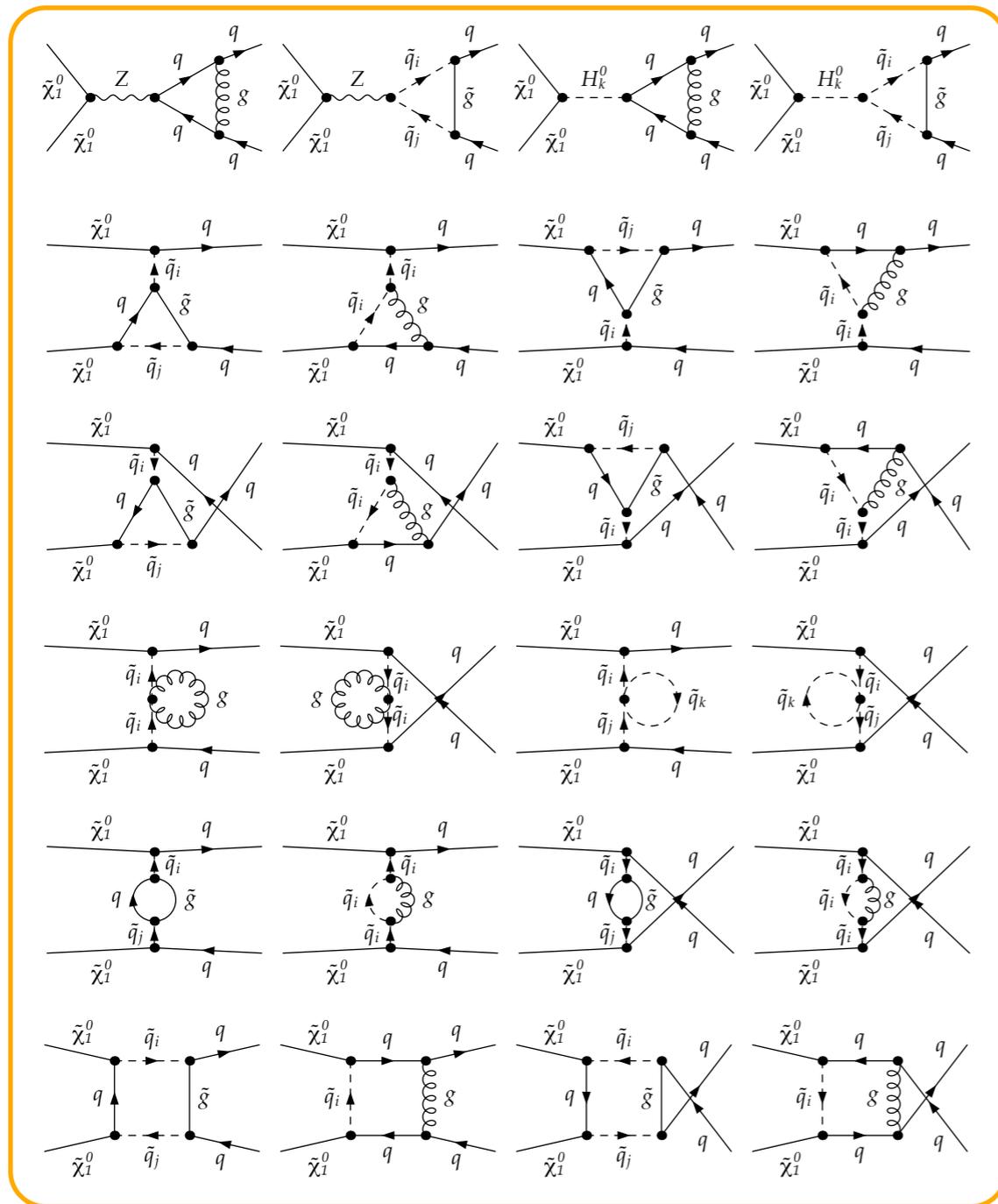
[Herrmann, Klasen, Kovařík, PRD 80 (2009)]

Corrections of order α_s

One-loop contributions combined with real gluon emission using dipole subtraction method

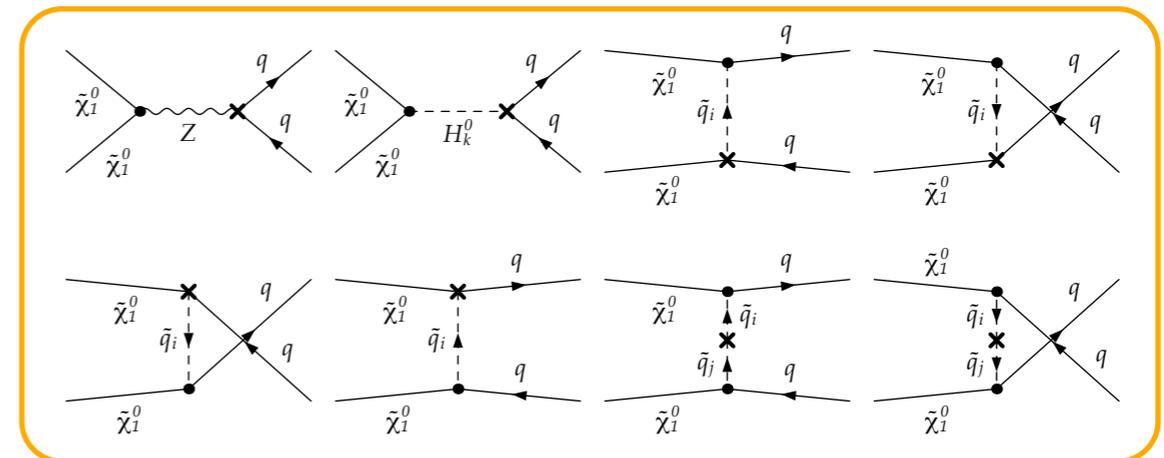
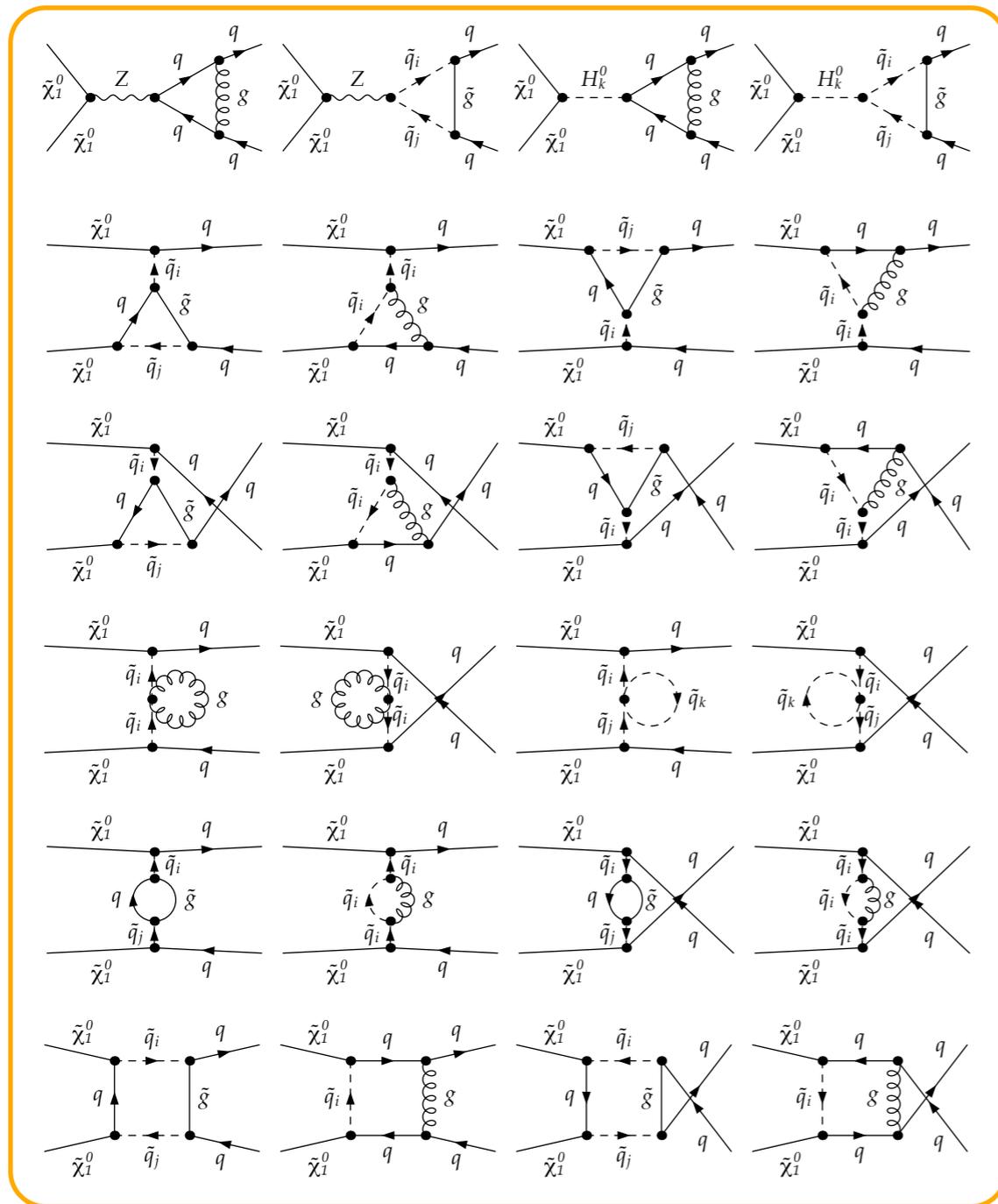
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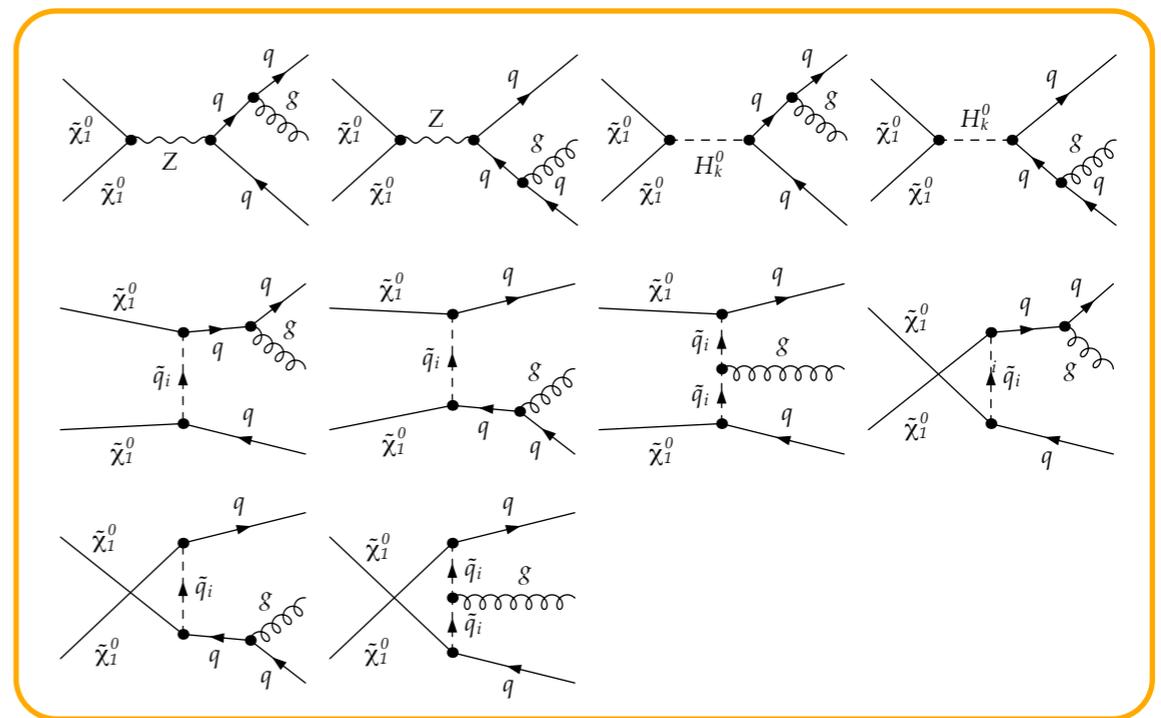
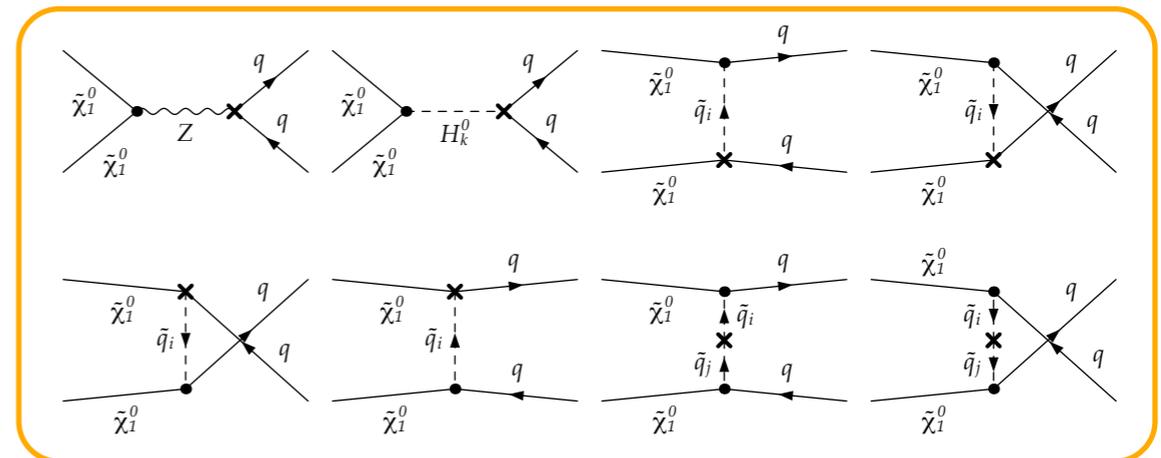
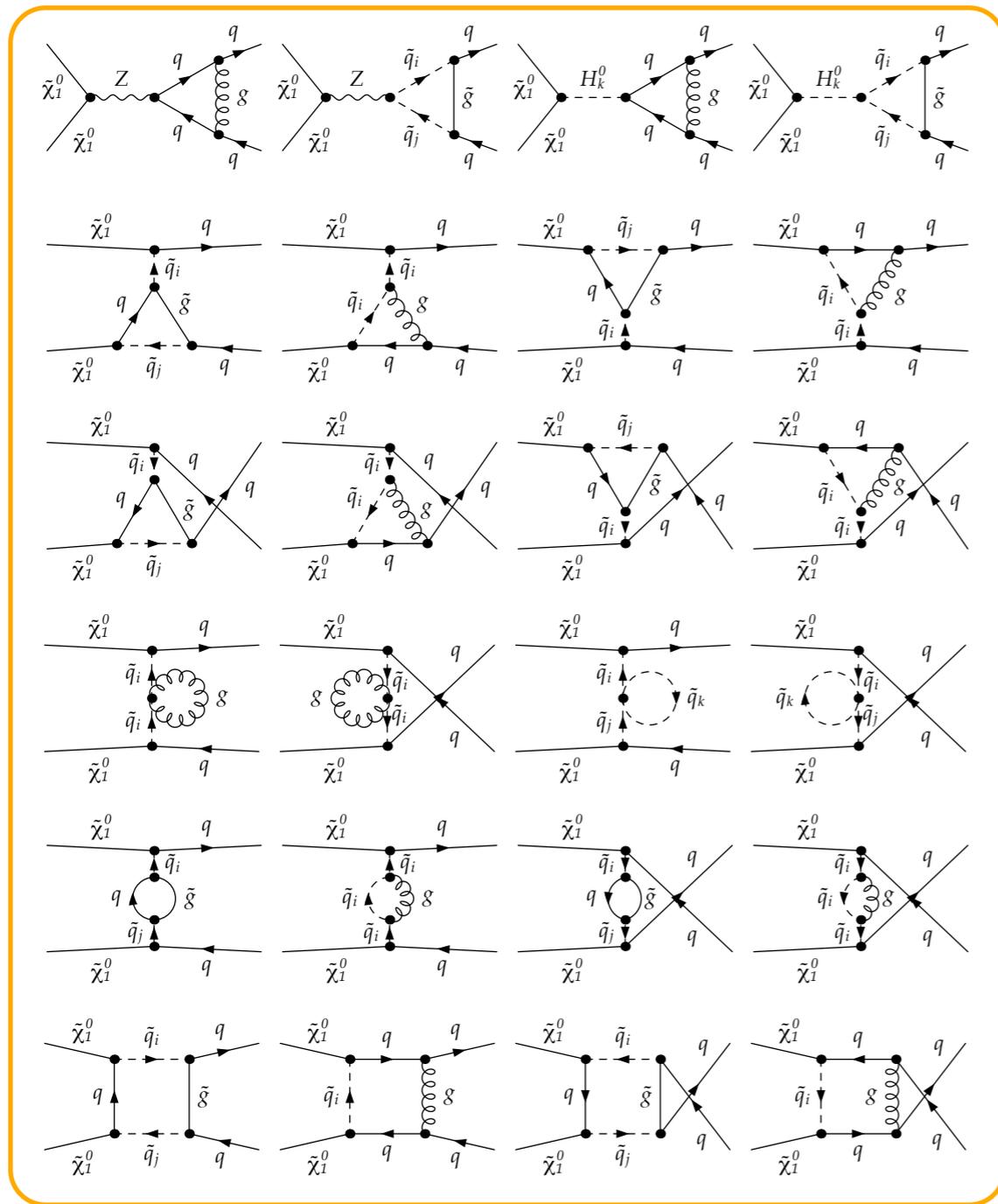
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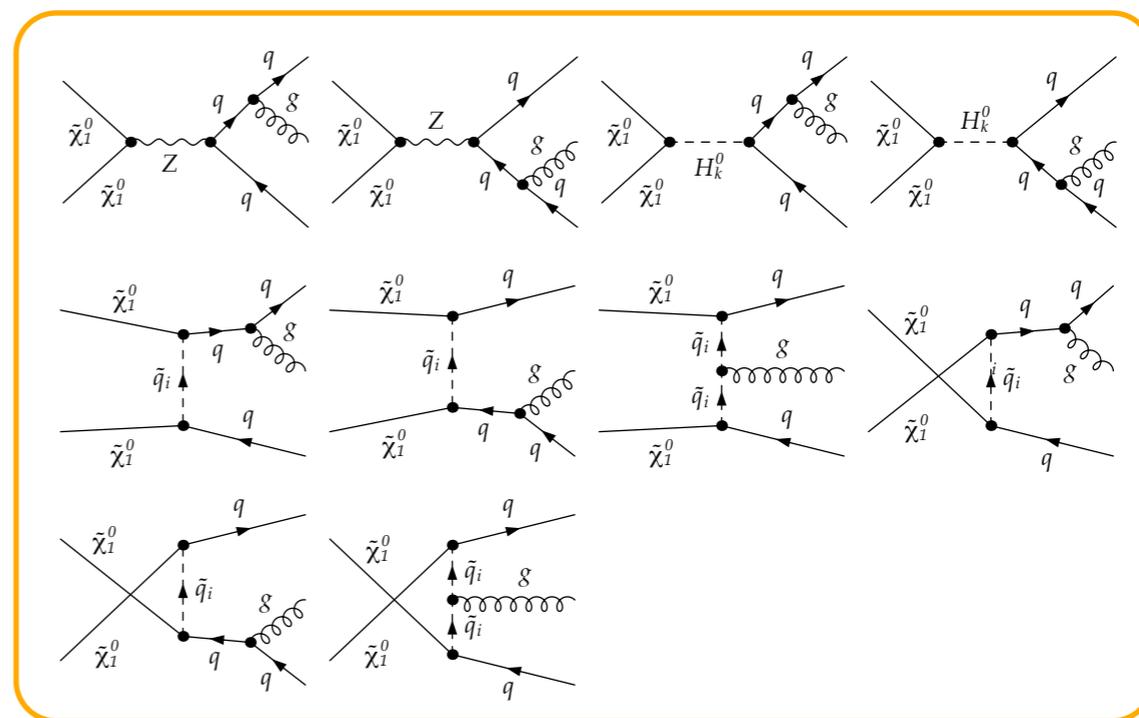
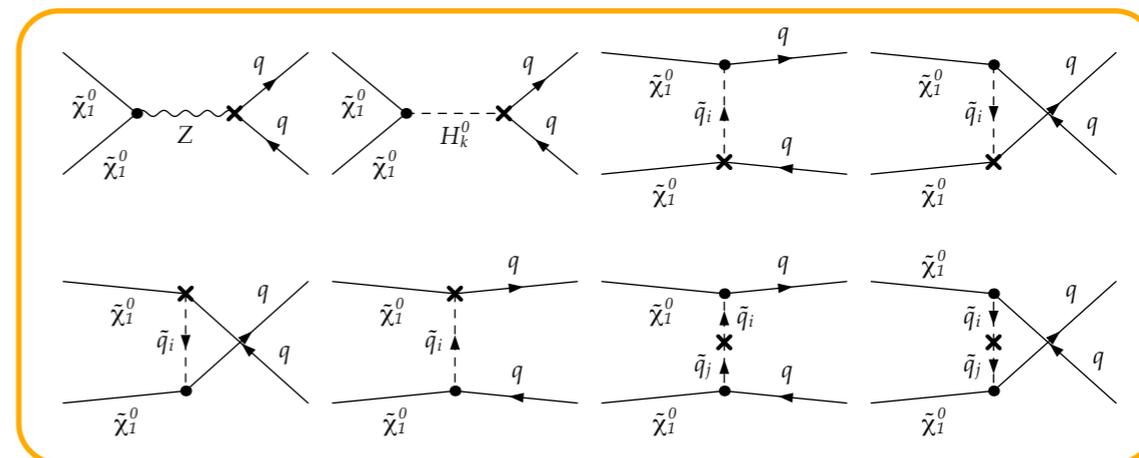
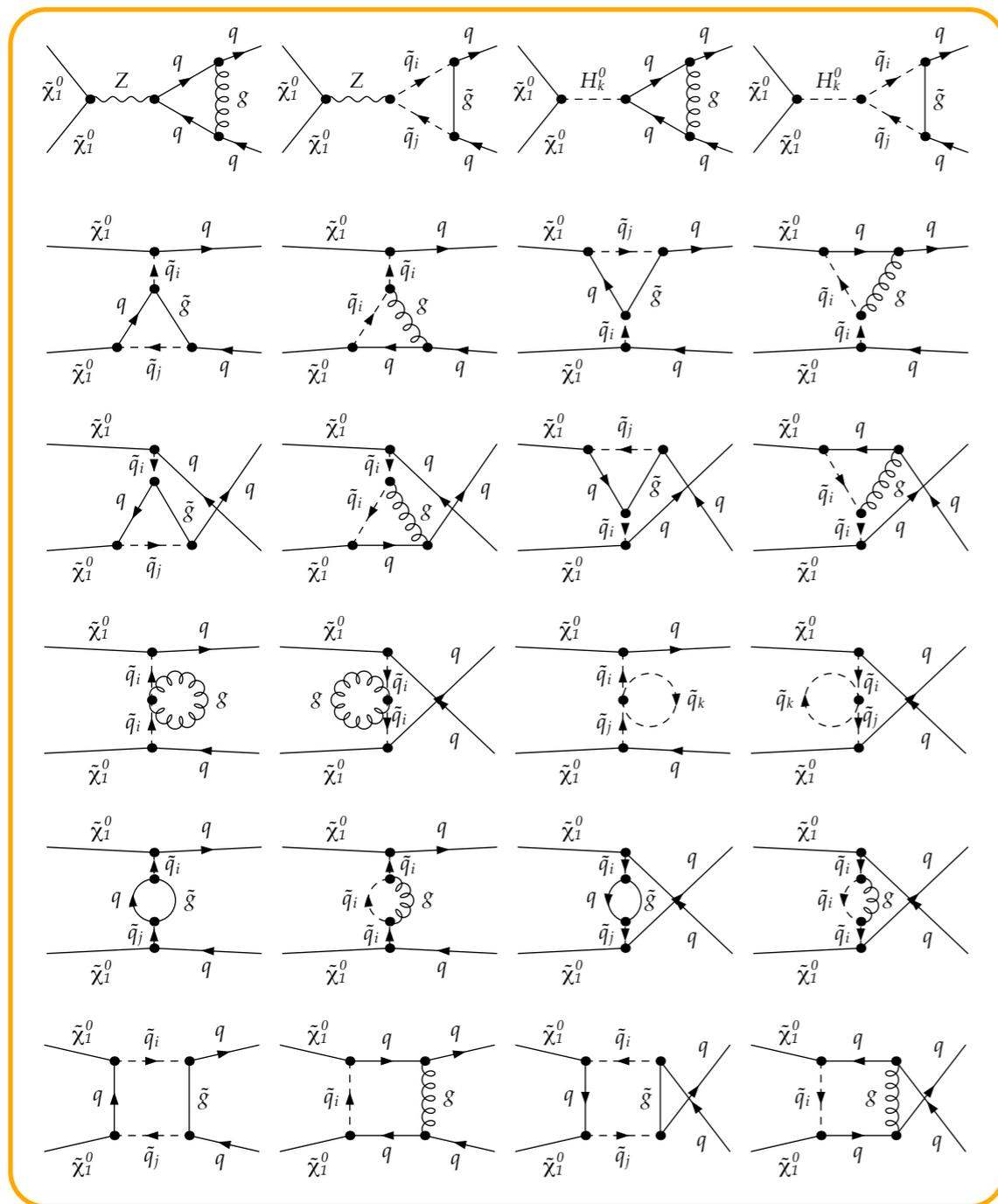
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Numerical implementation can serve as extension for micrOMEGAs or DarkSUSY

Corrections of order α_s

Virtual corrections at the one-loop level

Diagrams calculated in the $\overline{\text{DR}}$ renormalization scheme (preserving Supersymmetry)

Ultraviolet divergences (showing up as poles) removed by on-shell renormalization

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Real gluon emission from final state quarks

Poles in gluon emission diagrams cancel remaining infrared divergences

Dipole subtraction method allows for separate numerical integration

$$\sigma_{\text{NLO}} = \int_2 \left[d\sigma^{\text{V}} + \int_1 d\sigma^{\text{A}} \right] + \int_3 \left[d\sigma^{\text{R}} - d\sigma^{\text{A}} \right]$$

[Catani *et al.* 2000-2002]

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Improvements for Yukawa couplings relevant for Higgs-exchanges

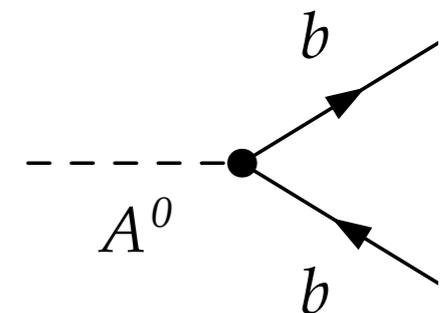
Decays of Higgs-bosons to quarks known up to $\mathcal{O}(\alpha_s^4)$

SUSY-QCD resummation known to be relevant at large $\tan\beta$

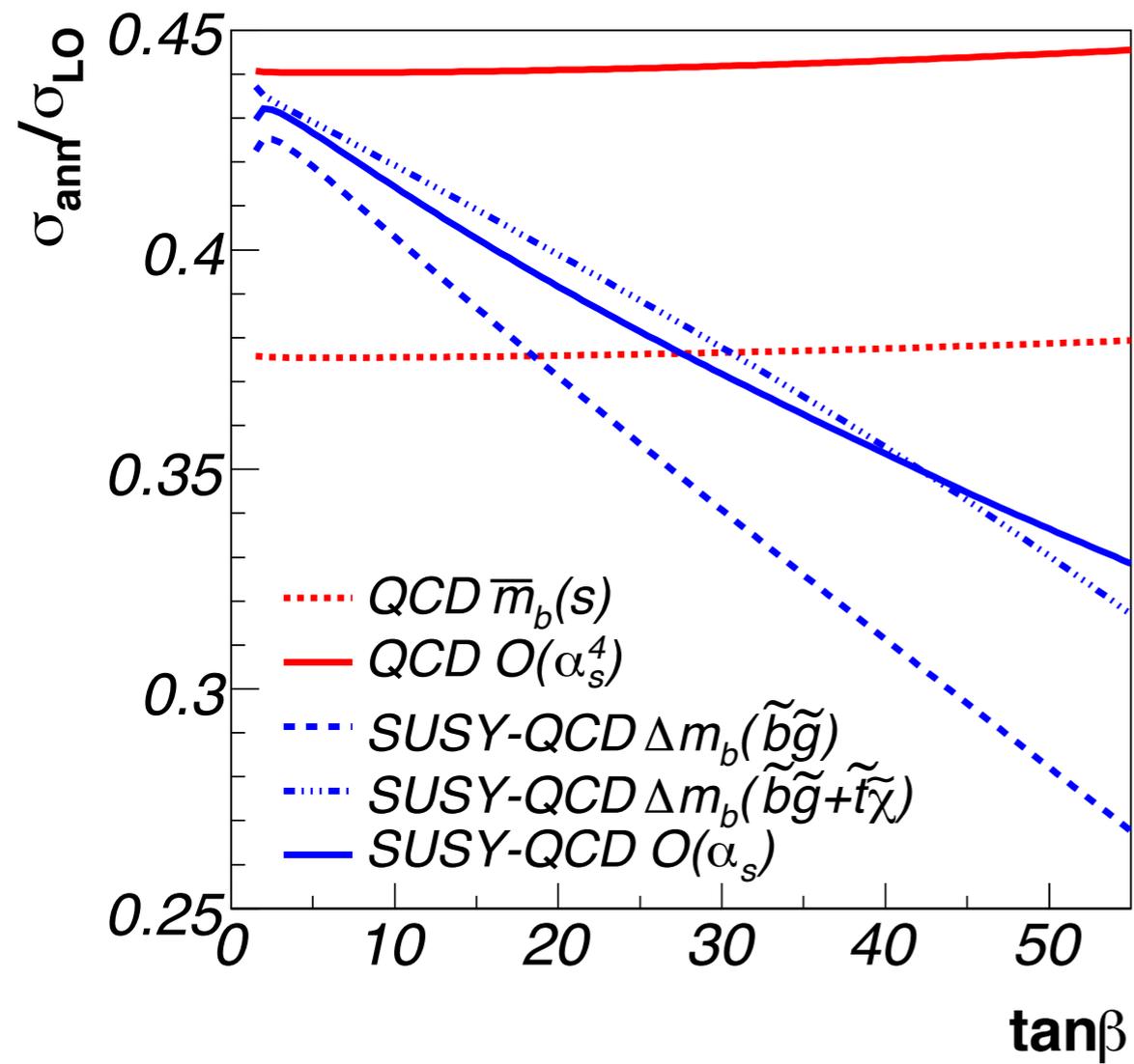
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[Braaten & Leveille 1980, Chetyrkin et al. 1995, Chetyrkin 1997, Chetyrkin et al. 2005]

[Carena et al. 2000, Guasch et al. 2003]

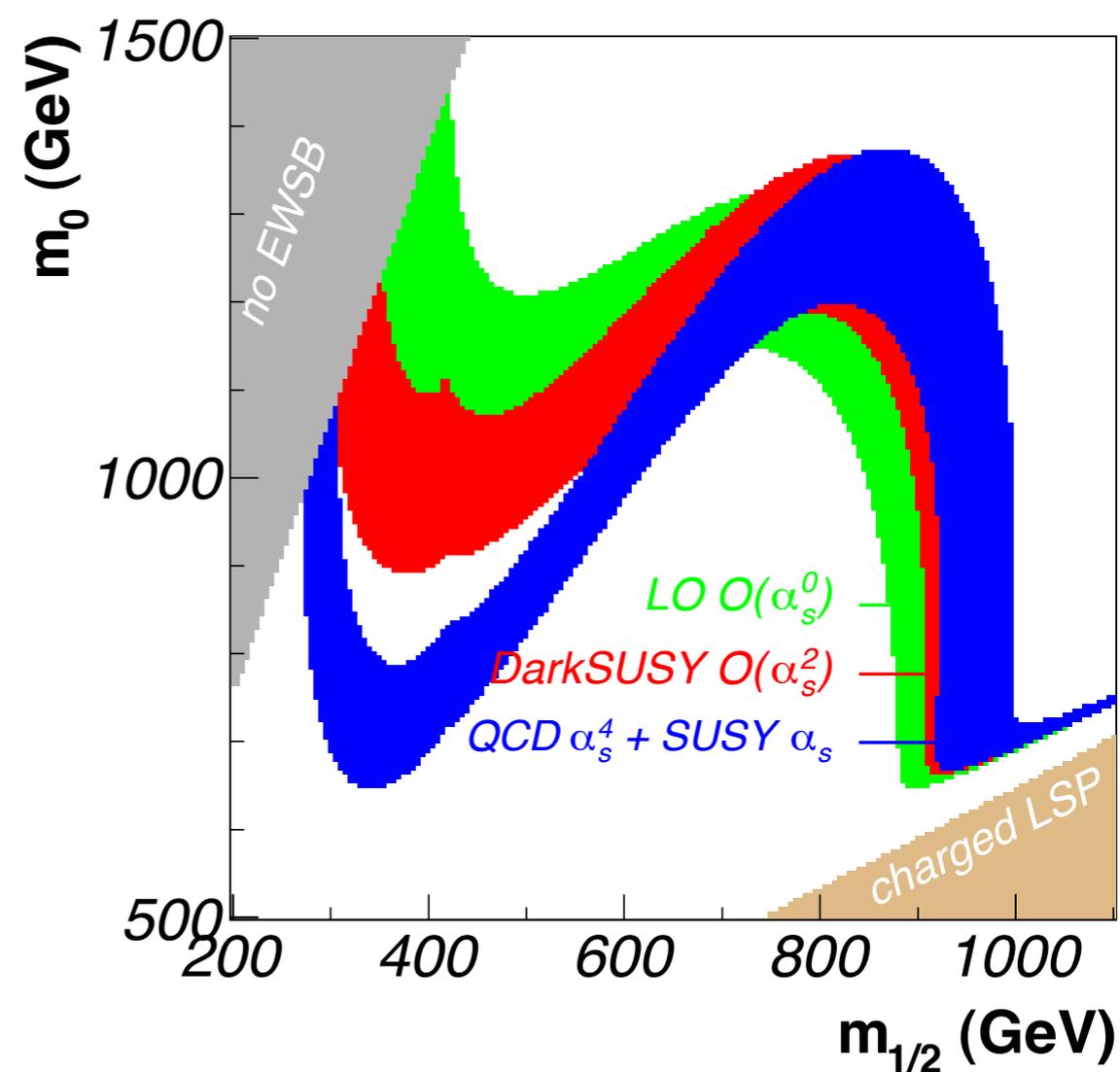
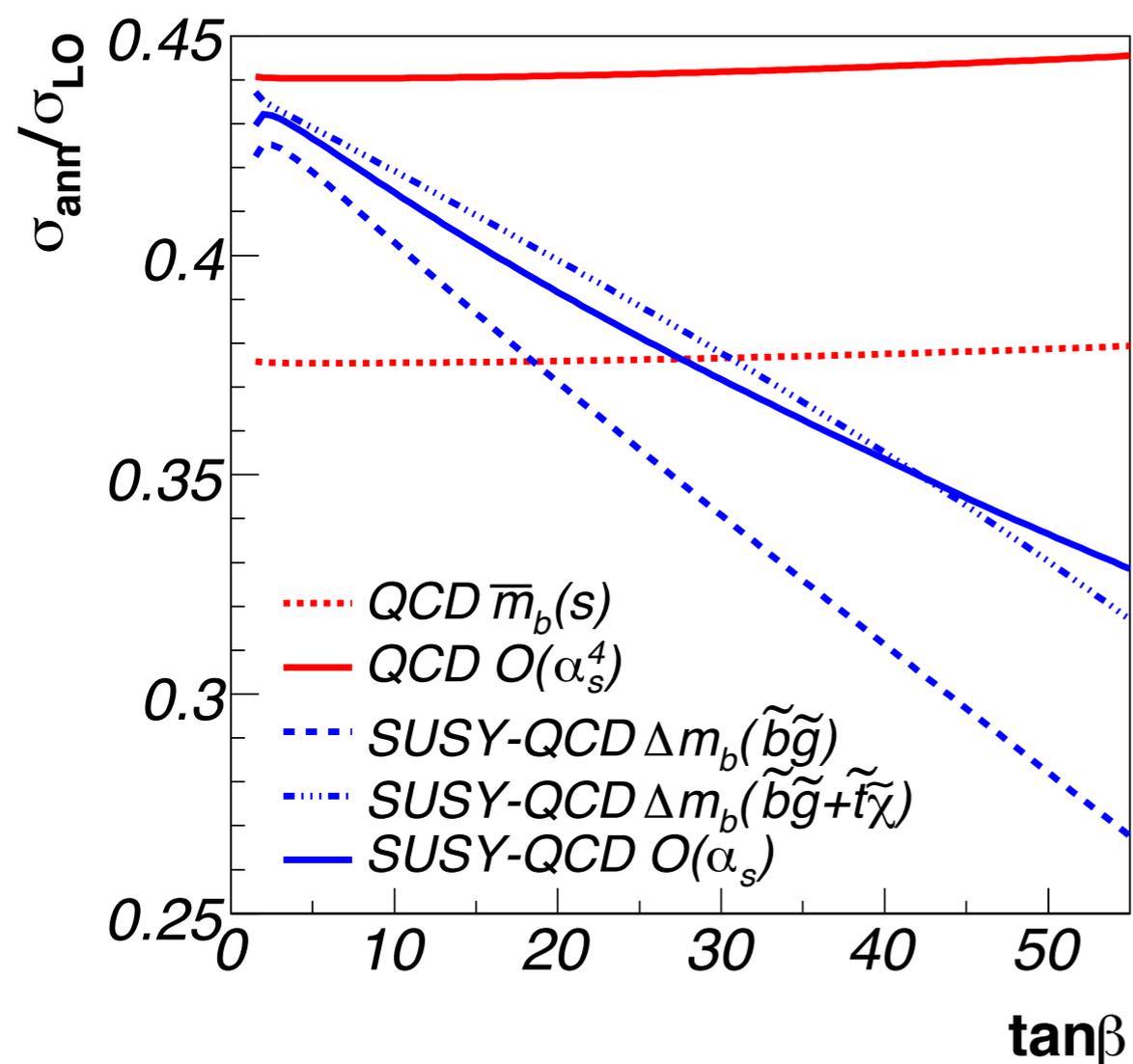


Higgs-Funnel at high $\tan\beta$



Main effects on cross-section here due to QCD and SUSY-QCD mass resummation

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Impact of corrections more important than experimental uncertainty

Favoured region shifted to smaller masses in order to compensate effect on cross-section

Effect reversed around the Higgs-pole due to corrections to decay width

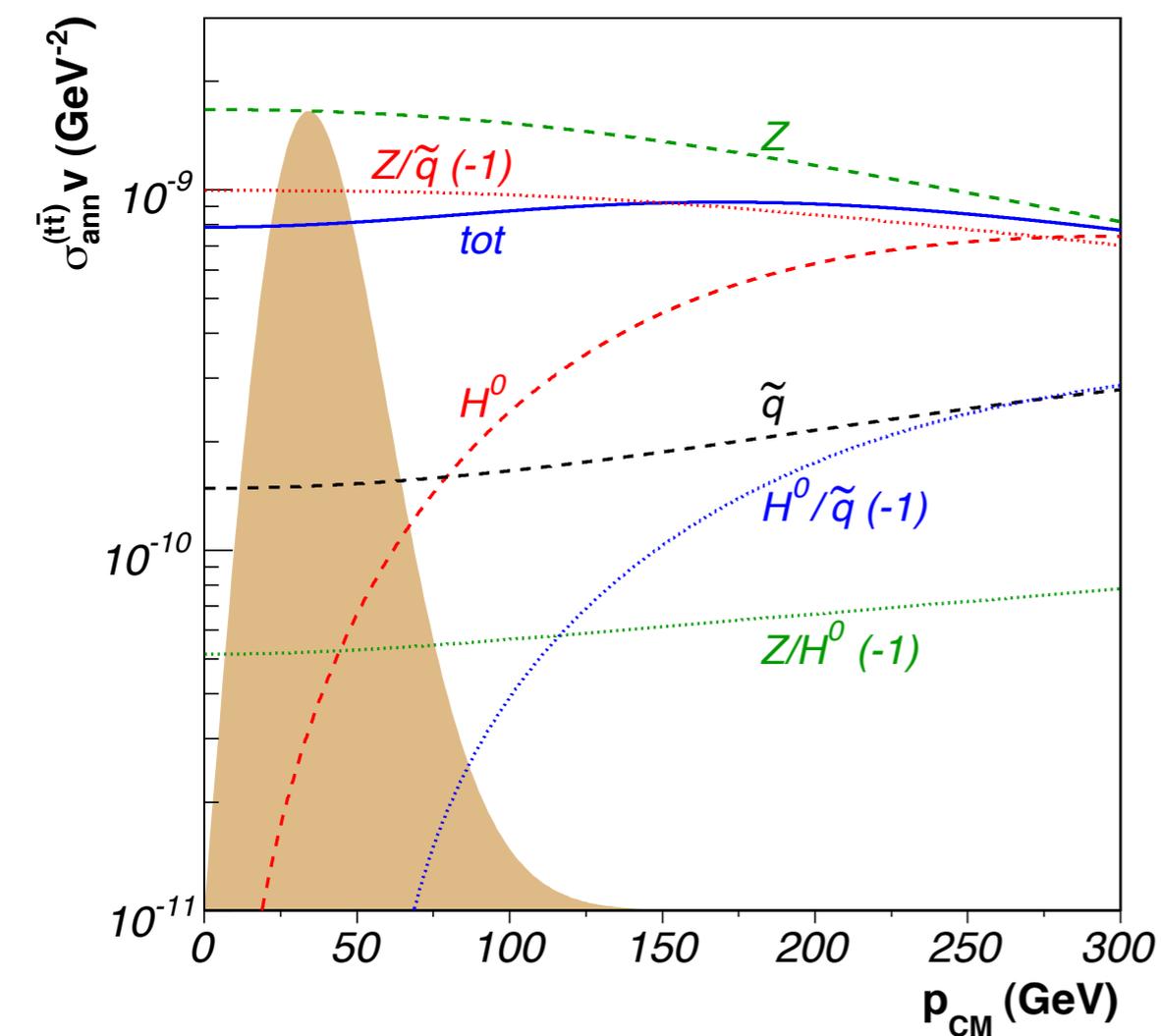
[Herrmann and Klasen, PRD 76: 117704 (2007)]

Case of dominant Z- or squark-exchange

m_0	1500 GeV
$M_{1,2}$	600 GeV
M_3	266 GeV
A_0	0
$\tan \beta$	10
$\text{sgn}(\mu)$	+

$\Omega_{\tilde{\chi}} h^2$	0.104
$t\bar{t}$	50.4%
$m_{\tilde{\chi}}$	235.6 GeV
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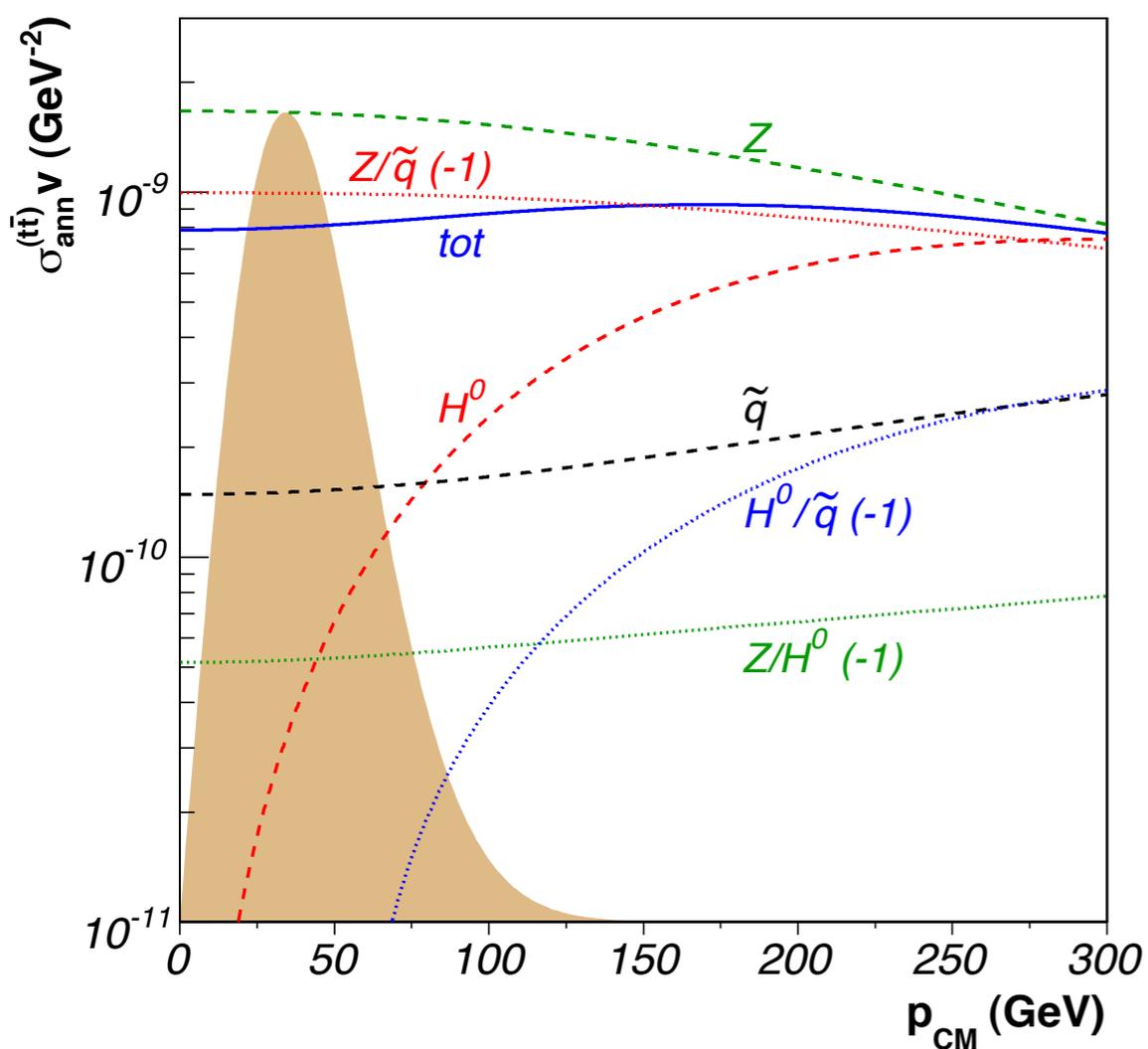


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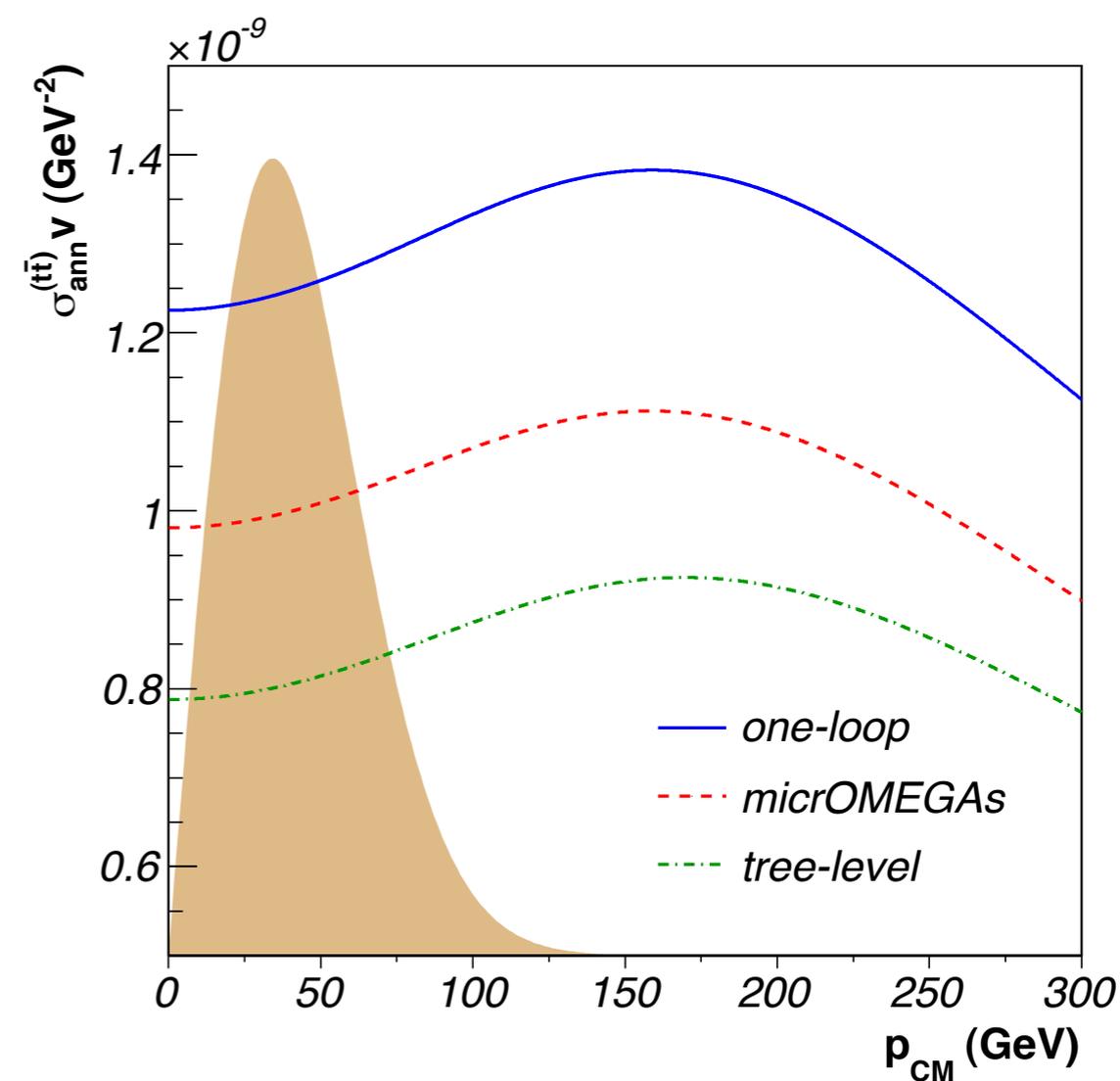
Z^0 - and squark-exchanges related by important interference term

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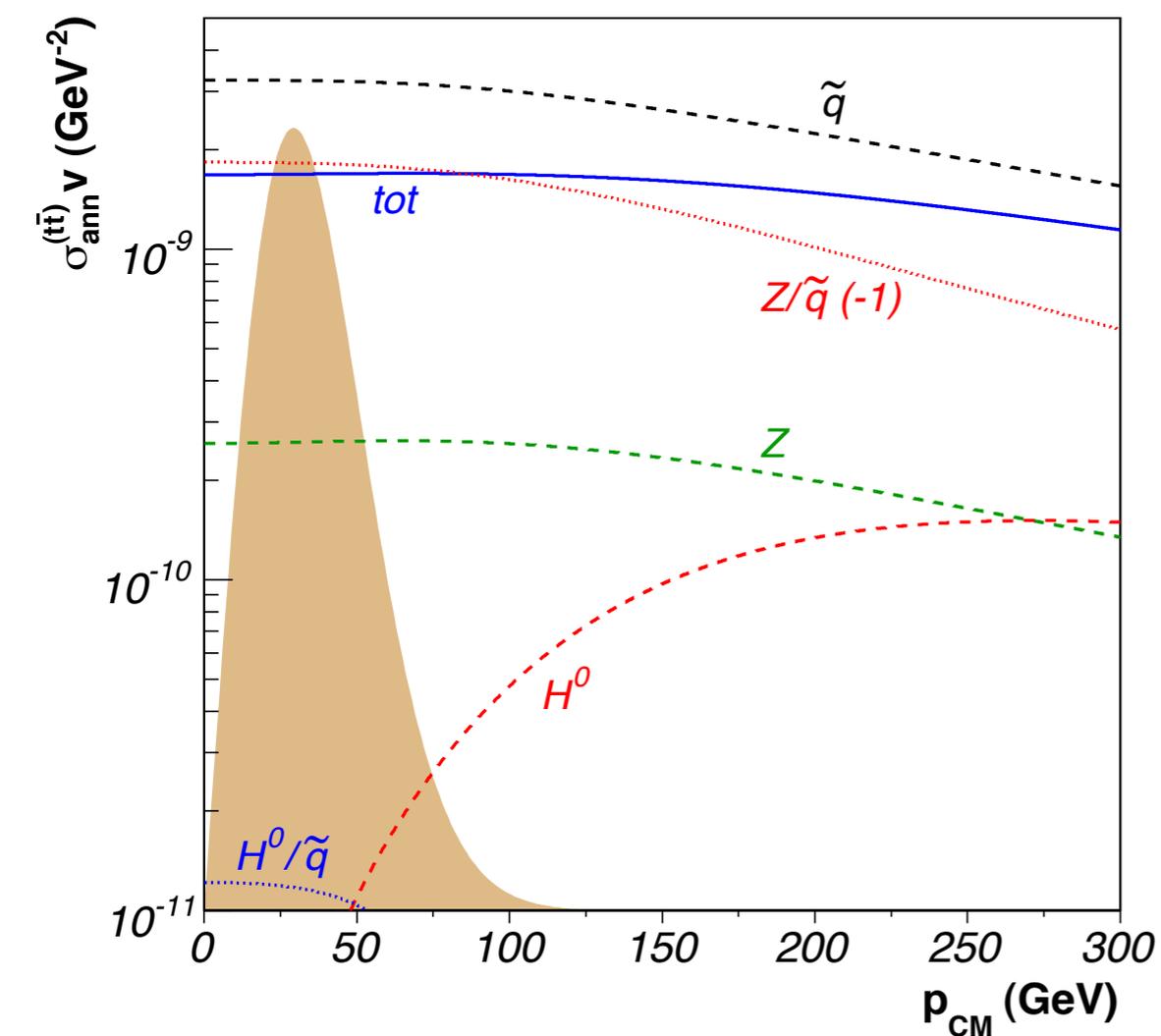


Z^0 - and squark-exchanges related by important interference term

One-loop corrections increase cross-section by about 50% w.r.t. tree-level approximation

[Herrmann, Klasen, Kovařik, PRD 80: 085025 (2009)]

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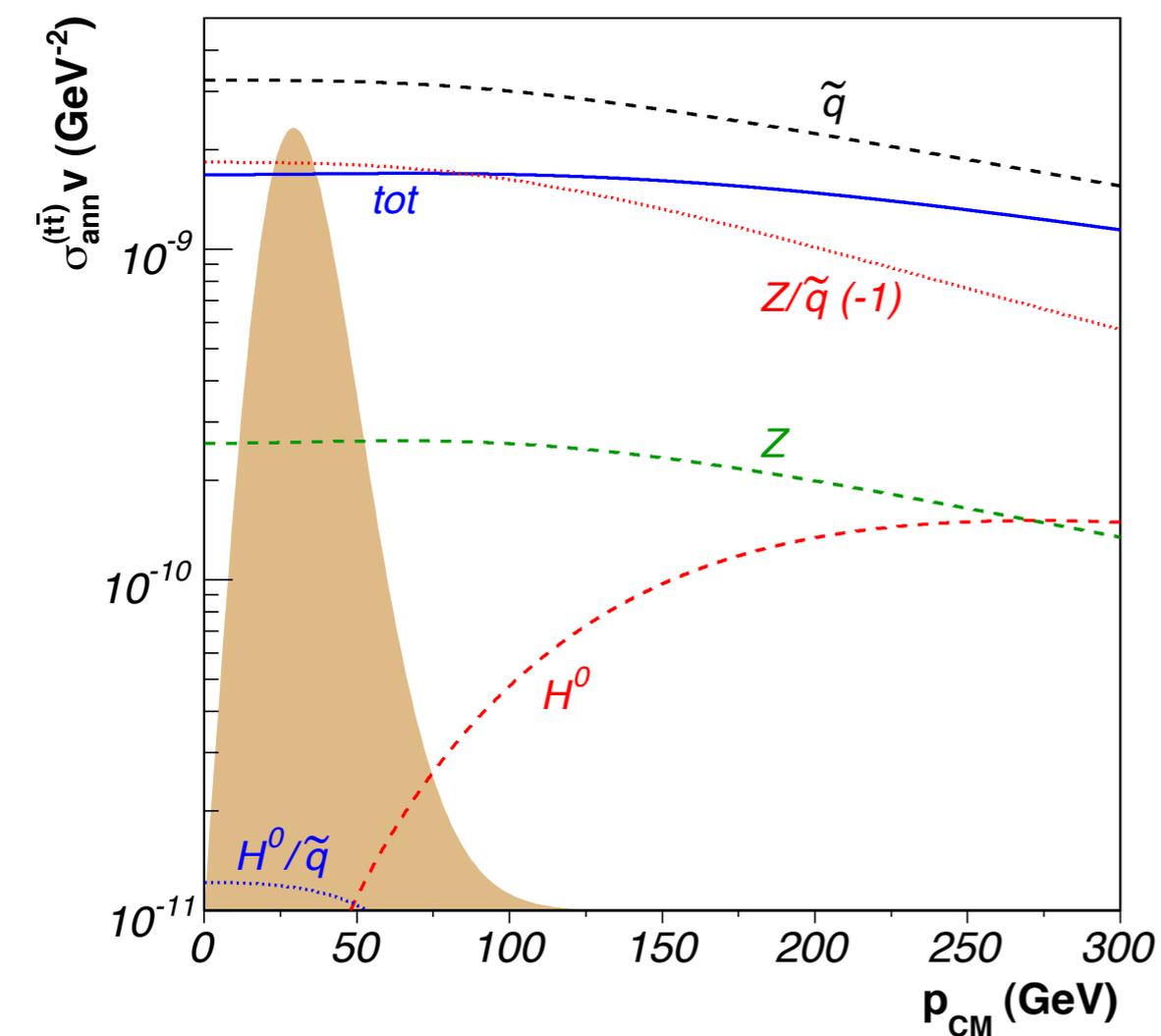


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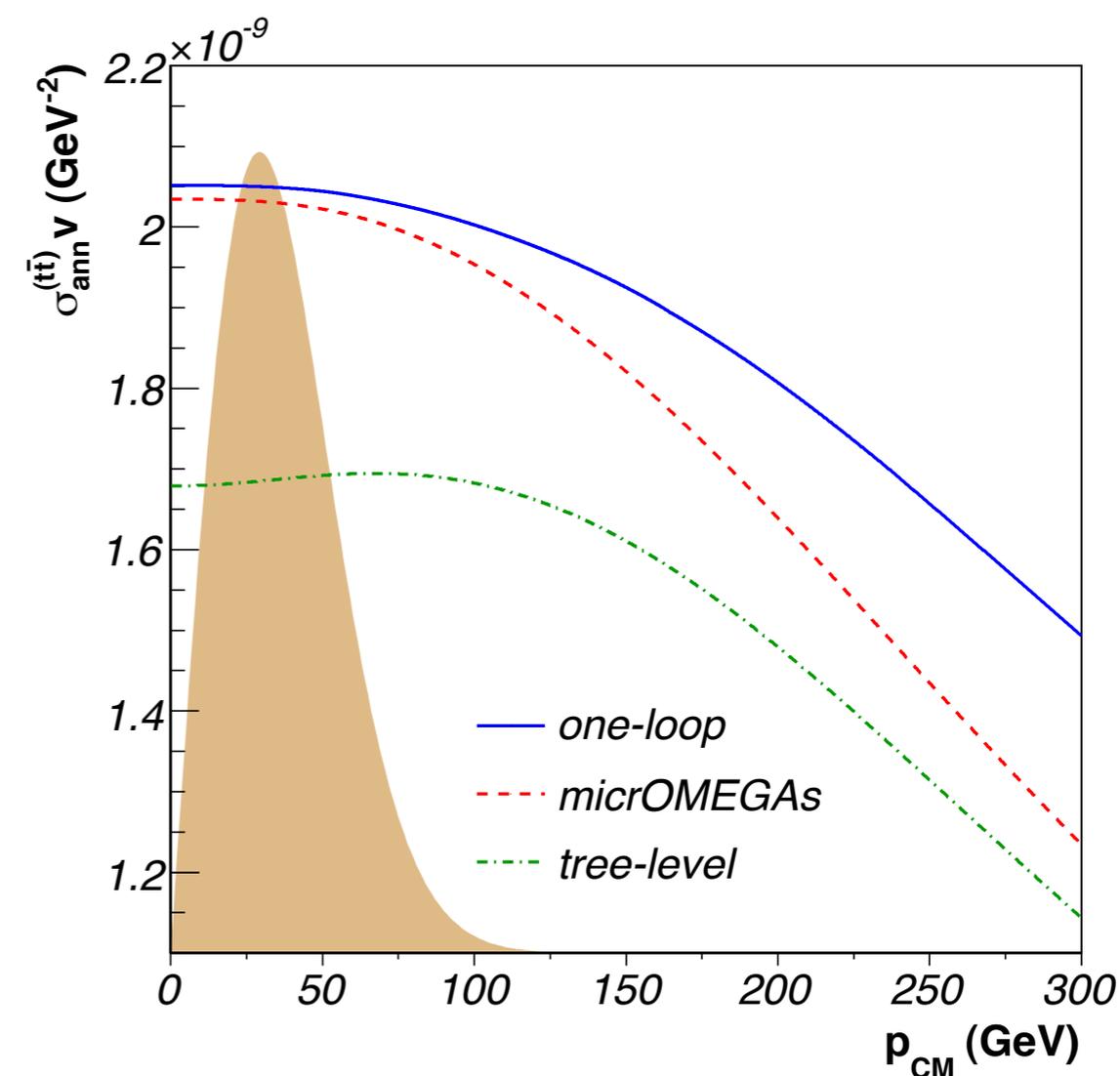
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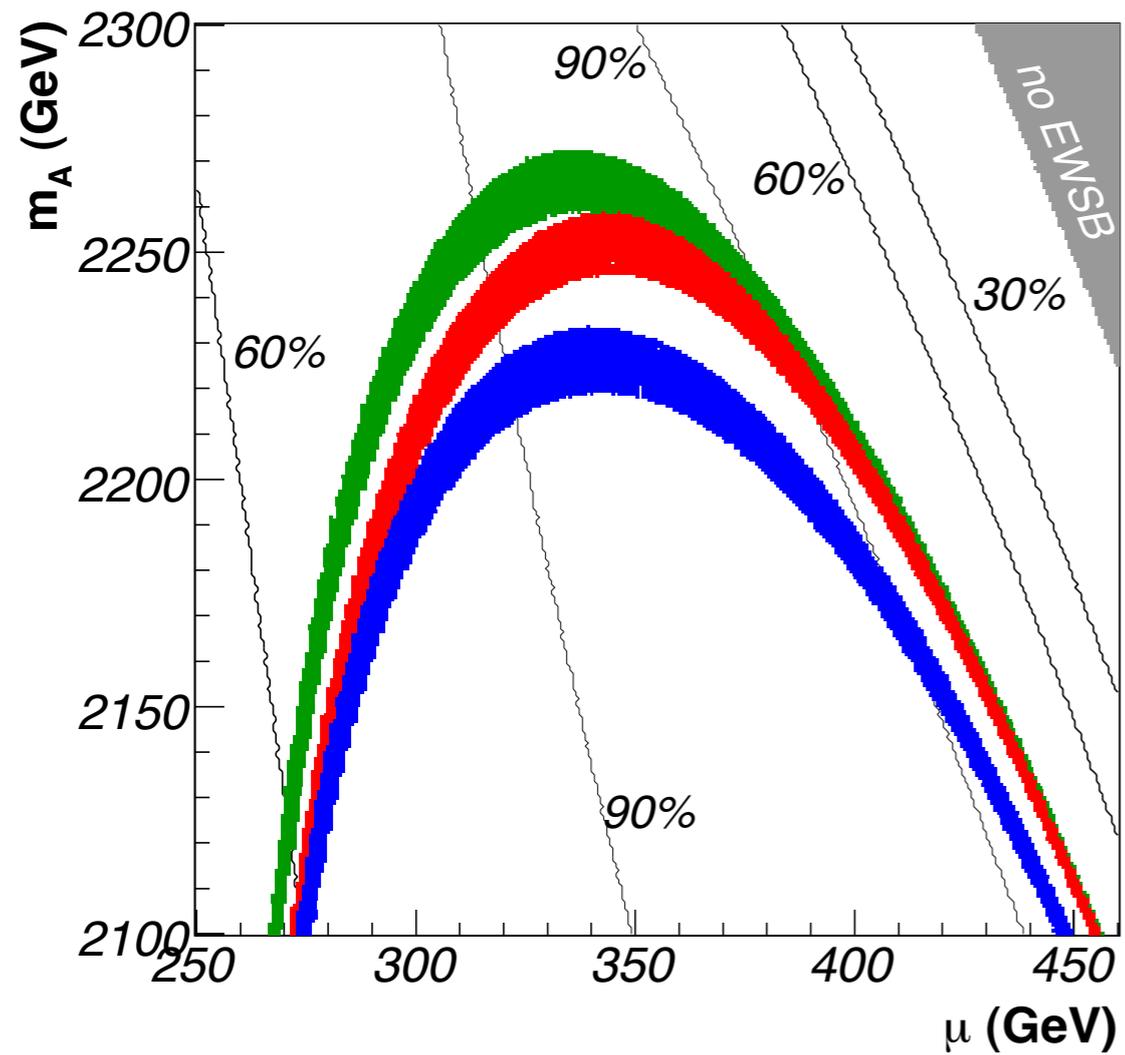


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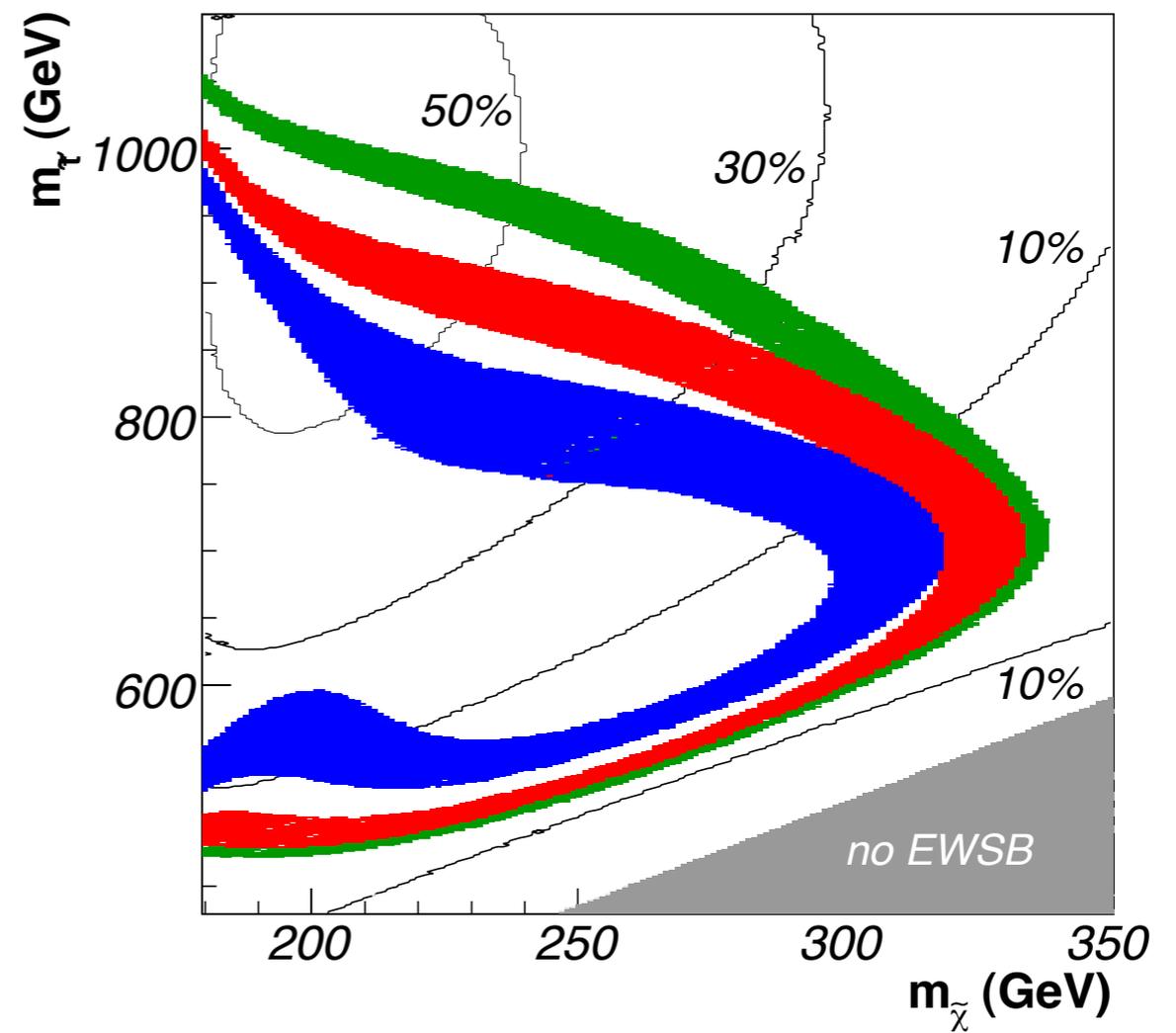
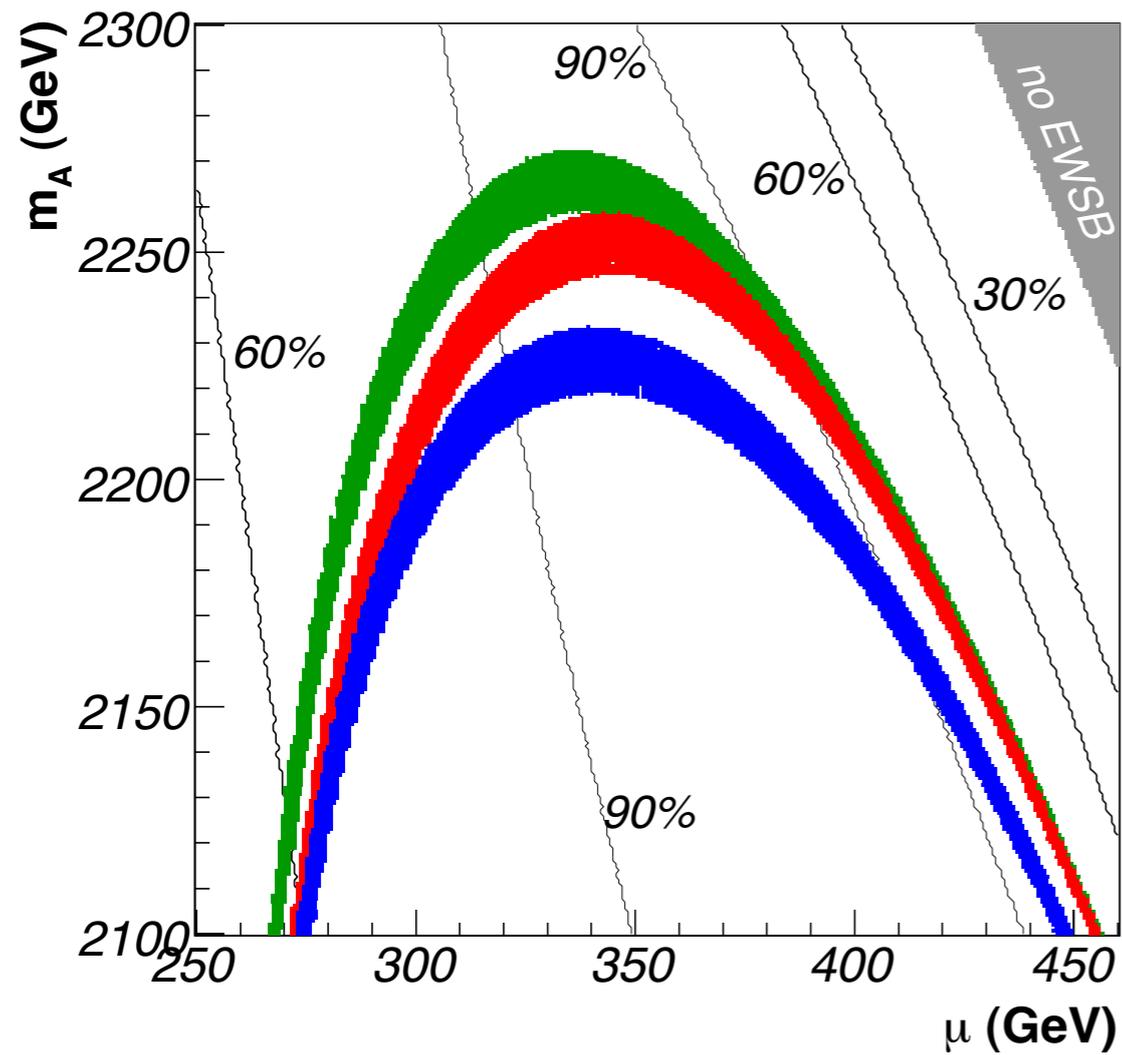
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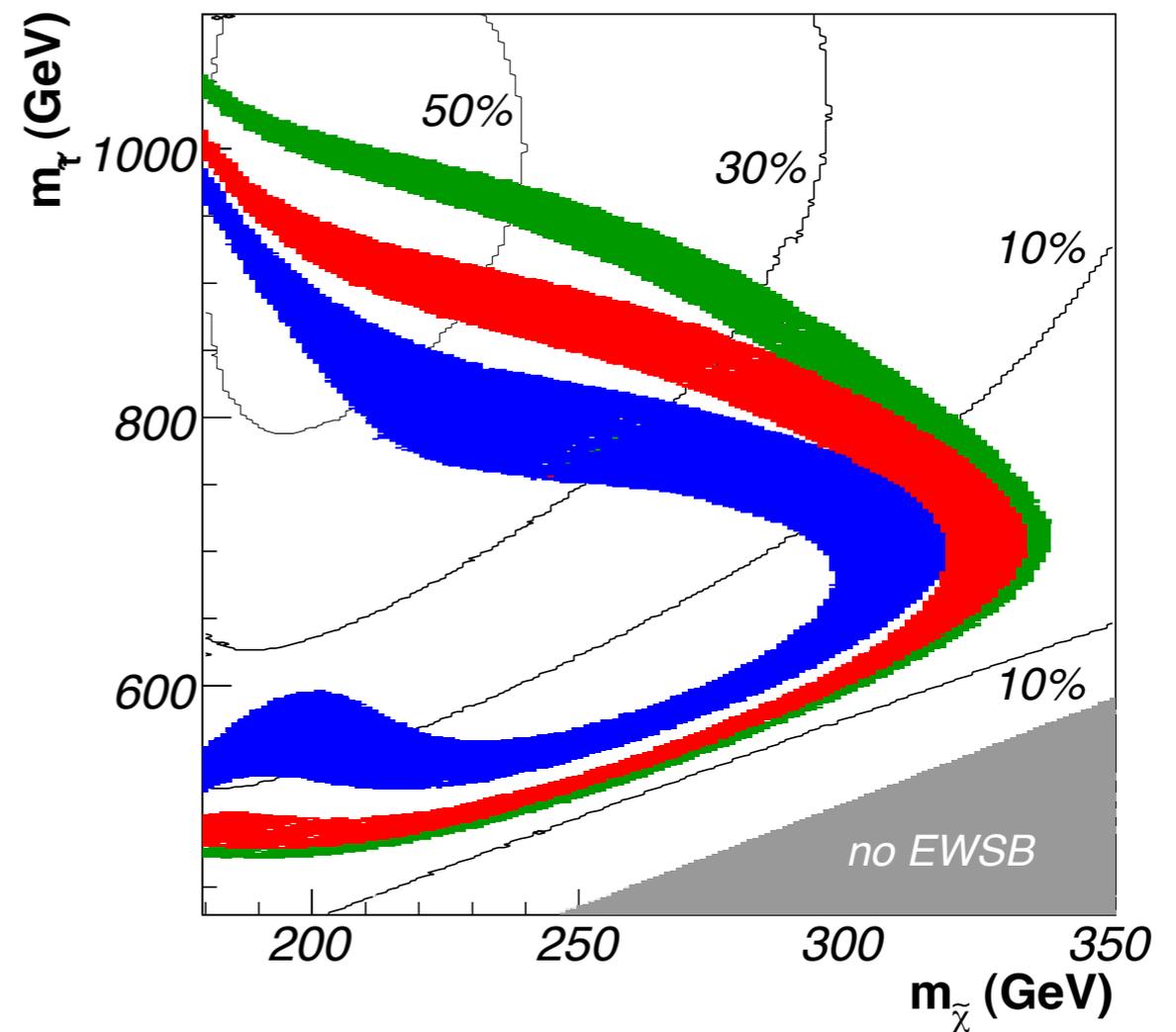
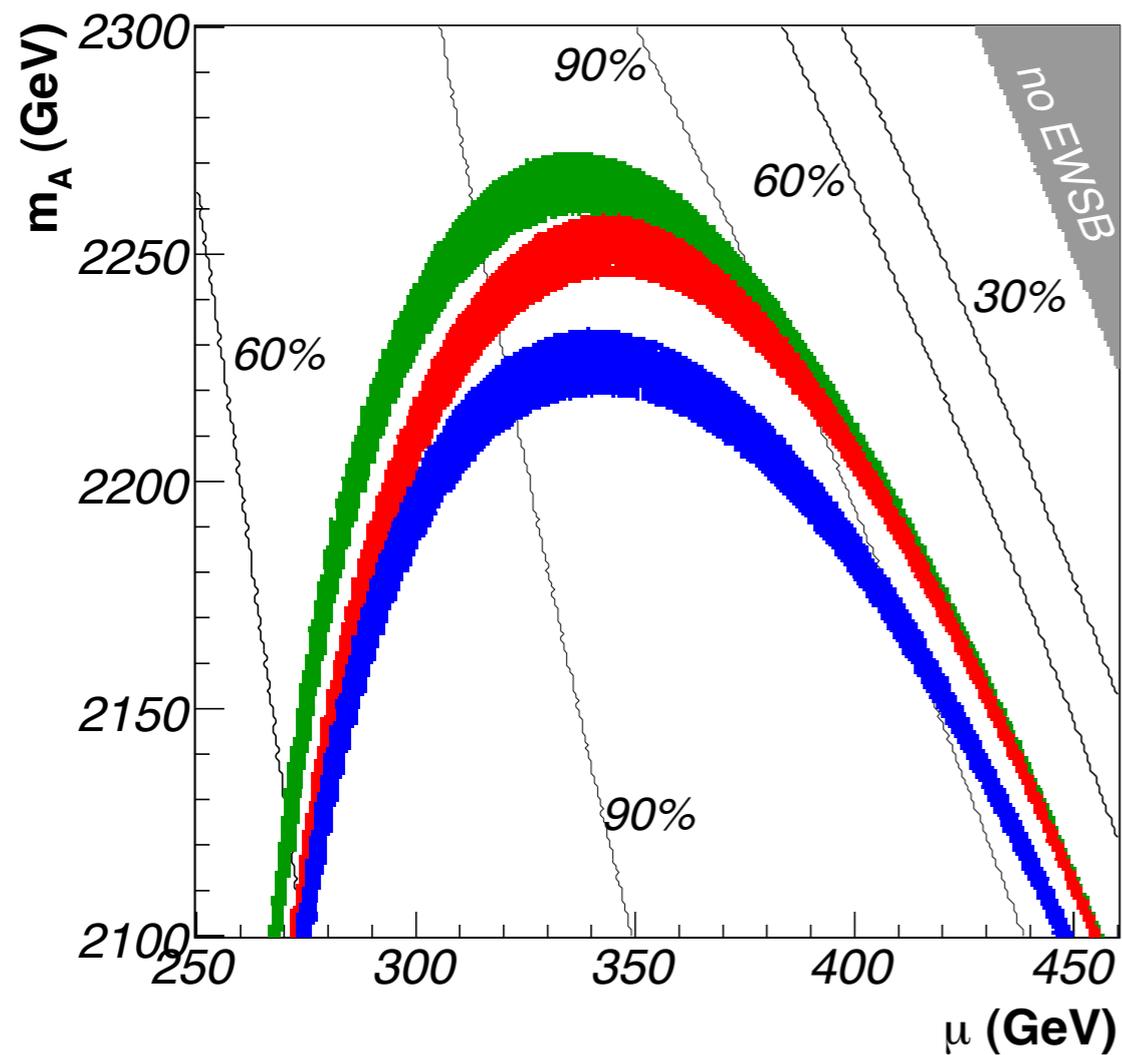
Favoured regions of parameter space



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Favoured regions of parameter space



Important impact of the cosmologically favoured regions of parameter space, e.g. shift of about 50 GeV for m_A or almost 200 GeV for m_{stop}

Numerical effect larger than experimental uncertainty in relevant regions of parameter space

[Herrmann, Klasen, Kovařík, PRD 80: 085025 (2009)]

Conclusion and Perspectives

Relic density calculation allows to obtain constraints on the MSSM parameter space, that are complementary w.r.t. collider data and precision measurements

Impact of SUSY-QCD corrections to neutralino annihilation into (heavy) quarks more important than the experimental uncertainty on the relic density of dark matter

Results to be generalized to 1st and 2nd generation quarks [Herrmann, Klasen, Kovařík (in progress)]

QCD corrections also relevant for co-annihilation

with neutralinos or charginos [Herrmann, Klasen, Kovařík (to be published)]

with lightest squark [Freitas 2007, Herrmann *et al.* (in preparation)]

Corrections potentially interesting for indirect dark matter detection [Herrmann *et al.* (in preparation)]

Correction due to light boson exchange before annihilation [Drees *et al.* 2009, **previous talk...**]

Electroweak corrections can also be relevant [Baro *et al.* 2008, Baro *et al.* 2010, **next talk...**]

Effective coupling approaches [Kulkarni *et al.*, **next-to-next talk...**]