Classification of BPS Objects in $\mathcal{N} = 6$ Chern-Simons Matter Theory

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Introduction

ABJM model [Aharony-Bergman-Jafferis-Maldacena (2008)]

Model

- $(2+1)$ dimensional superconformal Chern-Simons-Higgs model with level $(k, -k)$
- $U(N) \times U(N)$ gauge symmetry with gauge fields $A_\mu, \hat{A}_\mu$
- Matter fields Y^A, ψ_A $(A = 1, \cdots, 4)$ bi-fundamental repr of the gauge group, $SU(4)_R$ (anti) fundamental representation.

Action of the model,

$$\mathcal{L} = \mathcal{L}_{CS} + \mathcal{L}_{kin} + \mathcal{L}_{pot} + \mathcal{L}_{ferm}$$

Three-dimensional $\mathcal{N} = 6$ supersymmetry. Low-energy effective theory of N coincident M2-branes probing $\mathbf{C}^4/\mathbf{Z}_k$ orbifold. Dual to the M-theory on $AdS_4 \times S^7/\mathbf{Z}_k$ at large-N.

1/2 BPS conditions

Projection conditions on SUSY parameters

Projection conditions on $\mathcal{N} = 6$ Majorana SUSY parameters ϵ_i

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In general, we find 1/2 BPS projection condition is given by

$$\gamma\Xi_{ij}\epsilon_j = \epsilon_i, \quad (i,j=1,2,\cdots,6), \quad \mathrm{Tr}\gamma\otimes\Xi = 0,\ (\gamma\otimes\Xi)^2 = \mathbf{1}_2\otimes\mathbf{1}_6$$

γ : 2×2 matrix with $SO(2,1)$ spinor indices, Ξ : 6×6 matrix acting on $SO(6)_R$ vector indices.

$$\gamma\Xi_{ij} = \begin{cases} \gamma_0\otimes B \\ \gamma_2\otimes C^{(m,n)} \end{cases}$$

$$B \equiv \pm\begin{pmatrix} i\sigma_2 & & \\ & i\sigma_2 & \\ & & i\sigma_2 \end{pmatrix}, \quad C^{(m,n)} \equiv \begin{pmatrix} \mathbf{1}_m & \\ & -\mathbf{1}_n \end{pmatrix}, \quad m+n=6$$

1/2 BPS equations

SUSY transformation of fermions

$$\delta\psi_A = \left(\gamma^\mu D_\mu Y^B \delta^C_A + \Upsilon^{BC}_A\right)(\Gamma_i)_{BC}\,\epsilon_i$$

$$\Upsilon^{BC}_A \equiv Y^B Y^\dagger_A Y^C + \frac{1}{2}\delta^B_A \left(Y^C Y^\dagger_D Y^D - Y^D Y^\dagger_D Y^C\right) - (B \leftrightarrow C)$$

with projection condition B leads to

1/2 BPS equations

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Vortex type
$$\begin{cases} 0 = D_0 Y^B (\Gamma_j)_{BA} B_{ji} + \Upsilon_A^{BC}(\Gamma_i)_{BC}, \\ 0 = D_1 Y^B (\Gamma_i)_{BA} - D_2 Y^B (\Gamma_j)_{BA} B_{ji} \end{cases}$$

while for $C^{(m,n)}$, we have

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Vertex type $\begin{cases} 0 = D_0 Y^B (\Gamma_j)_{BA} B_{ji} + \Upsilon^{BC}_A (\Gamma_i)_{BC}, \\ 0 = D_1 Y^B (\Gamma_i)_{BA} - D_2 Y^B (\Gamma_j)_{BA} B_{ji} \end{cases}$

Fuzzy funnel type $\begin{cases} 0 = D_2 Y^B (\Gamma_j)_{BA} C^{(m,n)}_{ji} + \Upsilon^{BC}_A (\Gamma_i)_{BC}, \\ 0 = D_0 Y^B (\Gamma_i)_{BA} - D_1 Y^B (\Gamma_j)_{BA} C^{(m,n)}_{ji} \end{cases}$

Also the Gauss' law conditions should be satisfied

Physical interpretation

(BPS conditions in ABJM $\longrightarrow$ What kind of M-theory objects?)

BPS conditions in eleven dimensions

P : A map between 11d projection and that in ABJM model
$\longrightarrow$ Physical interpretation of BPS conditions in ABJM model.

11 dim		M2-brane world-volume
ξ : 16 SUSY	$\overset{\mathbf{Z}_k \text{ orbifold}}{\Longrightarrow}$	$\epsilon = P\xi$: 12 SUSY,
Projection condition : $\hat{\Gamma}\xi = \xi$	$\overset{P}{\Longrightarrow}$	BPS conditions in ABJM :
$\hat{\Gamma}_{012}\xi = \xi$		$\mathcal{A}\epsilon = \epsilon, \ \mathcal{A} \equiv P\hat{\Gamma}P^\dagger$,
		$\tilde{\mathcal{A}}\epsilon = 0, \ \tilde{\mathcal{A}} = \tilde{P}\hat{\Gamma}P^\dagger$
M-theory objects	$\Longrightarrow$	M-theory objects intersect with M2-branes

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There are three types of the 11d projection $\hat{\Gamma} = \underbrace{\hat{\Gamma}^{(0)}, \hat{\Gamma}^{(2)}, \hat{\Gamma}^{(4)}}_{\mathbf{Z}_k\,invariant}$.

Physical interpretation of the BPS equations

• $\hat{\Gamma} = \hat{\Gamma}^{(0)}$

We can set $\hat{\Gamma}^{(0)} = \hat{\Gamma}_2$ by $SO(2,1)$

$$\begin{cases} \hat{\Gamma}_{01}\xi = \xi \\ \hat{\Gamma}_{0134\cdots10}\xi = \xi \end{cases} \stackrel{P}{\Longrightarrow} \mathcal{A} = \gamma_2 \otimes \mathbf{1}_6, \ \tilde{\mathcal{A}} = 0$$

1/2 BPS condition in ABJM model

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M-waves, M9-branes (D8-branes in type IIA string theory) [Bergshoeff-Schaar (1999), M. de Roo (1997)]

	0	1	2	3	4	5	6	7	8	9	10
M2	•	•	•								
M9	•	•		•	•	•	•	•	•	•	•
M-wave	•	•									

Table: A possible configuration that corresponds to the condition.

$\bullet \hat{\Gamma} = \hat{\Gamma}^{(2)}$

$$\begin{cases} \frac{1}{2}\omega_{IJ}\hat{\Gamma}_{0IJ}\xi = \xi \\ \frac{1}{6!}\tilde{\omega}_{IJKLMN}\hat{\Gamma}_{0IJKLMN}\xi = -\xi \end{cases} \Longrightarrow \begin{array}{l} \mathcal{A} = \gamma_0 \otimes B, \ \tilde{\mathcal{A}} = 0, \\ \text{1/2 BPS condition in ABJM model} \end{array}$$

$\tilde{\omega} = *\omega$ on $\mathbf{C}^4/\mathbf{Z}_k$

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1/2 BPS condition in ABJM model

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M2-branes, KK-monopoles (D6-branes in type IIA string theory)

	0	1	2	3	4	5	6	7	8	9	10
M2	•	•	•								
M2	•									•	•
KK	•			•	•	•	•	•	•		

Table: Intersecting M2-branes and KK-monopoles.

• $\hat{\Gamma} = \hat{\Gamma}^{(4)}$

$$\hat{\Gamma}^{(4)} = \frac{1}{4!}\omega_{IJKL}\hat{\Gamma}_{2IJKL}, \ \omega \notin \mathbf{6}_2 \oplus \mathbf{6}_{-2}$$

$$\Longrightarrow \mathcal{A} = \gamma_2 \otimes C^{(m,n)}, \quad \tilde{\mathcal{A}} = 0,$$

1/2 BPS condition in ABJM model

M-theory objects – M5-branes with non-trivial volume form.

	0	1	2	3	4	5	6	7	8	9	10
M2	•	•	•								
M5	•	•		•	•			•	•		
M5	•	•				•	•			•	•

Table: M2-M5 configuration : $(m, n) = (4, 2)$.

BPS conditions with lower SUSYs

M2-M2 intersections with angles

Consider M2-branes specified by a 11 dim projector,

$$\hat{\Gamma} = \hat{\Gamma}_0(\sin\theta\,\hat{\Gamma}_8 + \cos\theta\,\hat{\Gamma}_9)\hat{\Gamma}_{10}$$

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$$\hat{\Gamma} = \hat{\Gamma}_0(\sin\theta\,\hat{\Gamma}_8 + \cos\theta\,\hat{\Gamma}_9)\hat{\Gamma}_{10}$$

	0	1	2	3	4	5	6	7	8	9	10
M2	●	●	●								
M2	●								○ $\sin\theta$	○ $\cos\theta$	●

Table: M2-branes configuration.

The M2-brane extend over two different orbifold planes.

Corresponding ABJM projector is $\mathcal{A} = \gamma_0 \otimes \Xi$ and $\tilde{\mathcal{A}} = \gamma_0 \otimes \tilde{\Xi}$ with

$$\begin{aligned} \Xi &= -g^T \,\mathrm{diag}(\, i\sigma_2 \,,\, i\sigma_2 \,,\, i\cos\theta\sigma_2 \,)\, g, \\ \tilde{\Xi} &= \begin{pmatrix} 0 & 0 & 0 & 0 & -\sin\theta & 0 \\ 0 & 0 & 0 & 0 & 0 & \sin\theta \end{pmatrix} g \end{aligned}$$

$g \in SO(6)_R$.

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$\mathcal{A}\epsilon = \epsilon,\ \tilde{\mathcal{A}}\epsilon = 0 \Longrightarrow$ 4 SUSY among 12 SUSY (1/3 BPS condition)

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$\theta = 0, \pi \Longrightarrow$ 4 SUSY (1/3 BPS) is enhanced to 6 SUSY (1/2 BPS).

Other BPS conditions

Other BPS conditions

- 1/3 and 5/12 BPS conditions for M5-M2-branes with angles
- 1/3, 1/4, 1/6, 1/12 BPS conditions for multiple stacks of M2 and M5-branes

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- 1/3, 1/4, 1/6, 1/12 BPS conditions for multiple stacks of M2 and M5-branes

	0	1	2	3	4	5	6	7	8	9	10
M2	●	●	●								
M5	●	●		●		●			●		●
$\overline{\mathrm{M5}}$	●	●		●			●	●			●
$\overline{\mathrm{M5}}$	●	●			●	●		●			●
$\overline{\mathrm{M5}}$	●	●			●		●		●		●
M5	●	●		●	●					●	●
M5	●	●				●	●			●	●
$\overline{\mathrm{M5}}$	●	●						●	●	●	●

Table: An example of 1/12 BPS configuration. The Hodge-dual branes are omitted.

Conclusions and discussion

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Conclusions in this talk

1. We found $n/12$ ($n = 1, \cdots, 6$) BPS conditions in $\mathcal{N} = 6$ ABJM model
2. The map from eleven-dimensional projection conditions to the BPS conditions in ABJM model were analyzed
3. Configurations of M-theory objects were studied
4. Reduction to type IIA – brane configurations with D2-branes are studied

Future researches

- Existence of solutions
- Explicit solutions
- Dynamics of various M-theory objects
- And so on...

$\mathcal{N}$	Residual symmetry	Intersecting branes
6	$SU(3) \times U(1)^2$	M2, KK-monopoles
6	$SU(4)$	M2, M9, M-waves
6	$SU(2) \times SU(2) \times \mathrm{U}(1)^2$	M5
5	$SU(2) \times SU(2)$	M5 with angles
4	$U(1)^3$	M5 with angles
4	$SU(2) \times U(1)^2$	M2, KK-monopoles
4	$SU(2) \times U(1)^2$	M2 with angles
3	$SO(3)$	M2 ending on M5
2	$SU(2) \times SU(2) \times U(1)^2$	M2, KK-monopoles
2	$U(1) \times U(1)$	M2 ending on M5
1	$SU(2) \times SU(2)$	M2 ending on M5, M9, M-waves
$n+m$	$Spin(n) \times Spin(m) \times Spin(6-n-m)$	M5, M9, M-waves

Table: Classification of the BPS equations in the number of preserved supercharges, the symmetry of BPS equations and the corresponding M-theoretical objects. $\mathcal{N}$ is the number of preserved supercharges.