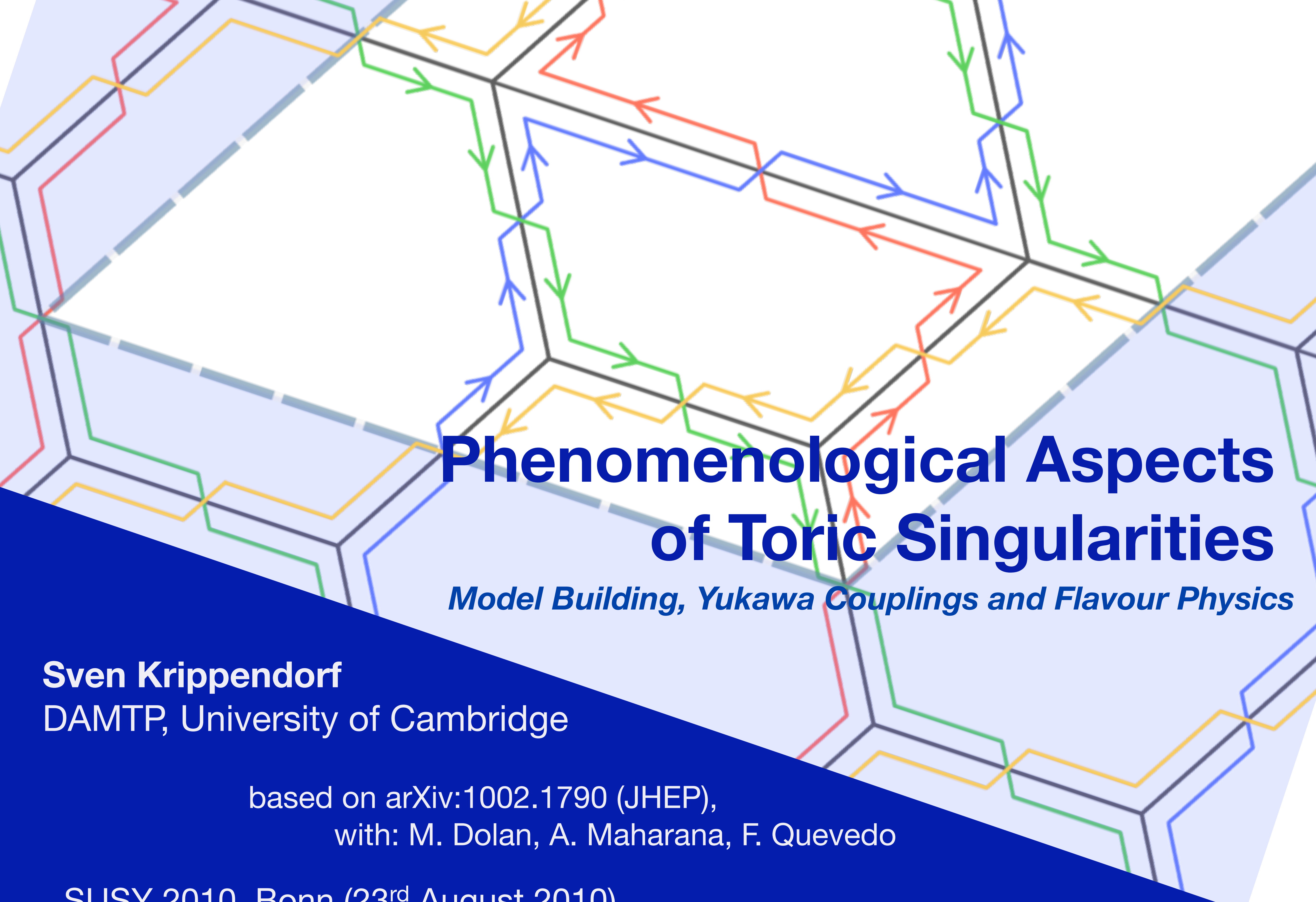


Phenomenological Aspects of Toric Singularities

Model Building, Yukawa Couplings and Flavour Physics

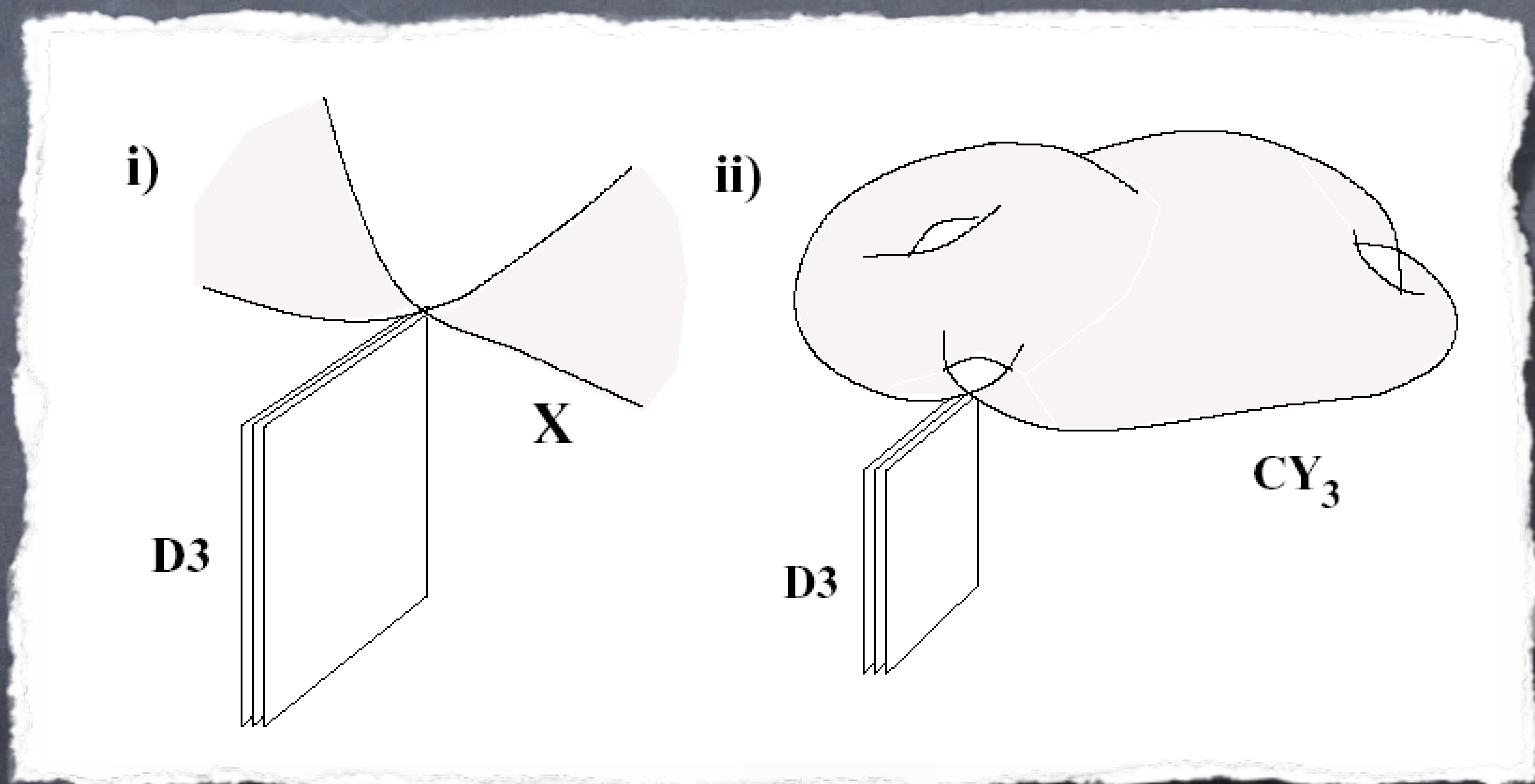
Sven Krippendorf
DAMTP, University of Cambridge

based on arXiv:1002.1790 (JHEP),
with: M. Dolan, A. Maharana, F. Quevedo

SUSY 2010, Bonn (23rd August 2010)

Perspective: Local Brane Models (within IIB)

... bottom-up model building



e.g.: aldazabal, ibanez, quevedo, uranga (10 years ago),
verlinde, wijnholt (5 years ago)

and what can we get?

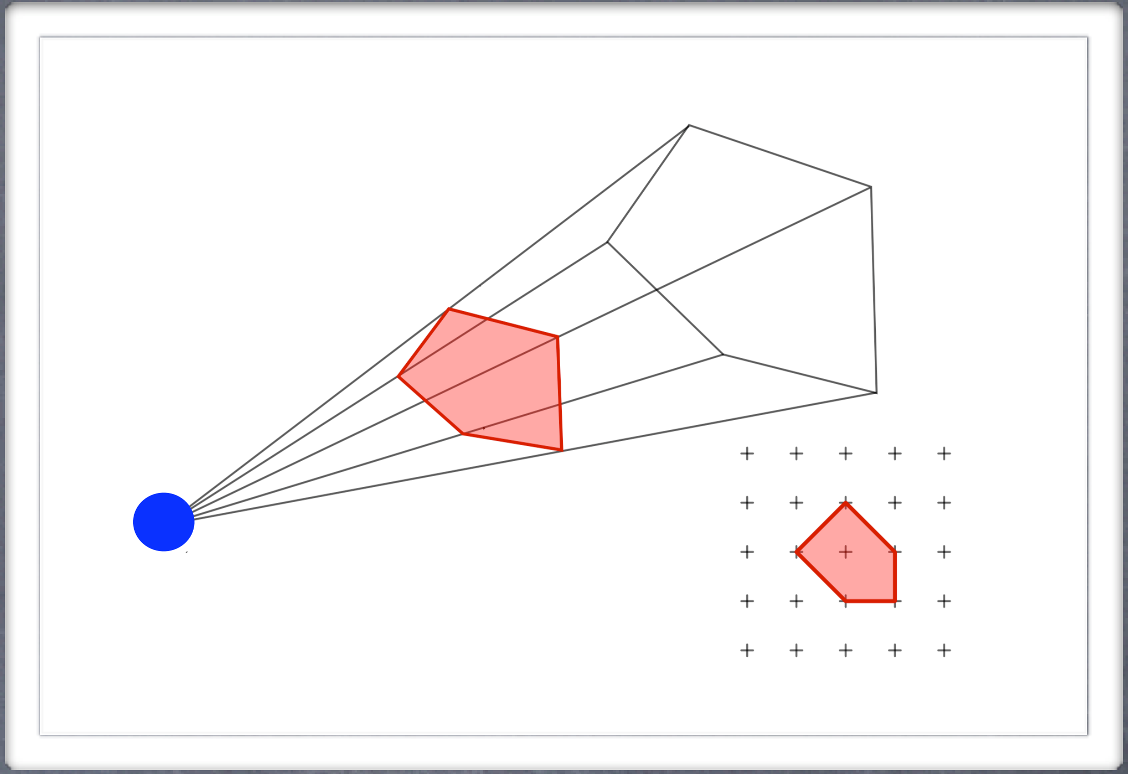
1. Model Building: Standard (like) Models with fractional (D3/D7) branes at singularities.

2. General properties for gauge theories of toric singularities via dimer techniques.

Motivation for branes at singularities

- Local models -> a lot of information without addressing moduli stabilisation
- Effective field theory well under control
  (tree-level Kähler potential can be flavour diagonal)
- Gauge coupling unification (in principle)
- Powerful (dimer) techniques for toric singularities
- Gauge theories highly restricted (unlike intersecting branes in IIA)

Gauge theories probing toric singularities



- Toric CY-cone: represented as T^3 fibration over rational polyhedral cone (-> toric diagram) [non-compact but embeddable in global manifold]
- Gauge theory of branes at tip of cone (bound states of D7, D5, D3) is a quiver gauge theory
- Gauge theory of bound states is always a quiver gauge theory (rank of gauge group only freedom)
- Gauge theory obtained via T-dual D5/NS5 brane system wrapping T^2. This system on T^2 is the dimer and encodes the whole gauge theory.

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0503149, 0504110, 0505211, 0511063, 0511287, 0604136, 0706.1660, 0803.4474

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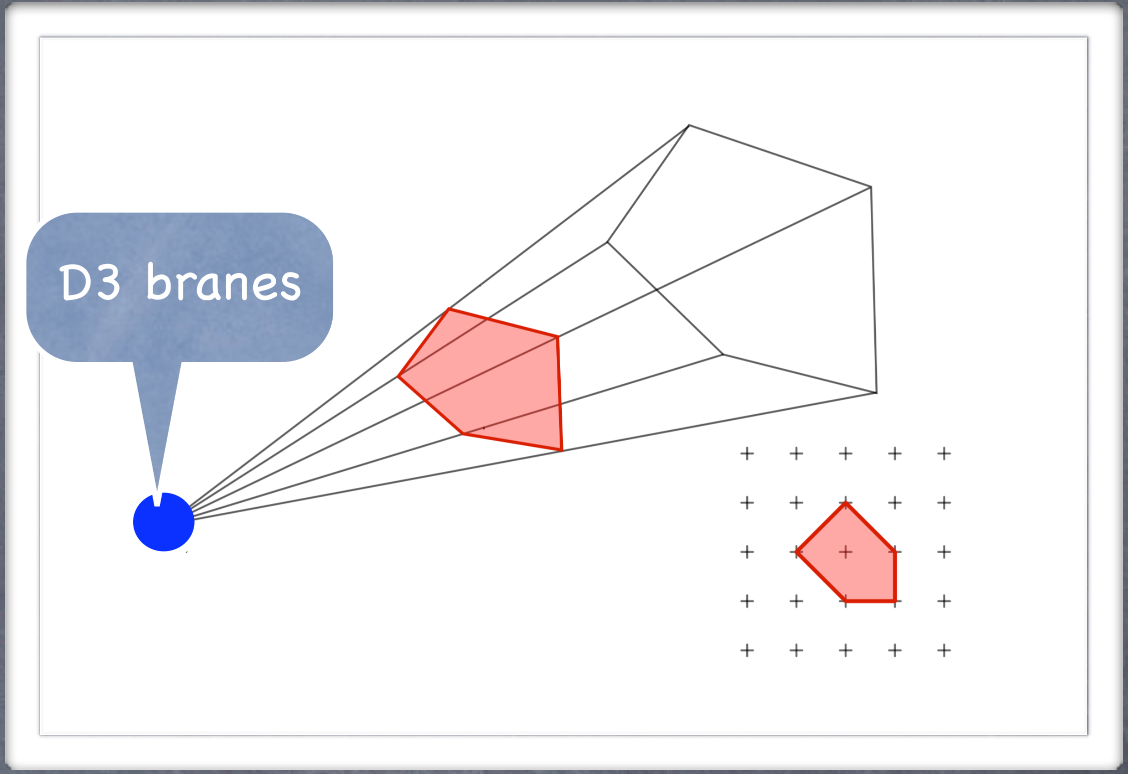

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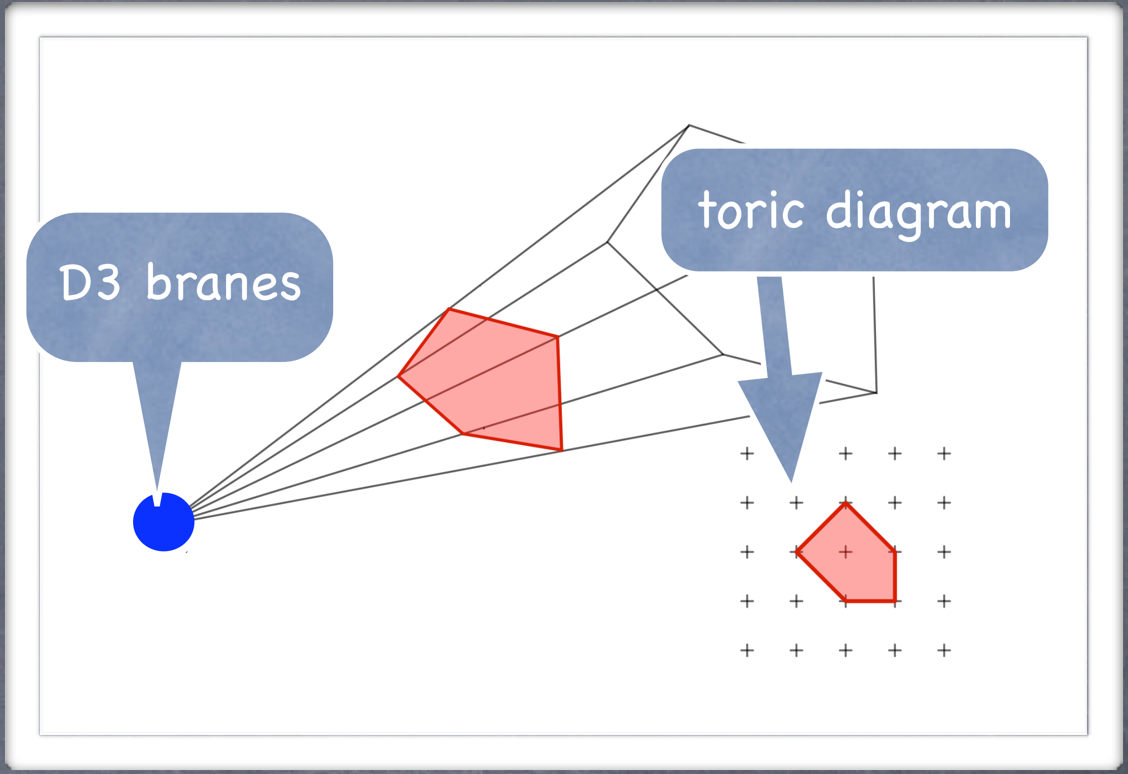

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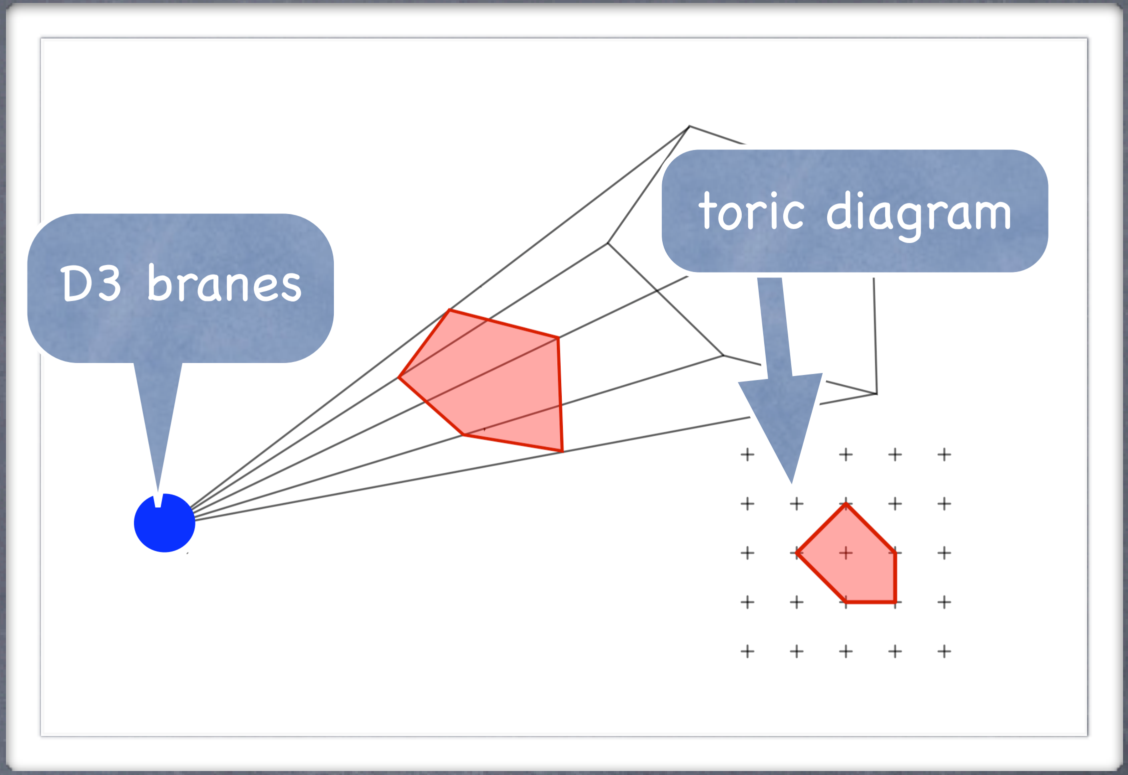

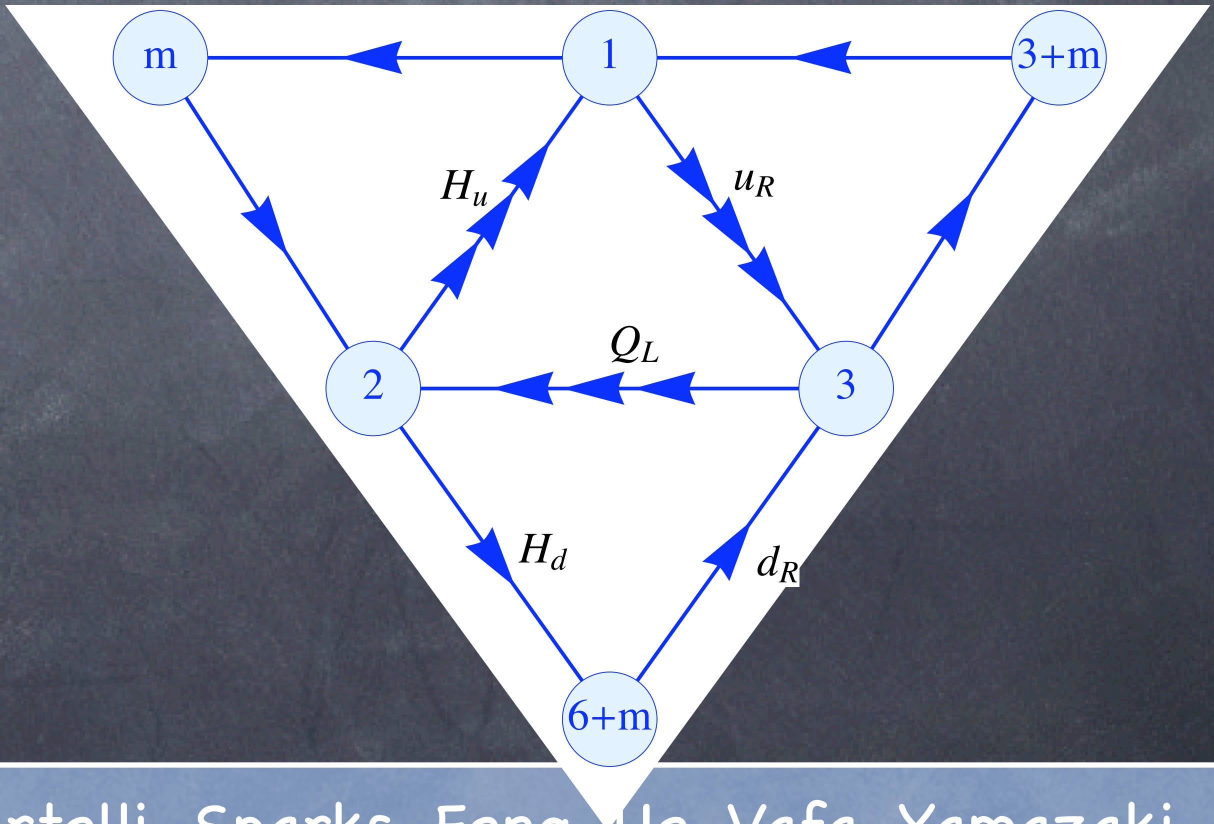

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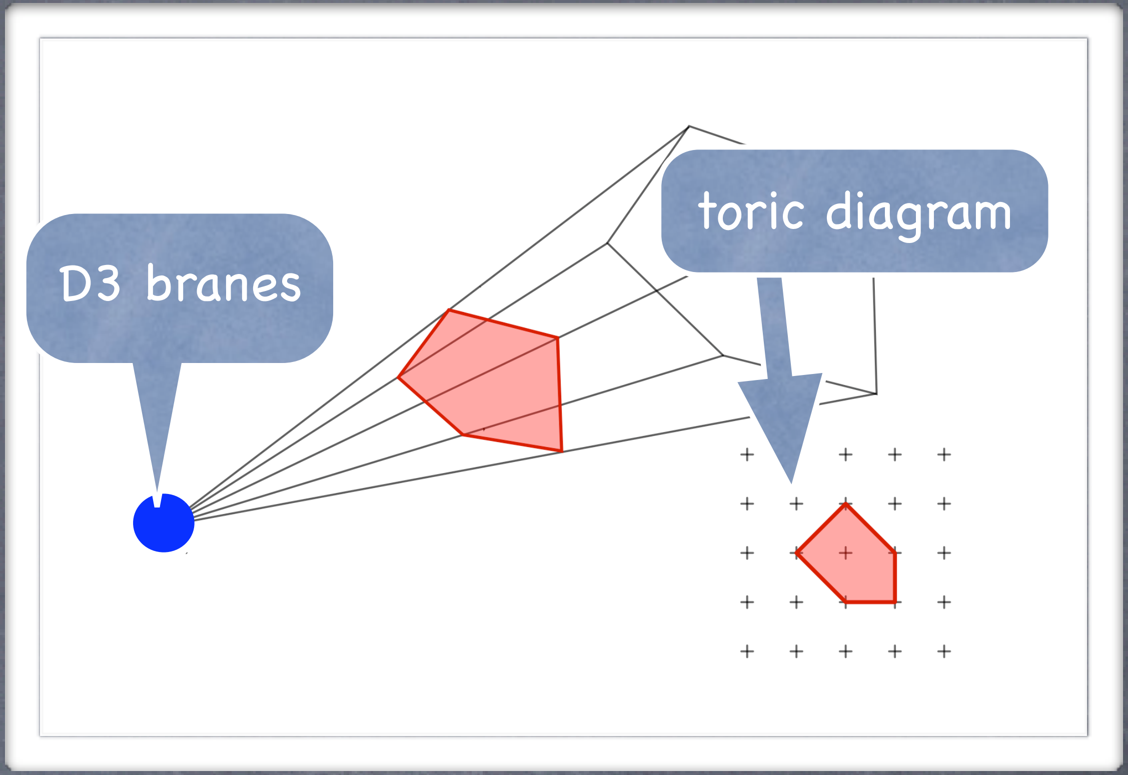

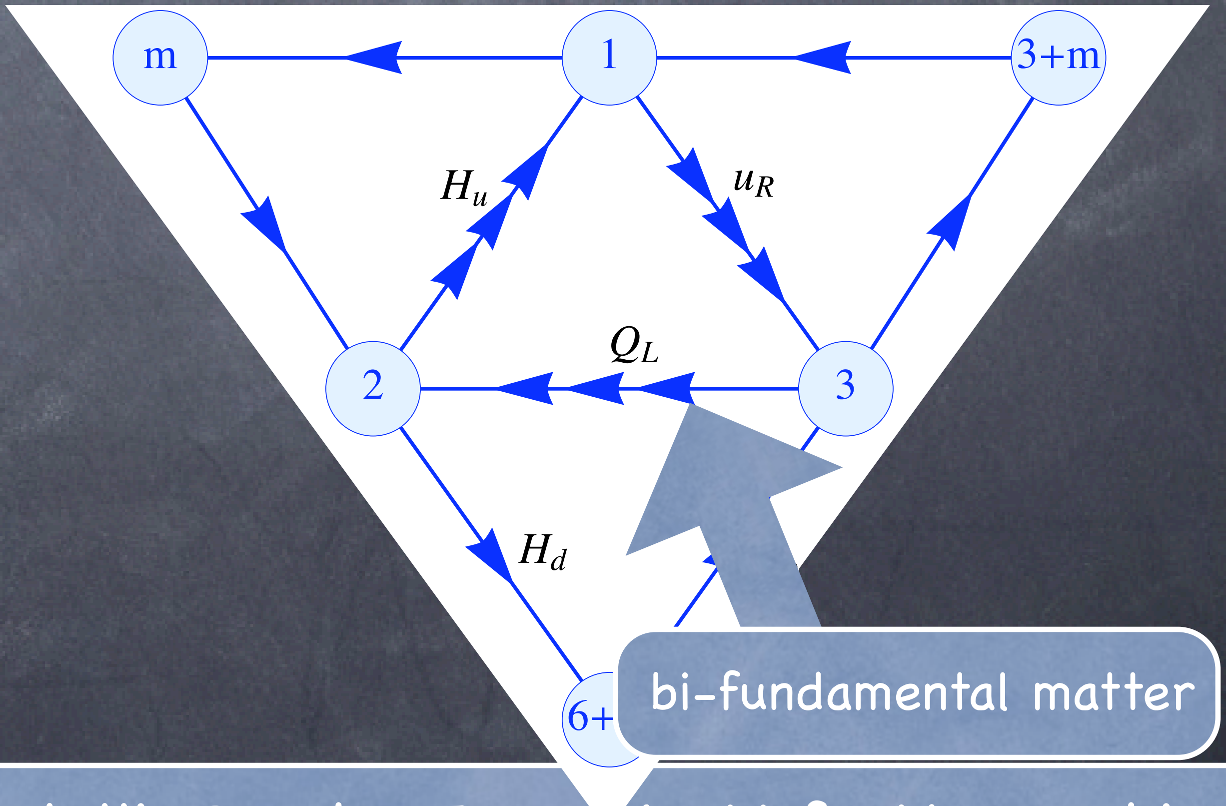

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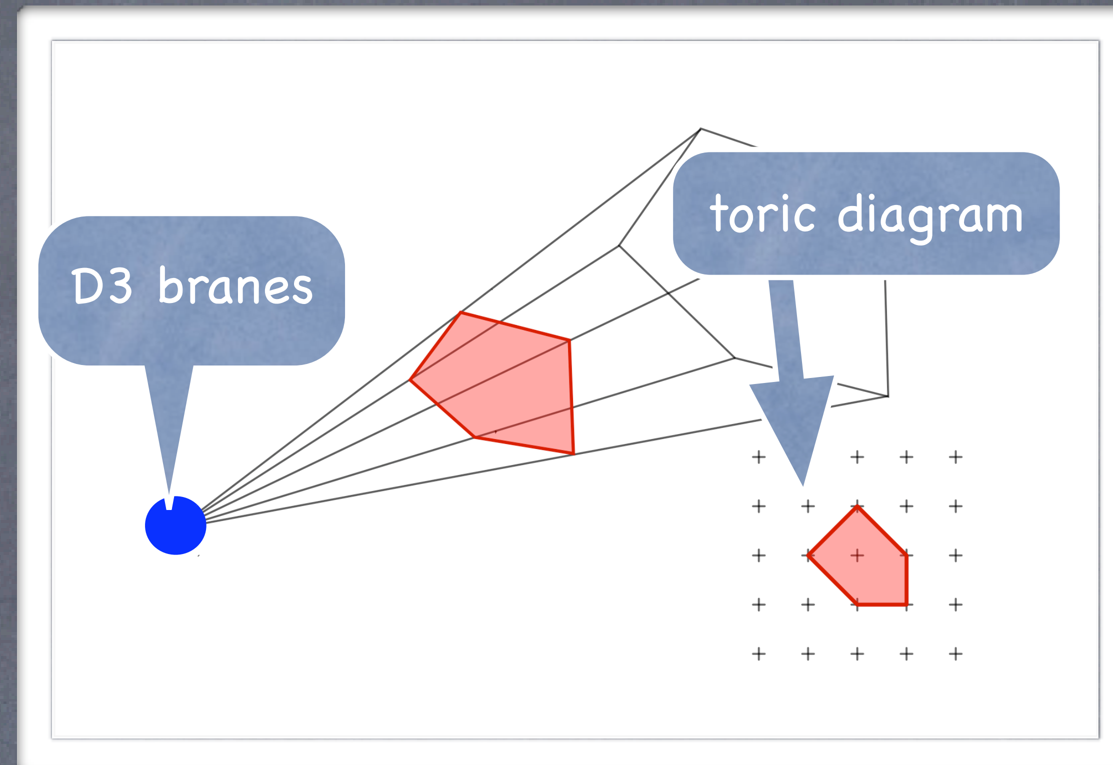

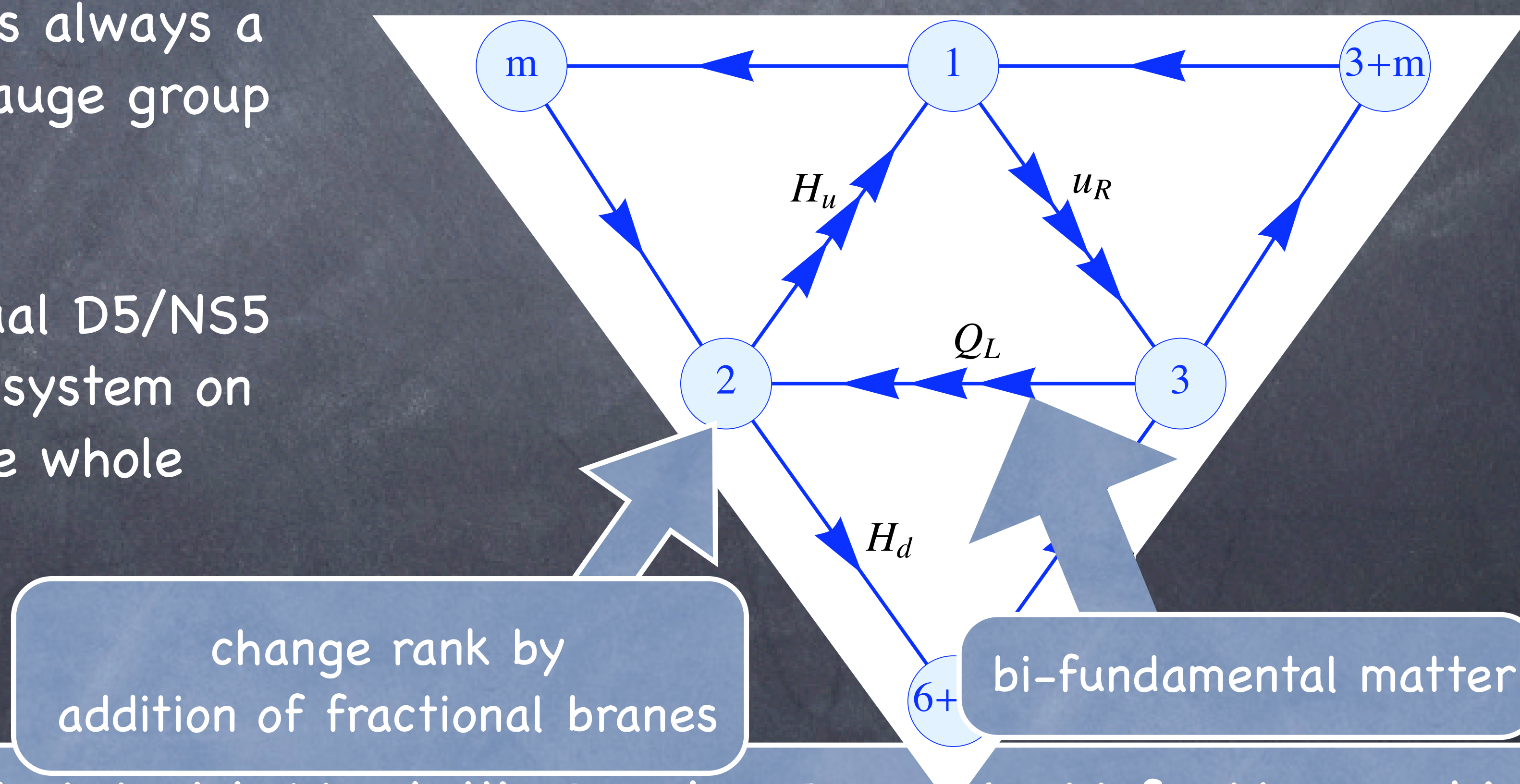

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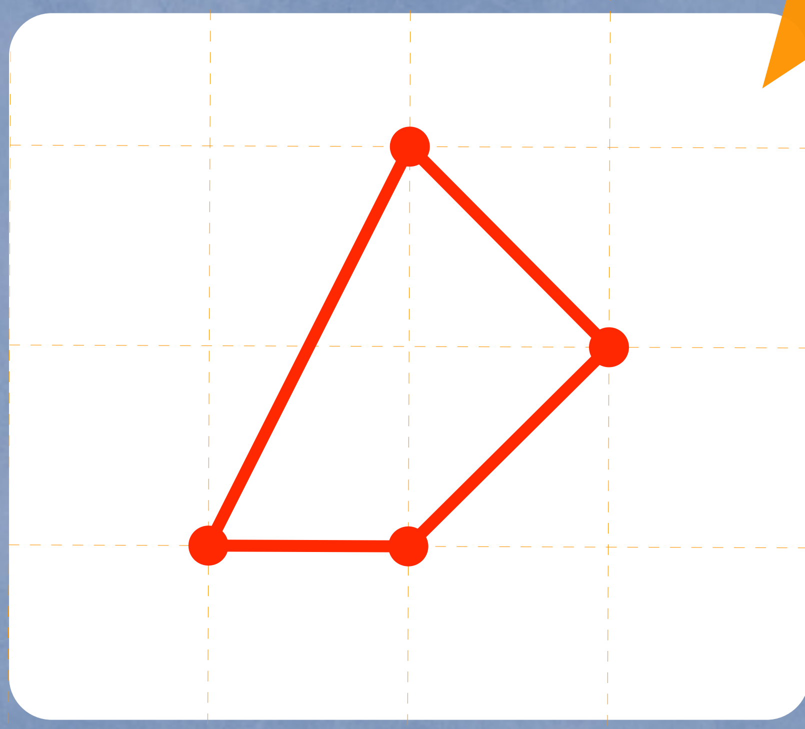
The Dimer Language

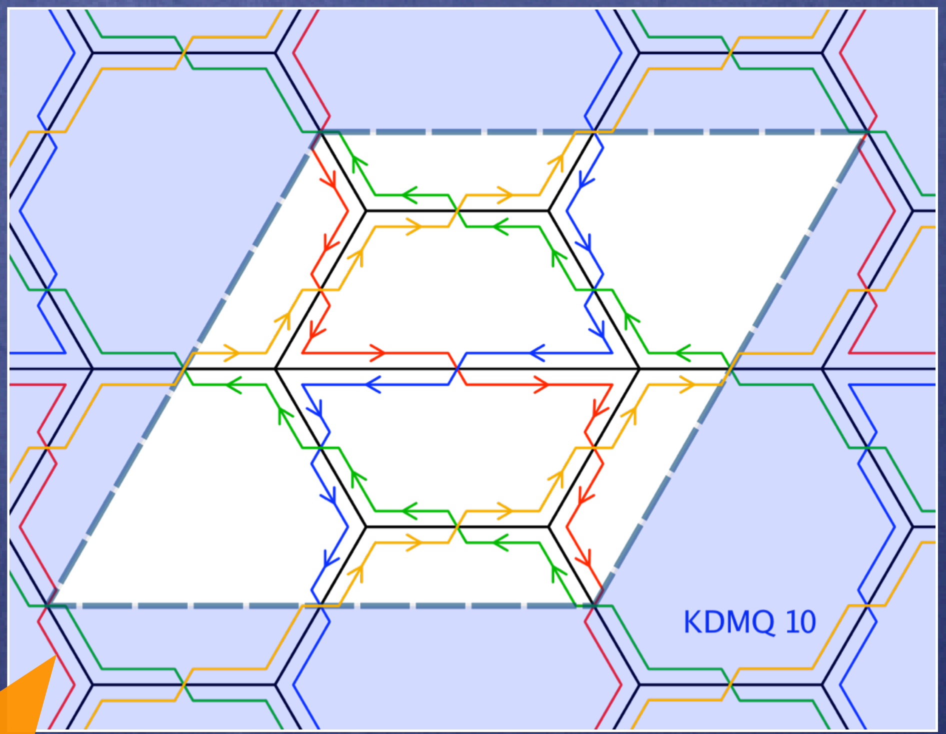
Dimers visualise the gauge theory of toric singularities.

Geometry: Toric Diagram

inverse slopes in toric diagram



Gauge Theory: Dimer



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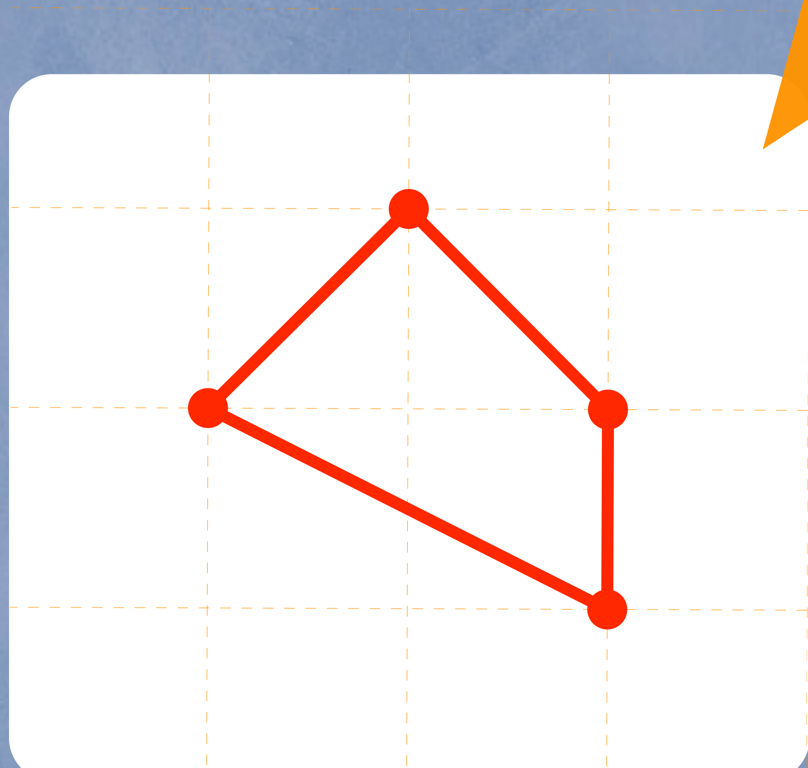

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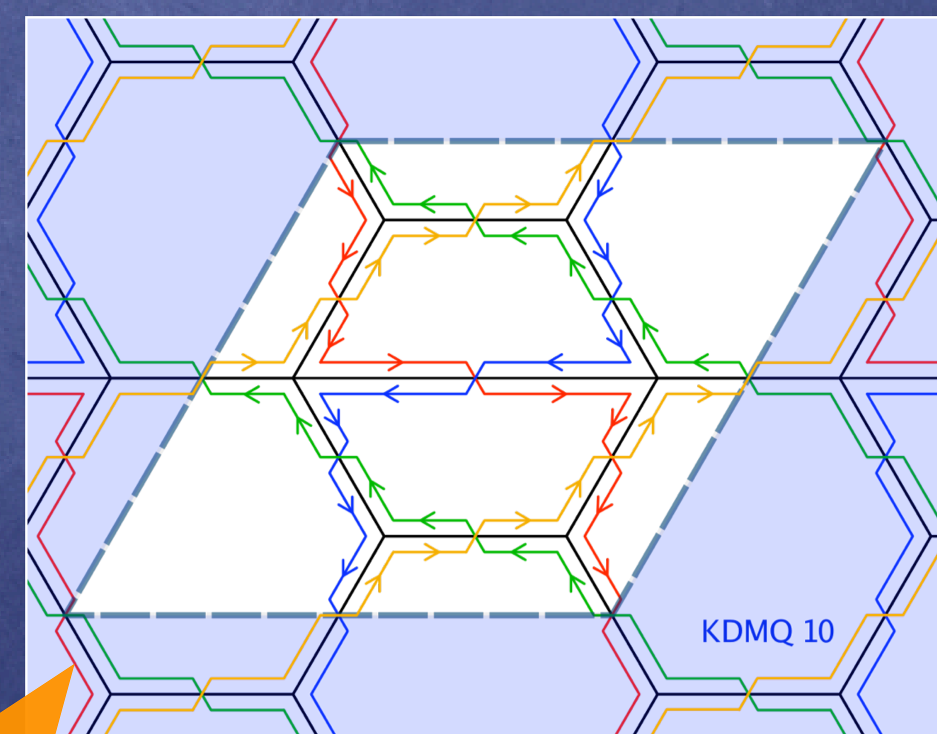

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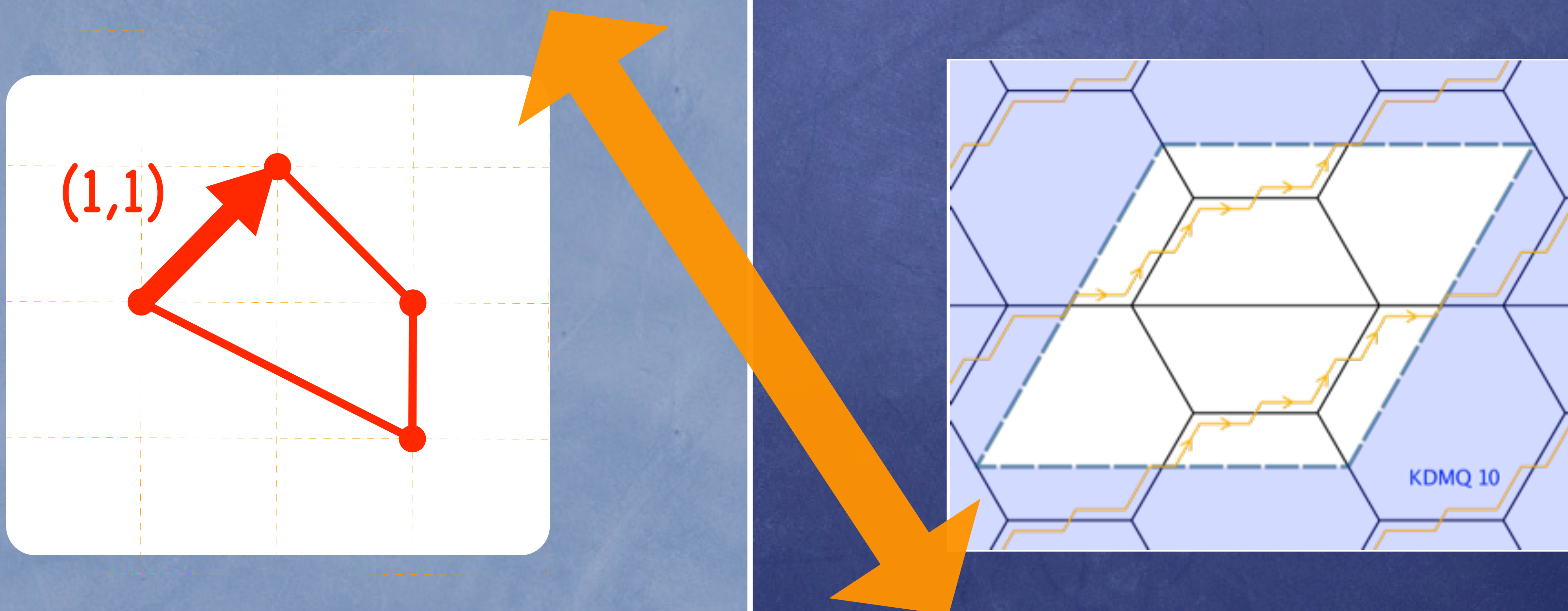

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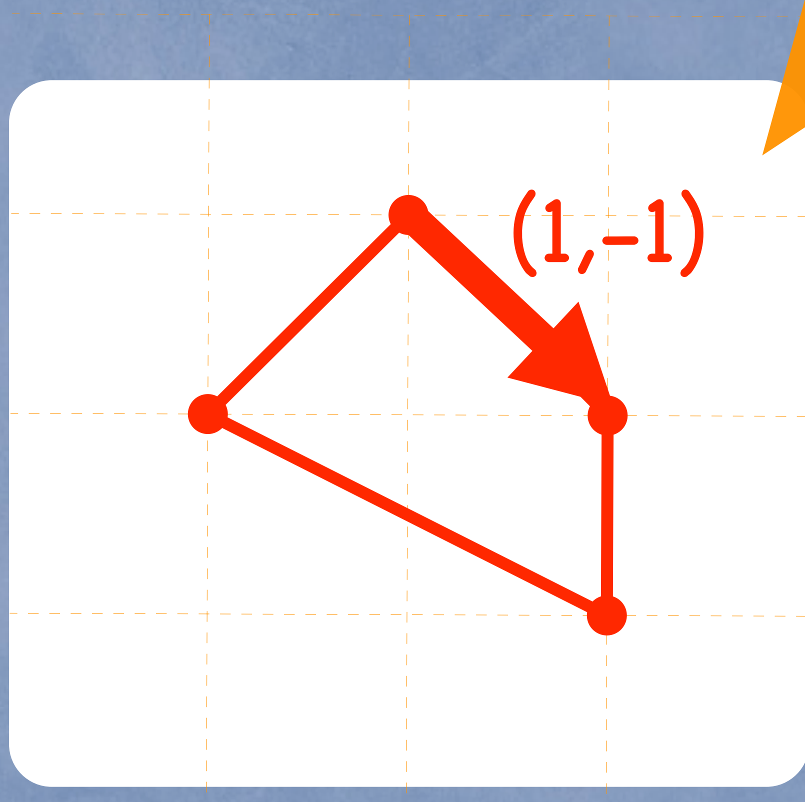

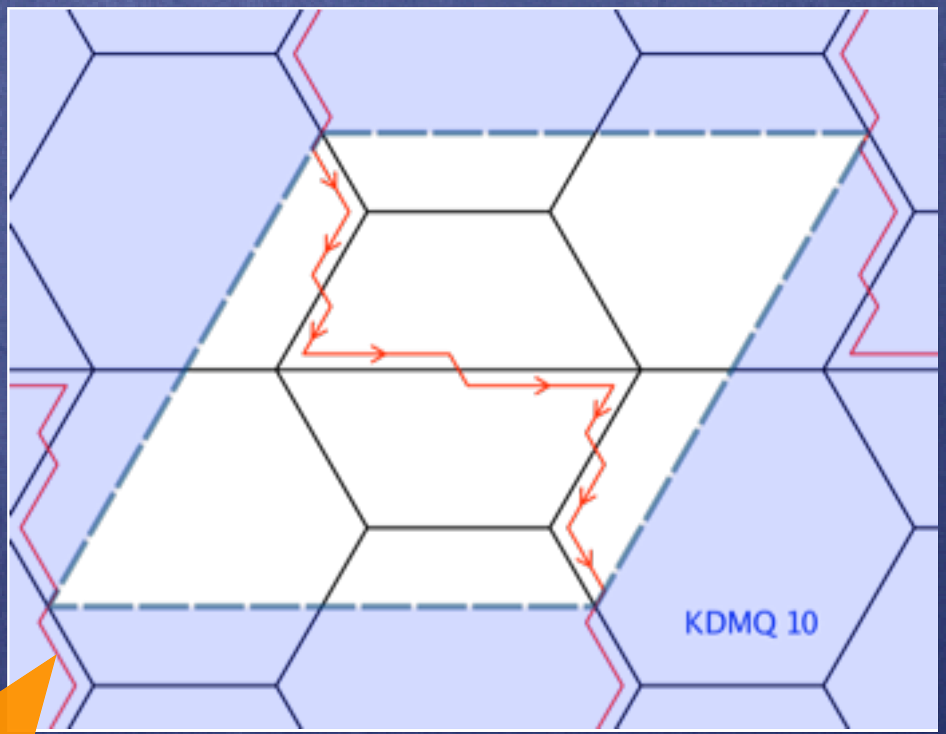

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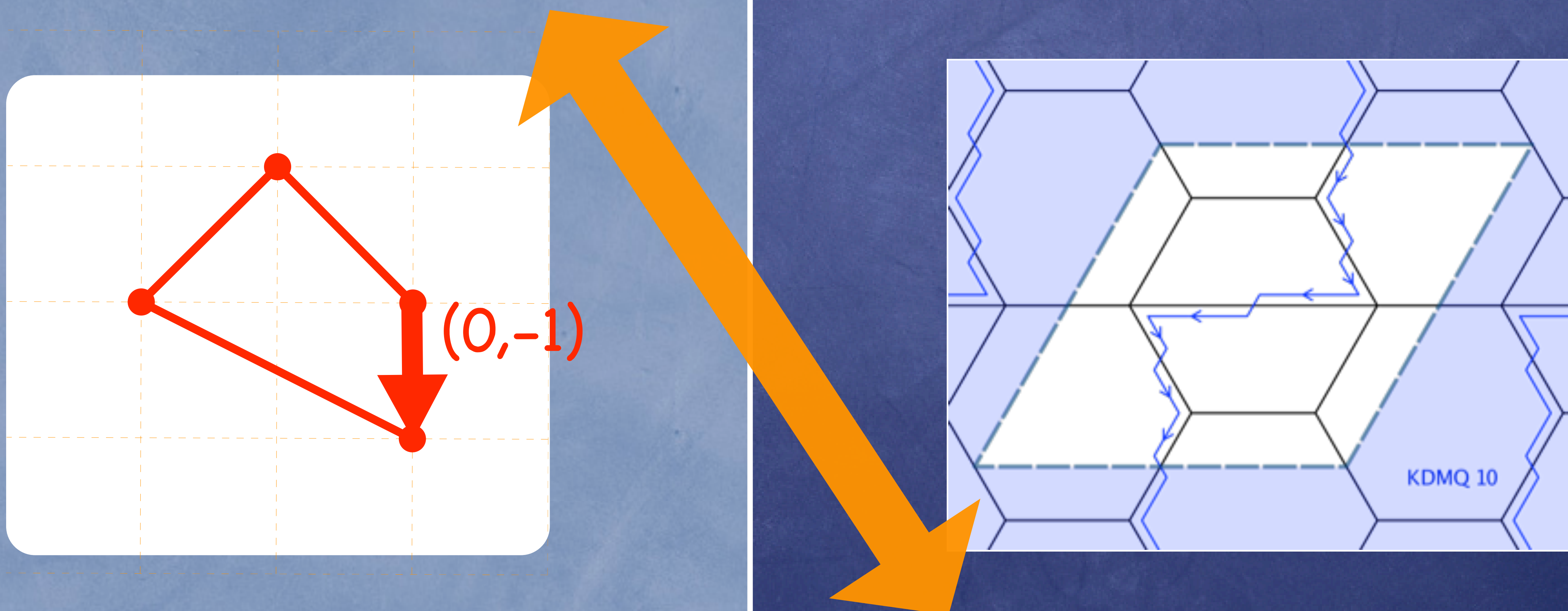

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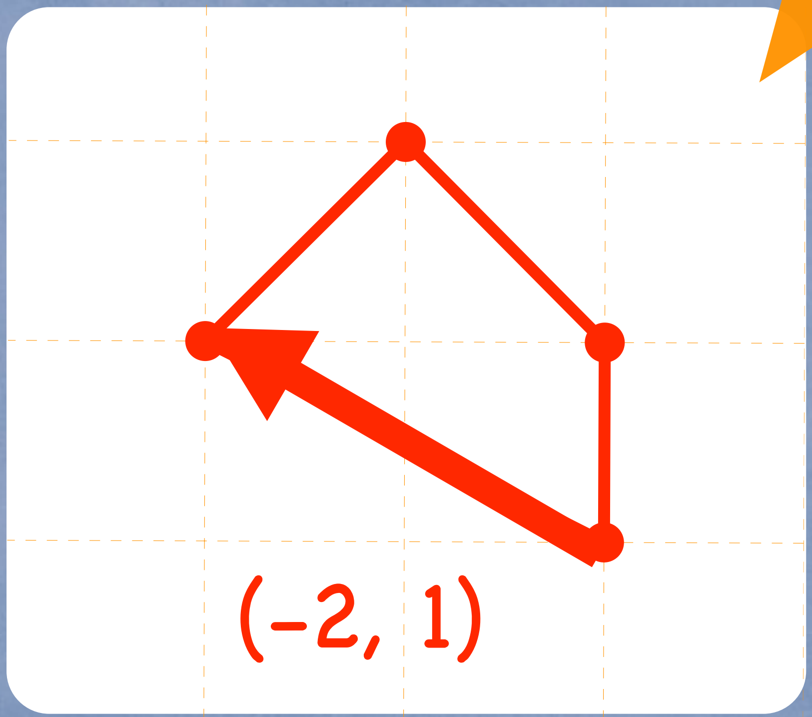

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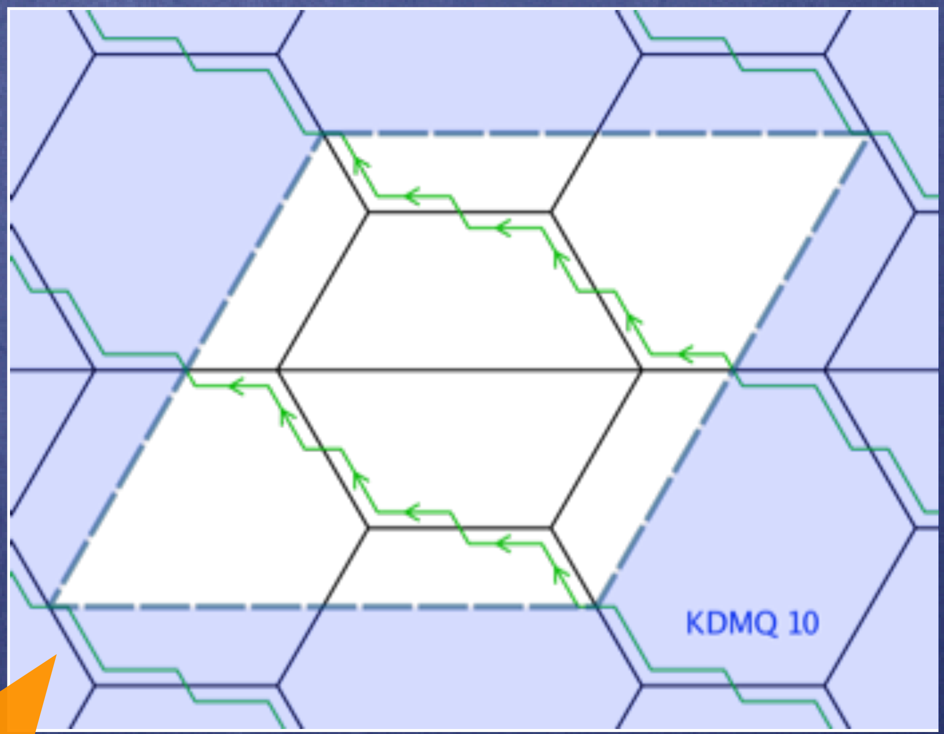

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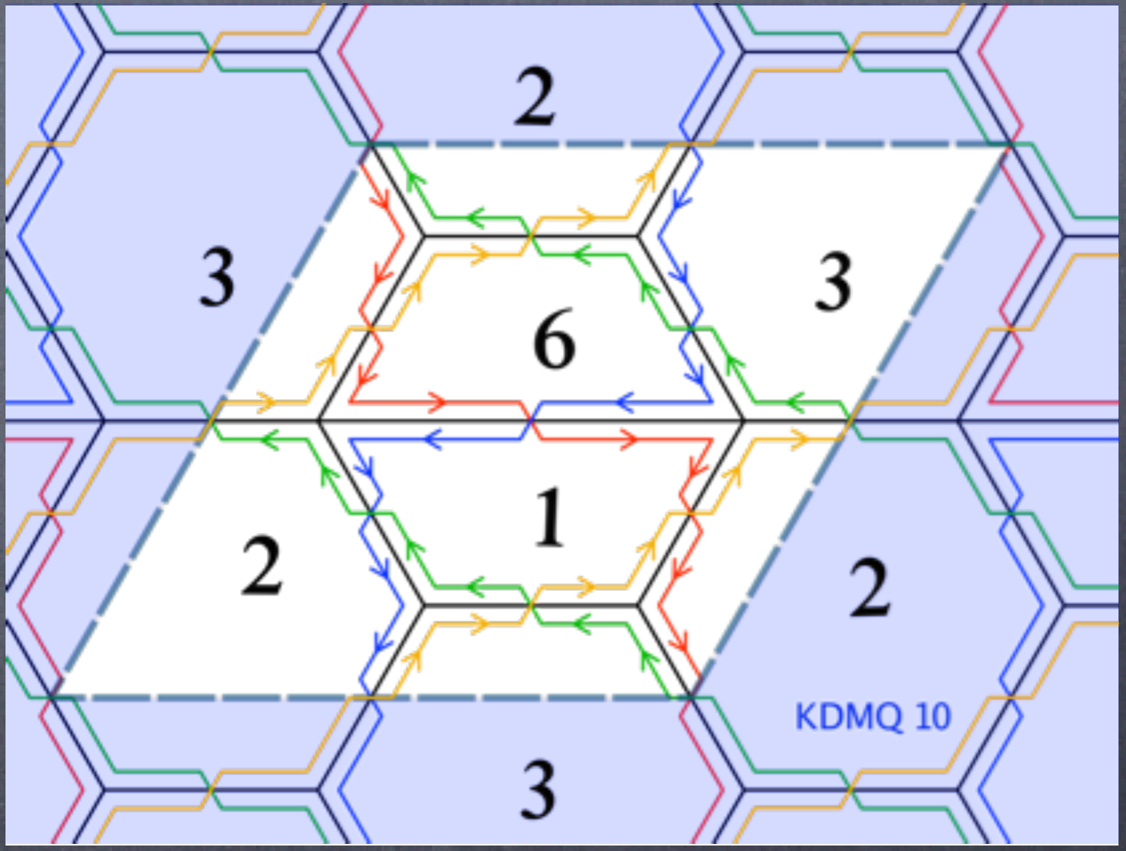
Dimer Language II

Reading off the gauge theory

- Faces
  = gauge groups
- Intersection of zigzag paths
  = bi-fundamental matter
- Vertices (faces orbited by zigzag paths)
  = superpotential terms



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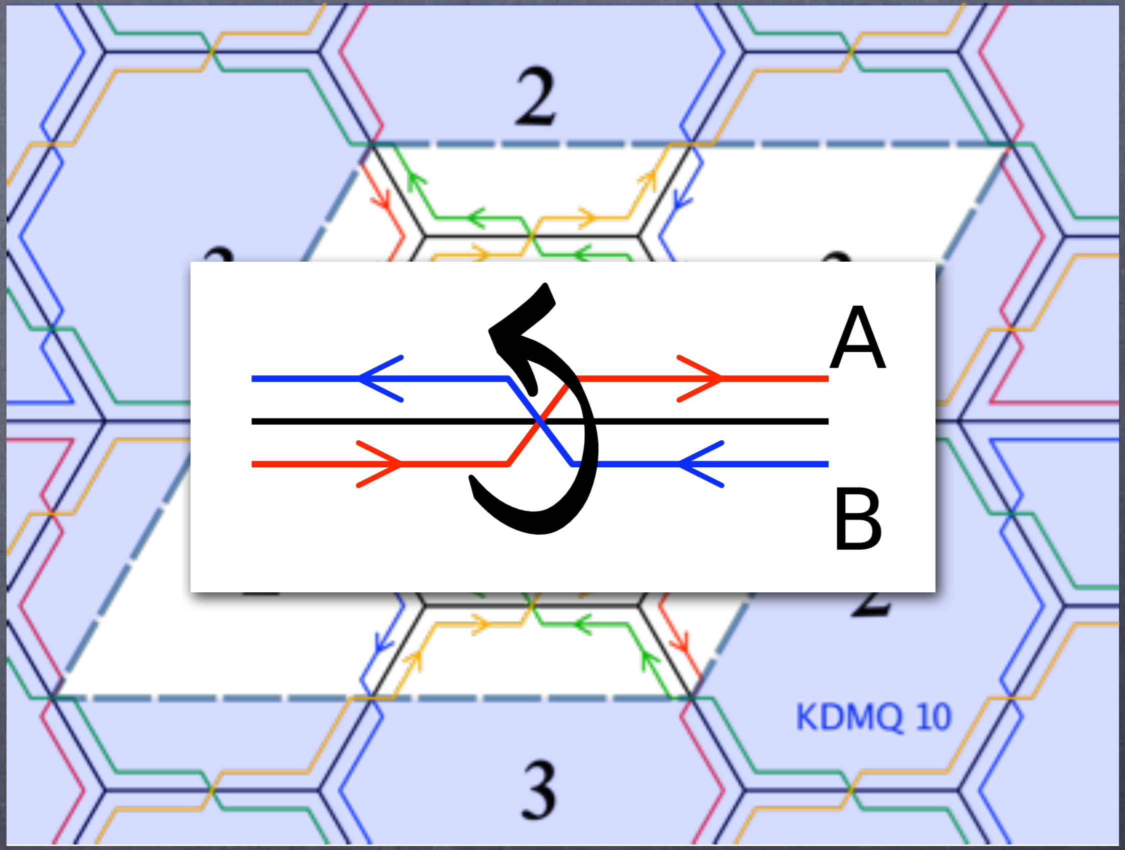

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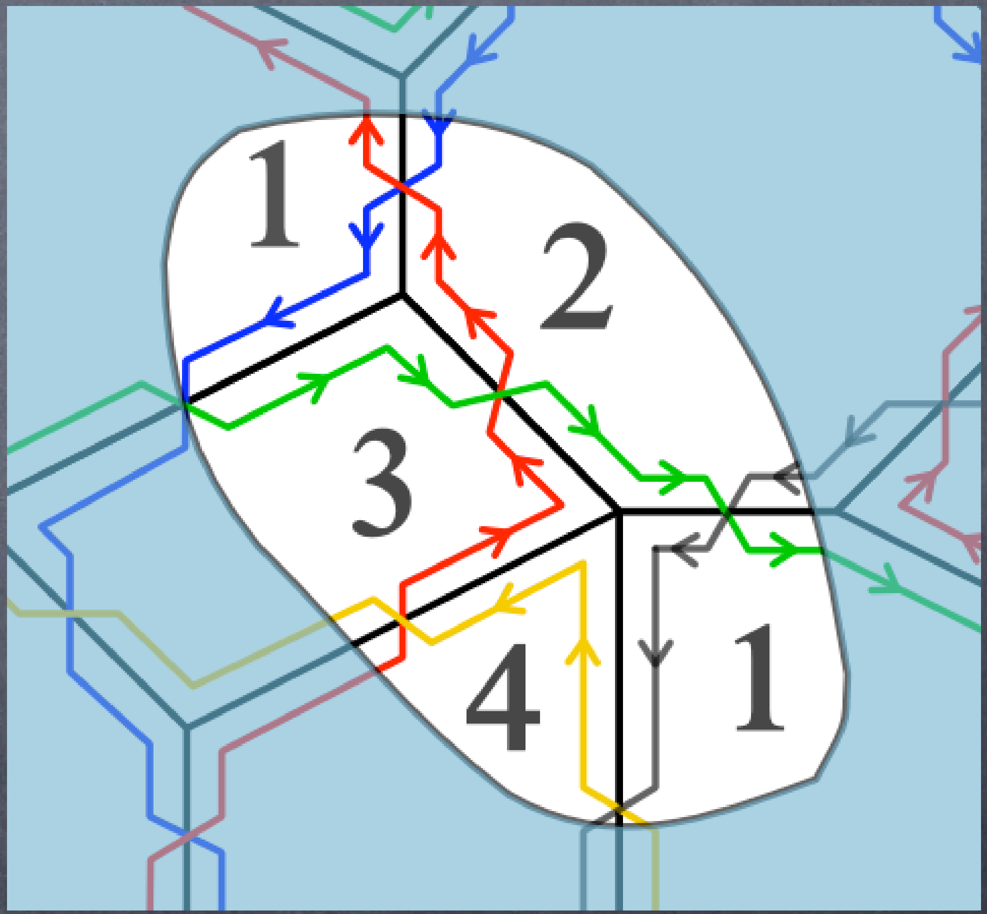

$$W = X_{13}X_{32}X_{21} - X_{14}X_{43}X_{32}X_{21}$$

Gulotta 0807.3012

Dimer Language III: How do I get a dimer?

- this is the inverse problem (following Gulotta)
- embed toric singularity in orbifold of conifold whose dimer is known (chess-board).
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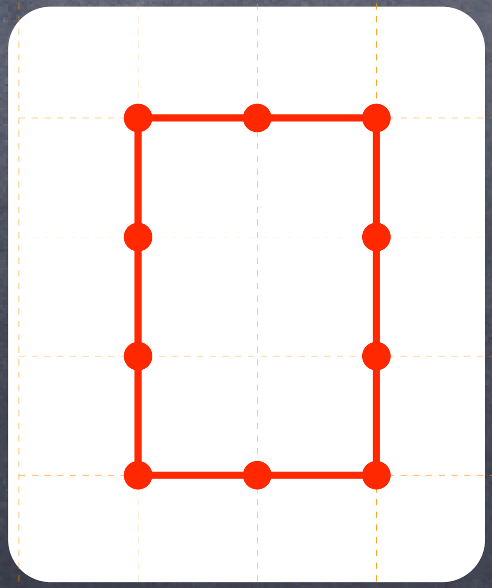

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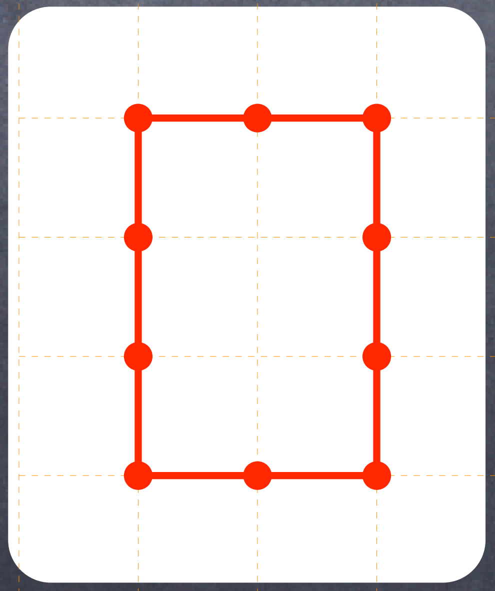

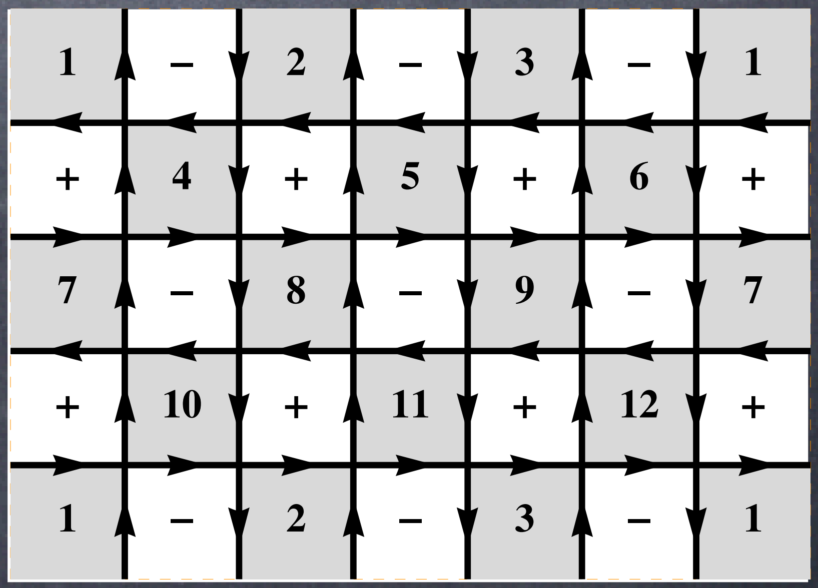

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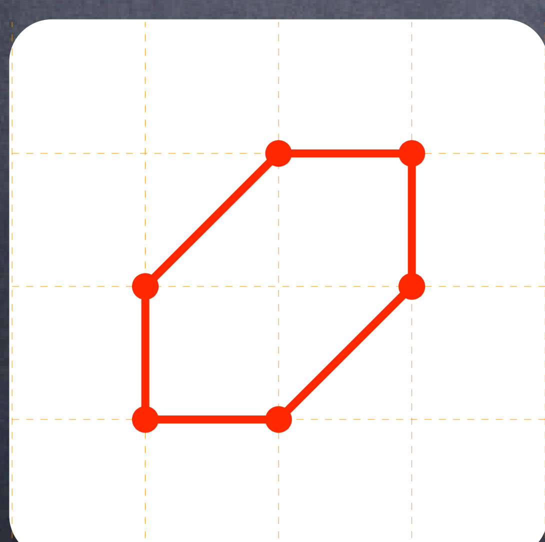

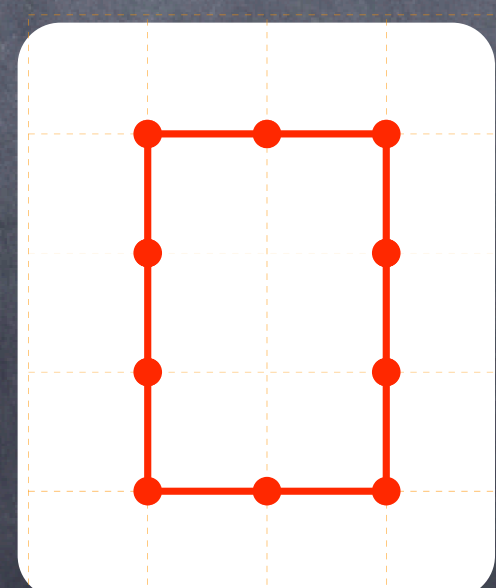

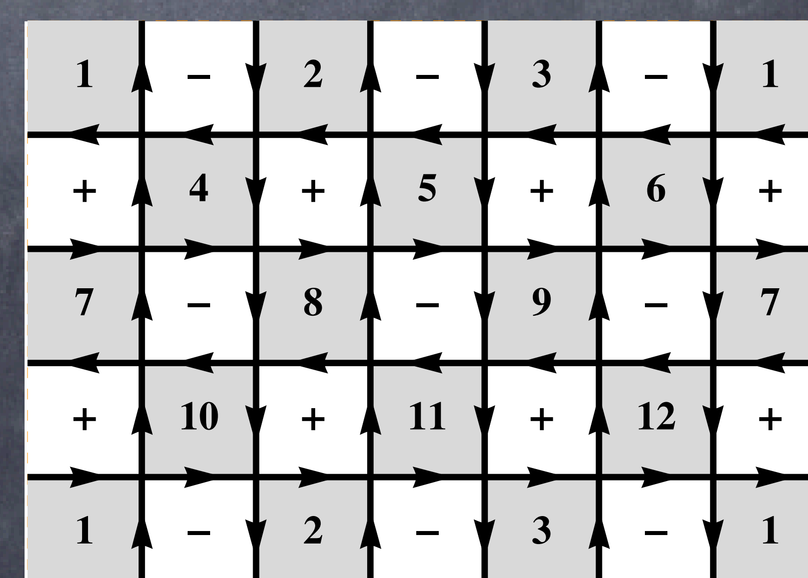

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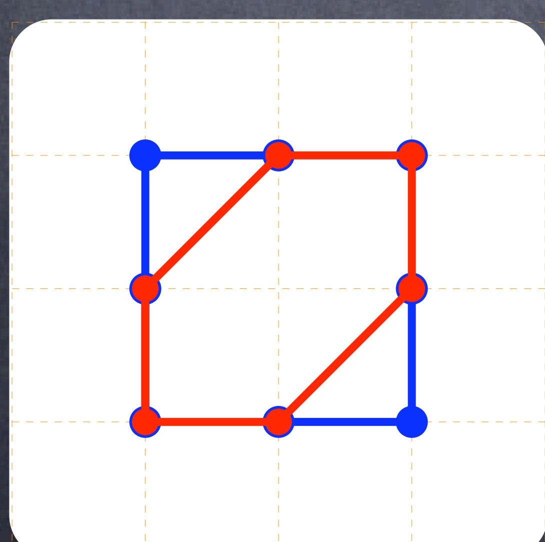

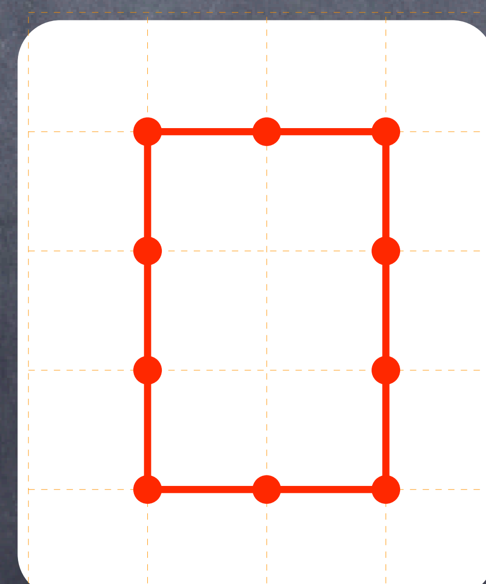

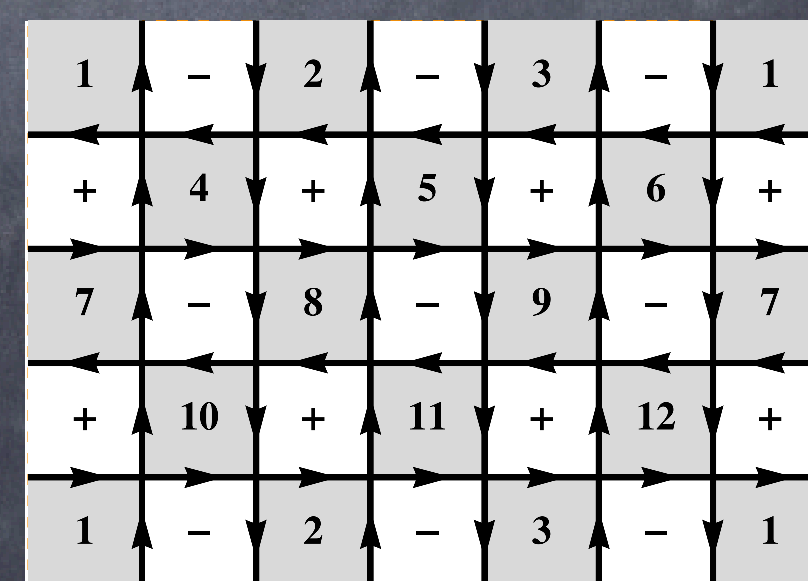

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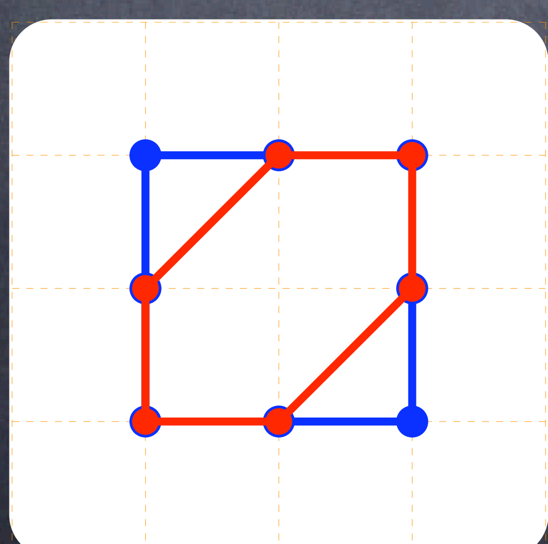

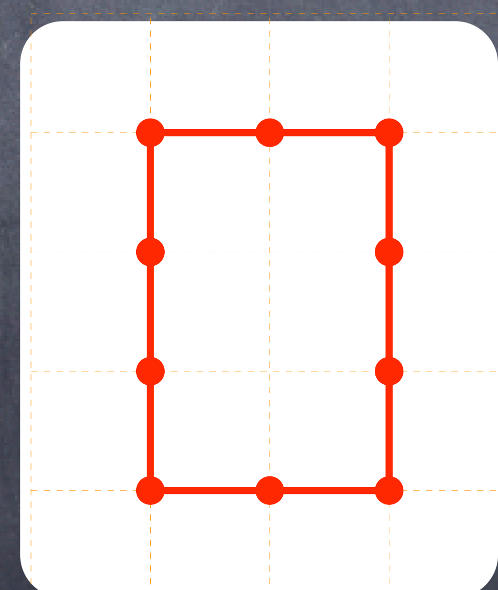

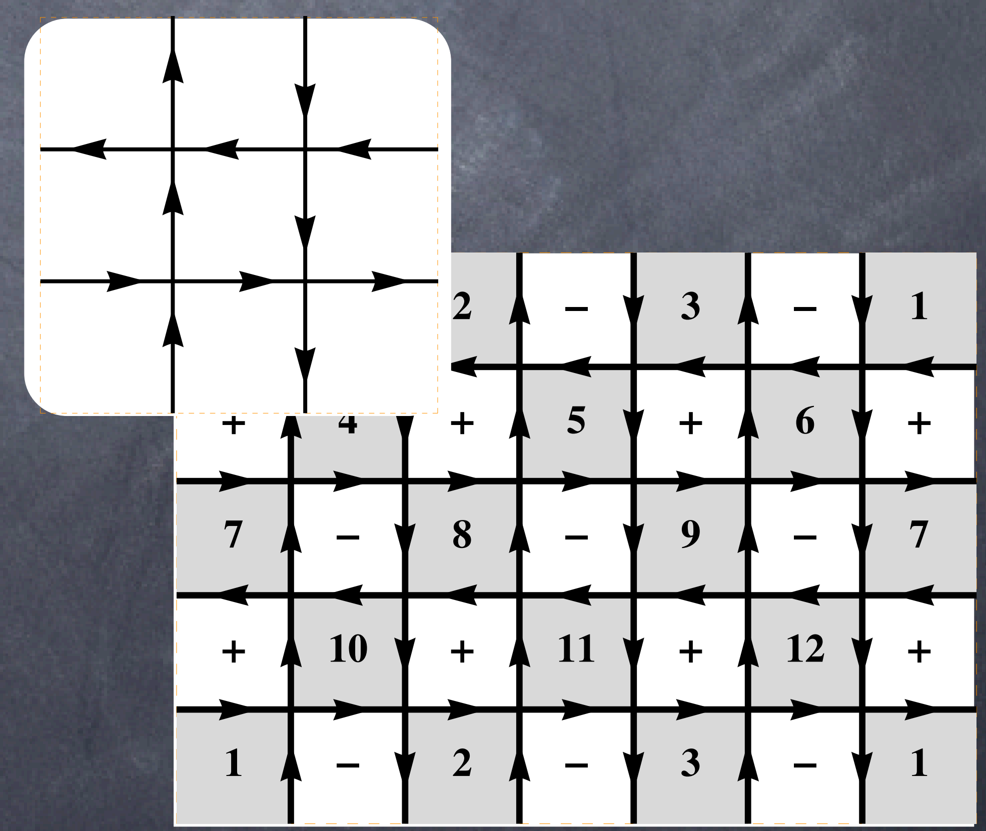

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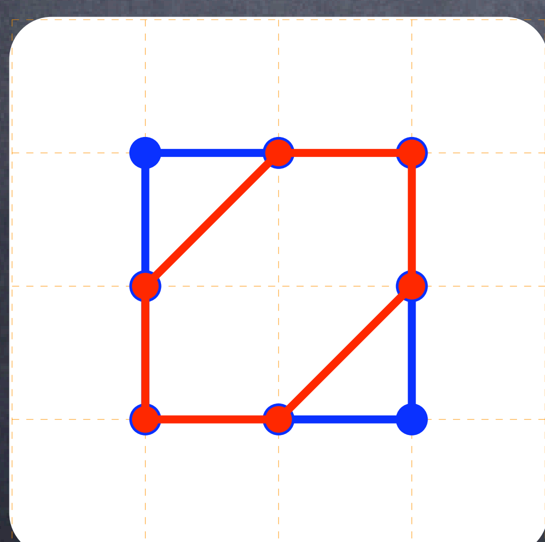
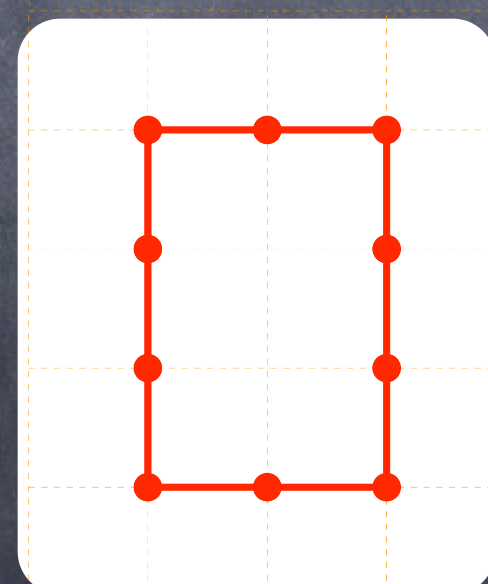
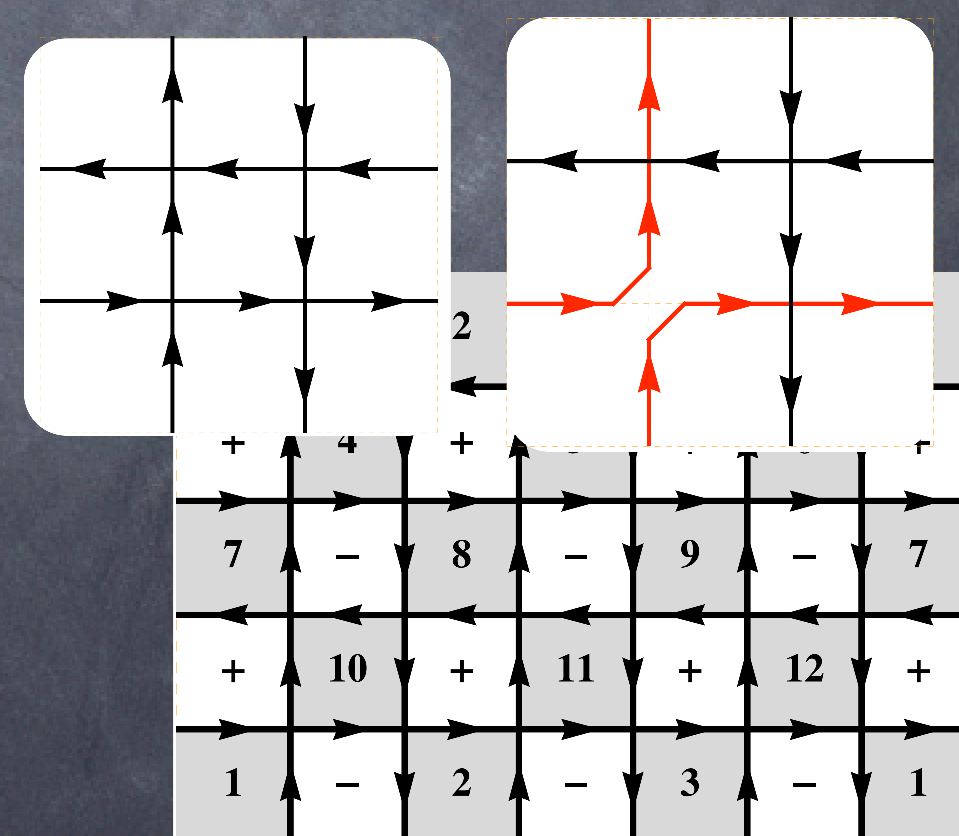
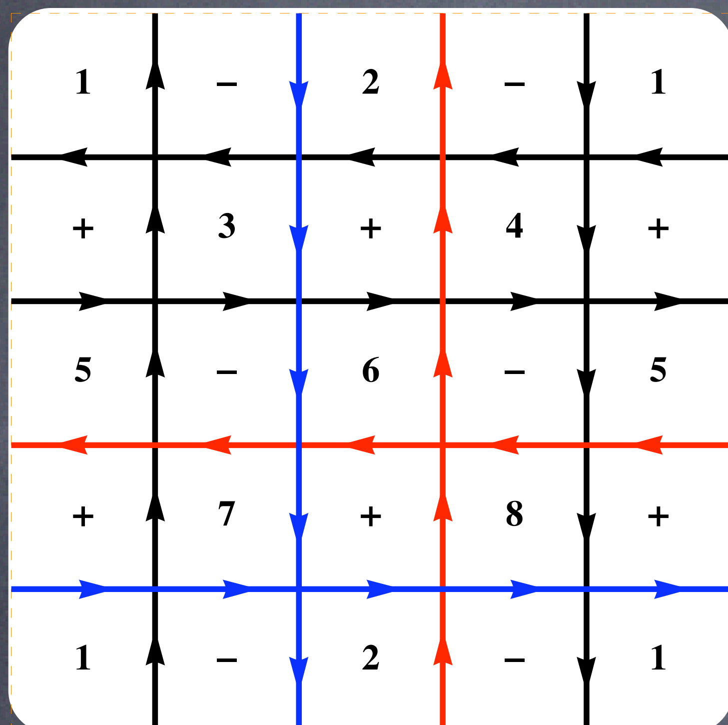
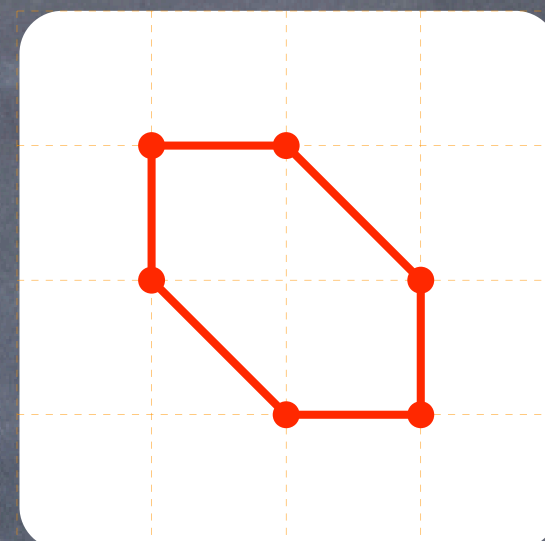


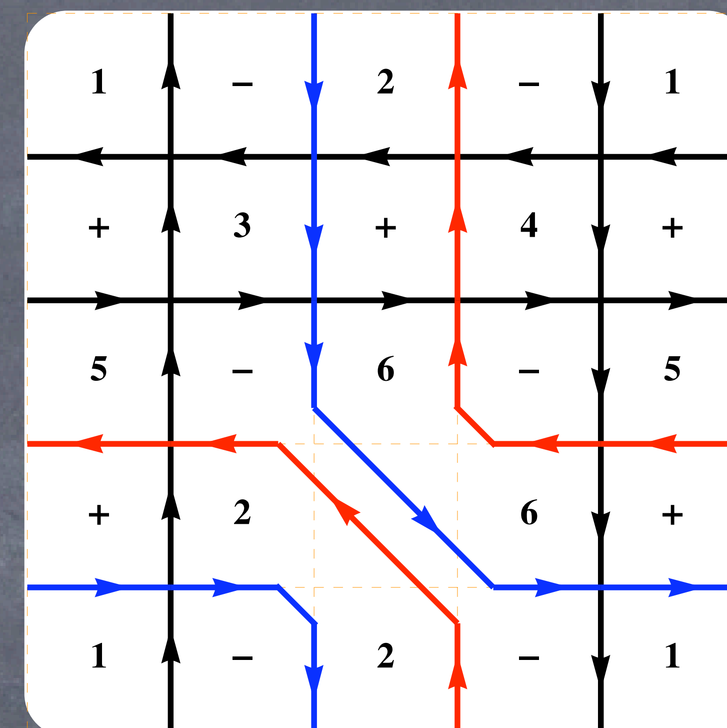




e.g. del Pezzo 3



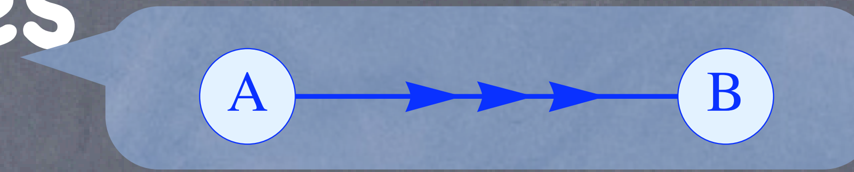




$$
\begin{aligned}
W_{dP_3} \quad &= \quad -X_{12}Y_{31}Z_{23} - X_{45}Y_{64}Z_{56} + X_{45}Y_{31}Z_{14}\rho_{53} + X_{12}Y_{25}Z_{56}\Phi_{61} \\
&\qquad + X_{36}Y_{64}Z_{23}\Psi_{42} - X_{36}Y_{25}Z_{14}\rho_{53}\Phi_{61}\Psi_{42}
\end{aligned}
$$

$$
= \begin{pmatrix} X_{45} \\ Y_{23} \\ Z_{25} \end{pmatrix} \begin{pmatrix} 0 & Z_{14}\rho_{53} & -Y_{64} \\ -Z_{14}\rho_{53}\Phi_{61}\Psi_{42} & 0 & X_{12}\Phi_{61} \\ Y_{64}\Psi_{42} & -X_{12} & 0 \end{pmatrix} \begin{pmatrix} X_{36} \\ Y_{31} \\ Z_{56} \end{pmatrix} .
$$

Restricting the # of families



- The dimers obtained with the operations of Gulotta are highly restricted (otherwise: inconsistent dimers)
- example: additional crossing (-> mass term)
- without add. crossings 3 (left), with add. crossings 4 but unique (right)

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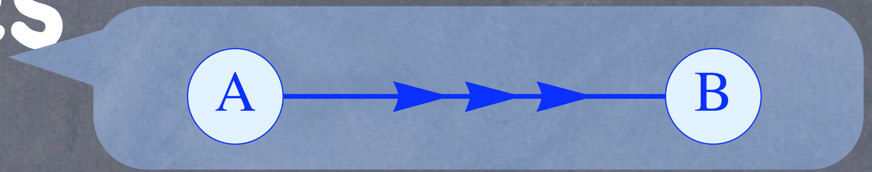

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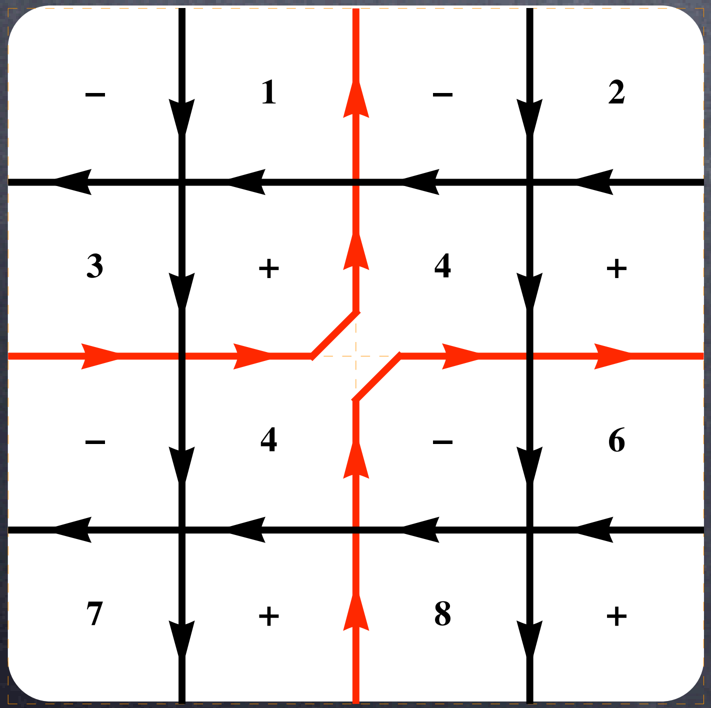
e.g.



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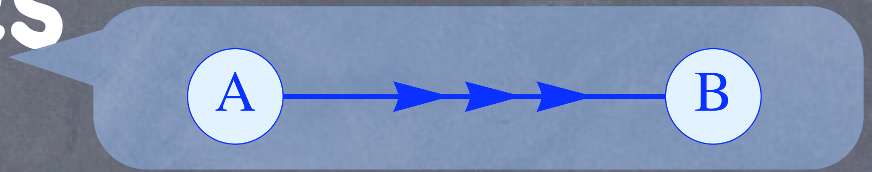

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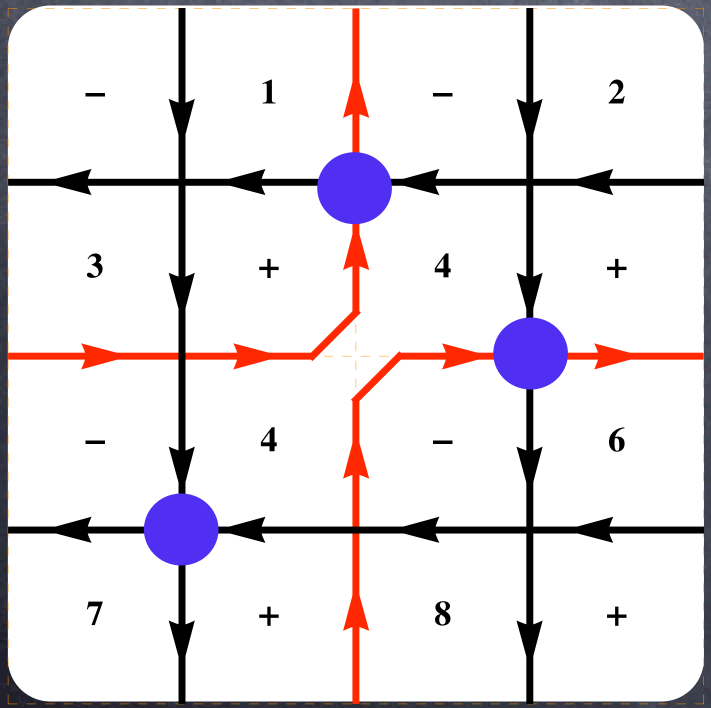
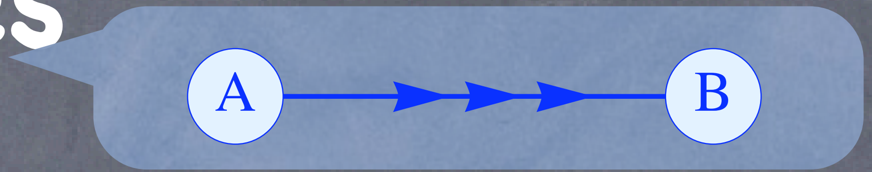
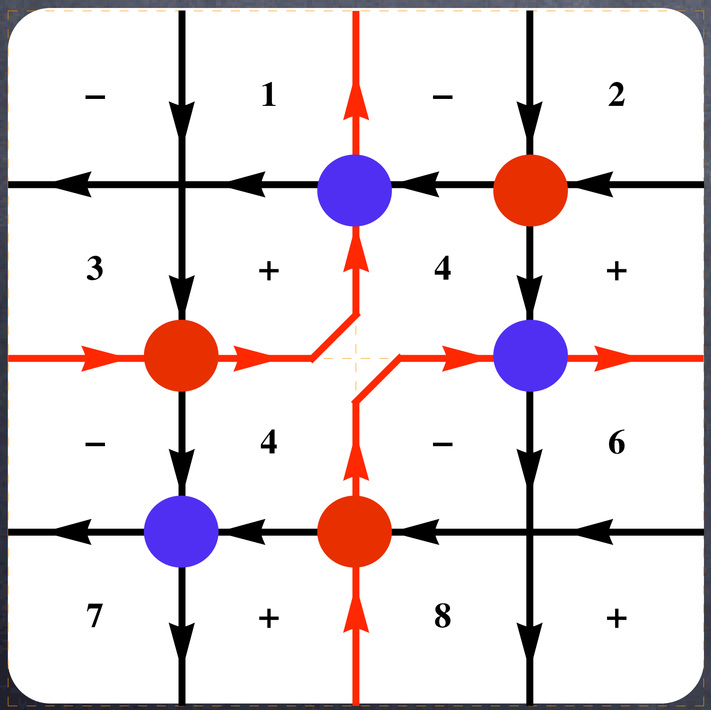


Restricting the # of families

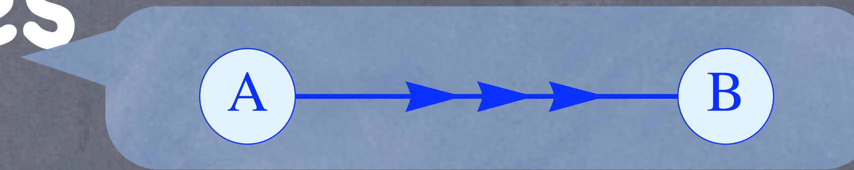


- The dimers obtained with the operations of Gulotta are highly restricted (otherwise: inconsistent dimers)
- example: additional crossing (-> mass term)
- without add. crossings 3 (left), with add. crossings 4 but unique (right)

e.g.



Restricting the # of families



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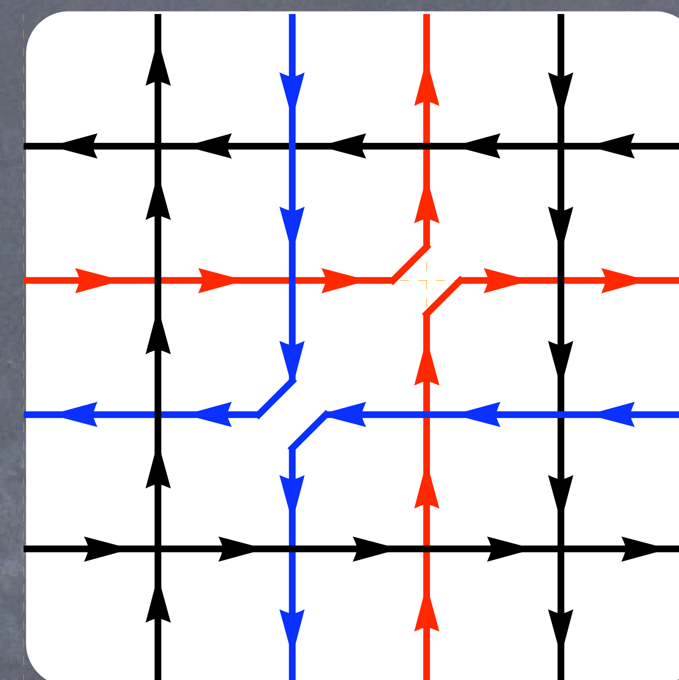
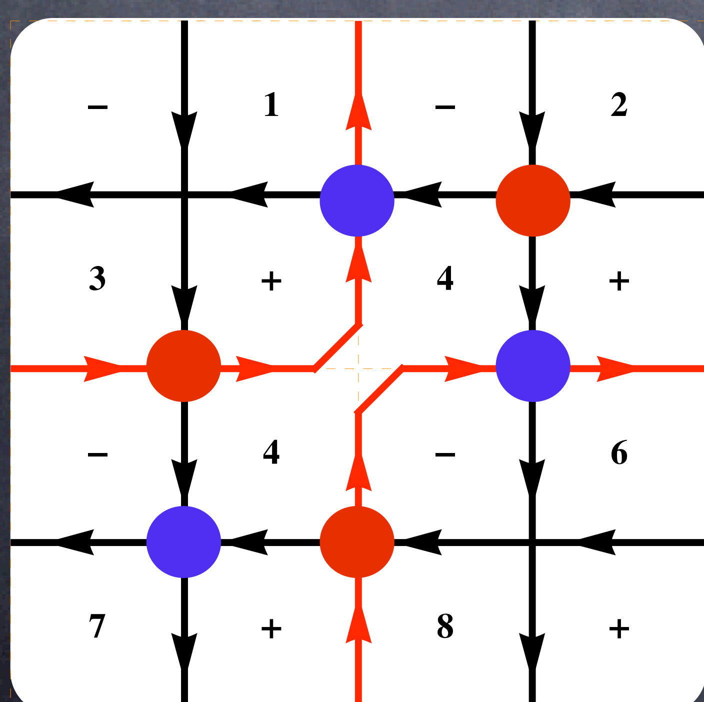
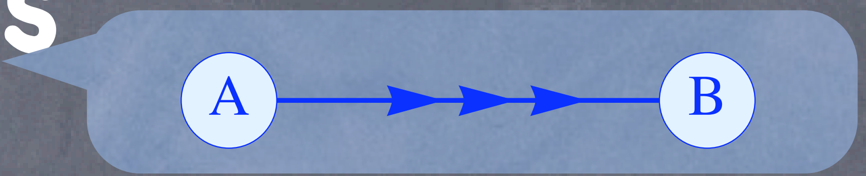
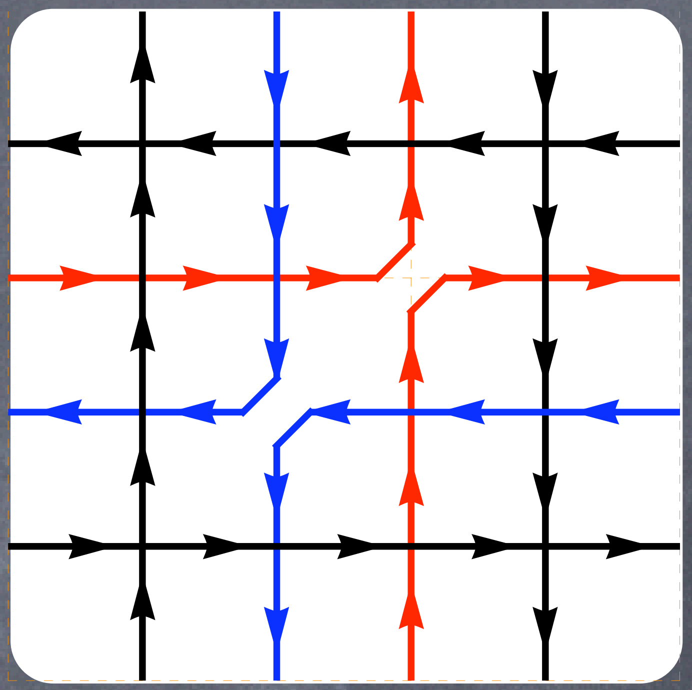


e.g.



Restricting the # of families





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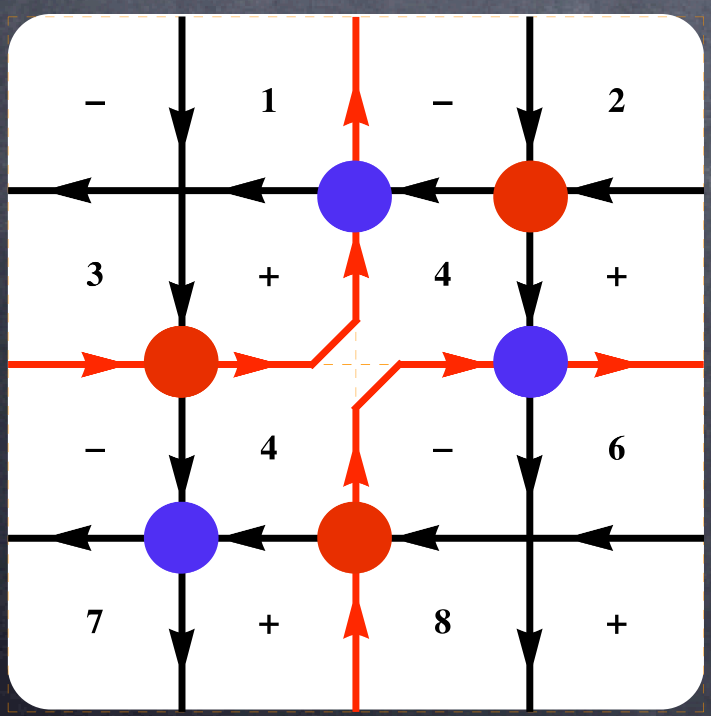


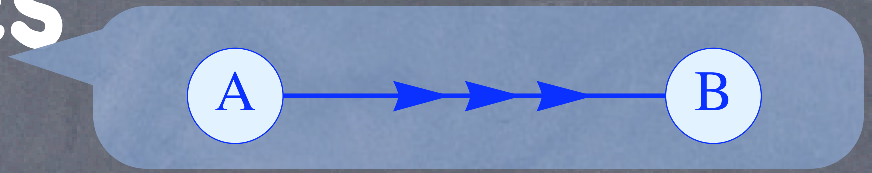
maximum of

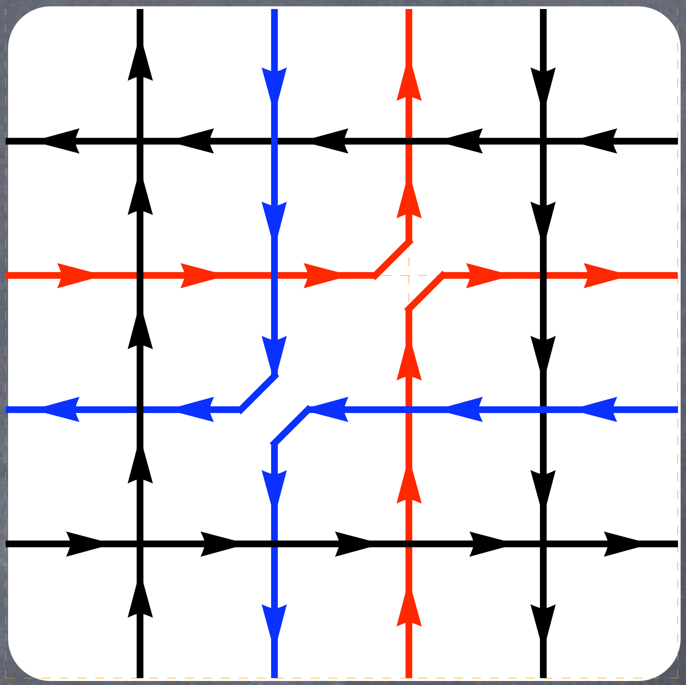
3 fields in/out for

any gauge group

-> 3 families

Restricting the # of families





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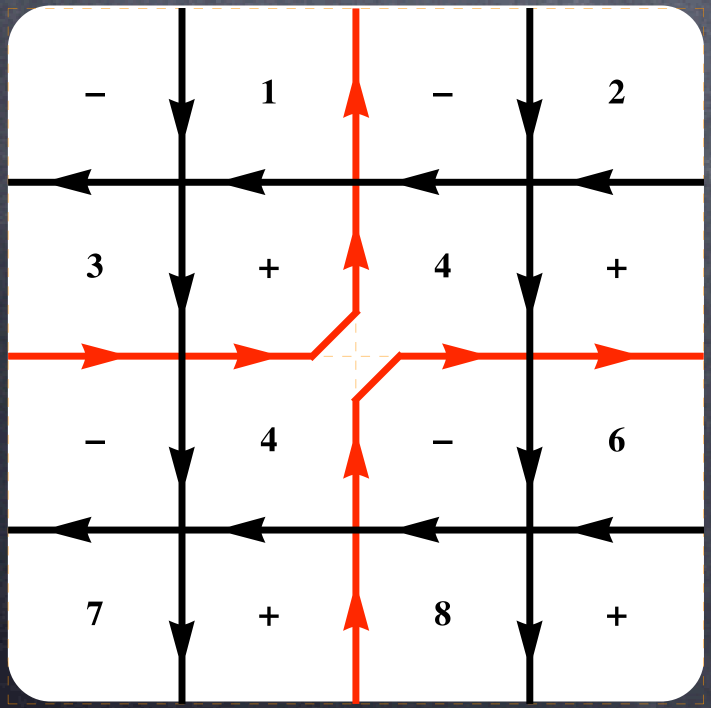
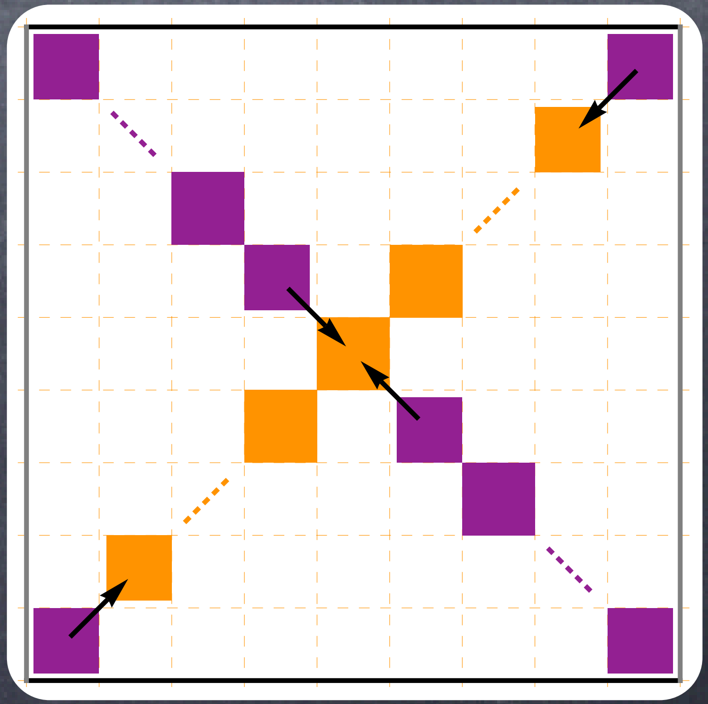
e.g.

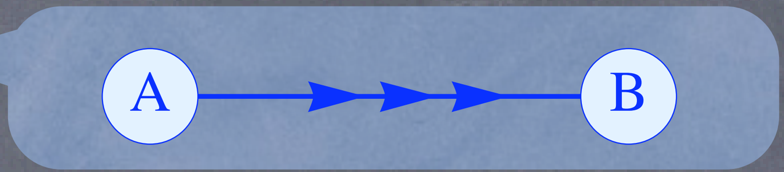


maximum of 4 fields (no add. branches) between 2 gauge groups

-> 4 families



Restricting the # of families



- The dimers obtained with the operations of Gulotta are highly restricted (otherwise: inconsistent dimers)
- example: additional crossing (-> mass term)
- without add. crossings 3 (left),
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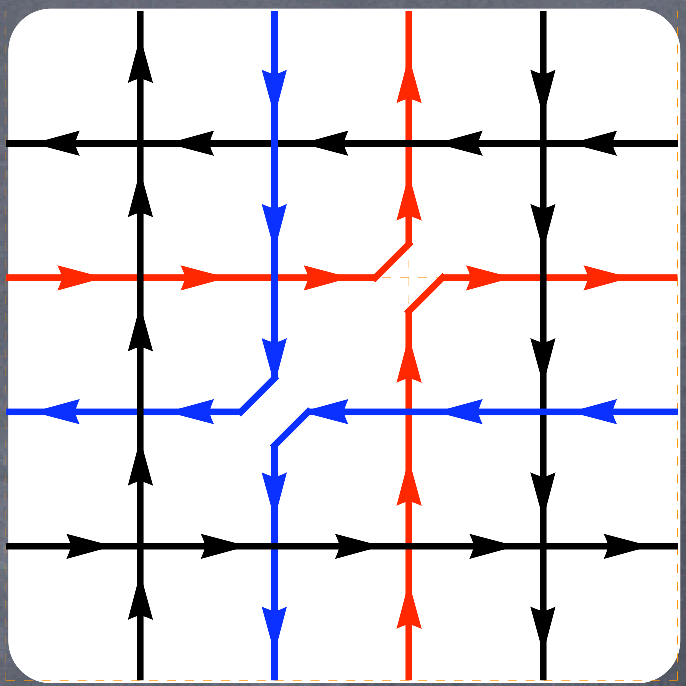
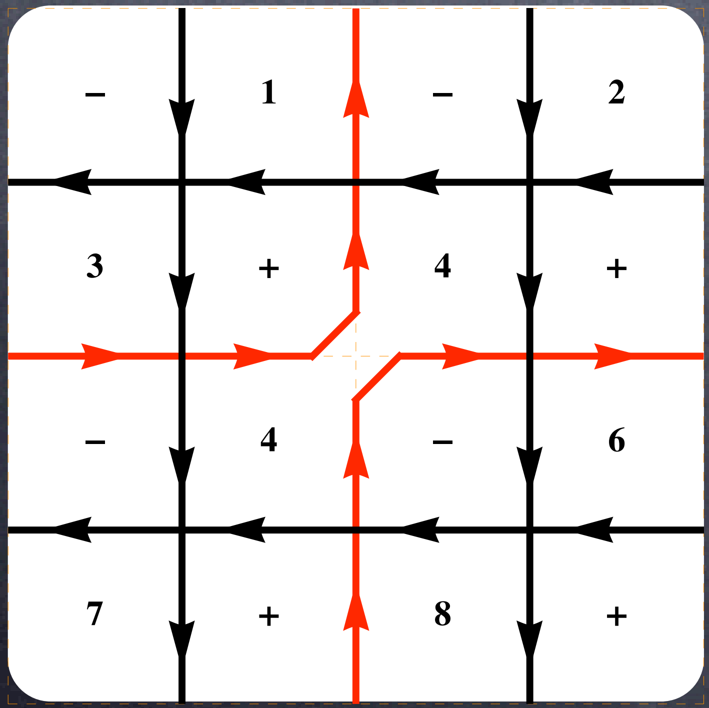
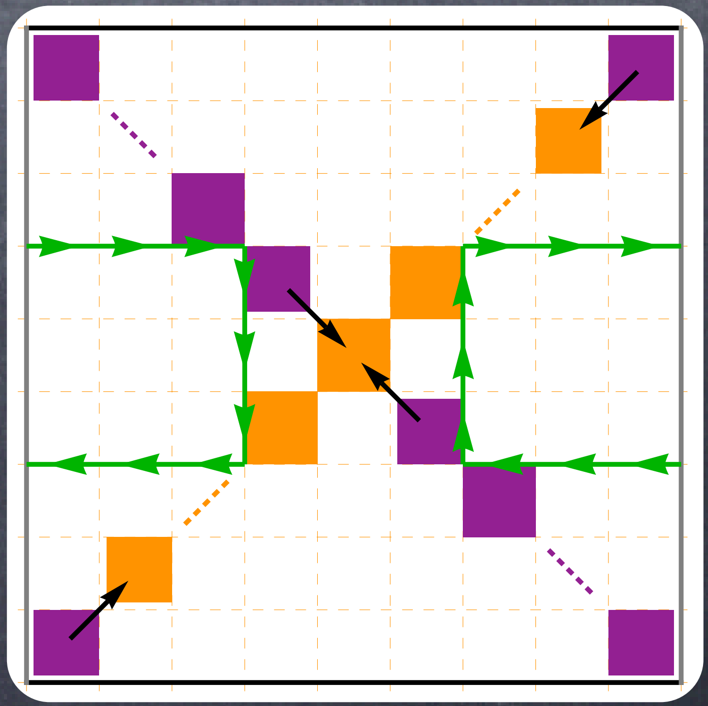


e.g.

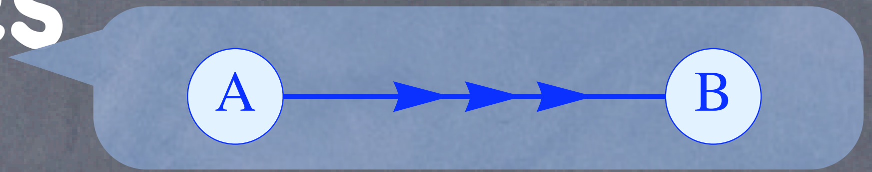


maximum of 4 fields (no add. branches) between 2 gauge groups

-> 4 families



Restricting the # of families



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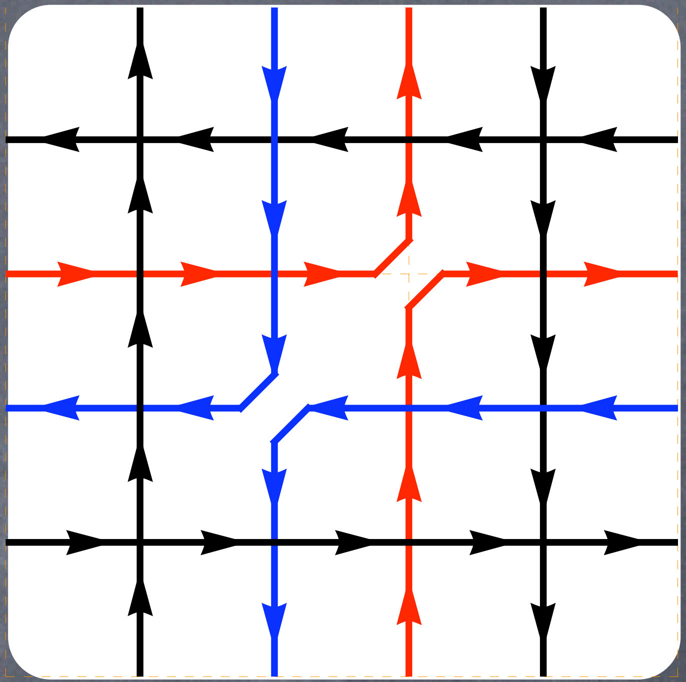
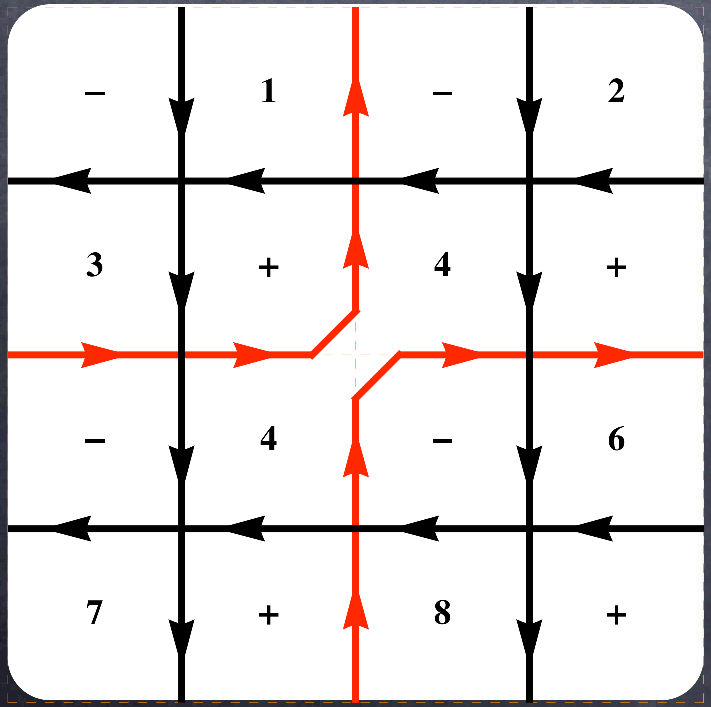
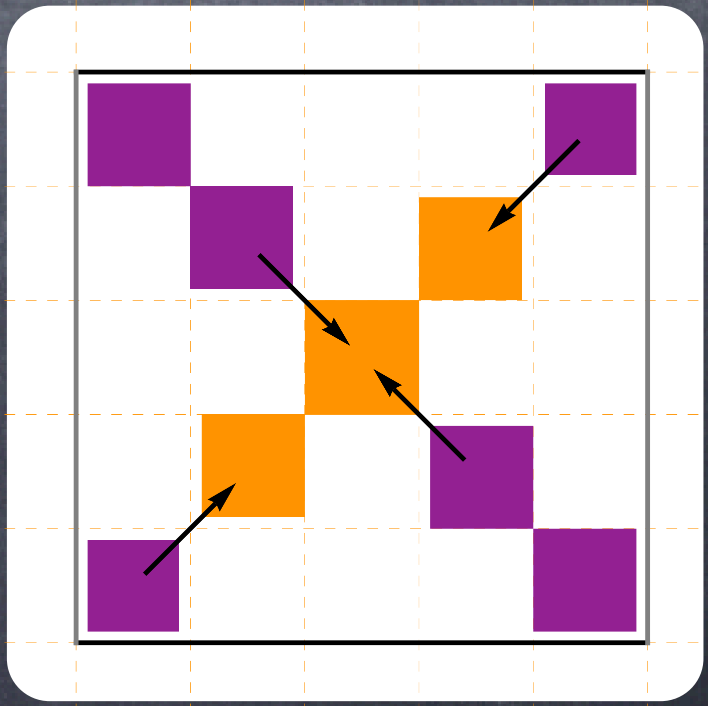


e.g.



maximum of 4 fields (no add. branches) between 2 gauge groups

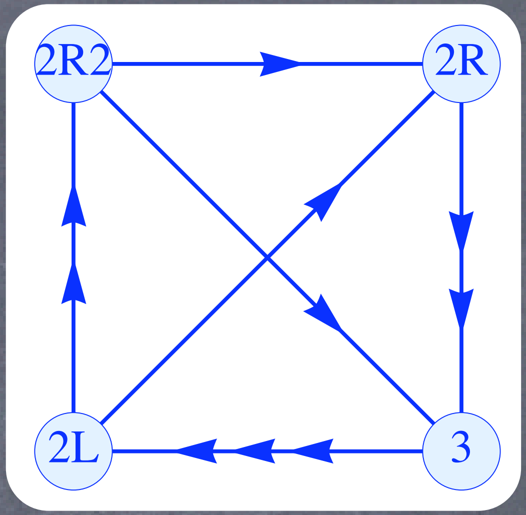
-> 4 families (unique F0)



Turning to model building

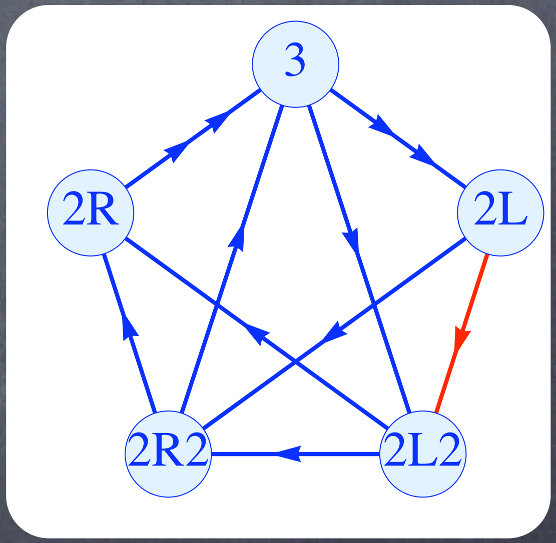
... we have a non-trivial superpotential (Yukawa structure) in these singularity models. What does this imply for model building?

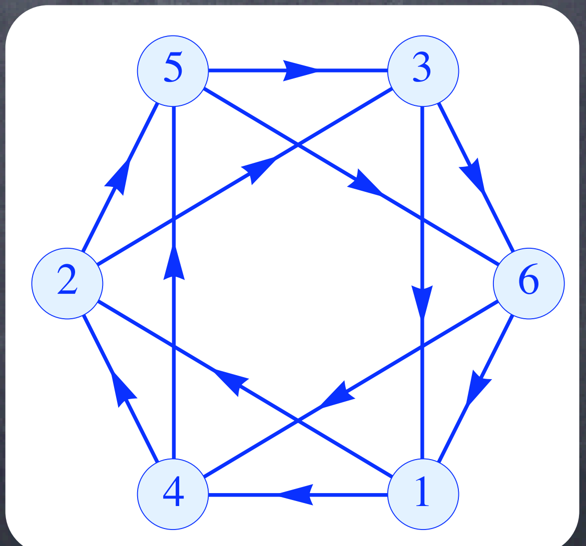
- focus on toric delPezzo singularities for concreteness
- Superpotential, matter content known via your favourite technique
- Kähler potential (hard): overall moduli weights and can be guaranteed to be flavour diagonal due to additional gauge groups (e.g. dP3)
- non-GUT (SU(5), SO(10)), e.g. left-right extensions of MSSM
- extended Higgs sector
- what can we say about fermion masses and flavour physics
- Disclaimer: no dynamical way of obtaining vevs yet...



dP1

dP2

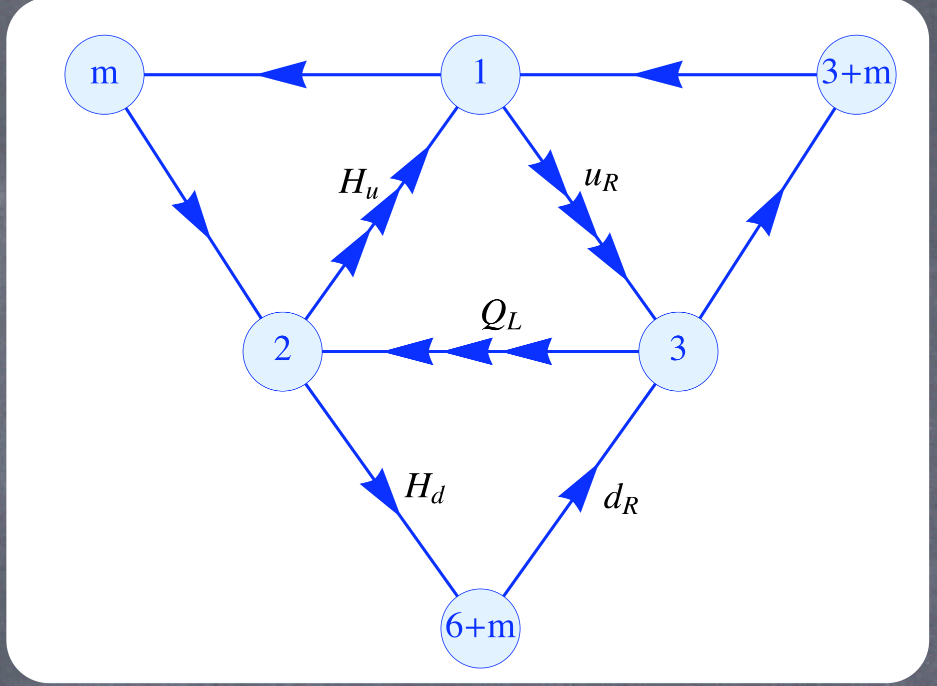




dP3

Mass Hierarchies

CMQ: 0810.5660



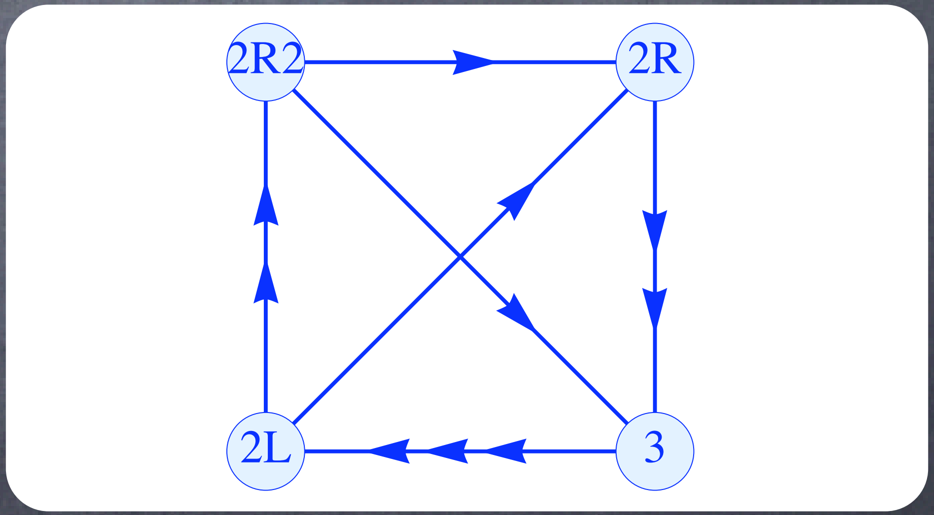
- Known result: dP0 (0, M, M);
  dP1 (0, m, M)

$$M = |X_{12}|^2 + |Y_{62}|^2 + |Z_{12}|^2$$

$$m = |Y_{62}|^2 + \frac{|\Phi_{61}|^2}{\Lambda^2}(|X_{12}|^2 + |Z_{12}|^2)$$

$$W = \epsilon_{ijk}\, Q_L^i H_u^j u_R^k$$

$$= \begin{pmatrix} Q_L^1 \\ Q_L^2 \\ Q_L^3 \end{pmatrix} \begin{pmatrix} 0 & Z_{12} & -Y_{12} \\ -Z_{12} & 0 & X_{12} \\ Y_{12} & -X_{12} & 0 \end{pmatrix} \begin{pmatrix} u_R^1 \\ u_R^2 \\ u_R^3 \end{pmatrix}.$$



- (0, m, M) generic to models at toric singularities

$$W = \begin{pmatrix} X_{23} \\ Y_{23} \\ Z_{23} \end{pmatrix} \begin{pmatrix} 0 & Z_{12} & -Y_{62} \\ -Z_{12}\frac{\Phi_{61}}{\Lambda} & 0 & X_{12}\frac{\Phi_{61}}{\Lambda} \\ Y_{62} & -X_{12} & 0 \end{pmatrix} \begin{pmatrix} X_{36} \\ Y_{31} \\ Z_{36} \end{pmatrix}.$$

Flavour mixing: CKM

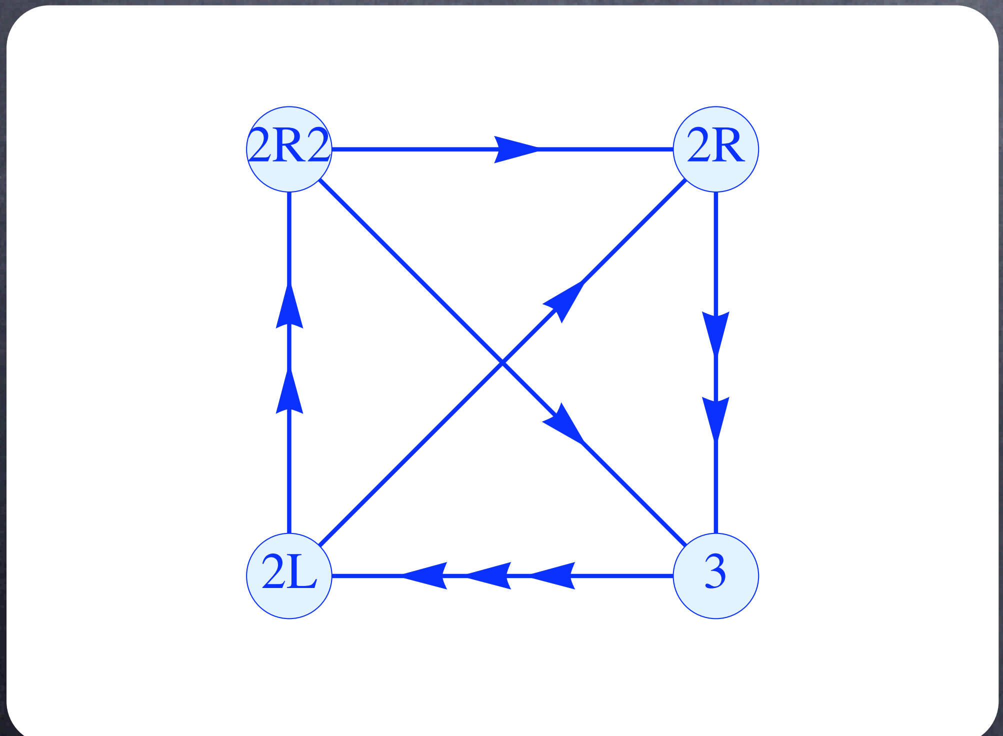
... we have a non-trivial superpotential (Yukawa structure) in these singularity models. What does this imply for model building?

Aim: construct models with the correct flavour mixing among quarks

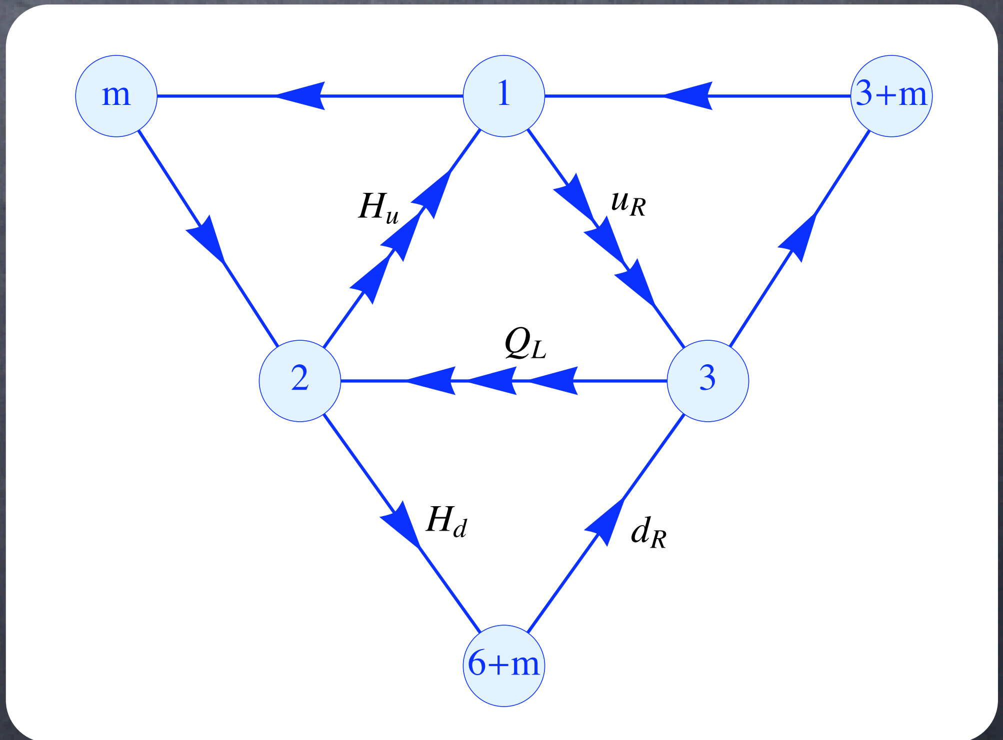
$$V_{\rm CKM} = \begin{pmatrix} 1 & \epsilon & \epsilon^3 \\ \epsilon & 1 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix}.$$

2 types of models:

a) up & down from D3D3 states

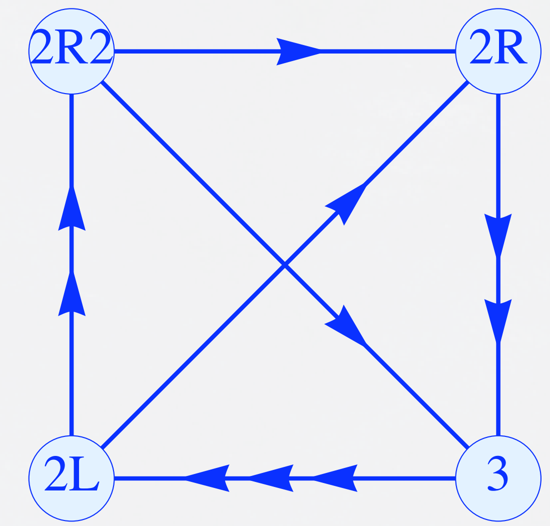


b) up from D3D3 states & down from D3D7 states



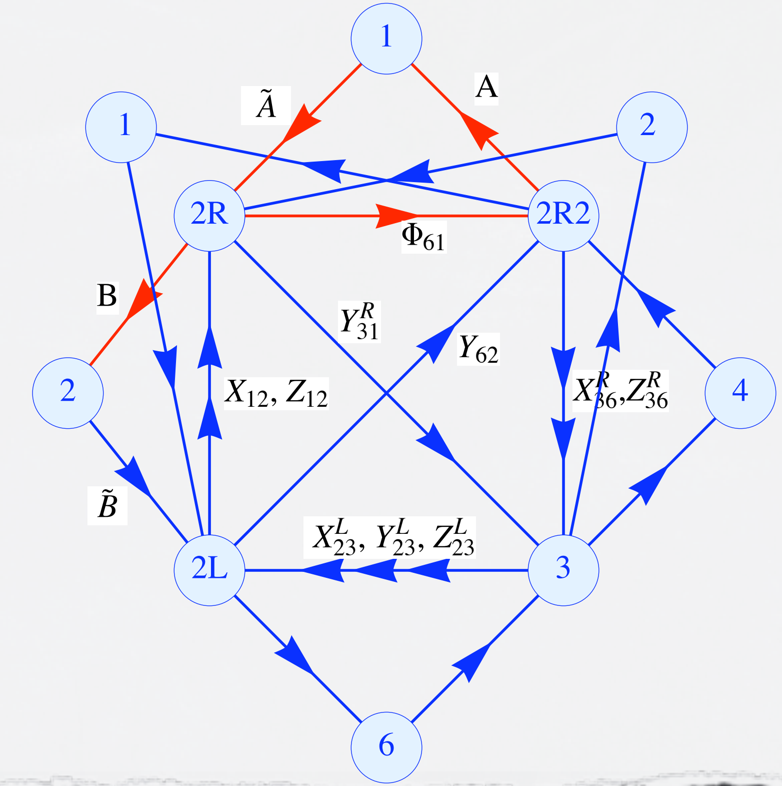
A left-right model with the right CKM

$$W = \begin{pmatrix} X_{23}^L \\ Y_{23}^L \\ Z_{23}^L \end{pmatrix} \begin{pmatrix} 0 & Z_{12} & -Y_{62} \\ -Z_{12}\frac{\Phi_{61}}{\Lambda} & 0 & X_{12}\frac{\Phi_{61}}{\Lambda} \\ Y_{62} & -X_{12} & 0 \end{pmatrix} \begin{pmatrix} X_{36} \\ Y_{31} \\ Z_{36} \end{pmatrix}.$$



A left-right model with the right CKM

$$W = \begin{pmatrix} X^L_{23} \\ Y^L_{23} \\ Z^L_{23} \end{pmatrix} \begin{pmatrix} 0 & Z_{12} & -Y_{62} \\ -Z_{12}\frac{\Phi_{61}}{\Lambda} & 0 & X_{12}\frac{\Phi_{61}}{\Lambda} \\ Y_{62} & -X_{12} & 0 \end{pmatrix} \begin{pmatrix} X_{36} \\ Y_{31} \\ Z_{36} \end{pmatrix}.$$



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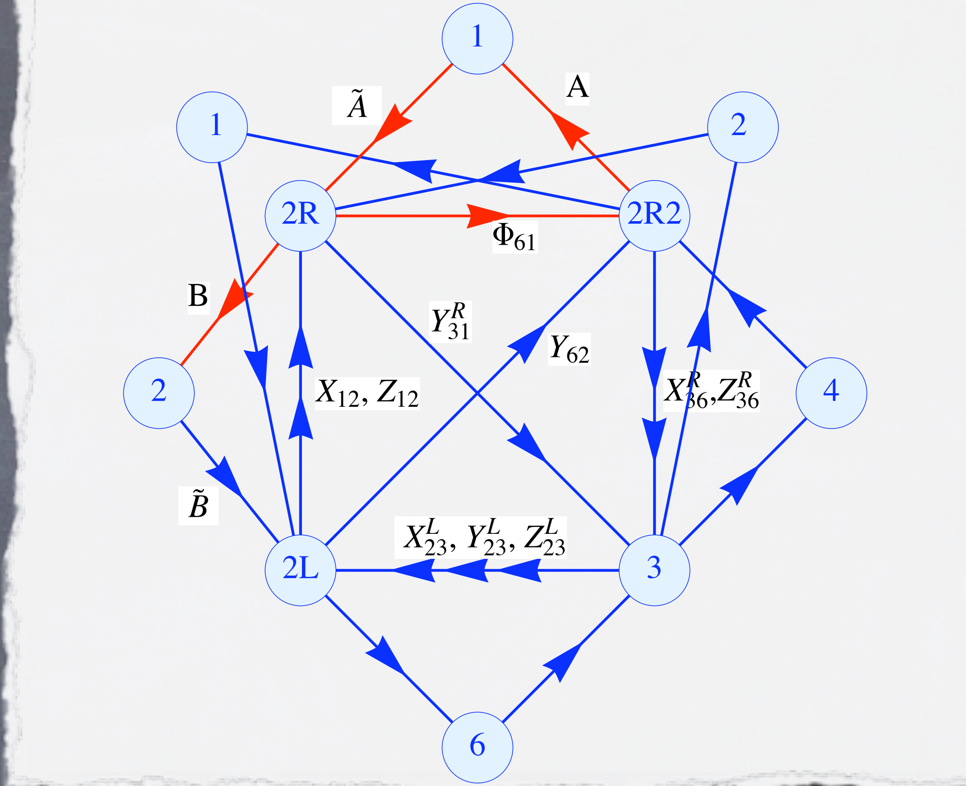
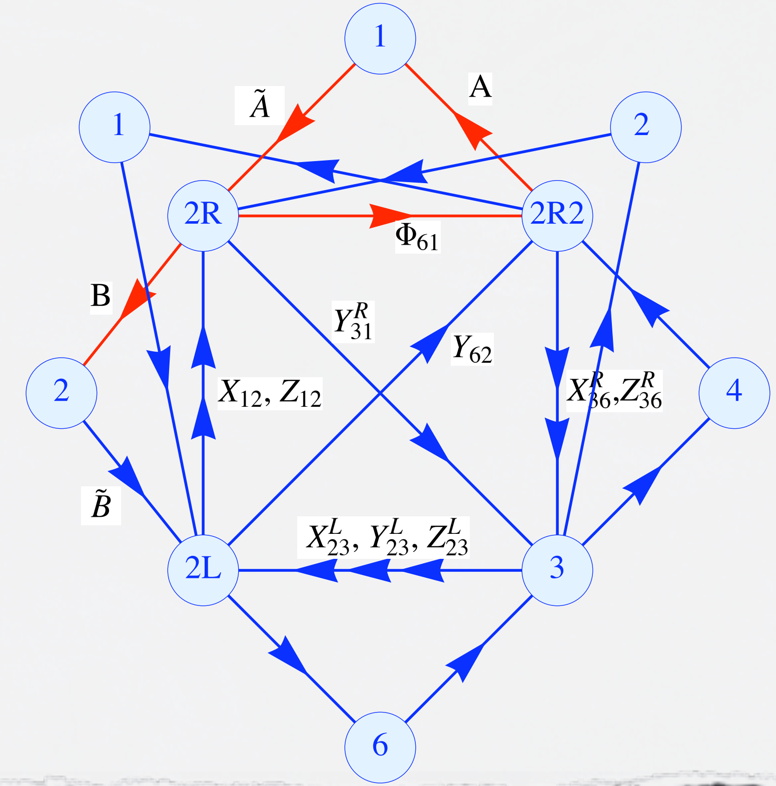


$$W = \begin{pmatrix} X^L_{23} \\ Y^L_{23} \\ Z^L_{23} \end{pmatrix} \begin{pmatrix} 0 & Z_{12} & -Y_{62} \\ -Z_{12}\frac{\Phi_{61}}{\Lambda} & 0 & X_{12}\frac{\Phi_{61}}{\Lambda} \\ Y_{62} & -X_{12} & 0 \end{pmatrix} \begin{pmatrix} X_{36} \\ Y_{31} \\ Z_{36} \end{pmatrix}.$$

After breaking of $U(2)_R$

$$W = \begin{pmatrix} X^L_{23} \\ Y^L_{23} \\ Z^L_{23} \end{pmatrix} \begin{pmatrix} 0 & Z^u_{12} & -Y^u_{62} \\ -Z^u_{12}\frac{\varphi}{\Lambda} & 0 & X^u_{12}\frac{\varphi}{\Lambda} \\ Y^u_{62} & -X^u_{12} & 0 \end{pmatrix} \begin{pmatrix} X^u_{36} \\ Y^u_{31} \\ Z^u_{36} \end{pmatrix} + \begin{pmatrix} X^d_{23} \\ Y^d_{23} \\ Z^d_{23} \end{pmatrix} \begin{pmatrix} 0 & Z^d_{12} & -Y^d_{62} \\ -Z^d_{12}\frac{v_d}{\Lambda} & 0 & 0 \\ Y^d_{62} & 0 & 0 \end{pmatrix} \begin{pmatrix} X^d_{36} \\ Y^d_{31} \\ Z^d_{36} \end{pmatrix}$$

A left-right model with the right CKM



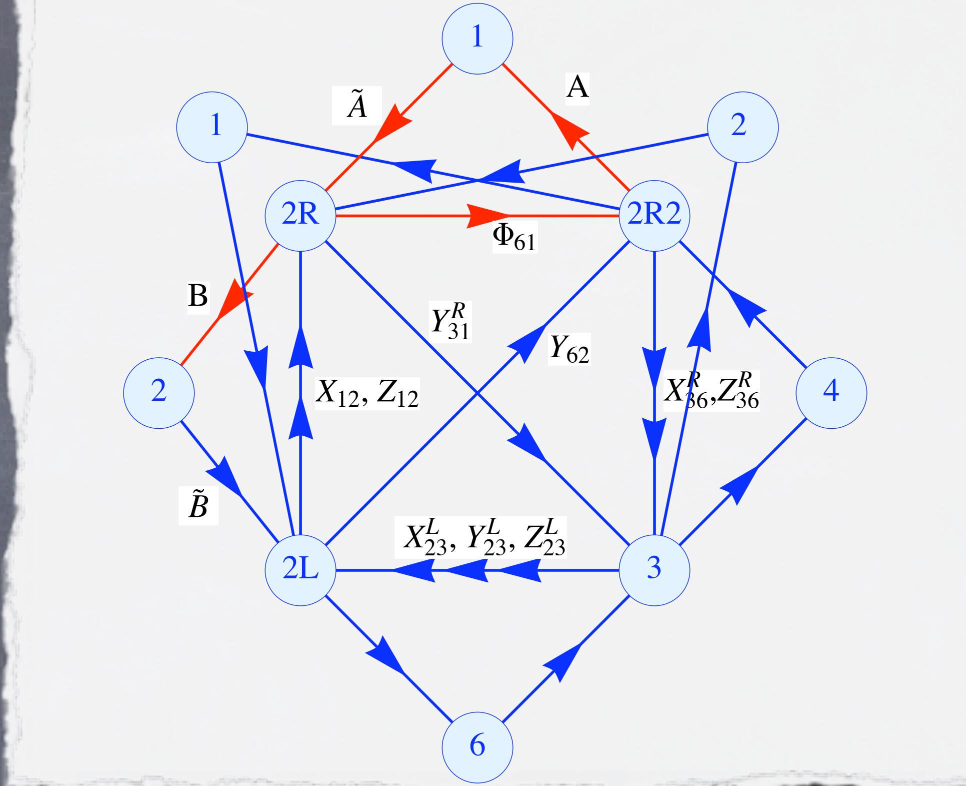
$$W = \begin{pmatrix} X_{23}^L \\ Y_{23}^L \\ Z_{23}^L \end{pmatrix} \begin{pmatrix} 0 & Z_{12} & -Y_{62} \\ -Z_{12}\frac{\Phi_{61}}{\Lambda} & 0 & X_{12}\frac{\Phi_{61}}{\Lambda} \\ Y_{62} & -X_{12} & 0 \end{pmatrix} \begin{pmatrix} X_{36} \\ Y_{31} \\ Z_{36} \end{pmatrix}.$$

After breaking of $U(2)_R$

$$W = \begin{pmatrix} X_{23}^L \\ Y_{23}^L \\ Z_{23}^L \end{pmatrix} \begin{pmatrix} 0 & Z_{12}^u & -Y_{62}^u \\ -Z_{12}^u\frac{\varphi}{\Lambda} & 0 & X_{12}^u\frac{\varphi}{\Lambda} \\ Y_{62}^u & -X_{12}^u & 0 \end{pmatrix} \begin{pmatrix} X_{36}^u \\ Y_{31}^u \\ Z_{36}^u \end{pmatrix} + \begin{pmatrix} X_{23}^d \\ Y_{23}^d \\ Z_{23}^d \end{pmatrix} \begin{pmatrix} 0 & Z_{12}^d & -Y_{62}^d \\ -Z_{12}^d\frac{v_d}{\Lambda} & 0 & 0 \\ Y_{62}^d & 0 & 0 \end{pmatrix} \begin{pmatrix} X_{36}^d \\ Y_{31}^d \\ Z_{36}^d \end{pmatrix}$$

The CKM is given in terms of ratios of Higgs vevs. dP1:

A left-right model with the right CKM



$$W = \begin{pmatrix} X^L_{23} \\ Y^L_{23} \\ Z^L_{23} \end{pmatrix} \begin{pmatrix} 0 & Z_{12} & -Y_{62} \\ -Z_{12}\frac{\Phi_{61}}{\Lambda} & 0 & X_{12}\frac{\Phi_{61}}{\Lambda} \\ Y_{62} & -X_{12} & 0 \end{pmatrix} \begin{pmatrix} X_{36} \\ Y_{31} \\ Z_{36} \end{pmatrix}.$$

After breaking of $U(2)_R$

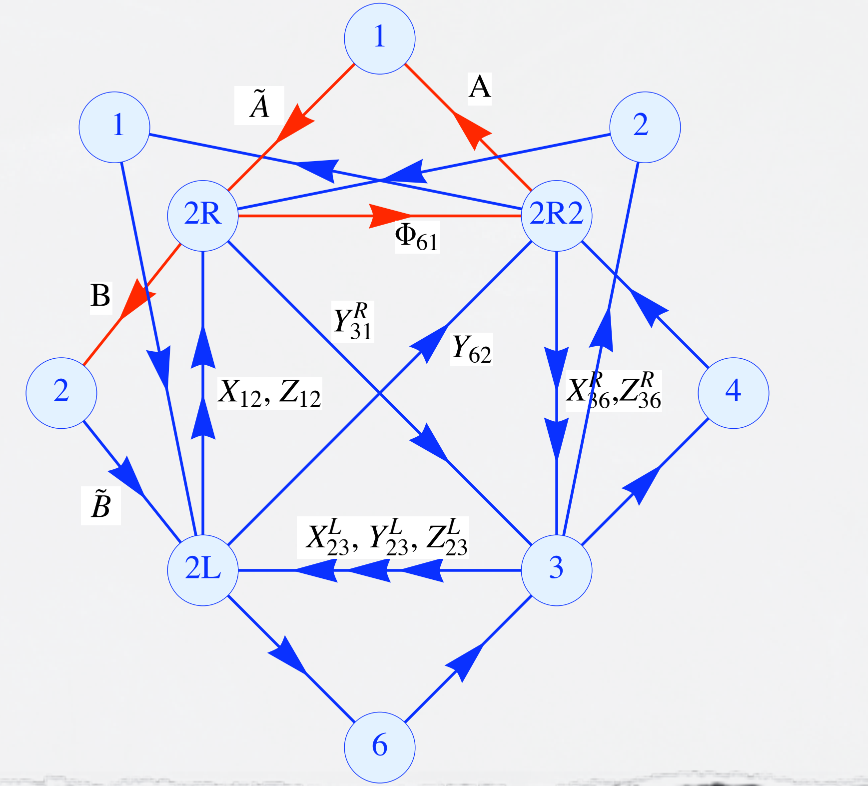
$$W = \begin{pmatrix} X^L_{23} \\ Y^L_{23} \\ Z^L_{23} \end{pmatrix} \begin{pmatrix} 0 & Z^u_{12} & -Y^u_{62} \\ -Z^u_{12}\frac{\varphi}{\Lambda} & 0 & X^u_{12}\frac{\varphi}{\Lambda} \\ Y^u_{62} & -X^u_{12} & 0 \end{pmatrix} \begin{pmatrix} X^u_{36} \\ Y^u_{31} \\ Z^u_{36} \end{pmatrix} + \begin{pmatrix} X^d_{23} \\ Y^d_{23} \\ Z^d_{23} \end{pmatrix} \begin{pmatrix} 0 & Z^d_{12} & -Y^d_{62} \\ -Z^d_{12}\frac{v_d}{\Lambda} & 0 & 0 \\ Y^d_{62} & 0 & 0 \end{pmatrix} \begin{pmatrix} X^d_{36} \\ Y^d_{31} \\ Z^d_{36} \end{pmatrix}$$

The CKM is given in terms of ratios of Higgs vevs. dP1:

$$V_{\rm CKM} = V_u^\dagger V_d = \begin{pmatrix} \frac{X^u_{12}}{Z^u_{12}} & \frac{\Lambda Y^u_{12}}{Z^u_{12}\Phi_{61}} & 1 \\ \frac{\Lambda Y^u_{12} X^u_{12}}{\left(Z^u_{12}\right)^2 \Phi_{61}} & 1 & -\frac{\Lambda Y^u_{12}}{Z^u_{12}\Phi_{61}} \\ 1 & 0 & -\frac{X^u_{12}}{Z^u_{12}} \end{pmatrix} \cdot \begin{pmatrix} 0 & 0 & 1 \\ a\frac{\Lambda Y^d_{12}}{Z^d_{12}\Phi_{61}} & a & 0 \\ a & -a\frac{\Lambda Y^d_{12}}{Z^d_{12}\Phi_{61}} & 0 \end{pmatrix} . = \begin{pmatrix} 1 & \epsilon & \epsilon^3 \\ \epsilon & 1 & \epsilon^2 \\ \epsilon^3 & \epsilon^4 & 1 \end{pmatrix}.$$

normalisation

A left-right model with the right CKM



$$W = \begin{pmatrix} X_{23}^L \\ Y_{23}^L \\ Z_{23}^L \end{pmatrix} \begin{pmatrix} 0 & Z_{12} & -Y_{62} \\ -Z_{12}\frac{\Phi_{61}}{\Lambda} & 0 & X_{12}\frac{\Phi_{61}}{\Lambda} \\ Y_{62} & -X_{12} & 0 \end{pmatrix} \begin{pmatrix} X_{36} \\ Y_{31} \\ Z_{36} \end{pmatrix}.$$

After breaking of $U(2)_R$

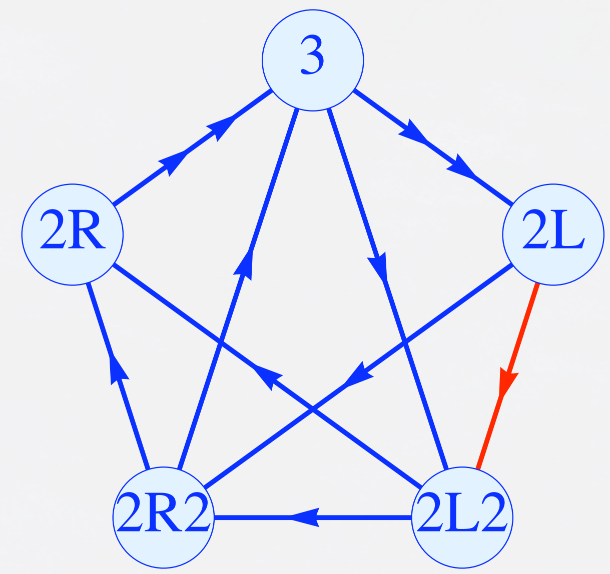
$$W = \begin{pmatrix} X_{23}^L \\ Y_{23}^L \\ Z_{23}^L \end{pmatrix} \begin{pmatrix} 0 & Z_{12}^u & -Y_{62}^u \\ -Z_{12}^u\frac{\varphi}{\Lambda} & 0 & X_{12}^u\frac{\varphi}{\Lambda} \\ Y_{62}^u & -X_{12}^u & 0 \end{pmatrix} \begin{pmatrix} X_{36}^u \\ Y_{31}^u \\ Z_{36}^u \end{pmatrix} + \begin{pmatrix} X_{23}^d \\ Y_{23}^d \\ Z_{23}^d \end{pmatrix} \begin{pmatrix} 0 & Z_{12}^d & -Y_{62}^d \\ -Z_{12}^d\frac{v_d}{\Lambda} & 0 & 0 \\ Y_{62}^d & 0 & 0 \end{pmatrix} \begin{pmatrix} X_{36}^d \\ Y_{31}^d \\ Z_{36}^d \end{pmatrix}$$

The CKM is given in terms of ratios of Higgs vevs. dP1:

$$V_{\mathrm{CKM}} = V_u^\dagger V_d = \begin{pmatrix} \frac{X_{12}^u}{Z_{12}^u} & \frac{\Lambda Y_{12}^u}{Z_{12}^u \Phi_{61}} & 1 \\ \frac{\Lambda Y_{12}^u X_{12}^u}{\left(Z_{12}^u\right)^2 \Phi_{61}} & 1 & -\frac{\Lambda Y_{12}^u}{Z_{12}^u \Phi_{61}} \\ 1 & 0 & -\frac{X_{12}^u}{Z_{12}^u} \end{pmatrix} \cdot \begin{pmatrix} 0 & 0 & 1 \\ a\frac{\Lambda Y_{12}^d}{Z_{12}^d \Phi_{61}} & a & 0 \\ a & -a\frac{\Lambda Y_{12}^d}{Z_{12}^d \Phi_{61}} & 0 \end{pmatrix} . = \begin{pmatrix} 1 & \epsilon & \epsilon^3 \\ \epsilon & 1 & \epsilon^2 \\ \epsilon^3 & \epsilon^4 & 1 \end{pmatrix}.$$

In dP1 we get almost the right CKM.

A left-right model with the right CKM



$$W = \begin{pmatrix} X^L_{43} \\ Y^L_{23} \\ Z^L_{23} \end{pmatrix} \begin{pmatrix} 0 & Z_{14} & -Y_{64} \\ -Z_{14}\frac{\Phi_{61}\Psi_{42}}{\Lambda^2} & 0 & X_{12}\frac{\Phi_{61}}{\Lambda} \\ Y_{64}\frac{\Psi_{42}}{\Lambda} & -X_{12} & 0 \end{pmatrix} \begin{pmatrix} X_{36} \\ Y_{31} \\ Z_{36} \end{pmatrix}.$$

After breaking of $U(2)_R$

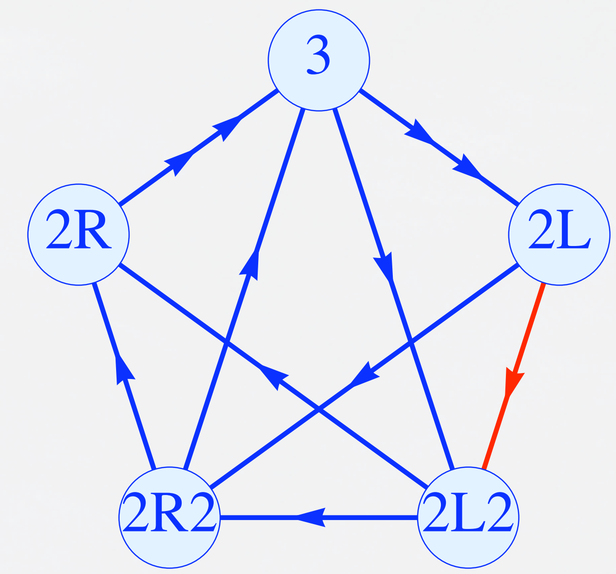
$$W = \begin{pmatrix} X^L_{23} \\ Y^L_{23} \\ Z^L_{23} \end{pmatrix} \begin{pmatrix} 0 & Z^u_{12} & -Y^u_{62} \\ -Z^u_{12}\frac{\varphi}{\Lambda} & 0 & X^u_{12}\frac{\varphi}{\Lambda} \\ Y^u_{62} & -X^u_{12} & 0 \end{pmatrix} \begin{pmatrix} X^u_{36} \\ Y^u_{31} \\ Z^u_{36} \end{pmatrix} + \begin{pmatrix} X^d_{23} \\ Y^d_{23} \\ Z^d_{23} \end{pmatrix} \begin{pmatrix} 0 & Z^d_{12} & -Y^d_{62} \\ -Z^d_{12}\frac{v_d}{\Lambda} & 0 & 0 \\ Y^d_{62} & 0 & 0 \end{pmatrix} \begin{pmatrix} X^d_{36} \\ Y^d_{31} \\ Z^d_{36} \end{pmatrix}$$

The CKM is given in terms of ratios of Higgs vevs.

$$V_{\mathrm{CKM}} = V_u^\dagger V_d$$

In dP1 we get almost the right CKM.

A left-right model with the right CKM



$$W = \begin{pmatrix} X^L_{43} \\ Y^L_{23} \\ Z^L_{23} \end{pmatrix} \begin{pmatrix} 0 & Z_{14} & -Y_{64} \\ -Z_{14}\frac{\Phi_{61}\Psi_{42}}{\Lambda^2} & 0 & X_{12}\frac{\Phi_{61}}{\Lambda} \\ Y_{64}\frac{\Psi_{42}}{\Lambda} & -X_{12} & 0 \end{pmatrix} \begin{pmatrix} X_{36} \\ Y_{31} \\ Z_{36} \end{pmatrix}.$$

After breaking of $U(2)_R$

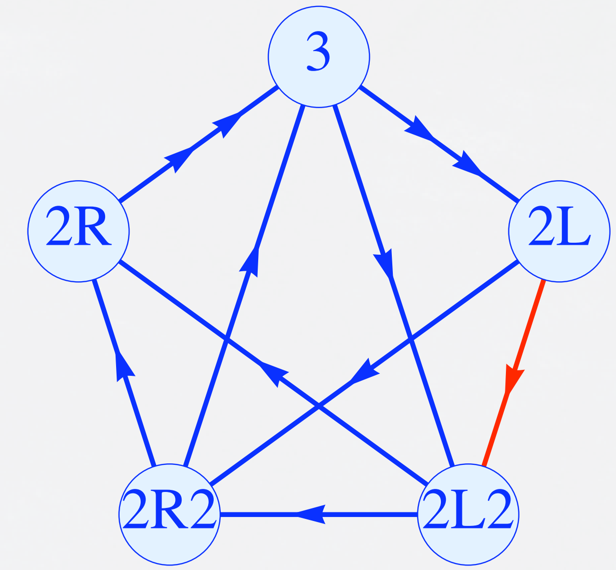
$$W = \begin{pmatrix} X^L_{23} \\ Y^L_{23} \\ Z^L_{23} \end{pmatrix} \begin{pmatrix} 0 & Z^u_{12} & -Y^u_{62} \\ -Z^u_{12}\frac{\varphi}{\Lambda} & 0 & X^u_{12}\frac{\varphi}{\Lambda} \\ Y^u_{62} & -X^u_{12} & 0 \end{pmatrix} \begin{pmatrix} X^u_{36} \\ Y^u_{31} \\ Z^u_{36} \end{pmatrix} + \begin{pmatrix} X^d_{23} \\ Y^d_{23} \\ Z^d_{23} \end{pmatrix} \begin{pmatrix} 0 & Z^d_{12} & -Y^d_{62} \\ -Z^d_{12}\frac{v_d}{\Lambda} & 0 & 0 \\ Y^d_{62} & 0 & 0 \end{pmatrix} \begin{pmatrix} X^d_{36} \\ Y^d_{31} \\ Z^d_{36} \end{pmatrix}$$

The CKM is given in terms of ratios of Higgs vevs. dP2:

$$V_{\rm CKM} = V_u^\dagger V_d \;\; = \begin{pmatrix} \frac{X^u_{12}\Phi_{61}}{Y^u_{64}\Lambda} & \frac{X^u_{12}Z^u_{14}}{\left(Y^u_{64}\right)^2} & 1 \\ 1 & -\frac{Z^u_{14}\Phi_{61}}{\Lambda Y^u_{64}} & -\frac{X^u_{12}\Phi_{61}}{\Lambda Y^u_{64}} \\ \frac{Z^u_{14}\Phi_{61}}{Y^u_{64}\Lambda} & 1 & -\frac{X^u_{12}Z^u_{14}}{\left(Y^u_{64}\right)^2} \end{pmatrix} \cdot \begin{pmatrix} 0 & 0 & 1 \\ a\frac{\Lambda Y^d_{64}}{Z^d_{14}\Phi_{61}} & -b\frac{Z^d_{14}\Phi_{61}}{\Lambda Y^d_{64}} & 0 \\ a & b & 0 \end{pmatrix} \doteq \begin{pmatrix} 1 & \epsilon & \epsilon^3 \\ \epsilon & 1 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix}.$$

In dP1 we get almost the right CKM.

A left-right model with the right CKM



$$W = \begin{pmatrix} X^L_{43} \\ Y^L_{23} \\ Z^L_{23} \end{pmatrix} \begin{pmatrix} 0 & Z_{14} & -Y_{64} \\ -Z_{14}\frac{\Phi_{61}\Psi_{42}}{\Lambda^2} & 0 & X_{12}\frac{\Phi_{61}}{\Lambda} \\ Y_{64}\frac{\Psi_{42}}{\Lambda} & -X_{12} & 0 \end{pmatrix} \begin{pmatrix} X_{36} \\ Y_{31} \\ Z_{36} \end{pmatrix}.$$

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The CKM is given in terms of ratios of Higgs vevs. dP2:

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In dP1 we get almost the right CKM. **In dP2 we get the right CKM.**

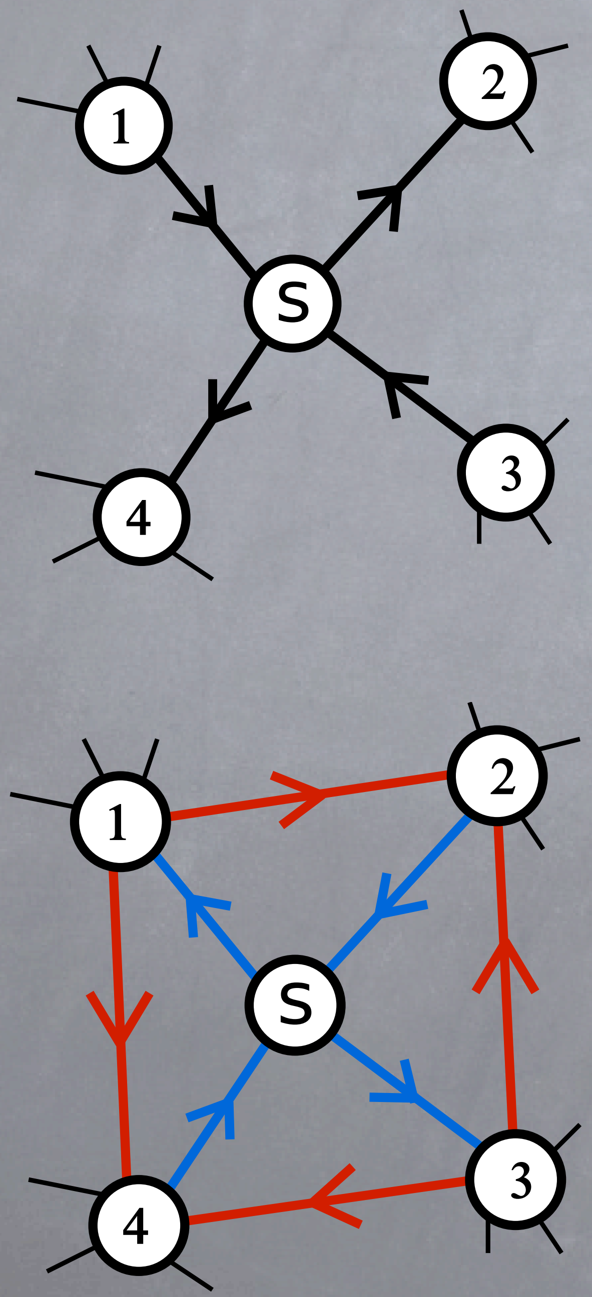
CP violation: with correct CKM, Jarlskog invariant $J \approx \epsilon^6$

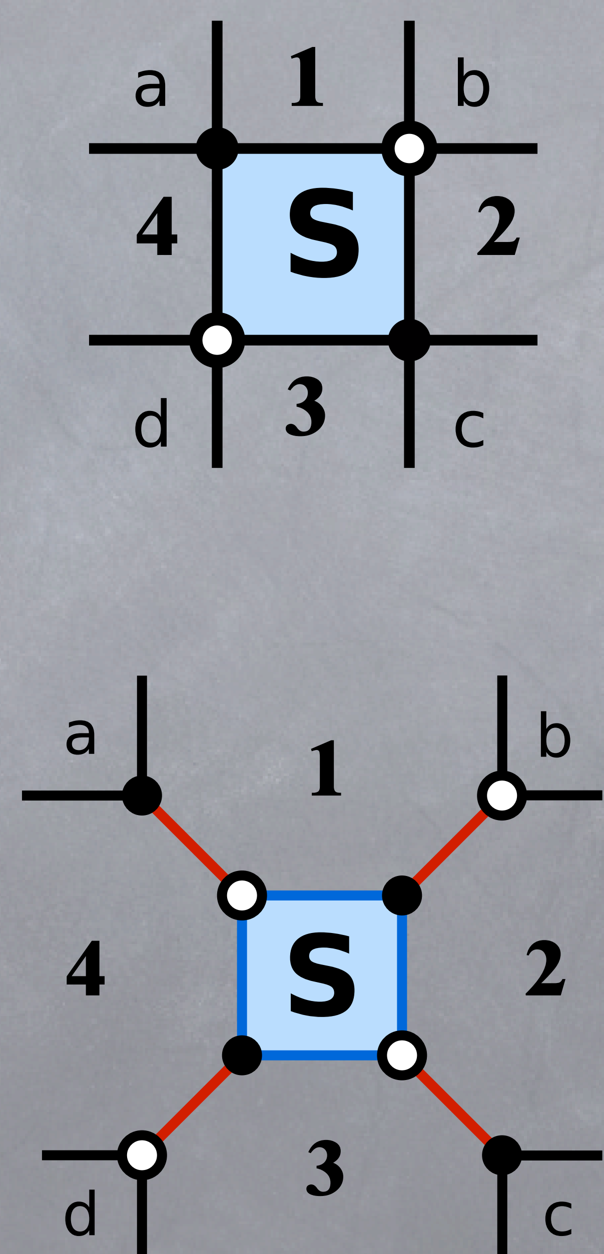
Summary

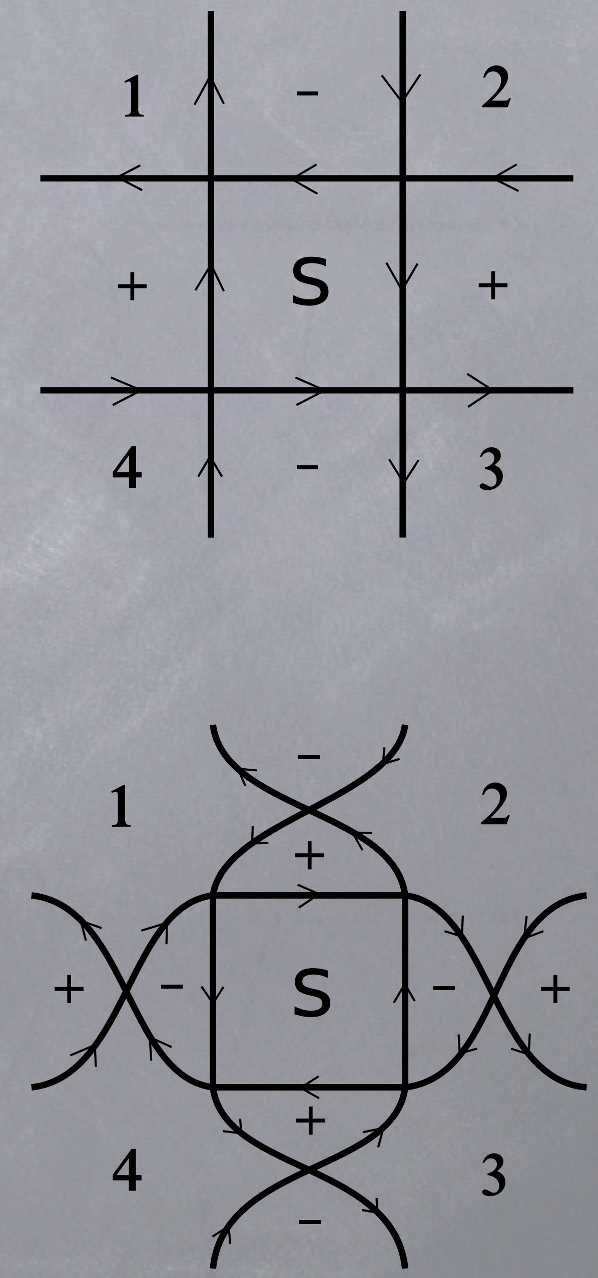
- D-branes at toric singularities interesting class of models:
- Upper bound of 3 families in toric singularities
- Mass Hierarchies are possible, generic structure (0, m, M).
- Sufficient structure for realistic CKM-matrix & CP-violation (concrete models with this structure)
- Open questions: compact models,
  a completely realistic local model...

Additional Slides

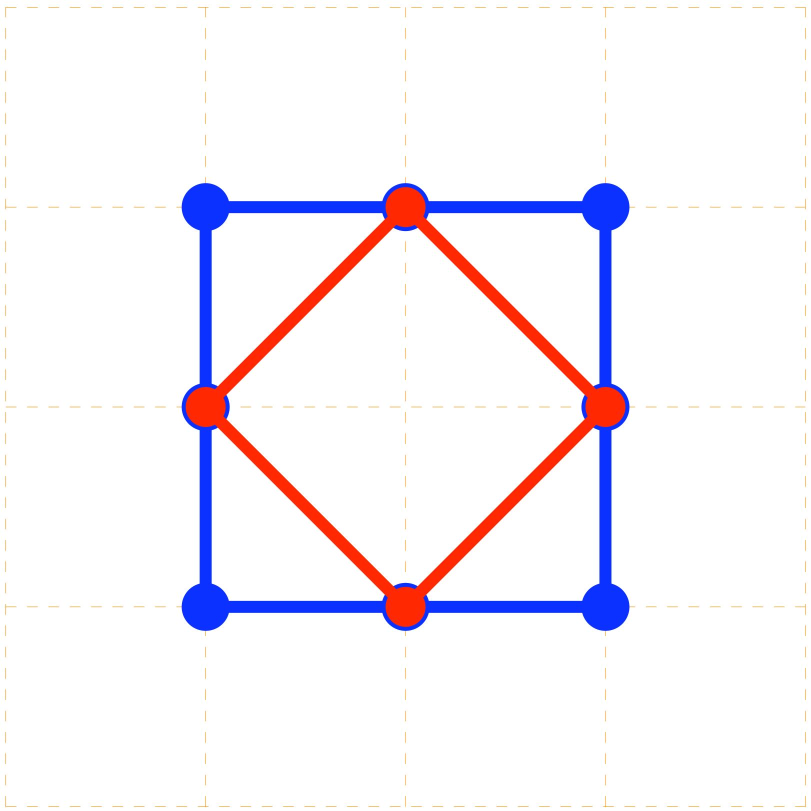
Seiberg duality in quivers and dimers

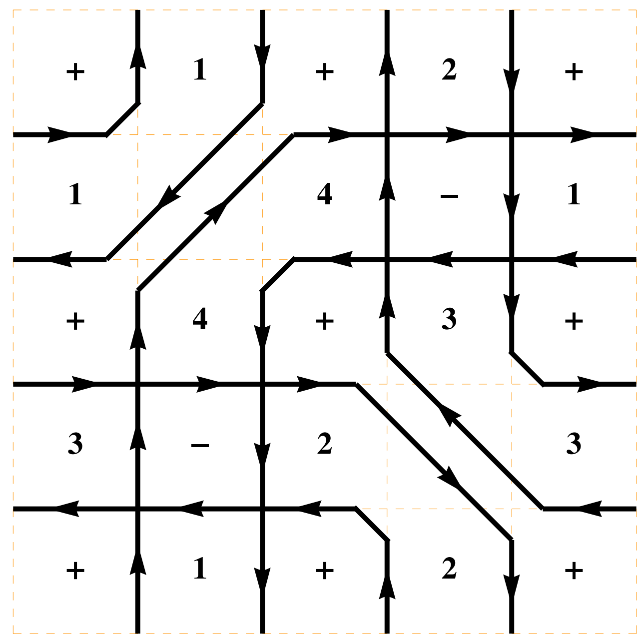


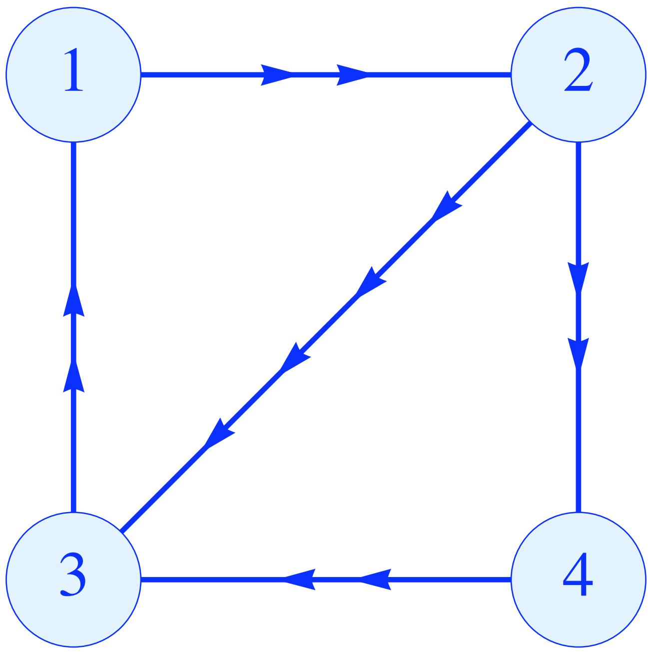


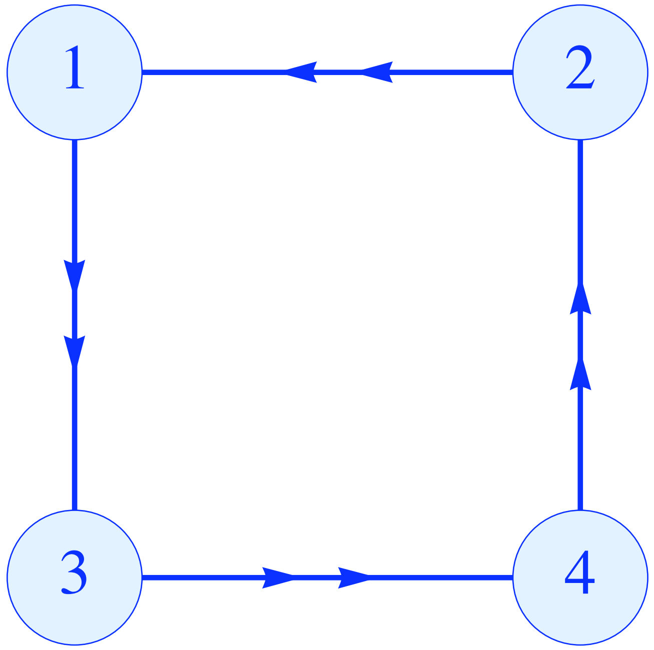


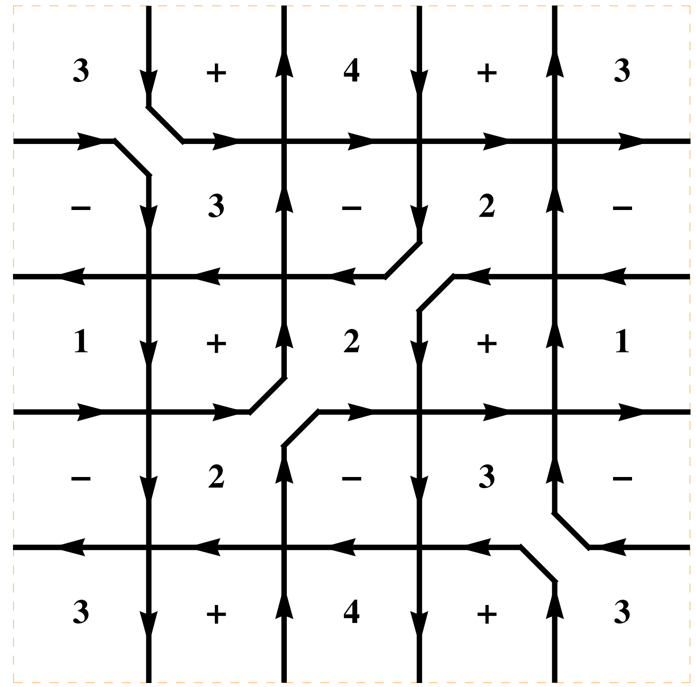
The zeroth Hirzebruch surface











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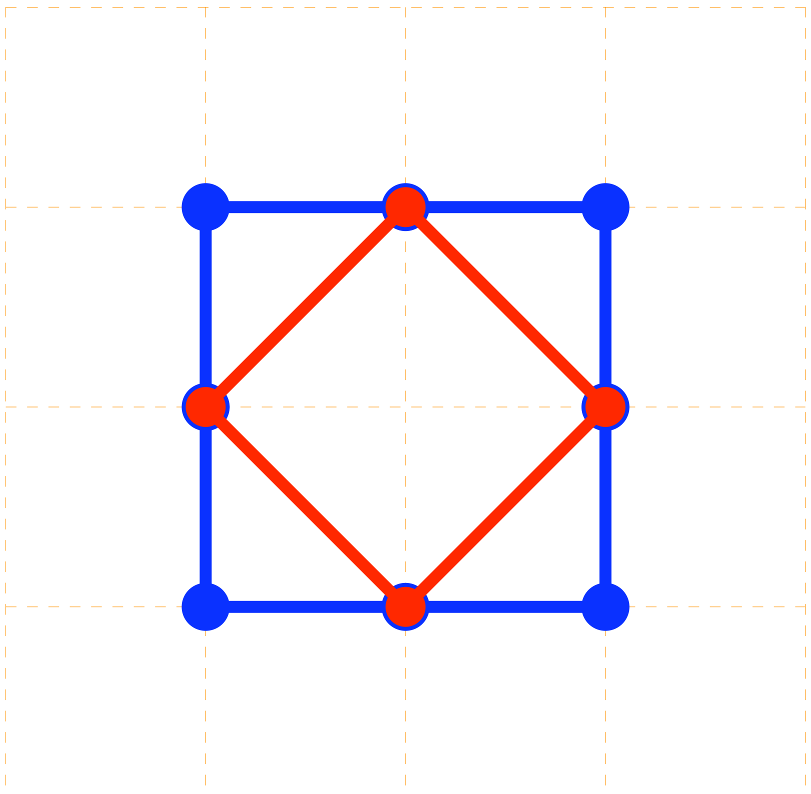
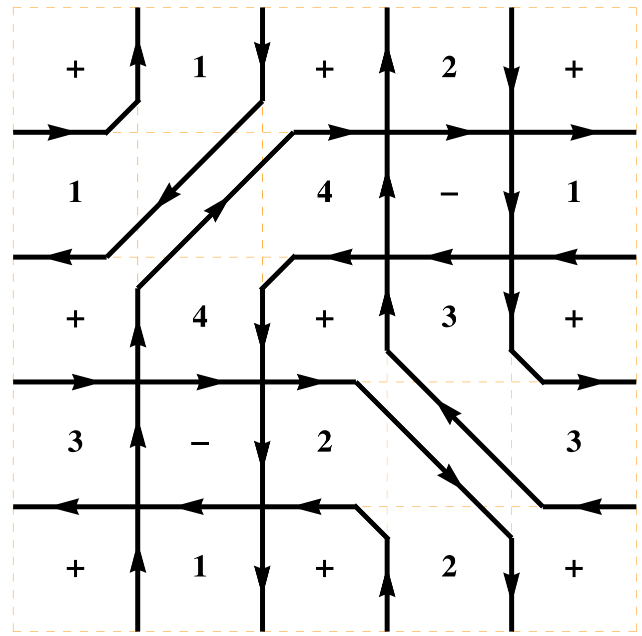
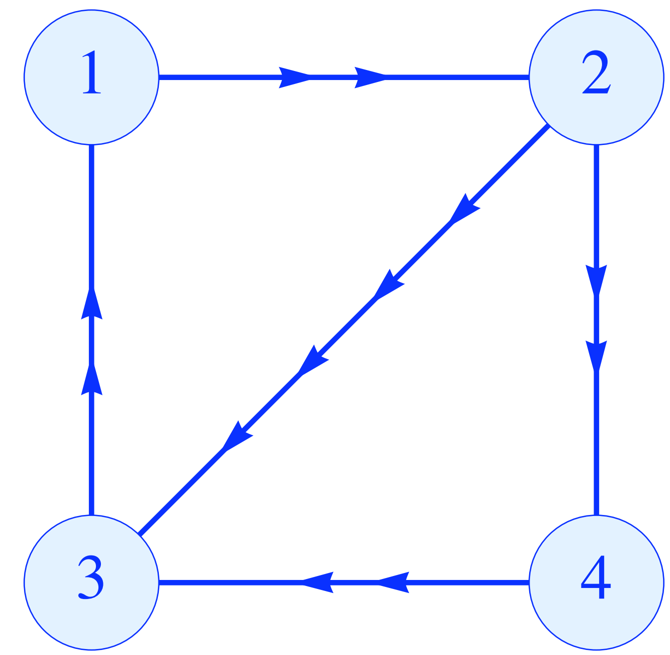
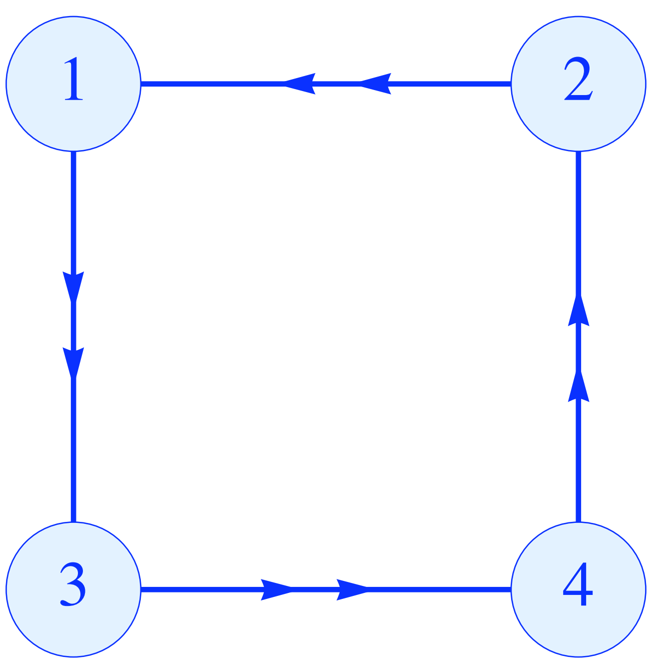
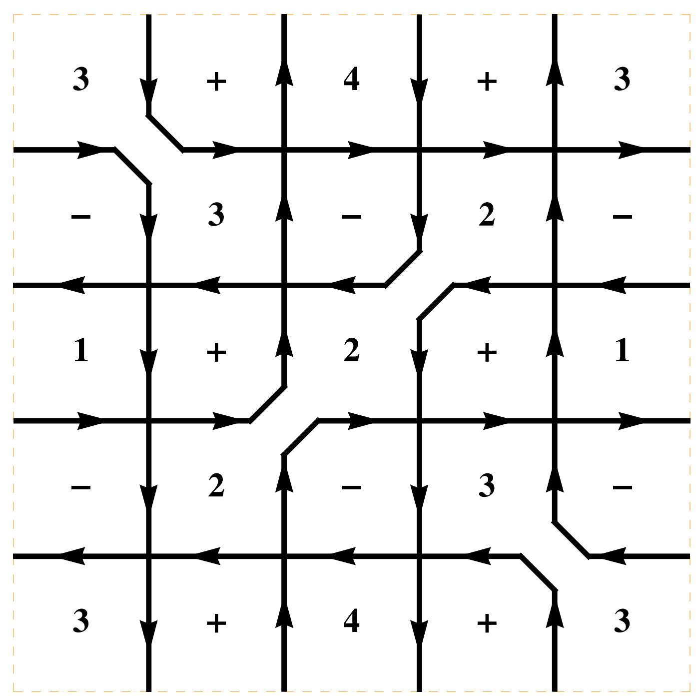
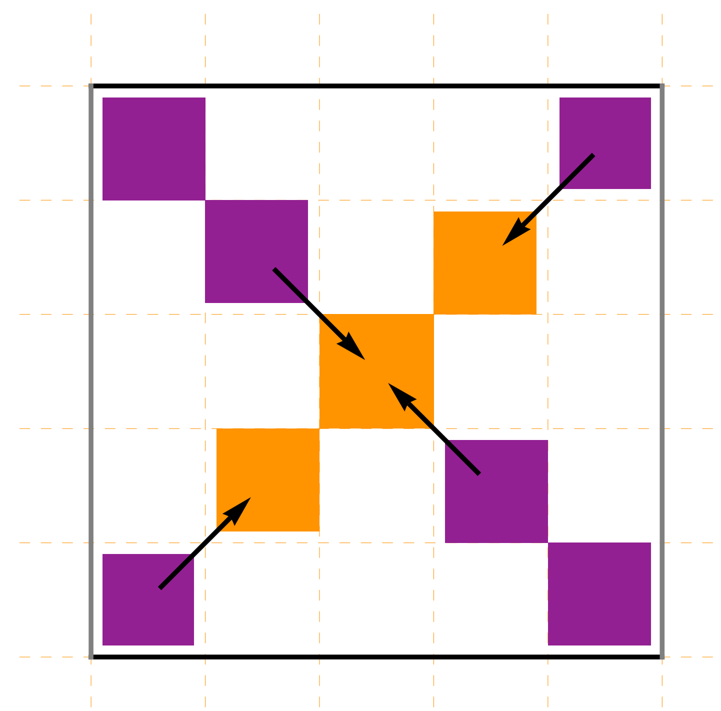




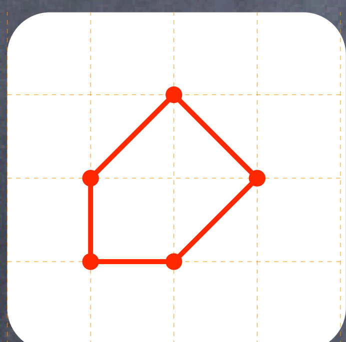


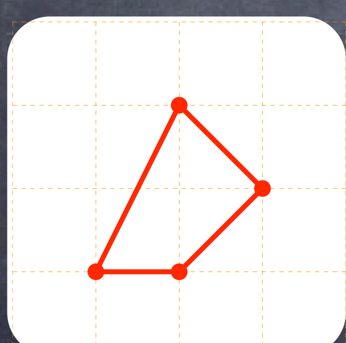


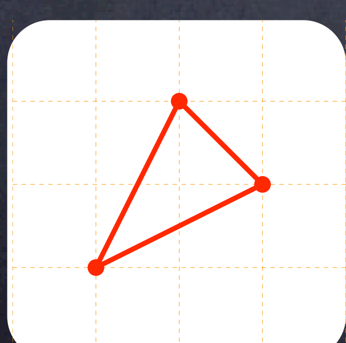


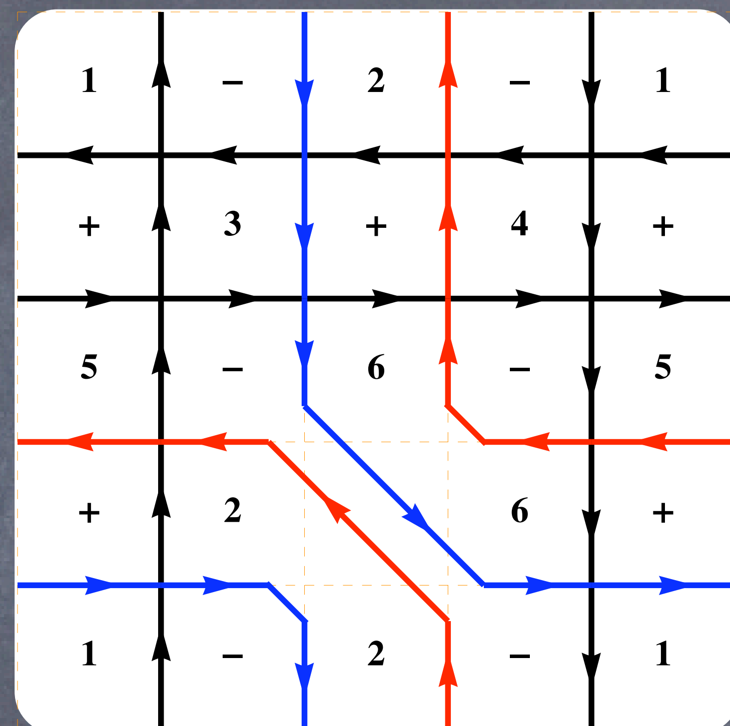


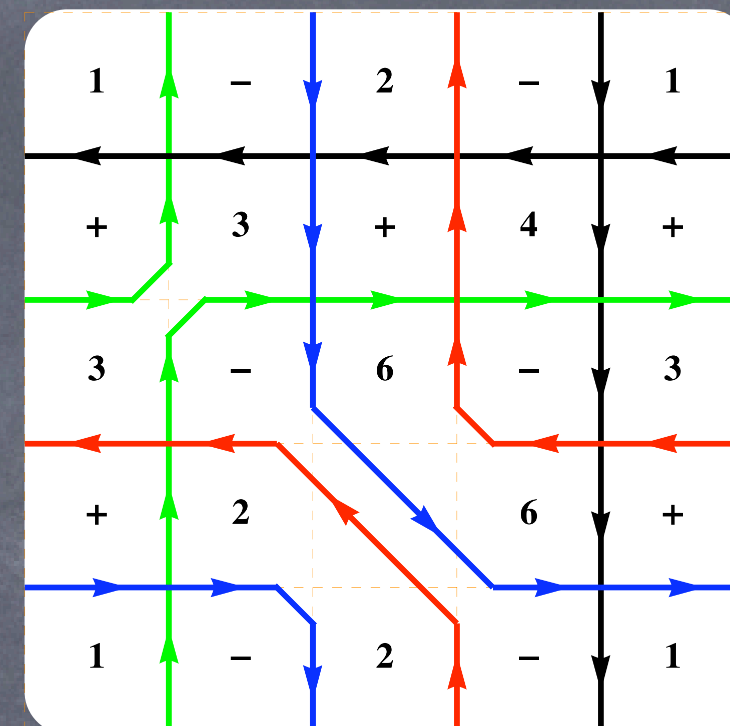
Application of Gulotta's algorithm to toric del-Pezzo surfaces

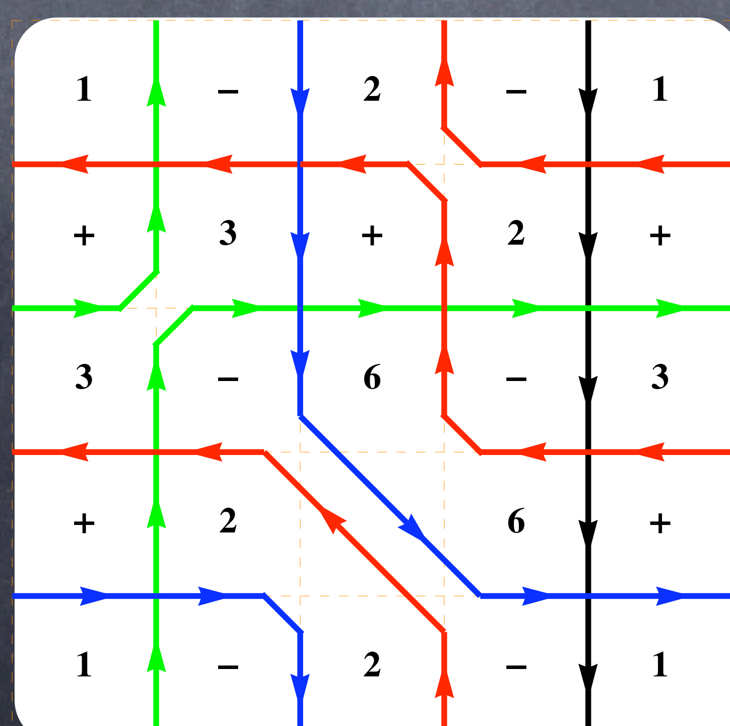


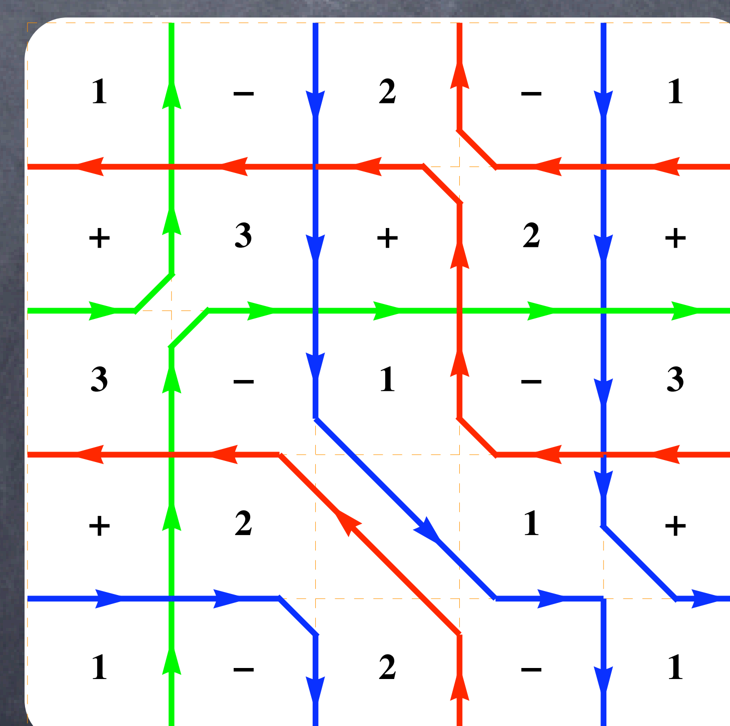












Operations on the dimer

Operation 1:

(1,0) + (0,1) -> (1,1)

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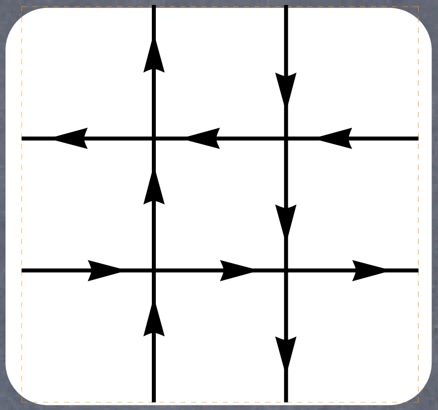

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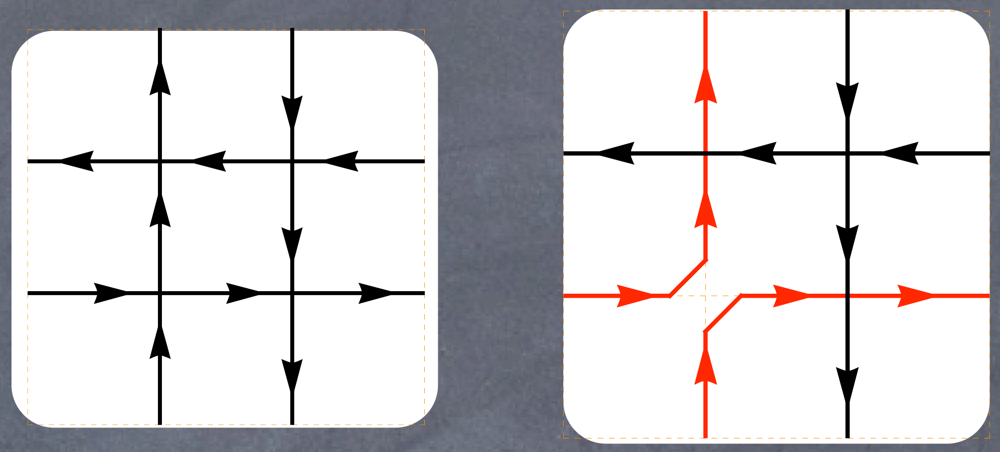

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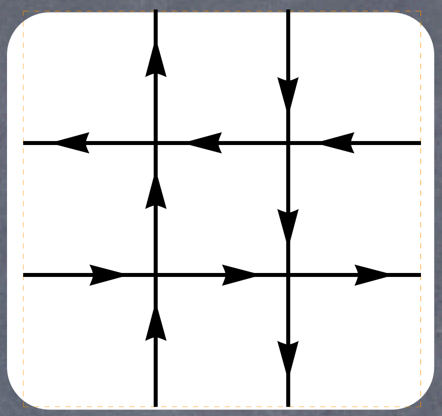

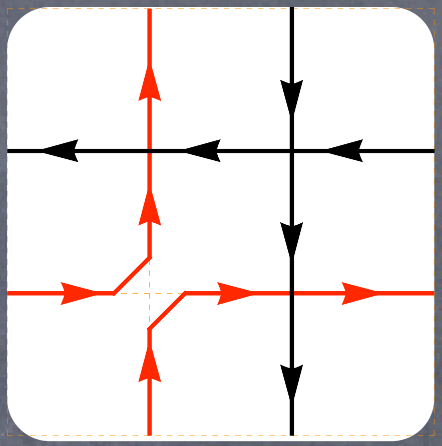

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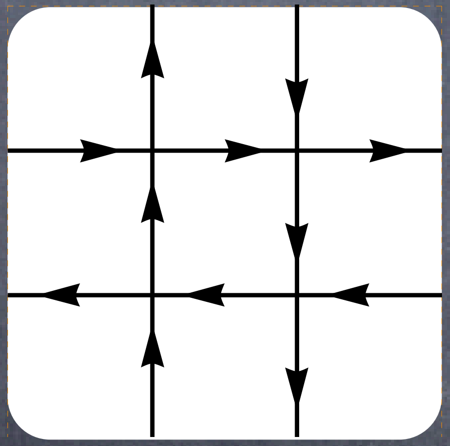

Operation 2:

(1,0) + (0,1) &
(-1,0) + (0,-1) -> (1,1) + (-1,-1)

$$I_{(1,1),(-1,-1)} = 0$$

$$I_{ab} = n_a m_b - m_a n_b$$

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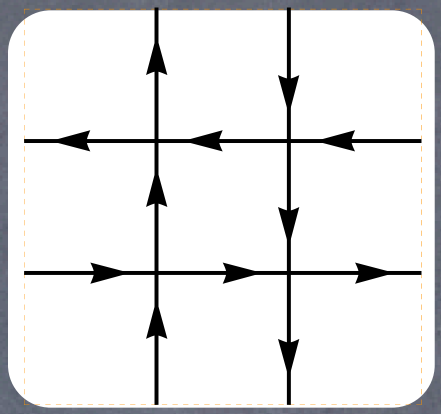

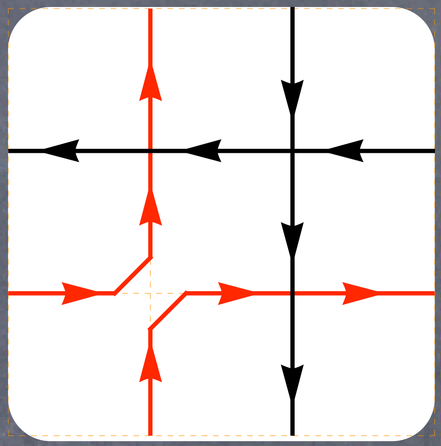

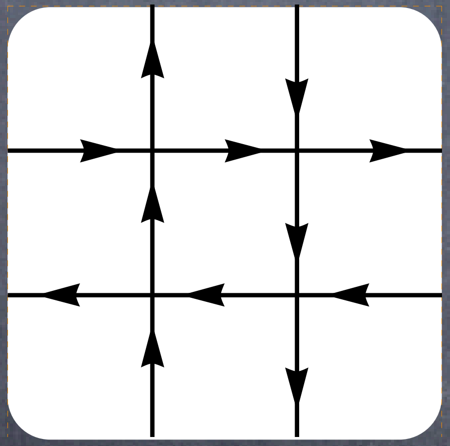

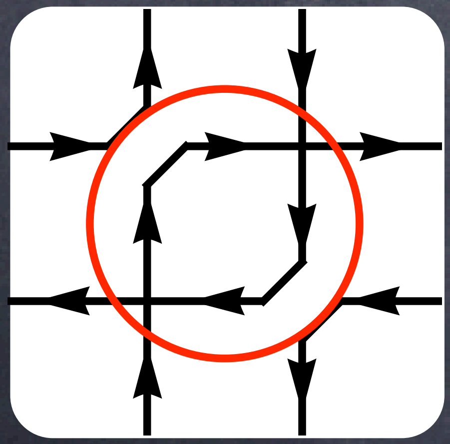

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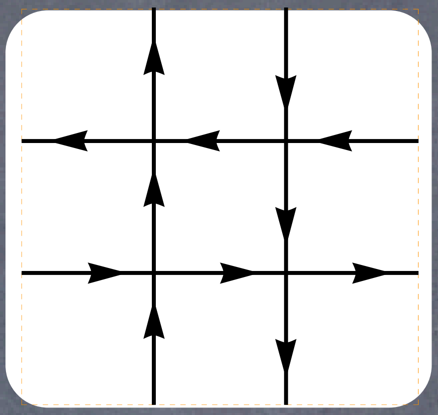

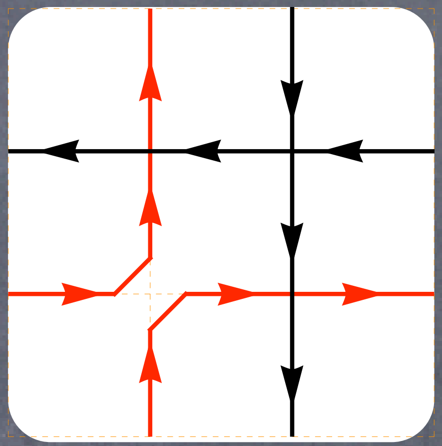

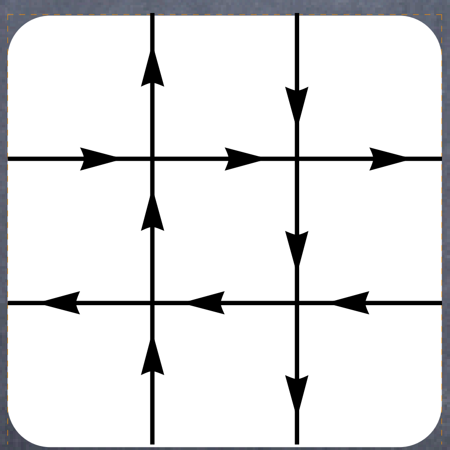

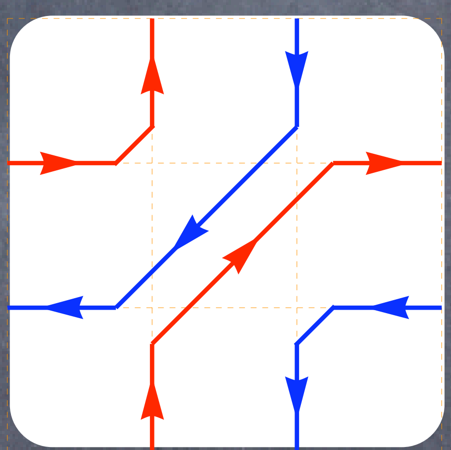

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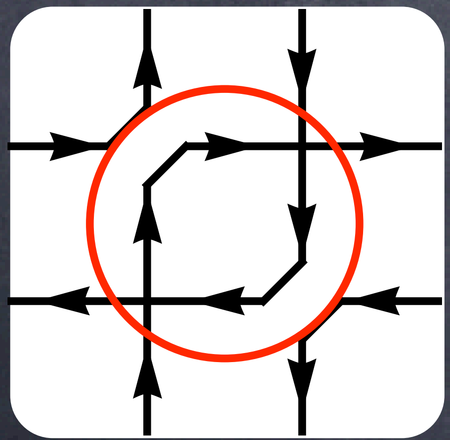

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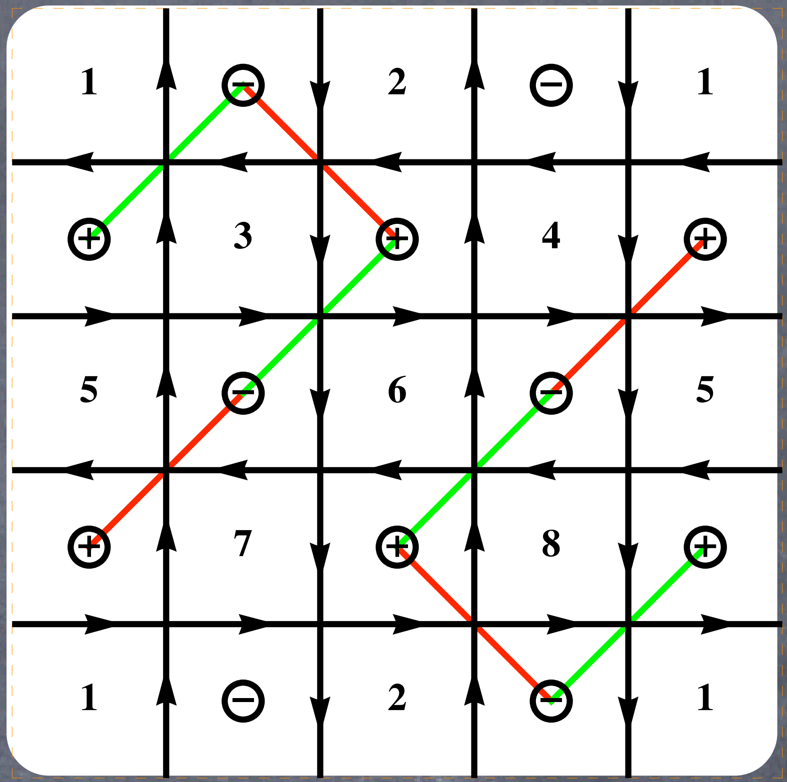
Mass hierarchies II



- How do we choose "quarks"?
  one left & right handed quark in every coupling with quarks.
  -> every superpotential term has two quarks
  -> quarks aligned in closed lines
- Connected or disconnected lines?
  connected to be able to higgs to common gauge group
- Maximal or non-maximal number of quarks?
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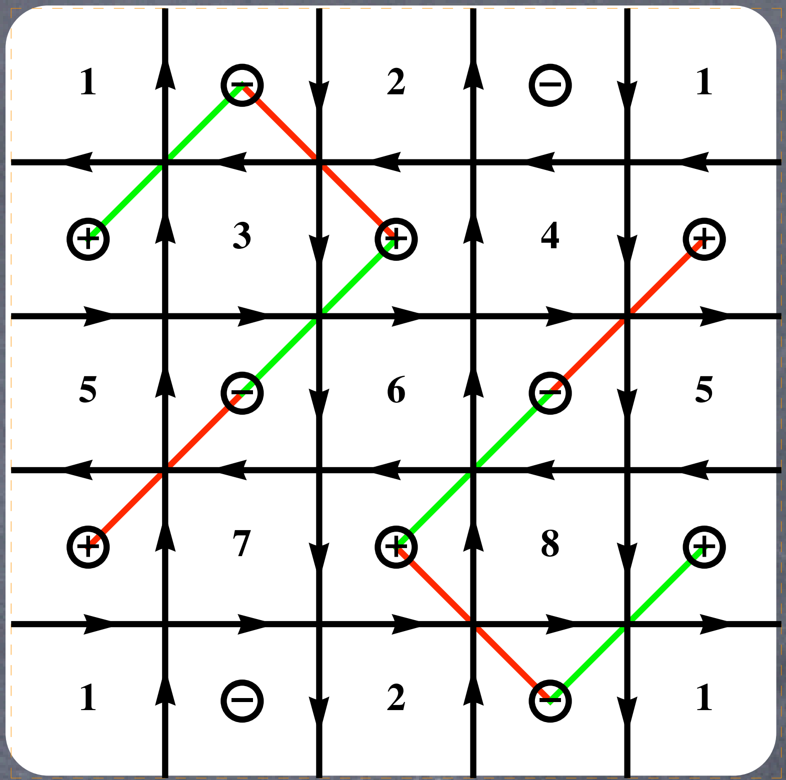

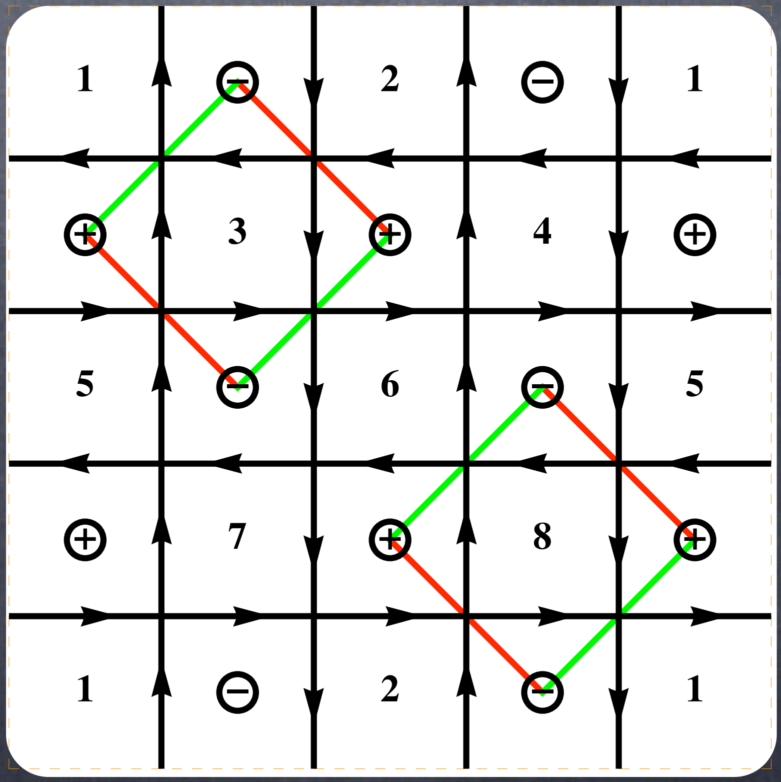

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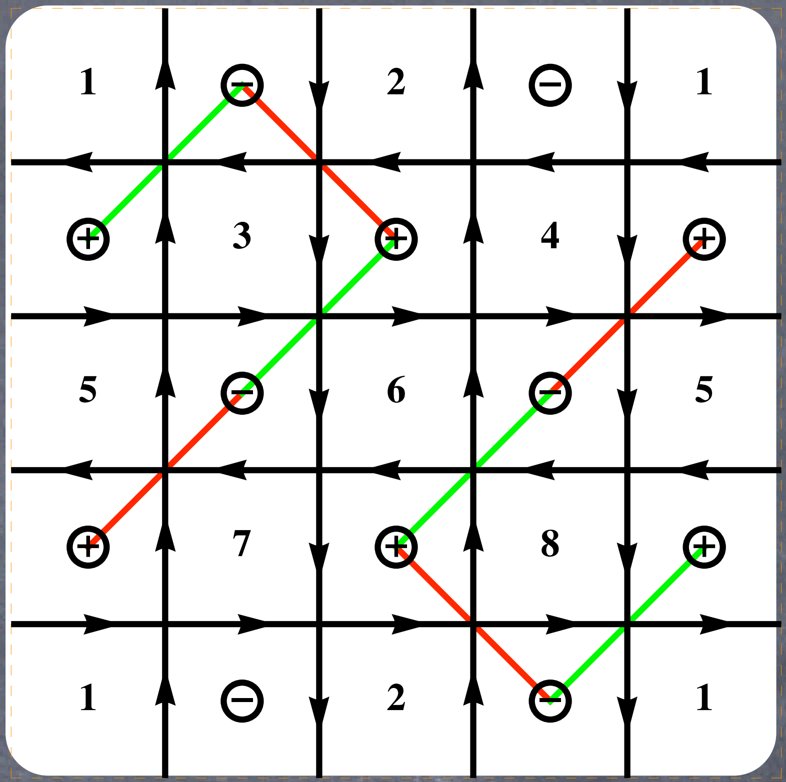

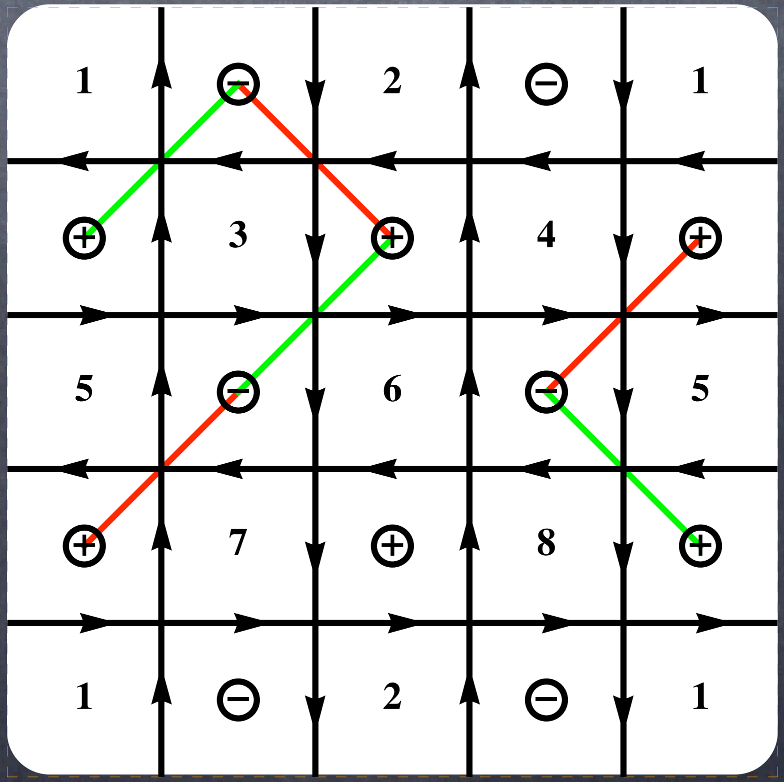

Gulotta's dimers = Traditional dimers