Generalized Gaugino Condensation: Discrete R-Symmetries and Supersymmetric Vacua

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[1] "Generalized Gaugino Condensation in Super Yang-Mills Theories: Discrete R-Symmetries and Vacua," J.K.
arXiv:1005.46865

See also [2] "Discrete R Symmetries and Low Energy Supersymmetry," Michael Dine and J.K.
arXiv:0909.1615

Gaugino Condensation

Gaugino Condensation (written as $\langle\lambda\lambda\rangle$) is a non-perturbative effect in supersymmetric Yang-Mills theories.

This effect breaks an R-symmetry while preserving supersymmetry, and can be used to generate a scale through dimensional transmutation (and then “retrofitted” [3] to another theory).

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The Setup

We consider supersymmetric Yang-Mills, with gauge group $SU(N_c), SO(N_c), Sp(2N_c)$, or G_2 (although the results should hold for any simple Lie group).

The matter content consists of N_f quarks Q in the fundamental and N_f quarks $\overline{Q}$ in the anti-fundamental (or as appropriate for the group), and N_f^2 singlets (possibly more if there are baryons).

The superpotential is

$$W = yS_{ij}M_{ij} + \frac{\gamma}{3}\mathrm{Tr}\, S^3 + W_{\mathrm{dyn}}, \qquad (1)$$

where $M_{ij} \equiv Q_i\overline{Q}_j$.

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Finding the Discrete R-Symmetry

First, consider the theory to have a $U(1)_R$ symmetry where:

$$\lambda \to \beta\lambda, \quad S_{ij} \to \beta^{2/3} S_{ij}, \quad Q_i \to \beta^{2/3} Q_i, \quad \psi_Q \to \beta^{-1/3}\psi_Q. \quad (2)$$

An instanton has a non-zero charge under this $U(1)_R$:

$$2C(A) - \frac{1}{3}\sum_i 2C(r_i) = \frac{2}{3}(3C_2(A) - \sum_i C(r_i)) = \frac{2}{3}b_0, \quad (3)$$

where $C(R)\delta_{ab} \equiv \mathrm{Tr}\ t_a t_b$ $(= 1/2$ for the fundamental)

The non-anomalous discrete R-symmetry is then $\mathbb{Z}_{2b_0 R}$, where b_0 is the coefficient of the one-loop beta function.

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Supersymmetric Vacua

We assume flavor-symmetric solutions, and take M and S to be proportional to the identity: their vevs are v^2 and s.

The vevs of M, S, and gaugino condensation break the discrete R-symmetry completely, to a (non-R) $\mathbb{Z}_2$.

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Generic Dynamical Superpotential and Counting

To study the vacua, we use the dynamical superpotential (when it exists, e.g. an $SU(N_c)$ theory with $N_f < N_c$):

$$W_{\mathrm{dyn}} = C \left(\frac{\Lambda^{b_0}}{M_{ij}^a} \right)^{1/b} . \tag{4}$$

Setting the first derivatives of W to zero and solving,

$$s = \left[\frac{-y^a}{\gamma^{a+b}} \left(C \frac{a}{b} \right)^b \right]^{1/b_0} \Lambda. \tag{5}$$

There are b_0 possible solutions, and so the number of supersymmetric vacua is b_0, exactly the number expected from breaking the R-symmetry.

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Larger N_f

When N_f is larger we can integrate out heavy flavors or use Seiberg's electric-magnetic duality [4].

Additional singlets are needed to lift the baryonic flat directions, and set the baryons to zero at the minimum.

With $\gamma \ll y$, the Q's are all very heavy and can be integrated out, yielding an effective W from gaugino condensation.

With $\gamma \gg y$ and using Seiberg duality, we can extend the dynamical superpotential for larger values of N_f.

Both methods extend the previous calculations, and we always find b_0 vacua.

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Explicit Examples

- $SU(N_c)$ with $N_f < N_c$ has a dynamical superpotential [5, 6] given by

$$W_{\text{dyn}} = (N_c - N_f)\left(\frac{\Lambda^{b_0}}{\det M}\right)^{1/(N_c - N_f)} . \quad (6)$$

- $Sp(2N_c)$ with $2N_f$ quarks also has a known dynamical superpotential [7]

$$W_{\text{dyn}} = A\left(\frac{\Lambda^{b_0}}{\mathrm{Pf}\, M}\right)^{1/(N_c + 1 - N_f)} . \quad (7)$$

- $SO(N_c)$ [8] and the exceptional group G_2 also fits into this analysis [9, 10], but exceptional groups generally have many gauge invariants. By breaking the group to some non-exceptional subgroup, we can use our results.

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Model Building and the μ Problem

"Retrofitting" is a procedure to dynamically generate small mass scales, by replacing mass parameters by interactions with e.g. $W_\alpha W^\alpha$ from a pure gauge theory (and then suppressed by powers of some high energy scale).

Like retrofitting, the singlets in the above theories can be used to generate a mass scale.

This can be directly applied to the μ problem of the MSSM (see also earlier work by Yanagida [11]):

$$\frac{S^2}{M_p} H_U H_D, \tag{8}$$

can be a μ term of the right size, with the B_μ term zero at tree level (the F component of S is small). B_μ is generated at one loop or through a similar operator with one more chiral superfield.

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Summary

- In super Yang-Mills theories with additional singlets (generalized gaugino condensation) to lift flat directions, the discrete R-symmetry is $\mathbb{Z}_{2b_0}$.
- By incorporating the dynamical superpotential (instanton or gaugino condensation effects), we find b_0 supersymmetric vacua.
- These results hold for larger N_f by integrating out heavy flavors or using electric-magnetic duality.
- We demonstrated the above results explicitly for $SU(N_c)$, $SO(N_c)$, $Sp(2N_c)$, and G_2. Besides some subtleties for the other exceptional groups, this is all of the simple Lie groups.
- One can use these theories as another means of retrofitting mass scales, including as a way of generating appropriate μ and B_μ terms.

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Selected References

Please note, this is only a partial list.

[1] J. Kehayias, "Generalized Gaugino Condensation in Super Yang-Mills Theories: Discrete R-Symmetries and Vacua," arXiv:1005.4686 [hep-th].

[2] M. Dine and J. Kehayias, "Discrete R Symmetries and Low Energy Supersymmetry," arXiv:0909.1615 [hep-ph].

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