Gravitational waves and Dark photon dark matter from Axion rotation

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Journal club's talk by Peera Simakachorn
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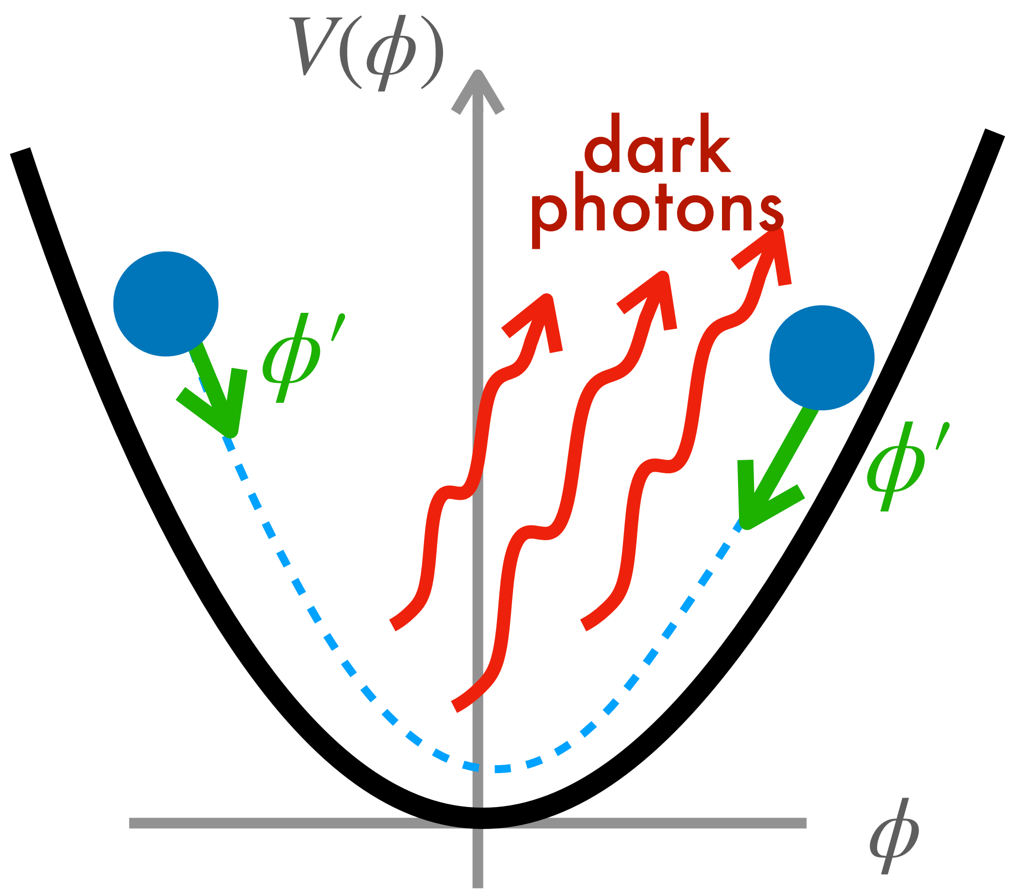
Axion and GW

Spontaneous-symmetry breaking of global symmetry
$\Rightarrow$ pNGB e.g. axion could acquire mass term.

If axions couple weakly to SM sector or only coupled to dark sector,
$\Rightarrow$ Difficult to probe $\Rightarrow$ should rely on gravitational effect

One interesting example: **"Audible Axion"**
[Machado, Ratzinger, Schwaller, Stefanek, '18]

Axion is first frozen,
then oscillates at $H \sim m$



Axion couples to dark photon.

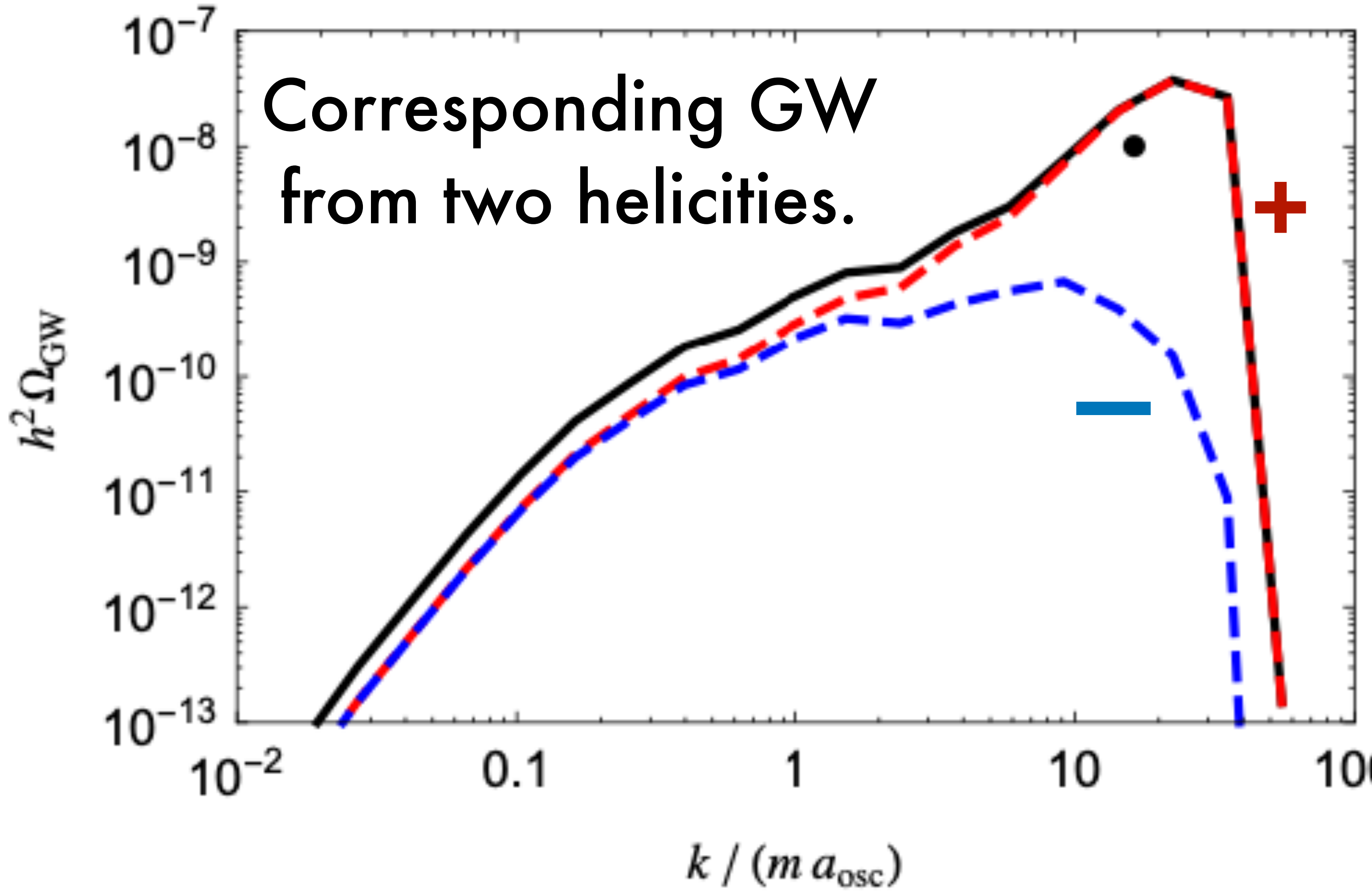
Tachyonic instability

$\Rightarrow$ explosive production of dark photons
(of two helicities)

$$v''_{\pm}(k,\tau) + \omega^2_{\pm}(k,\tau)\, v_{\pm}(k,\tau) = 0\,,$$

with $\omega^2_{\pm}(k,\tau) = k^2 \mp k\frac{\alpha}{f}\phi'\,.$

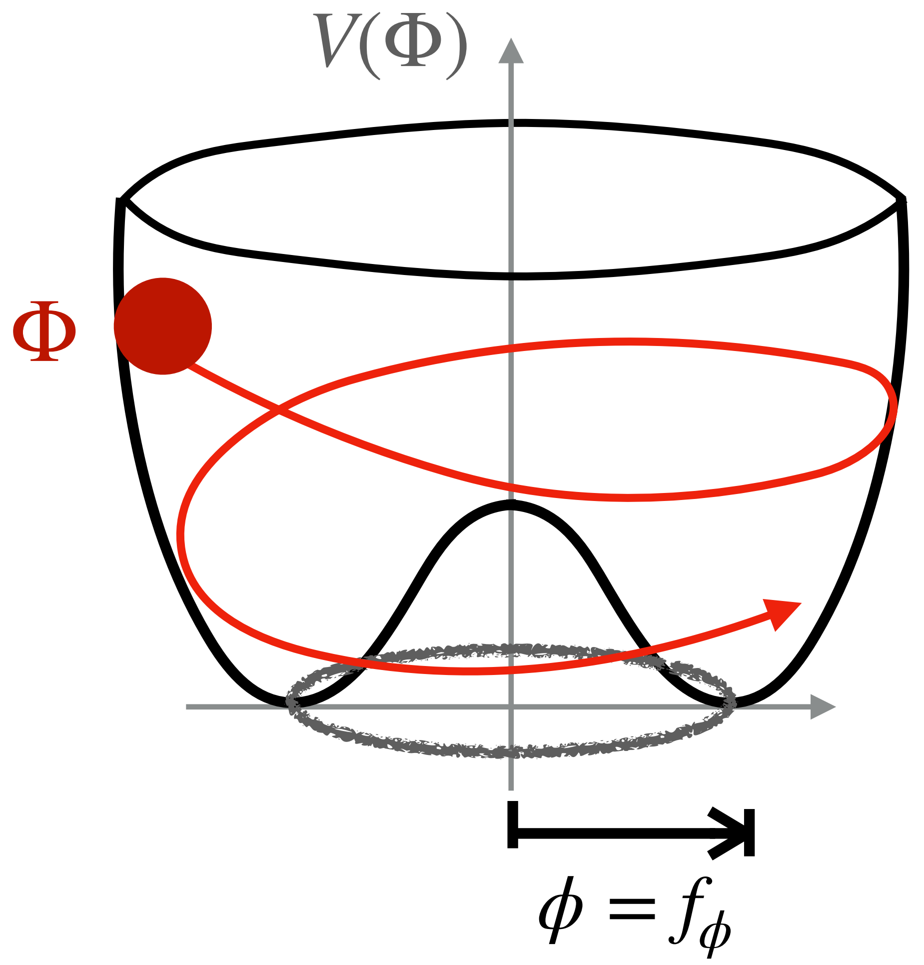
Scalar field oscillation with alternating ϕ'-sign



Another interesting dynamics: “Axion rotation”

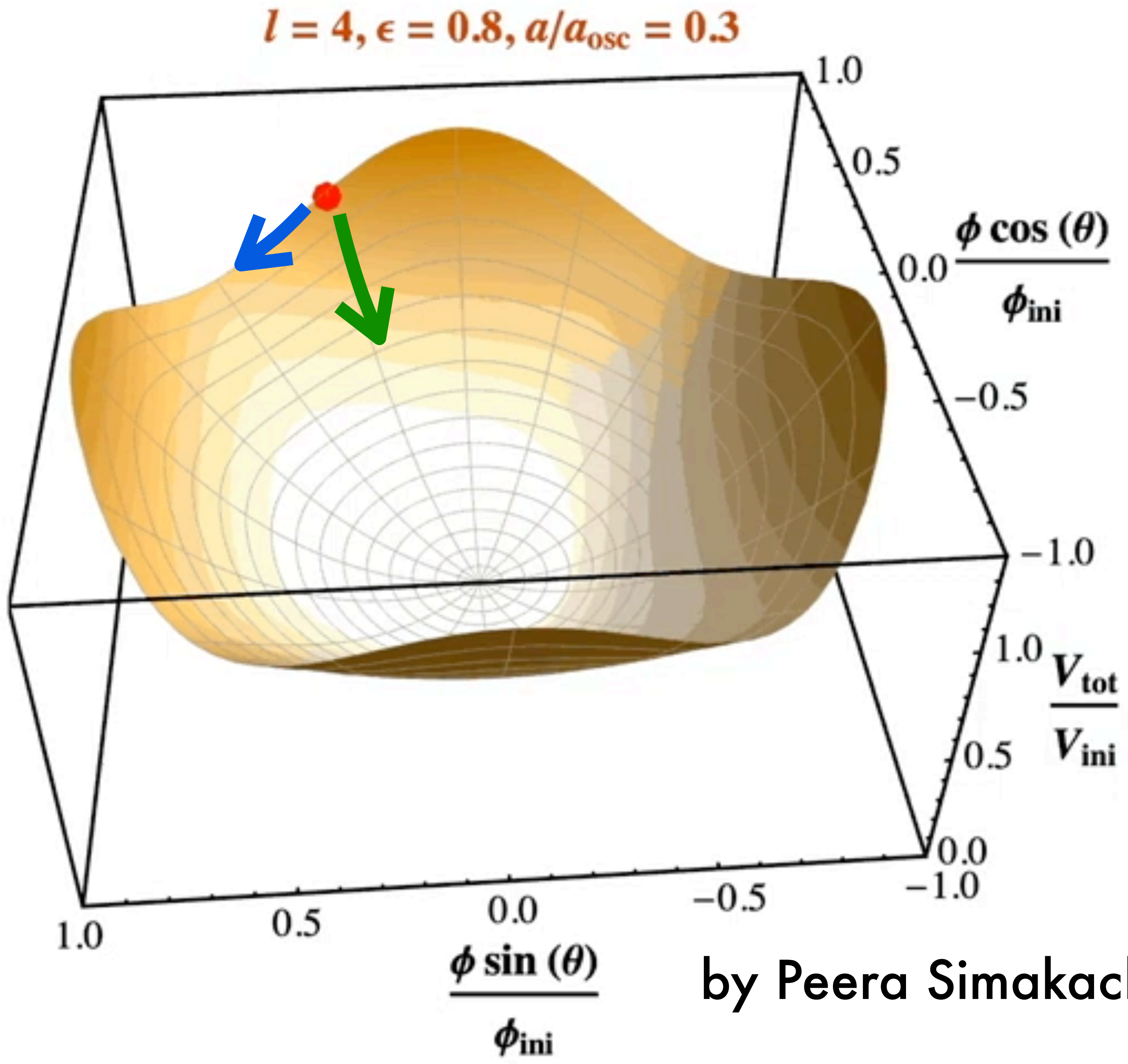
Axiogenesis [Co,Harigaya, ’19]
Kinetic misalignment
[Co,Harigaya, ’19] [Chang, Cui, ’19]

Peccei-Quinn scalar: $P \sim \phi e^{i\theta}$
with $U(1)$-symmetry and axion as phase, $a = \theta f_\phi$



The scalar field can be dynamically driven to large value, $\phi \gg f_\phi$.

a) de Sitter fluctuation during inflation [Wu & Petraki, ’20]
b) Driven by negative Hubble mass to global minimum [Dine-Randall-Thomas, 1995]



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Generating rotation via explicit $U(1)$-breaking term

$$V_{\rm break} \sim \frac{P^l}{M^{l-4}} + \mathrm{h.c.} \propto \cos(l\theta) \quad \text{(effective at large } \phi\text{)}$$

Both radial oscillation and angular rotation:
Elliptic orbit with red-shifting size

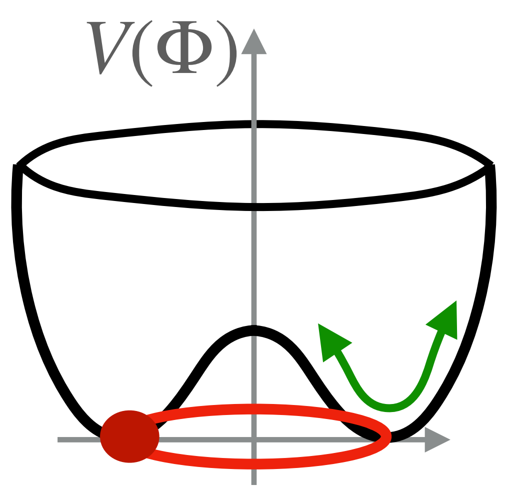
At later times, $V_{\rm break}$ becomes negligible.
There is a conserved charge (PQ-charge):

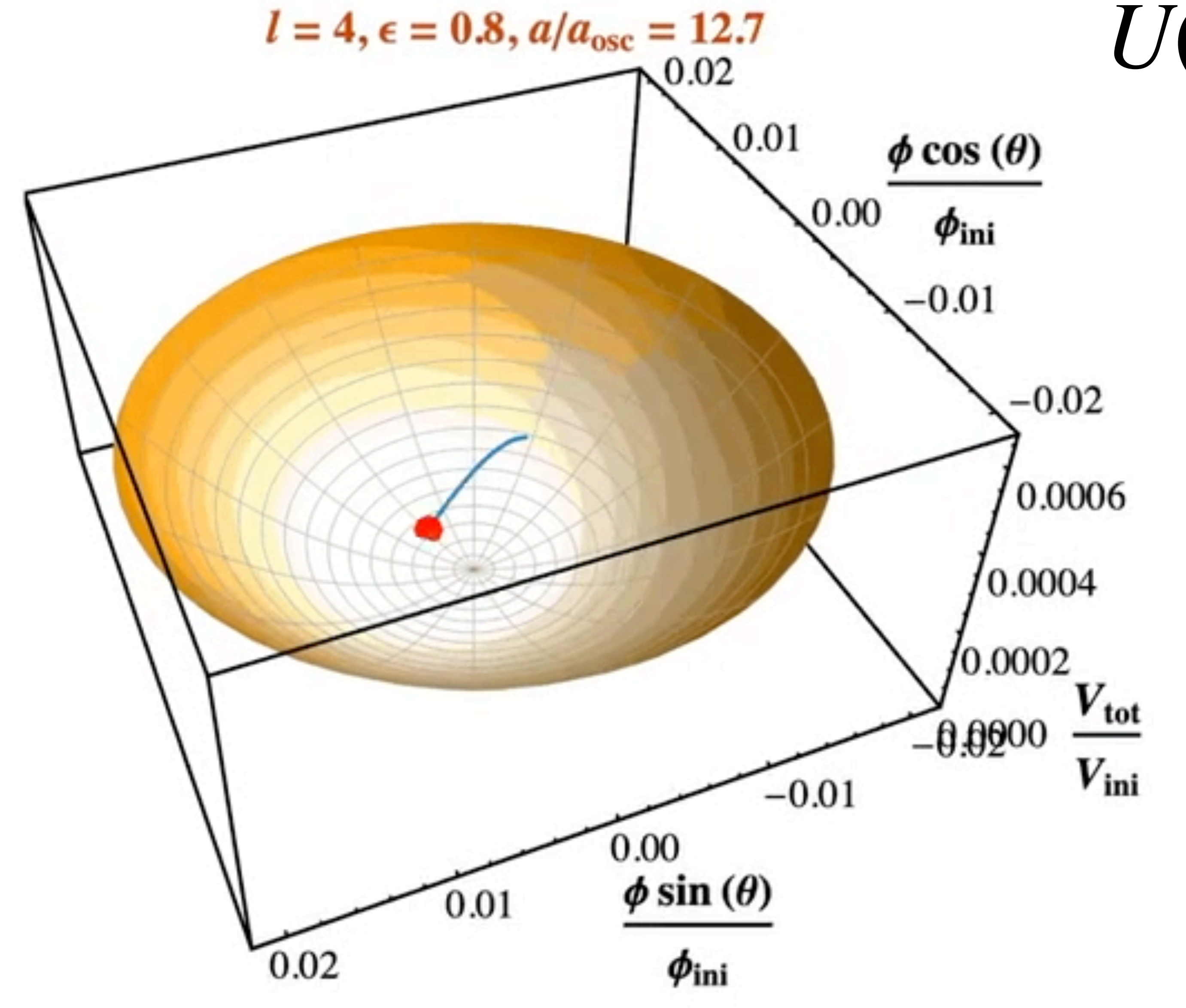
$$\frac{d}{dt}(R^3 \phi^2 \dot{\theta}) = 0$$

Another interesting dynamics: "Axion rotation" (cont.)

Elliptic orbit is dangerous.

Unwanted matter domination
"Cosmological moduli problem"





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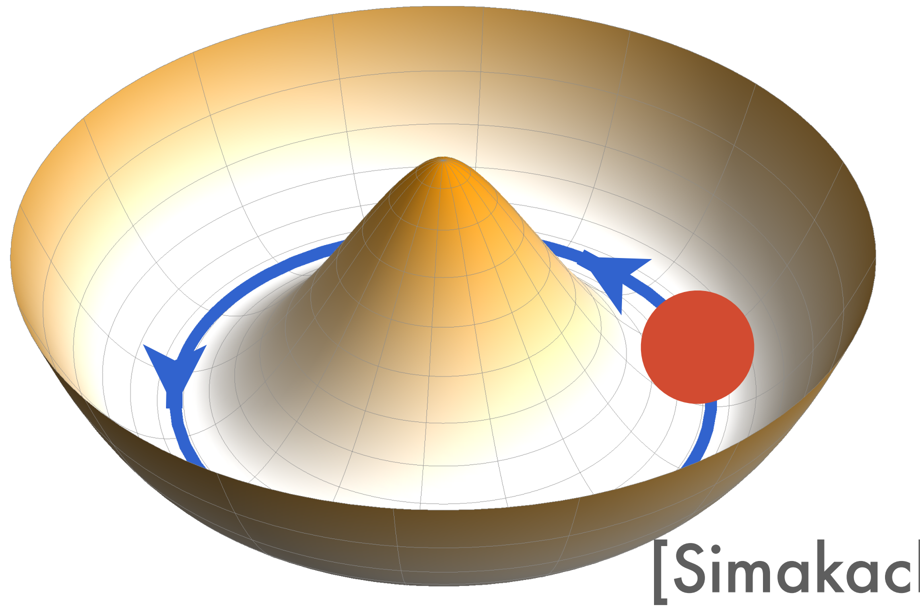
$U(1)$-conserving interactions can damp radial motion.

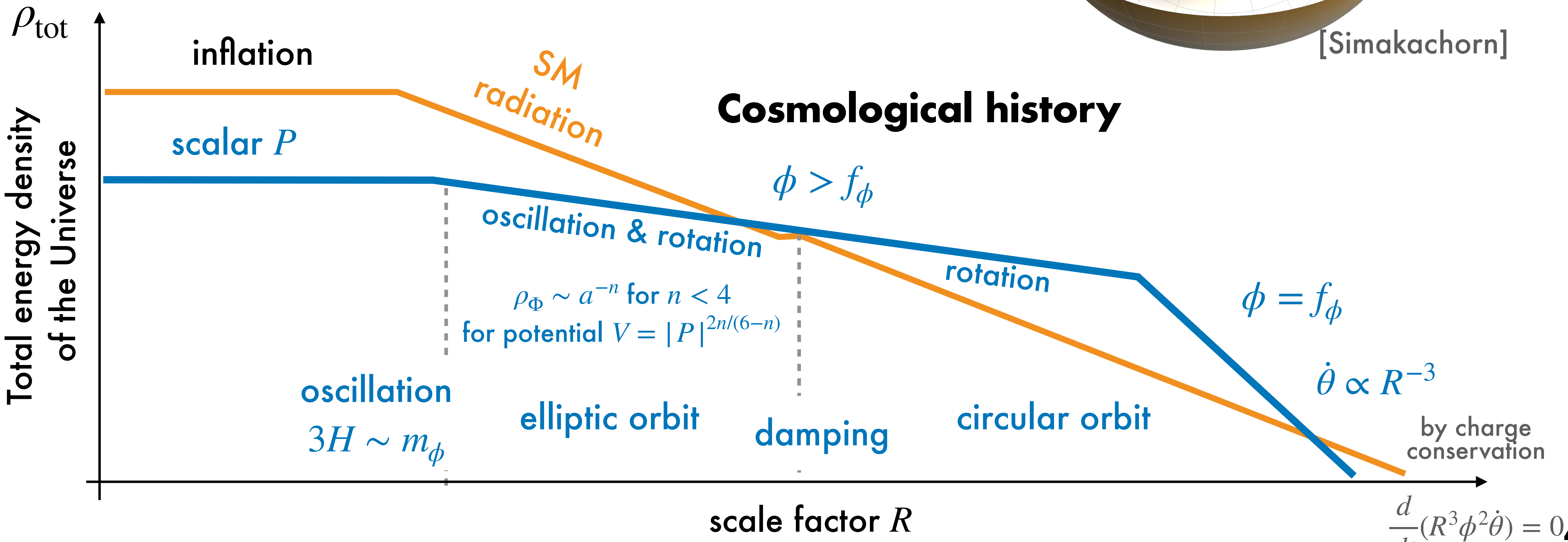
Angular rotation remains.

[Allahverdi-Mazumdar, '08, Co-Harigaya, '19]



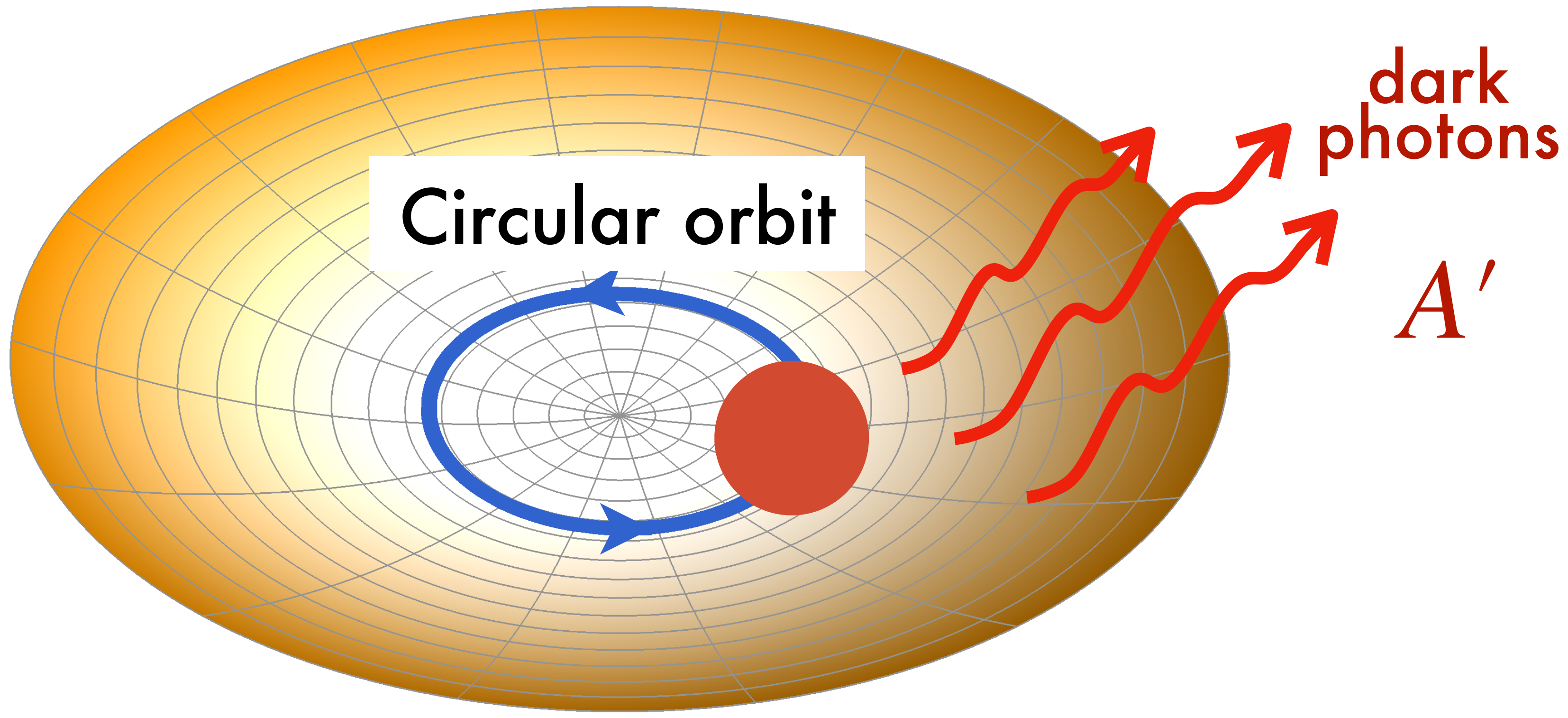
[Simakachorn]

Circular orbit red-shifts down to the bottom.



$\frac{d}{dt}(R^3 \phi^2 \dot{\theta}) = 0$

"Axion rotation" couples to dark photons



For simplicity, consider only circular orbit

$\dot{\theta}$ does not oscillate.

(similar result is expected for elliptic orbit)

Assume Yukawa interaction with

fermions $\psi, \bar{\psi}$ charged under $U(1)_{\rm PQ}$ and $U(1)_d$

$$\mathcal{L} \supset y_\psi P \bar{\psi} \psi + {\rm h.c.}.$$

assume to be heavy
(not causing e.g. dark electric conduct.)

induced anomaly and axion-dark photon coupling

$$\mathcal{L} \supset \frac{e_D^2}{64\pi^2} \theta \epsilon^{\mu\nu\rho\sigma} F'_{\mu\nu} F'_{\rho\sigma},$$

Equation-of-motion of two helicities of dark photon $A'_\pm$

$$\frac{\partial^2 A'_\pm}{\partial t^2} + H \frac{\partial A'_\pm}{\partial t} + \left(m_{A'}^2 + \underline{\frac{k^2}{R^2} \pm \frac{e_D^2}{8\pi^2} \frac{k}{R} \dot{\theta}} \right) A'_\pm = 0\,,$$

Tachyonic instability for modes

with $0 < \dfrac{k}{R} < \dfrac{e_D^2}{8\pi^2} \dot{\theta}$

if dark photon mass $m_{A'}$ is small.

Fastest growing mode:

$$\frac{k}{R} \simeq \frac{e_D^2}{16\pi^2} \dot{\theta} \equiv k_{\rm TI},$$

"PQ charges in rotation $\Rightarrow$ asymmetry in helicity of A'"

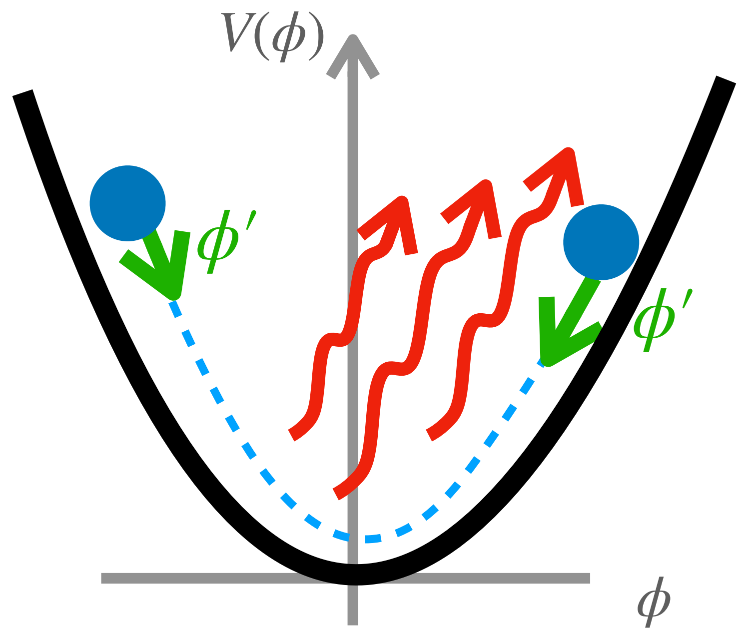
Effectiveness of tachyonic instability (TI)

Equation-of-motion of two helicities of dark photon $A'_\pm$

$$\frac{\partial^2 A'_\pm}{\partial t^2} + H\frac{\partial A'_\pm}{\partial t} + \left(m_{A'}^2 + \frac{k^2}{R^2} \pm \frac{e_D^2}{8\pi^2}\frac{k}{R}\dot{\theta}\right)A'_\pm = 0\,, \quad \Rightarrow \quad \frac{k}{R} \simeq \frac{e_D^2}{16\pi^2}\dot{\theta} \equiv k_{\rm TI},$$

The growth rate should exceed Hubble friction: $k_{\rm TI} \gtrsim H$

Oscillating axion



field oscillates at $H \sim m_\phi$

with alternating $\dot{\theta}$-sign.

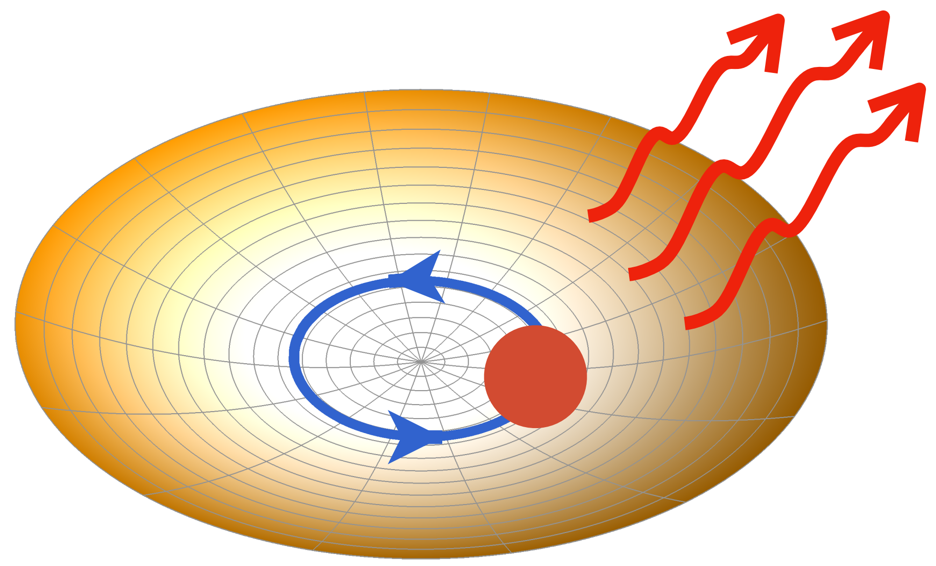
Two A'-helicities are produced.

For effective TI, $\frac{e_D^2}{16\pi^2}\dot{\theta} = k_{\rm TI} > m_\phi \sim \dot{\theta}$

Need large axion-dark photon coupling.

$e_D \gtrsim 4\pi$

Rotating axion



$\dot{\theta}$ does not change sign.

Helicity is highly asymmetric.

$\dot{\theta} \propto R^0$ (quadratic)

$\dot{\theta} \propto R^{-1} \sim T$ (quartic).

While $H \propto T^2$ (radiation era),

the explosive A'-production is effective

without large axion-dark photon coupling.

Stoping the dark-photon explosion

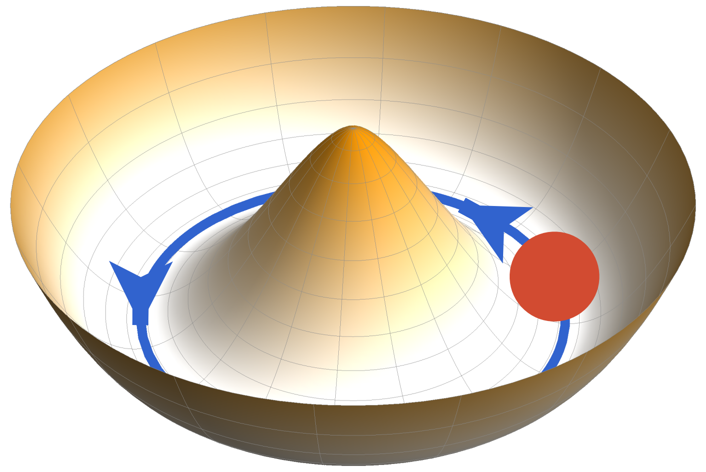
The production saturates when the back-reaction kicks in.

$\Rightarrow \dot{\theta}$ slows down by transferring charge to A'.

$\Rightarrow$ The produced A' scatters with the rotating field.

Both happen when A' obtains energy density comparable to P.

If ϕ is close to f_a , TI stops earlier because it becomes ineffective with $\dot{\theta} \propto R^{-3}$



Rough estimation, the production is complete when

$$H_P \equiv \frac{k_{\rm TI}}{r_p} \quad \leftarrow \ \mathcal{O}(10) \text{ factor}$$

and dark-photon energy density $\rho_{A'} \sim \rho_\theta$ @ production

and the A'-momentum distribution has a peak at $k_{\rm TI}$.

(To be confirmed by lattice results)

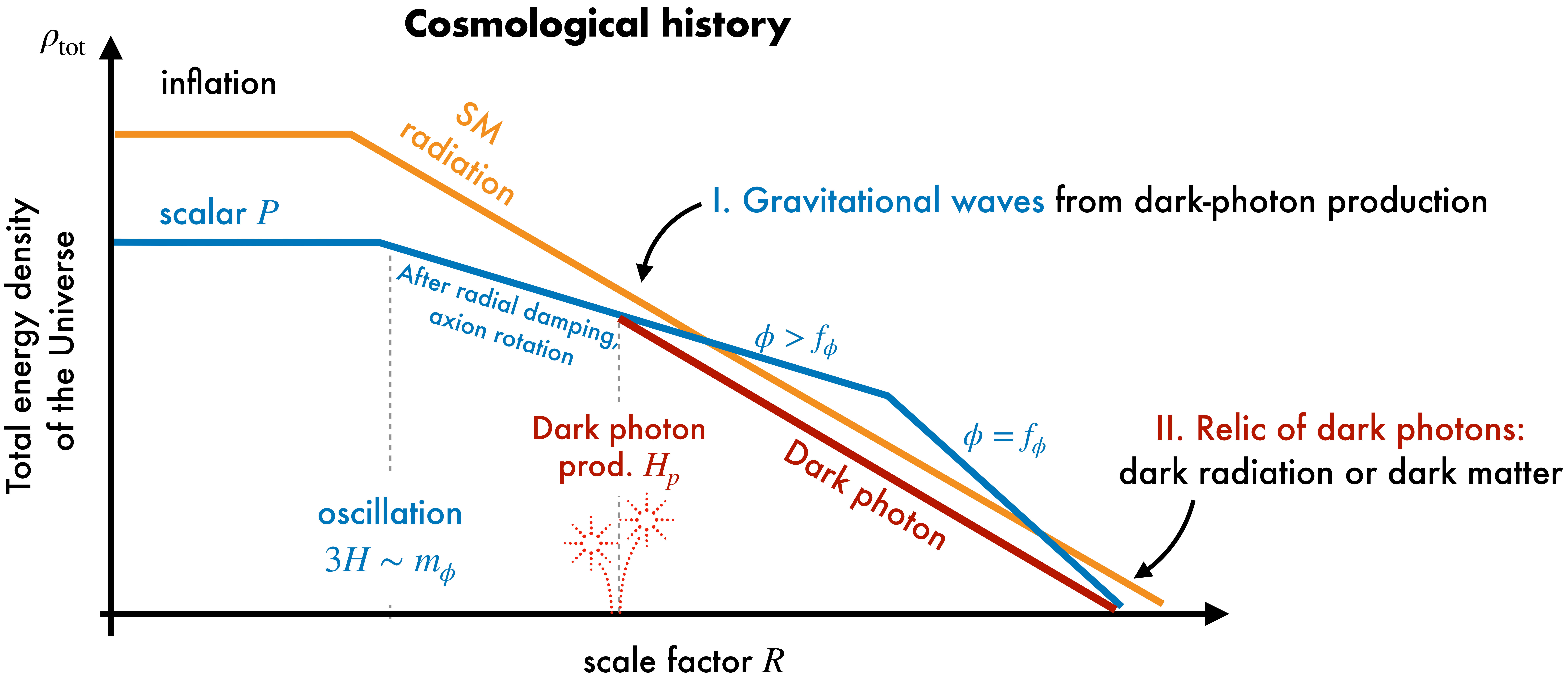
After the A'-production stops, there is a residual rotation.

$$Y_\theta = \frac{n_\theta}{s} = r_\theta Y_{\theta,i}, \quad \leftarrow \text{before TI}$$

$\uparrow$ $\mathcal{O}(1)$ free parameter

Must not violate some bounds: matter domination during BBN, or $N_{\rm eff}$ bound.

Consequences of dark-photon explosion



I. Gravitational waves from axion rotation

EOM for metric tensor perturbation (or GW) is

$$\ddot{h}_{ij} + 3H\dot{h}_{ij} - \frac{1}{a^2}\nabla^2 h_{ij} = \frac{2}{M_{\rm Pl}^2}\Pi_{ij}^{TT},$$

← anisotropic stress tensor is sourced by dark photon energy density $\sim \rho_{A'}$

short-lasting source ⇒ peak GW signal

A. Peak frequency: expect GW have a peak at wave number

$$k_{\rm GW} \approx k_{\rm TI} \equiv r_p H_p.$$

Today frequency:

$$\boxed{f_{\rm GW} = 2\ {\rm nHz}\ \left(\frac{r_p}{20}\right)^{\frac{1}{2}} \left(\frac{k_{\rm TI}}{3\times 10^{-13}\,{\rm eV}}\right)^{\frac{1}{2}} \left(\frac{10}{g_*(T_p)}\right)^{\frac{1}{12}}.}$$

(in range of pulsar-timing array)

Observation of a peak ⇒ Temperature of dark photon production

$$T_p \simeq 5.5\ {\rm MeV} \left(\frac{f_{\rm GW}}{2\ {\rm nHz}}\right)\left(\frac{20}{r_p}\right)\left(\frac{10}{g_*(T_p)}\right)^{\frac{1}{6}}.$$

I. Gravitational waves from axion rotation (cont.)

EOM for metric tensor perturbation (or GW) is $\ddot{h}_{ij} + 3H\dot{h}_{ij} - \frac{1}{a^2}\nabla^2 h_{ij} = \frac{2}{M_{\rm Pl}^2}\Pi_{ij}^{TT},$

B. GW energy density: $\rho_{\rm GW} = M_{\rm pl}^2 \left\langle \dot{h}_{ij}\dot{h}_{ij} \right\rangle /4$

$\dot{h}_{ij} \sim \frac{2\rho_{A'}}{3HM_{\rm pl}^2}$ ← sourced by dark photon $\sim \rho_{A'}$

$$\frac{\rho_{\rm GW}}{\rho_R} \simeq \frac{\rho_{A'}^2}{H_p^4 M_{\rm pl}^4} \simeq \frac{r_p^4 \rho_\theta^2}{k_{TI}^4 M_{\rm pl}^4}$$

GW signal is related to dark-photon and axion-rotation energy densities.

need lattice results to see energy ratio between dark photons and GW

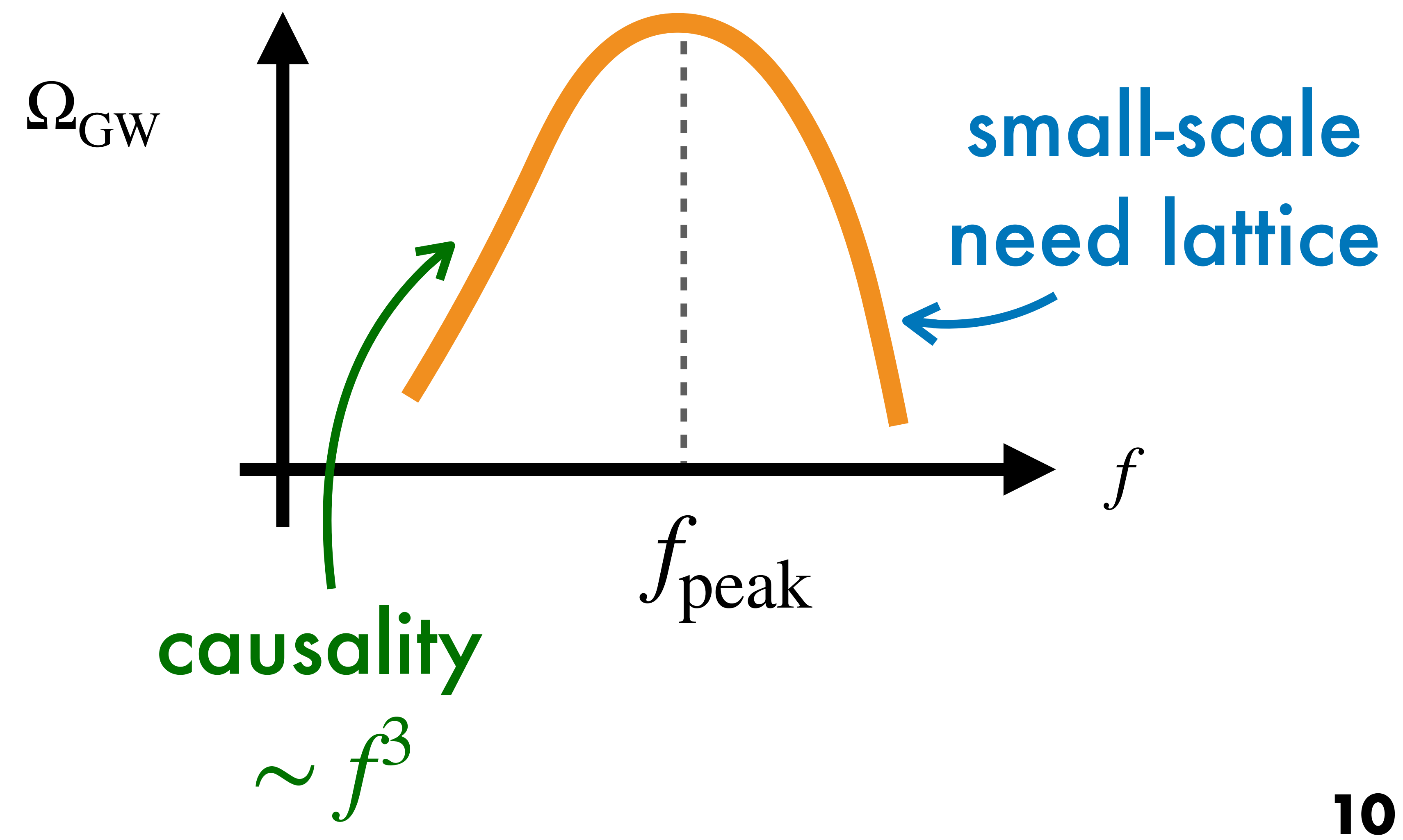
Today fraction on energy density in GW

$$\Omega_{\rm GW}h^2 \simeq 3\times 10^{-10}\left(\frac{2\ {\rm nHz}}{f_{\rm GW}}\right)^8 \left(\frac{\dot{\theta}_p}{\rm MeV}\right)^4 \left(\frac{\phi_p}{10\ {\rm MeV}}\right)^4 \left(\frac{r_p}{20}\right)^6 \left(\frac{10}{g_*(T_p)}\right) \quad \text{(PTA)}$$

$$\simeq 4\times 10^{-13}\left(\frac{0.1\ {\rm Hz}}{f_{\rm GW}}\right)^8 \left(\frac{\dot{\theta}_p}{100\ {\rm GeV}}\right)^4 \left(\frac{\phi_p}{10^8\ {\rm GeV}}\right)^4 \left(\frac{r_p}{20}\right)^6 \left(\frac{200}{g_*(T_p)}\right). \quad \text{(LISA)}$$

The shape could broadening due to scattering that change A'-momentum distribution.

Expect very polarized GW



II. Relic dark photons from axion rotation

The dark-photon mass determines whether they are non-relativistic or relativistic.

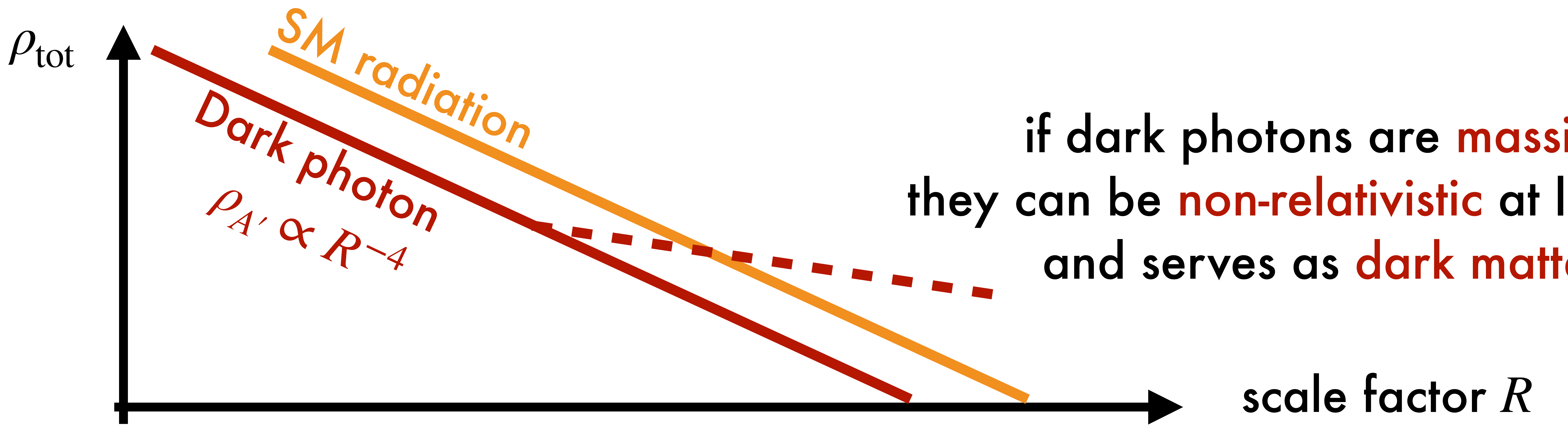
A. (nearly) massless case: $m_{A'} \approx 0$

Dark photons contributes to the dark radiation.

$\Delta N_{\rm eff}$ constraint bounds the energy density of dark photons $\rho_{A'}$

$$\frac{\rho_{\rm GW}}{\rho_R} \simeq \frac{\rho_{A'}^2}{H_p^4 M_{\rm pl}^4} \simeq \frac{r_p^4 \rho_\theta^2}{k_{TI}^4 M_{\rm pl}^4}$$

$$\Delta N_{\rm eff} = \frac{4 r_p g_*(T_p)}{7\sqrt{3}} \sqrt{\frac{\rho_{\rm GW}}{\rho_R}} \simeq 0.1 \left(\frac{\Omega_{\rm GW} h^2}{10^{-10}}\right)^{\frac{1}{2}} \left(\frac{r_p}{20}\right) \left(\frac{g_*(T_p)}{10}\right)^{\frac{7}{6}} .$$

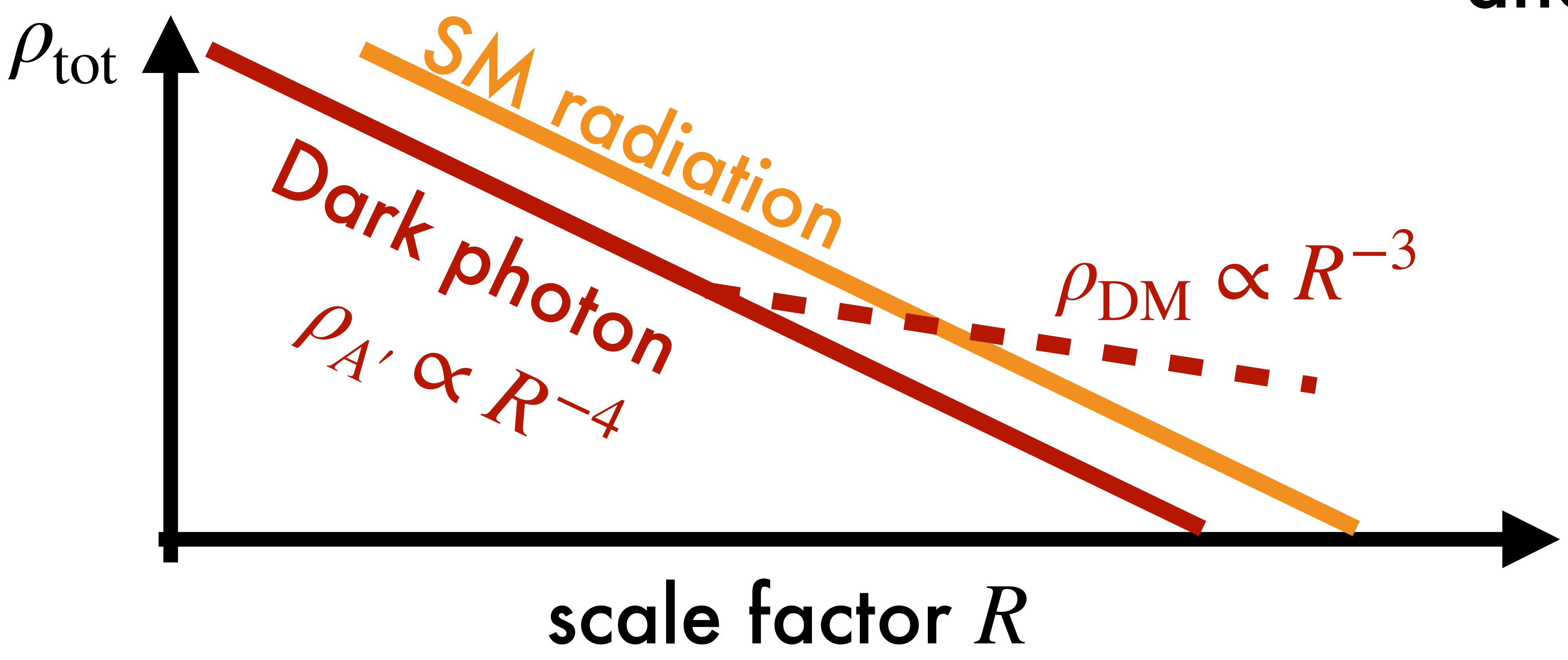


if dark photons are massive,
they can be non-relativistic at late-times
and serves as dark matter.

Note: axion fluctuation from dark photons will also contribute to dark radiation,
but will not exceed the above bound. Need lattice for precise results

II. Relic dark photons from axion rotation (cont.)

B. massive case: dark photons turns non-relativistic at temperature T_{NR} and could be dark matter.



$$\rho_{\rm DM}(T_{NR}) \simeq \rho_{A',p}\left(\frac{T_{NR}}{T_p}\right)^4$$

with observed value

$$\rho_{\rm DM}/s = 0.44 \text{ eV}$$

$$T_{\rm NR} = \frac{4}{\sqrt{3}}\left(\frac{\rho_{\rm DM}}{s}\right)\left(\frac{g_*(T_p)}{g_*(\text{eV})}\right)^{\frac{1}{3}}\left(\frac{\rho_R}{\rho_{\rm GW}}\right)^{\frac{1}{2}}$$

$$\simeq 4 \text{ keV}\left(\frac{10^{-14}}{\Omega_{\rm GW}h^2}\right)^{\frac{1}{2}}\left(\frac{20}{r_p}\right)\left(\frac{g_*(T_p)}{10}\right)^{\frac{1}{6}},$$

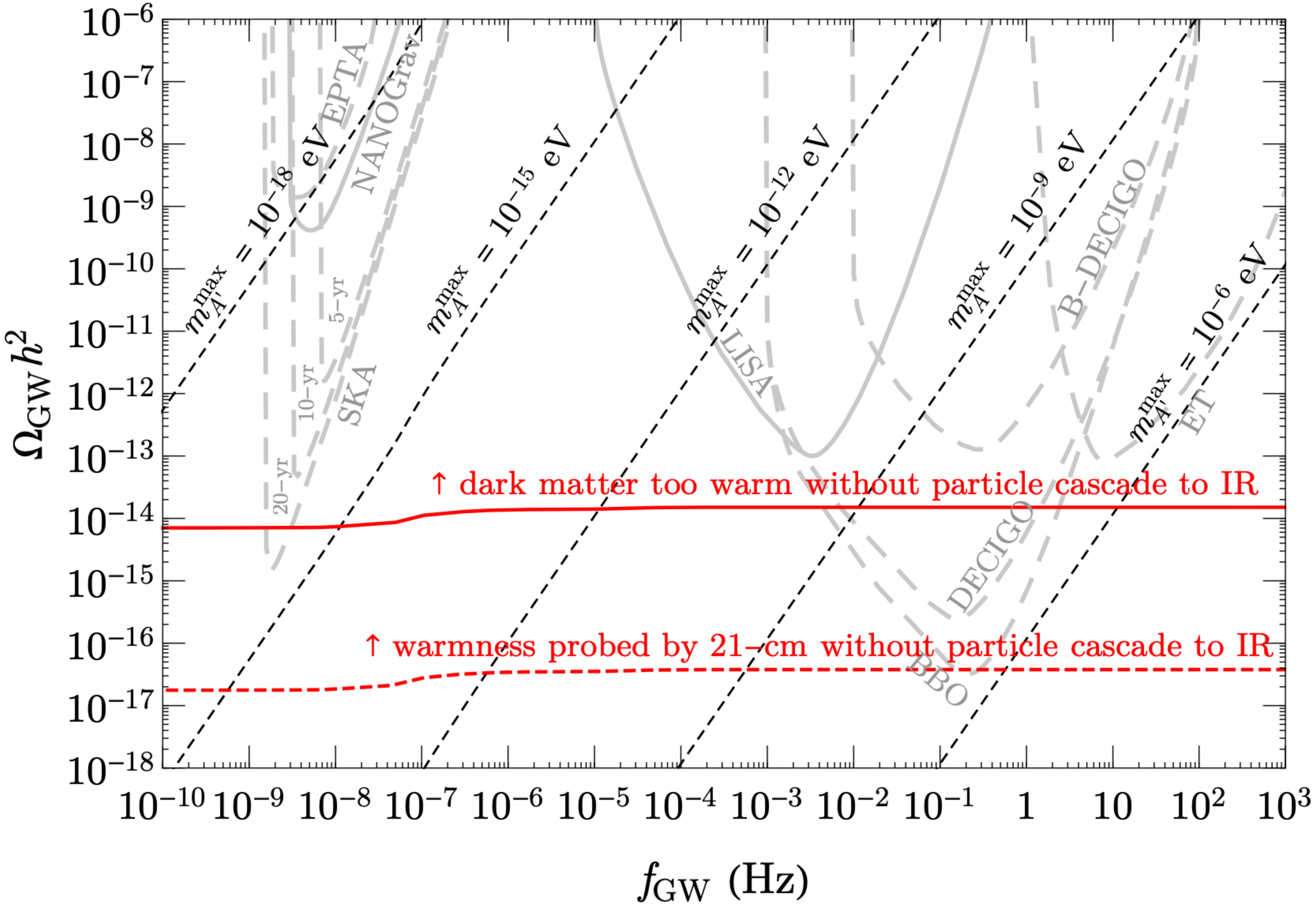
(stronger $\Omega_{\rm GW}$, larger $\rho_{A'}$, smaller T_{NR})

The warmness constraints: $T_{NR} > 5$ keV (Lyman-α), 100 keV (21-cm)

By knowing T_{NR}, we know the mass of dark photon dark matter.
Dark photon becomes non-relativistic when

$$k_{\rm peak}(T_{NR}) = k_{\rm TI}\left(\frac{T_{NR}}{Tp}\right) = m_{A'} \quad \Rightarrow \quad m_{A'} \simeq 10^{-8} \text{ eV}\left(\frac{10^{-14}}{\Omega_{\rm GW}h^2}\right)^{\frac{1}{2}}\left(\frac{f_{\rm GW}}{0.1 \text{ Hz}}\right)\left(\frac{20}{r_p}\right)\left(\frac{g_*(T_p)}{200}\right)^{\frac{1}{6}}.$$

GW signals and dark photon dark matter



Note: here the scattering via axion-dark photon coupling is neglected, but it could change the distribution of dark photons.
E.g. increasing in $\rho_{A'}$ and relaxing warmness constraints.

Concrete model (example)

So far, the story of radial damping/thermalization is neglected.

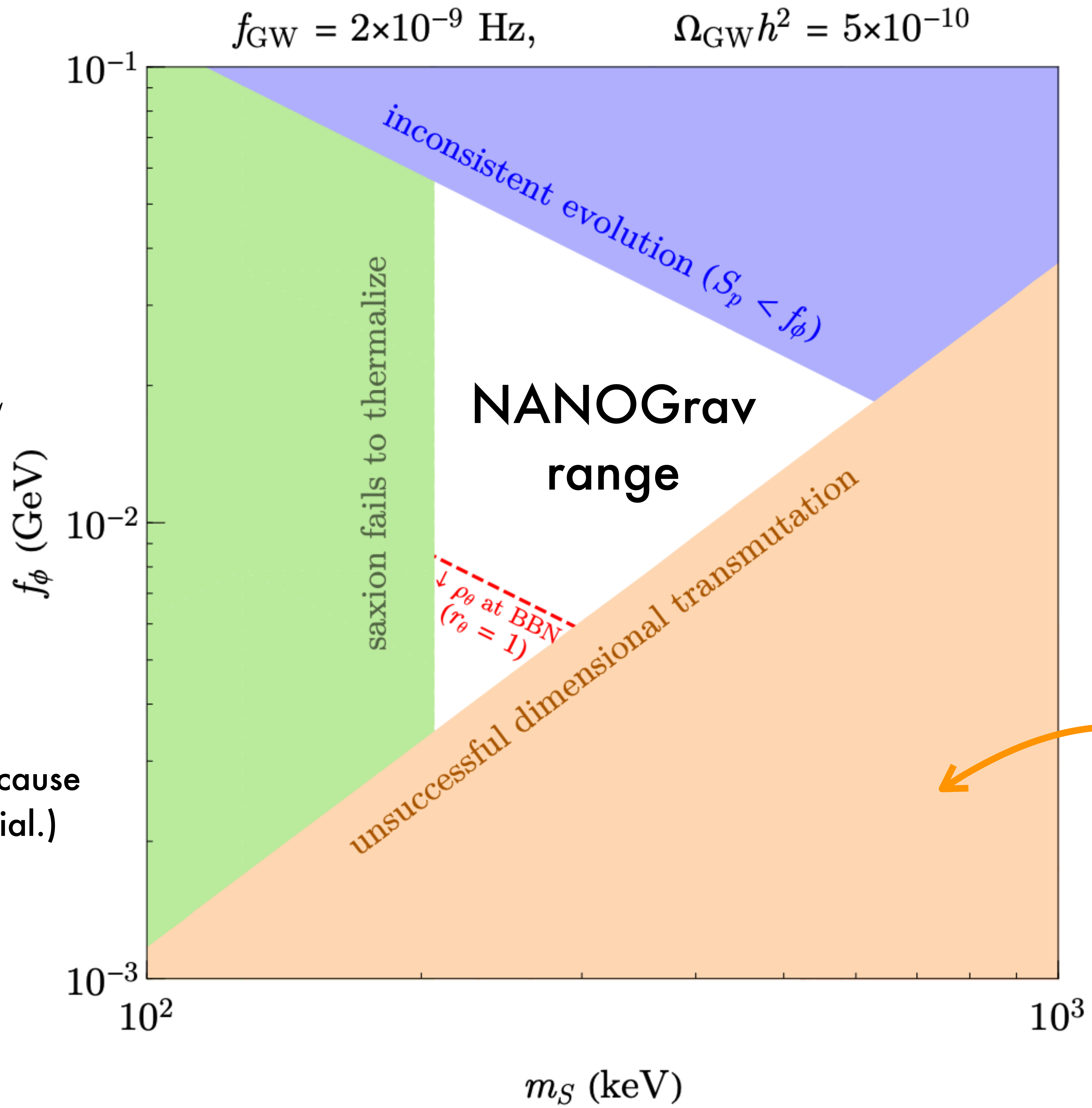
Assume PQ field interacts with SM charged leptons $L, \bar{L}$

$$W = (y_L P \ell + m_L L)\bar{L},$$

induces radial damping rate

$$\Gamma_{\rm th} \simeq \alpha_2 y_L^2 T,$$

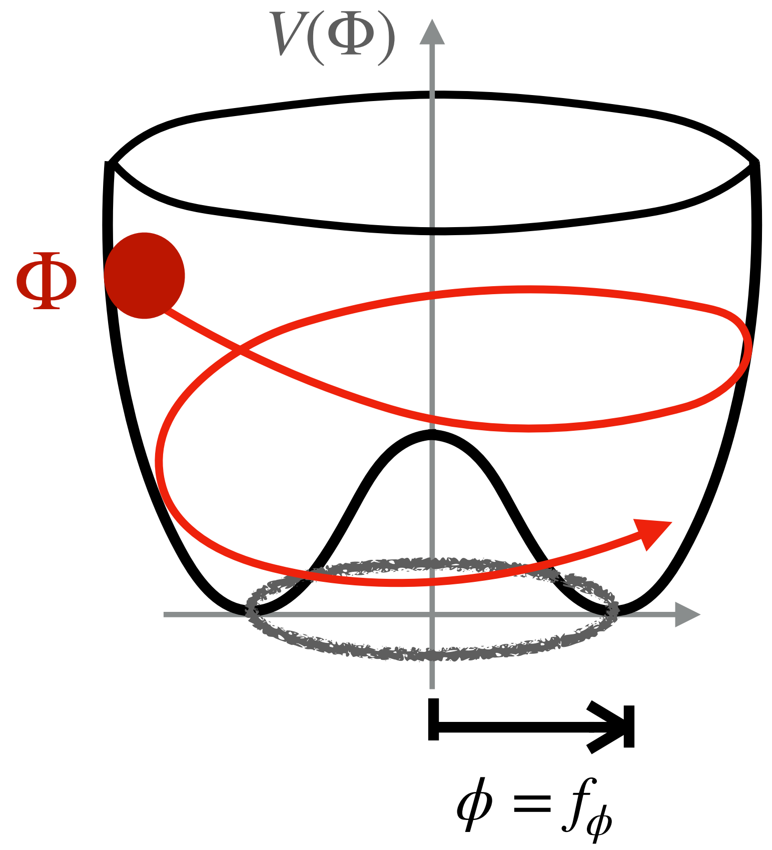
(fail to thermalize when large y_L is required because this could lead to thermal correction of potential.)



Using SUSY-model with a running soft-mass

$$V = m_P^2 |P|^2 + \frac{y_\psi^2 m_{\tilde{\psi}}^2}{8\pi^2} |P|^2 \log\left(\frac{y_\psi^2 |P|^2}{\mu^2}\right),$$

In summary...



A dynamics of rotating axion field provides interesting phenomenology:

baryon asymmetry by "Axiogenesis" model, "kinetic misalignment" mechanism

By coupled to dark photons, axion in circular orbit can induce the explosive dark-photon production via tachyonic instability.

In contrast to the oscillating axion, e.g. "Audible axion":

$\Rightarrow$ the rotational velocity $\dot{\theta}$ does not change sign:

dark photons (and GW) are highly asymmetric in helicity (and polarization).

$\Rightarrow$ the axion-dark photon coupling need not to be large for efficient production.

Expected peak GW signal

with $f_{\rm GW} \propto \sqrt{\dot{\theta}}$ and $\Omega_{\rm GW} \propto \rho_\theta^2$.

The relic dark photons can be dark radiation/matter and their properties can be related to GW signal.