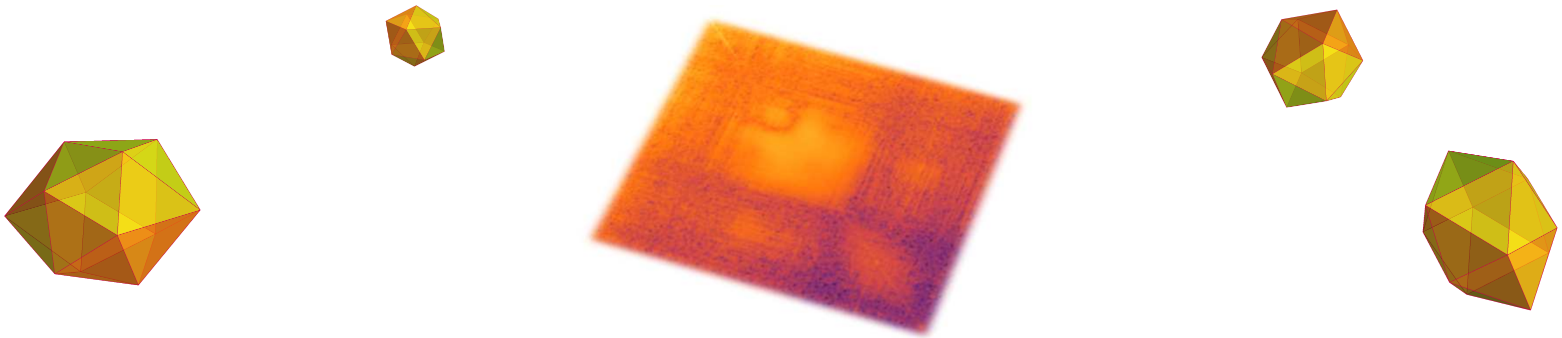
Introduction

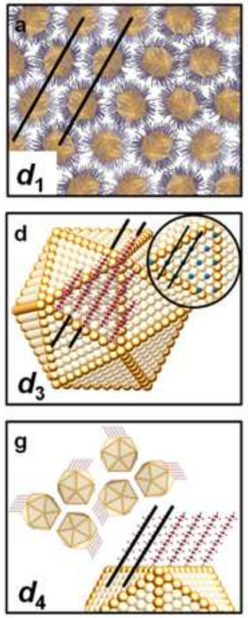
to Fluctuation X-ray Scattering

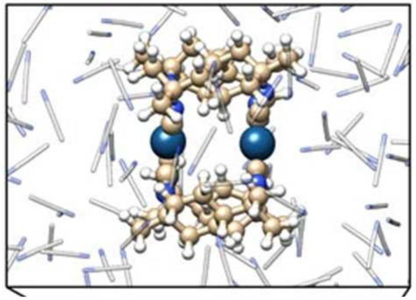
Ruslan Kurta

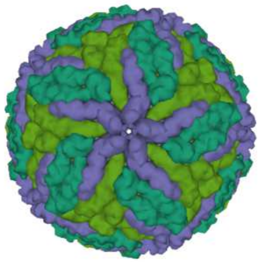
European XFEL



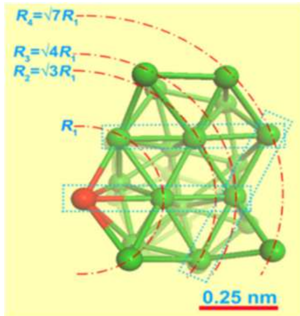
Structure and dynamics of non-crystalline materials



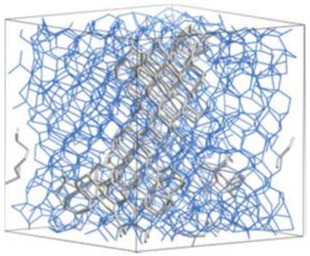


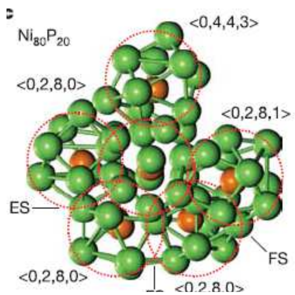


- Amorphous and partially ordered solids
- Solutions, colloids
- Nano- & bio-particles

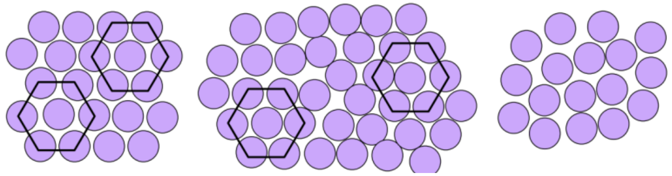


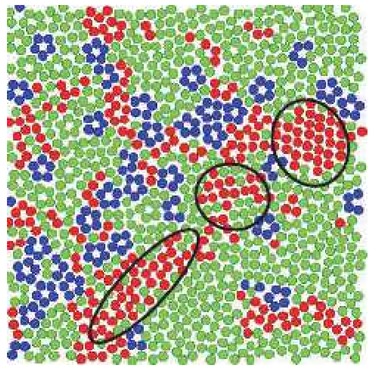
- Short and medium-range order
- Bond-orientational order
- Temporal & spatial heterogeneity
- Correlations between different length scales





- Phase transitions
- Structure dynamics

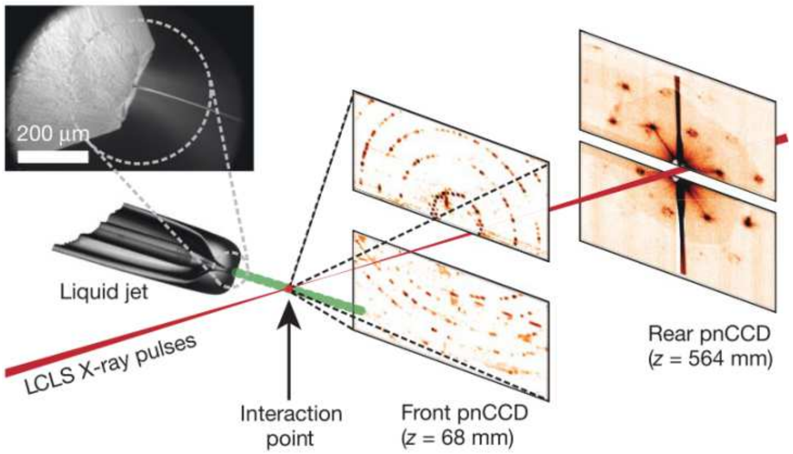




Y.Q. Cheng, E. Ma, Prog. Mat. Sci. 56, 379 (2011)
H.W. Sheng et al., Nature 439, 419 (2006)
H. Shintani, H. Tanaka, Nature Mat. 9, 324 (2010)
X.J. Liu et al., Phys. Rev. Lett. 105, 155501 (2010)
M.M.J. Treacy, K.B. Borisenko, Science 385, 950 (2012)
G.F. Mancini et al., Nano Lett. 16, 2705 (2016)
T. B. van Driel et al., Nat. Comm. 7, 13768 (2016)

X-ray scattering for biological structure determination

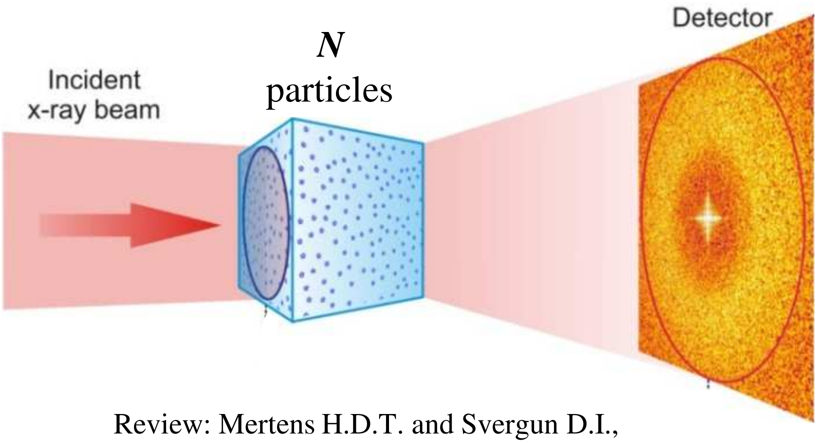
Serial Crystallography



H. N. Chapman *et al.*, Nature 470, 73 (2011)

- High resolution structure determination
- Requires crystalline samples

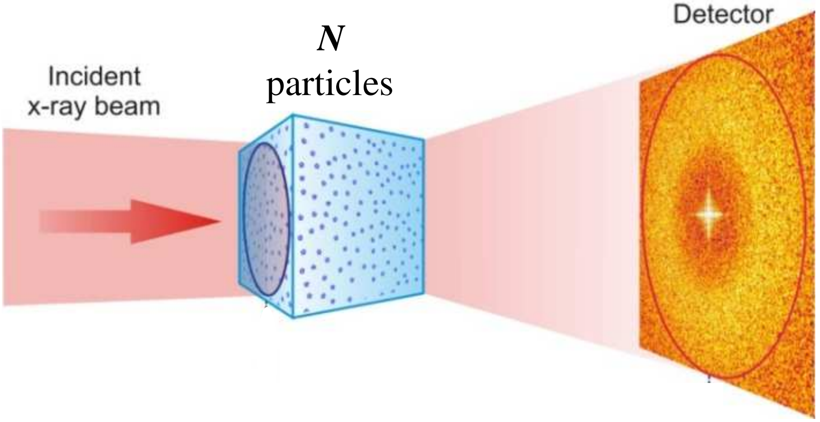
Small Angle X-ray Scattering (SAXS)



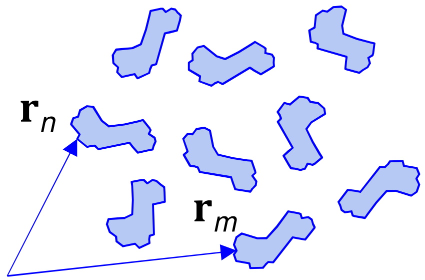
Review: Mertens H.D.T. and Svergun D.I.,
Journal of Structural Biology 172, 128-141 (2010)

- Low resolution structure determination
- Relatively simple sample preparation
- Near native environment

Biological structure determination from solution scattering: SAXS



- Disordered ensemble of reproducible particles (N >>1)



$$I(\mathbf{q}) = \sum_n \sum_m F_n(\mathbf{q}) F_m^*(\mathbf{q}) e^{i\mathbf{q}(\mathbf{r_n}-\mathbf{r_m})}$$

where F_n is the form-factor of the n-th particle.

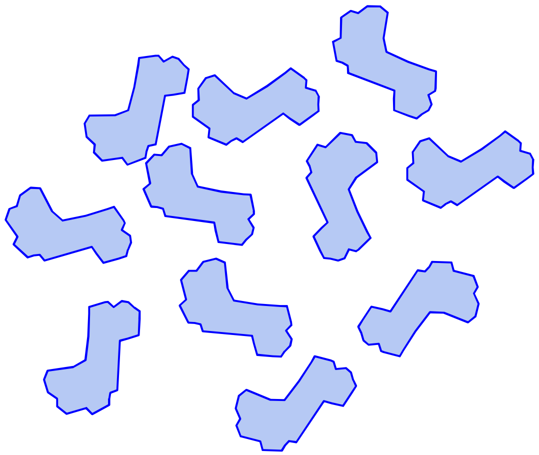
$$I(\mathbf{q}) = \sum_n |F_n(\mathbf{q})|^2 + \sum_n \sum_{m, m\neq n} F_n(\mathbf{q}) F_m^*(\mathbf{q}) e^{i\mathbf{q}(\mathbf{r_n}-\mathbf{r_m})}$$

- Dilute limit:

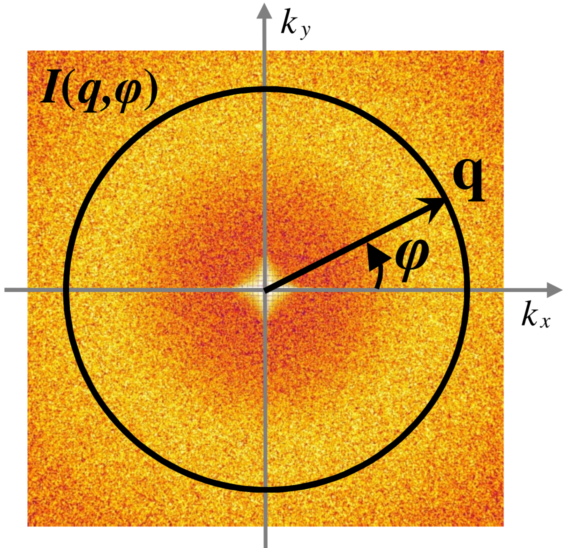
$$I_{\text{SAXS}}(q) \propto N \langle |F(\mathbf{q})|^2 \rangle,$$

where $\langle \ \rangle$ denotes orientational averaging.

Structure determination using biological SAXS

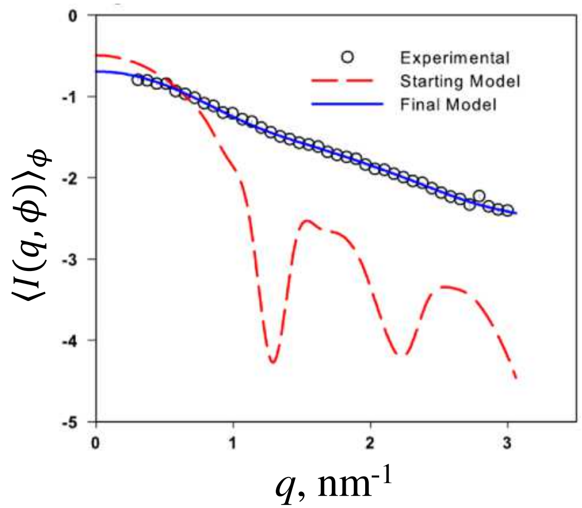


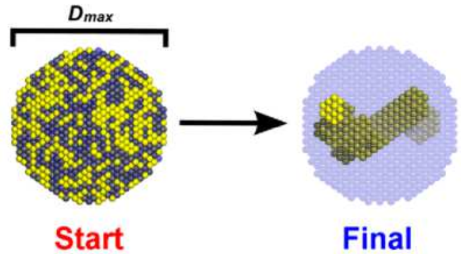
Diffraction pattern



theory: $I_{\mathrm{SAXS}}(q) \propto N \, \langle |F(\mathbf{q})|^2 \rangle,$

practice: $\langle I(q,\varphi) \rangle_{\varphi} = \frac{1}{2\pi} \int_0^{2\pi} I(q,\varphi) d\varphi$

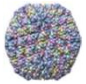




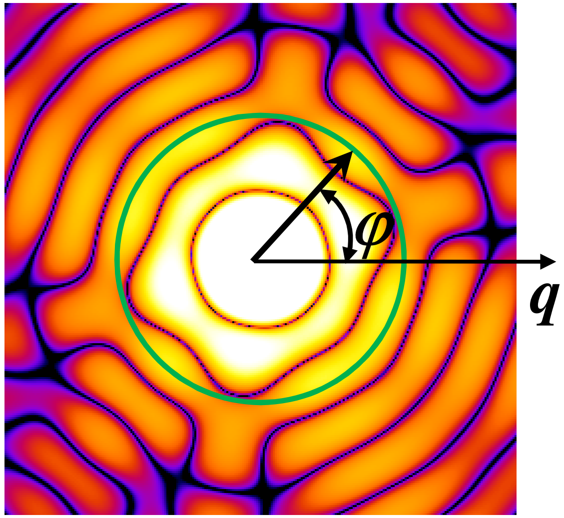
Bead modelling

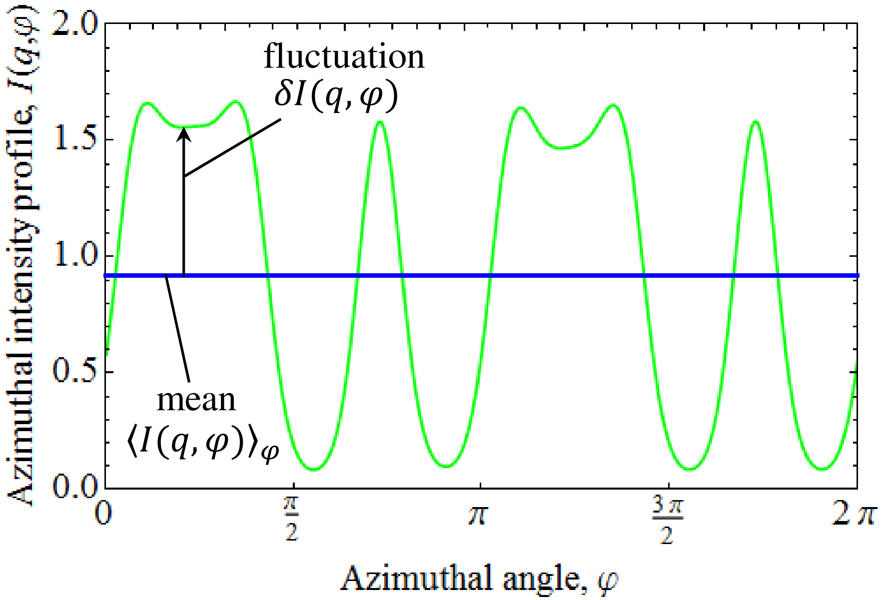
Mertens H.D.T. and Svergun D.I.,
Journal of Structural Biology 172, 128-141 (2010)

Fluctuation X-ray Scattering (FXS) : intensity fluctuations



Scattering pattern





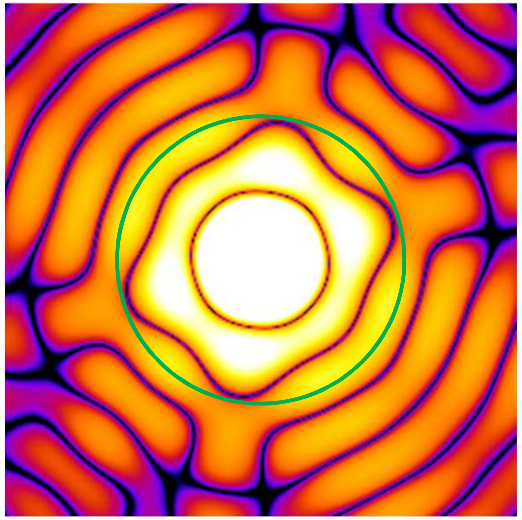
$I(q,\varphi) = \langle I(q,\varphi)\rangle_\varphi + \delta I(q,\varphi)$ - azimuthal intensity profile

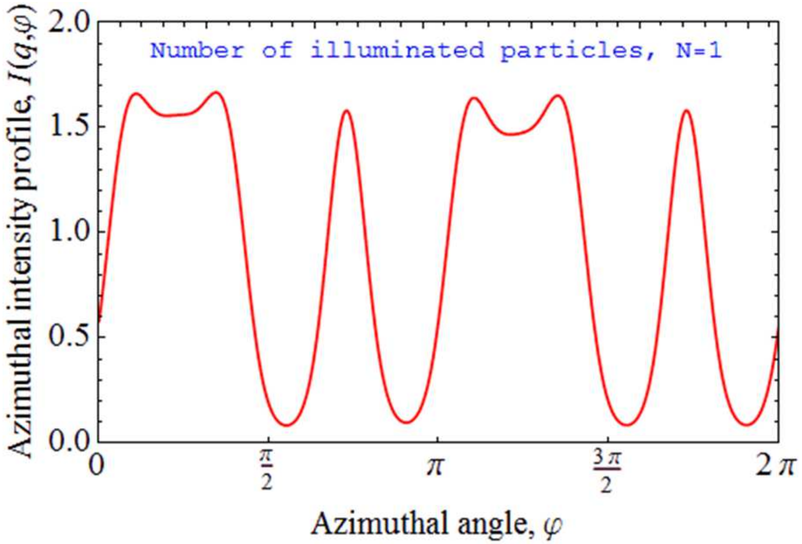
$\langle I(q,\varphi)\rangle_\varphi$ - average intensity

$\delta I(q,\varphi)$ - intensity fluctuation, $\langle \delta I(q,\varphi)\rangle_\varphi = 0$

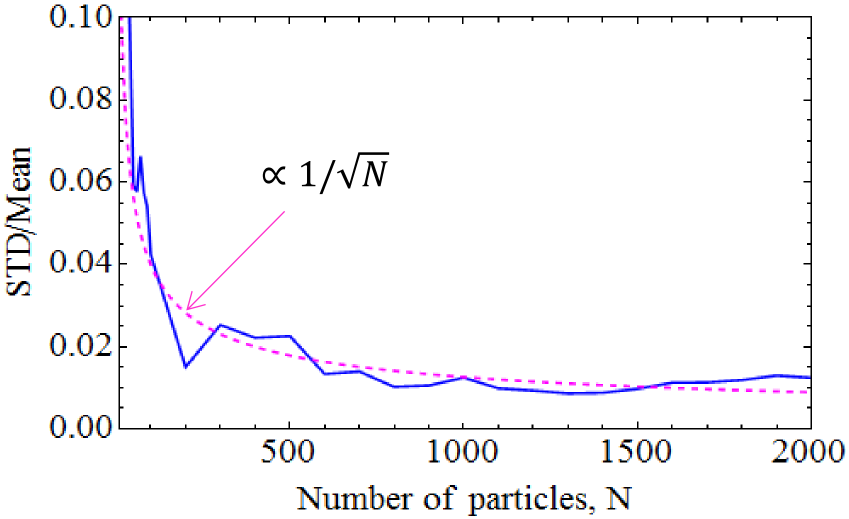
Intensity fluctuations are small for a many-particle system

Diffraction pattern

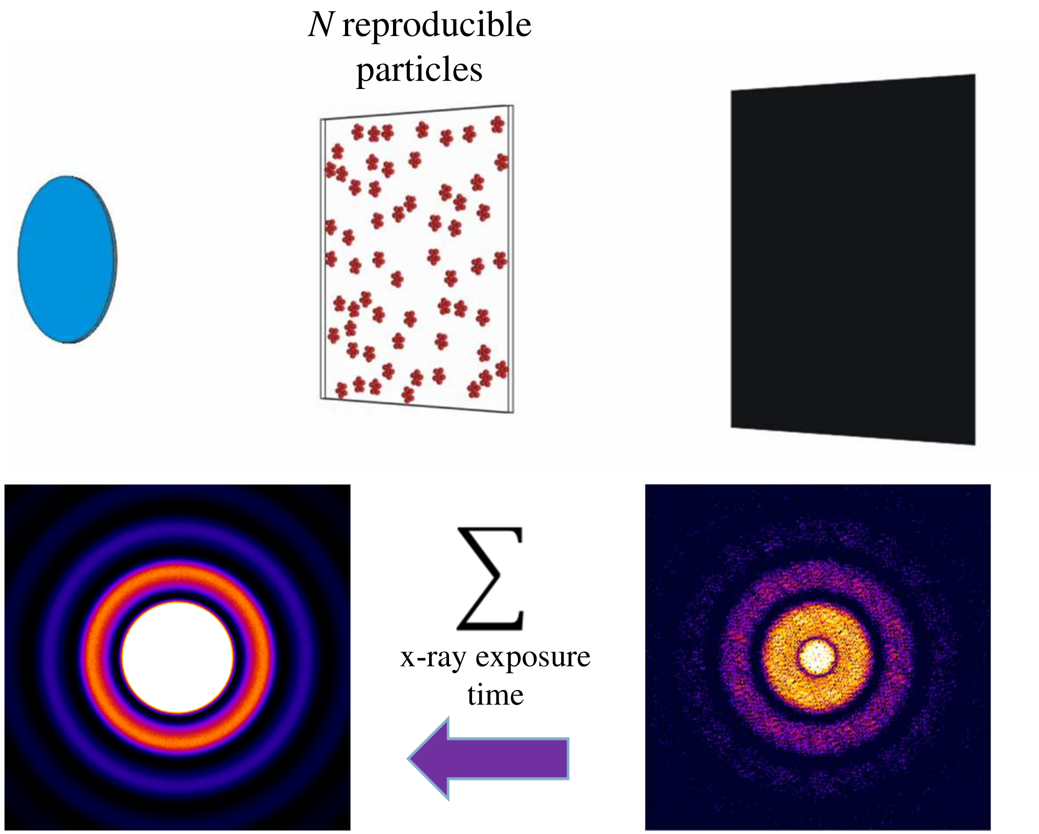




- Average intensity growth proportionally to N, while fluctuations are proportional to $\sqrt{N}$

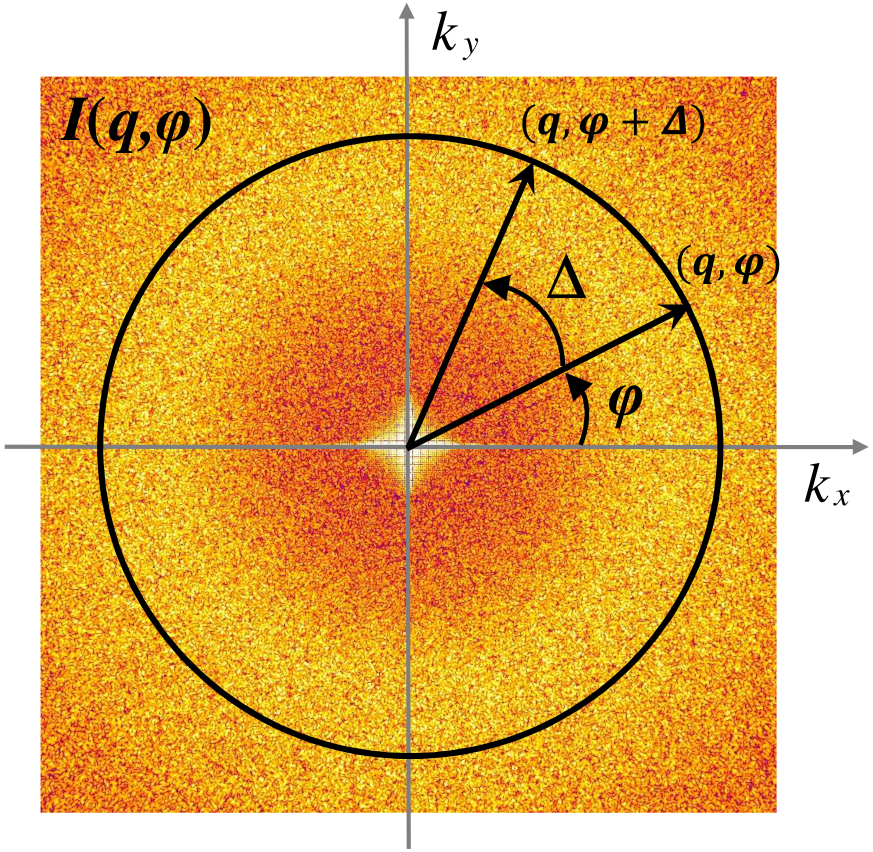


Scattering of ultrashort x-ray pulses from a disordered ensemble of particles



- Rotational diffusion of particles smears intensity fluctuations → this sets the limit for pulse duration.

Angular Cross-Correlation Function (CCF)



$|\mathbf{q}_1| = |\mathbf{q}_2| = q$

Angular CCF can be defined as:

$$C(q,\Delta) = \langle I(q,\varphi) I(q,\varphi+\Delta)\rangle_\varphi \quad (1)$$

$$= \frac{1}{2\pi}\int_0^{2\pi} I(q,\varphi) I(q,\varphi+\Delta)\, d\varphi,$$

where $I(q,\varphi)$ is the scattered intensity.

Assume $I(q,\varphi) = \langle I(q,\varphi)\rangle_\varphi + \delta I(q,\varphi)$, then

$$C(q,\Delta) = \langle \delta I(q,\varphi)\,\delta I(q,\varphi+\Delta)\rangle_\varphi + \langle I(q,\varphi)\rangle_\varphi^2 \quad (2)$$

Angular cross-correlation functions can be used to extract structural information from the measured intensity fluctuations.

Z. Kam, Macromolecules 10, 927 (1977)
Review: R.P. Kurta, M. Altarelli, I.A. Vartanyants, Adv. Chem. Phys. 161, Ch.1 (2016)

Analysis of the angular CCFs

- Model-based analyses
- Iterative Phase Retrieval (MTIP)
- Algebraic phasing

- Spherical harmonics

  Z. Kam, Macromolecules 10, 927 (1977)
  D. Starodub et al., Nat. Comm. 3, 1276 (2012)
  J. J. Donatelli et al., PNAS 112 (33), 10286 (2015)
  K. Pande et al., PNAS 115 (46), 11772 (2018)

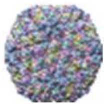
- Icosahedral harmonics

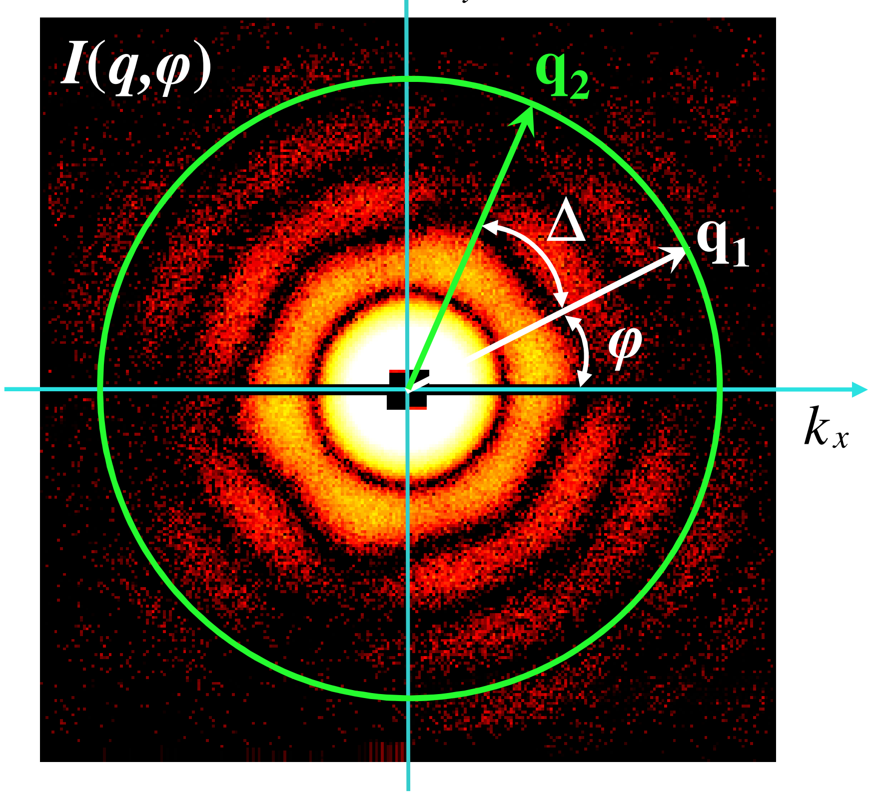
  D. K. Saldin et al., Opt. Express 19, 17318 (2011)

- Circular harmonics

  M. Altarelli, et al., Phys. Rev. B 82, 104207 (2010)
  D. K. Saldin et al., Phys. Rev. B 81, 174105 (2010)
  B. Pedrini et al., Nat. Comm. 4, 1647 (2013)
  R.P. Kurta et al., Phys. Rev. Lett. 119, 158102 (2017)

Fourier analysis of angular cross-correlation functions





$$|\mathbf{q}_1| = |\mathbf{q}_2| = q$$

Two-point cross-correlation function (CCF):

$$C(q,\Delta) = \langle I(q,\varphi) I(q,\varphi+\Delta) \rangle_{\varphi} \qquad (1)$$

Fourier components:

$$C^n(q) = \frac{1}{2\pi} \int_0^{2\pi} C(q,\Delta) \exp(-in\Delta) d\Delta \qquad (2)$$

$$I^n(q) = \frac{1}{2\pi} \int_0^{2\pi} I(q,\varphi) \exp(-in\varphi) d\varphi \qquad (3)$$

$$\langle I(q,\varphi) \rangle_{\varphi,M} = \langle I^0(q) \rangle_M$$ - conventional SAXS (4)

$$\langle C^n(q) \rangle_M = \left\langle |I^n(q)|^2 \right\rangle_M$$ -“higher-order SAXS” (5) for n>0

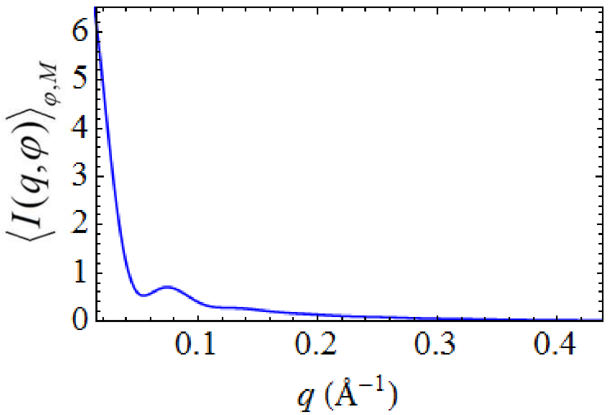
$\langle \ \rangle_M$ - averaging over diffraction patterns (orientational averaging).

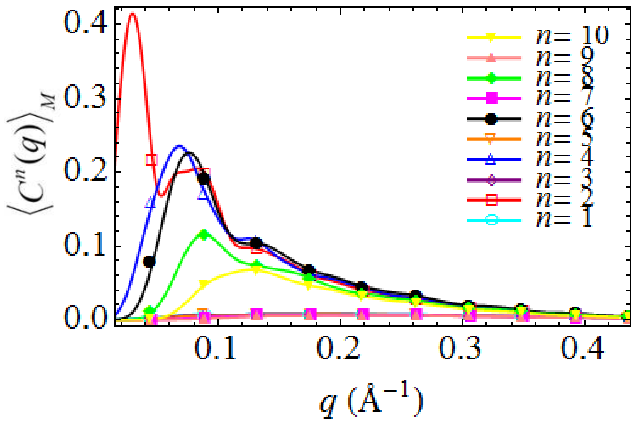
R.P. Kurta, M. Altarelli, I.A. Vartanyants, Adv. Chem. Phys. 161, Ch.1 (2016)

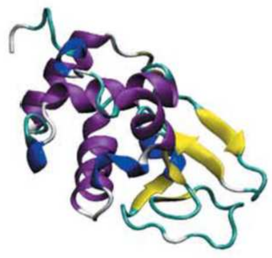
Extending conventional SAXS

$$\langle I(q,\varphi)\rangle_{\varphi,M} = \langle I^0(q)\rangle_M$$

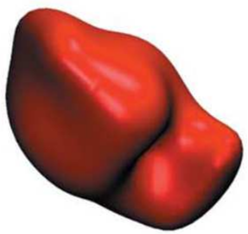
$$\langle C^n(q)\rangle_M \Rightarrow \langle |I^n(q)|^2 \rangle_M, n = 1,2,3...$$

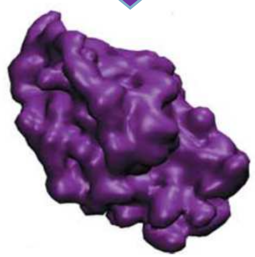






Bead model, *etc*

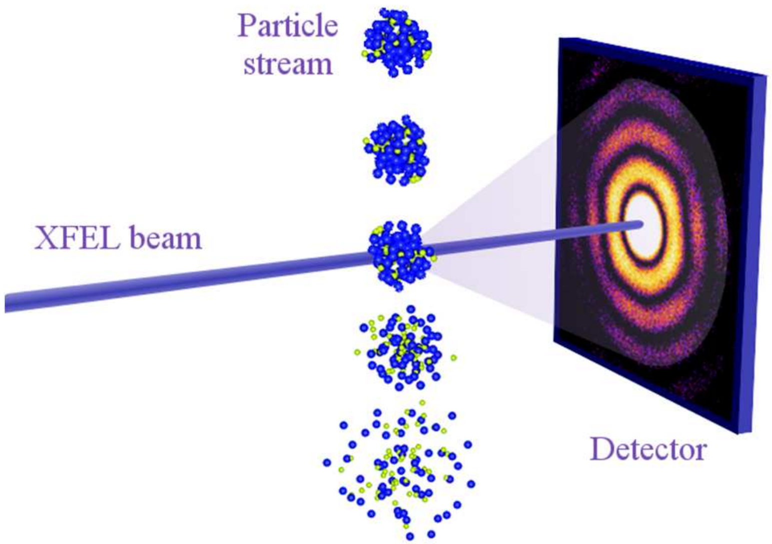




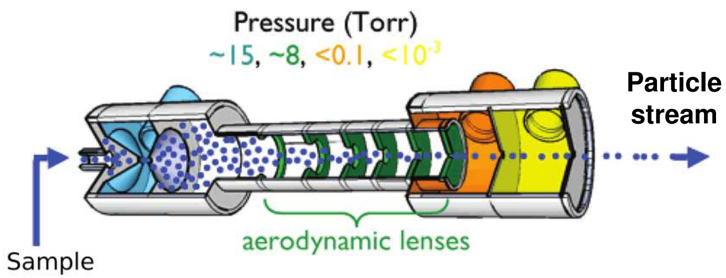
Conventional SAXS analysis

Higher-order SAXS analysis (FXS)

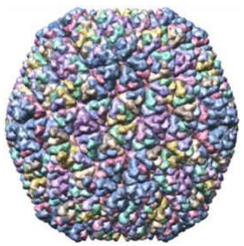
X-ray scattering experiment on aerosolized virus particles @ LCLS



AMO instrument @ LCLS
Photon energy: E=1.6keV
Sample-detector distance: 581 mm
Detector: pnCCD
Sample injection: aerodynamic lens stack system with a GDVN



RDV



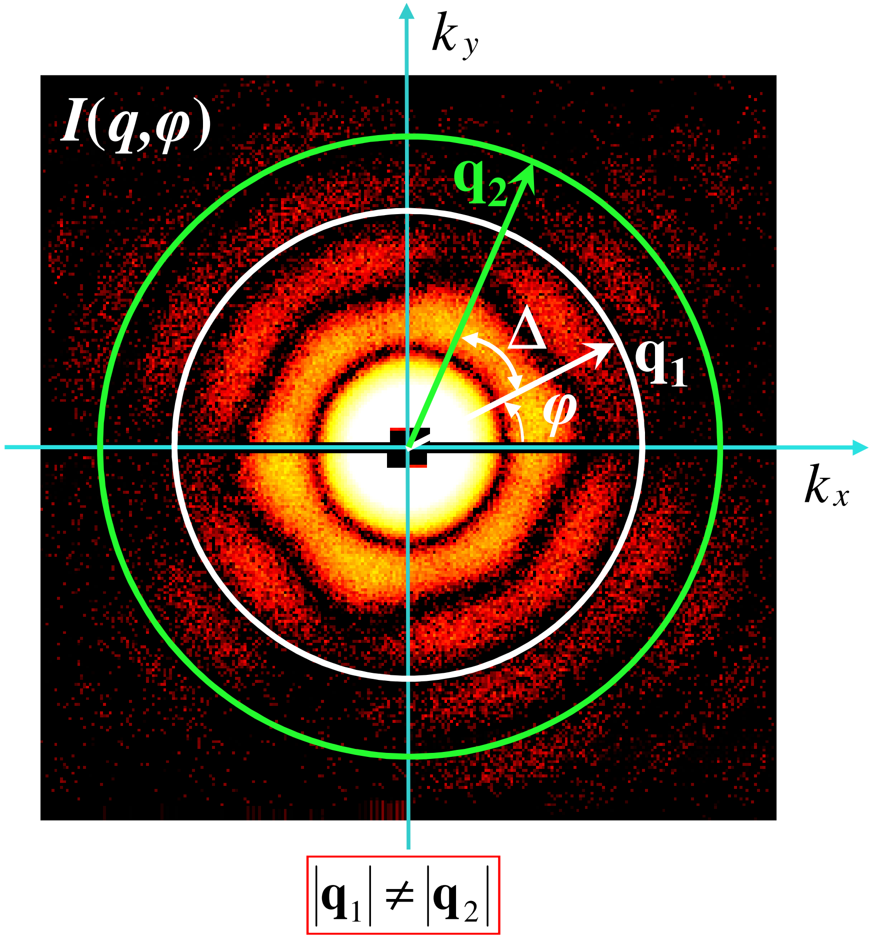
The Rice Dwarf Virus was the first studied plant pathogenic virus; an icosahedral double shelled virus, 70-75 nm in diameter.

PR772

PR772 is a bacteriophage, i.e. a virus that infects and replicates within bacteria. PR772 is used to standardize nomenclature for virus-retentive filters.

H. K. N. Reddy *et al.*, Scientific Data 4, 170079 (2017)
M. Bogan *et al.*, Nano Lett. 8, 310 (2008)

Extended set of cross-correlation functions



Two-point CCF defined at two different resolution rings:

$$C(q_1, q_2, \Delta) = \langle I(q_1, \varphi) I(q_2, \varphi + \Delta) \rangle_\varphi \qquad (1)$$

Fourier components (FCs) of the CCF:

$$C^n(q_1, q_2) = \frac{1}{2\pi} \int_0^{2\pi} C(q_1, q_2, \Delta) \exp(-in\Delta) d\Delta \qquad (2)$$

Orientational average:

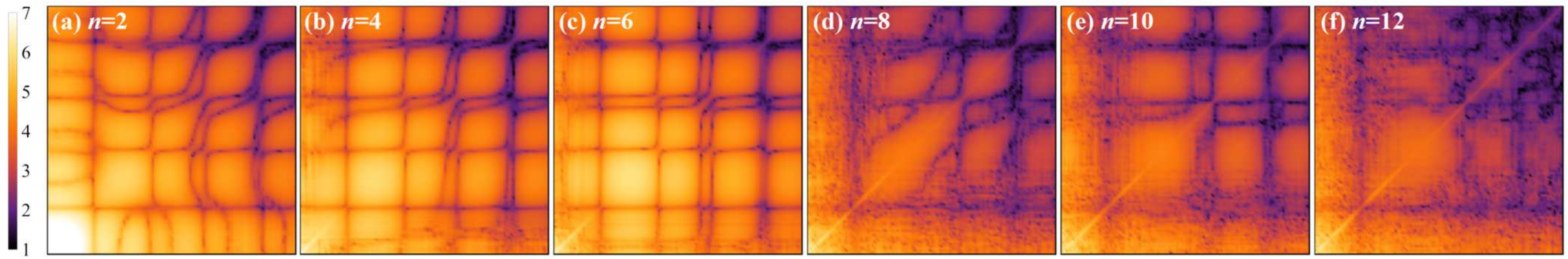
$$\langle C^n(q_1, q_2) \rangle \qquad (3)$$

Z. Kam, Macromolecules 10, 927 (1977)
R.P. Kurta, M. Altarelli, I.A. Vartanyants, Adv. Chem. Phys. 161, Ch.1 (2016)

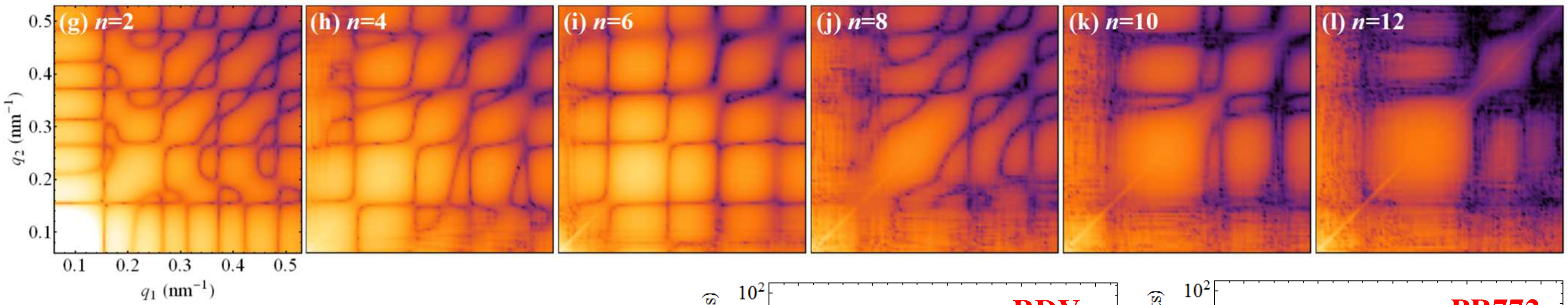
Experimental correlation maps for RDV and PR772 viruses

$|\langle C^n (q_1, q_2)\rangle|$, for n=2,4,6,8,10,12

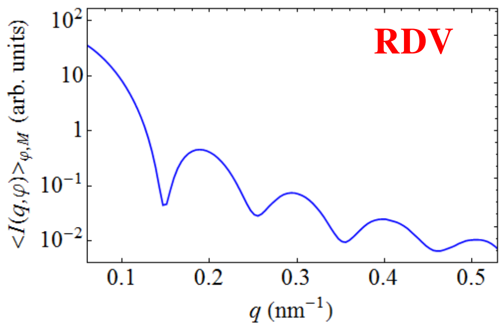
RDV

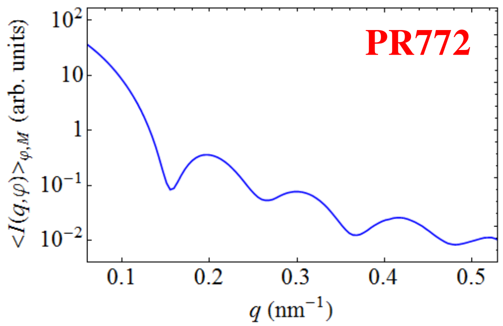


PR772



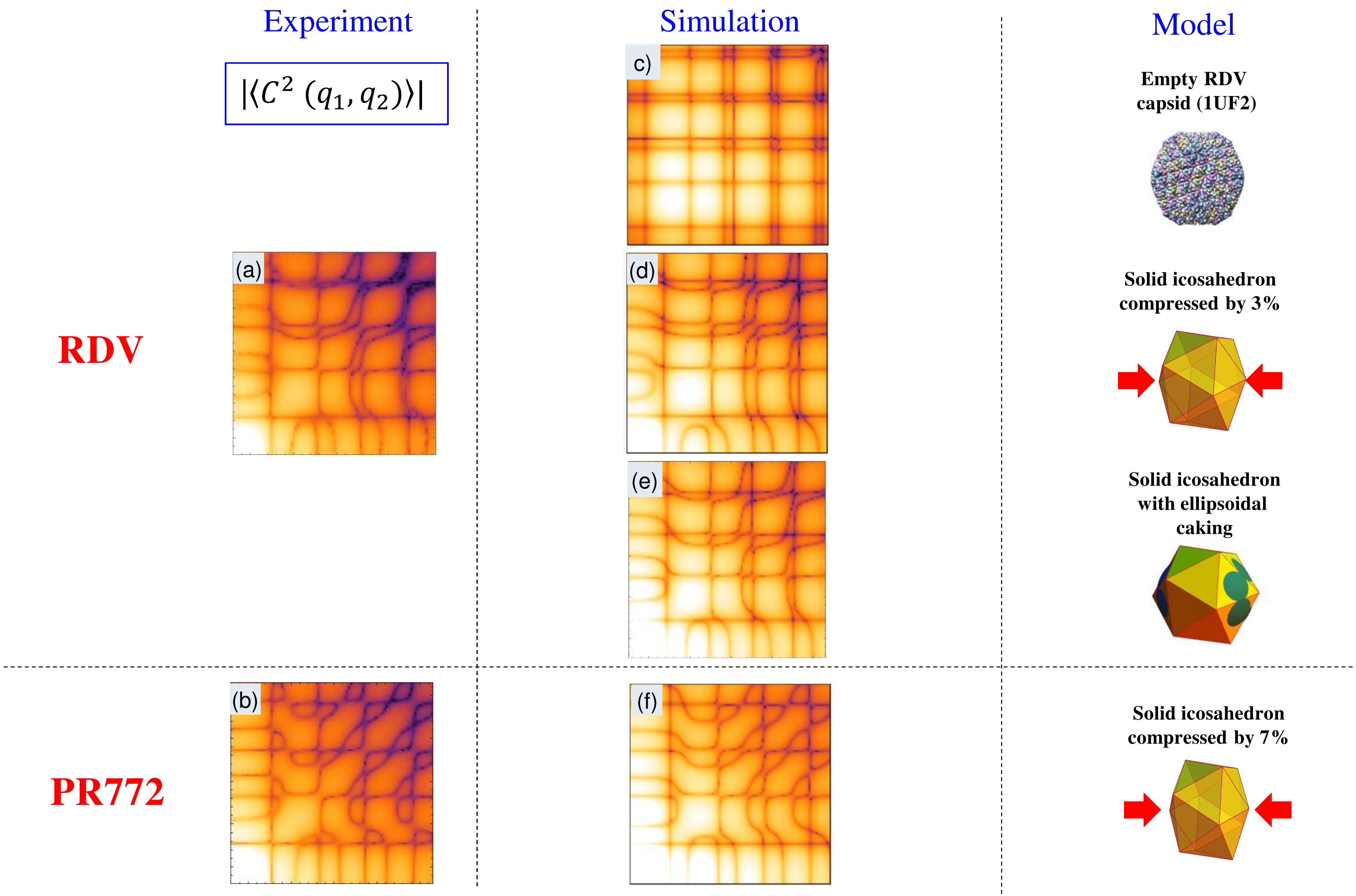
Conventional SAXS:



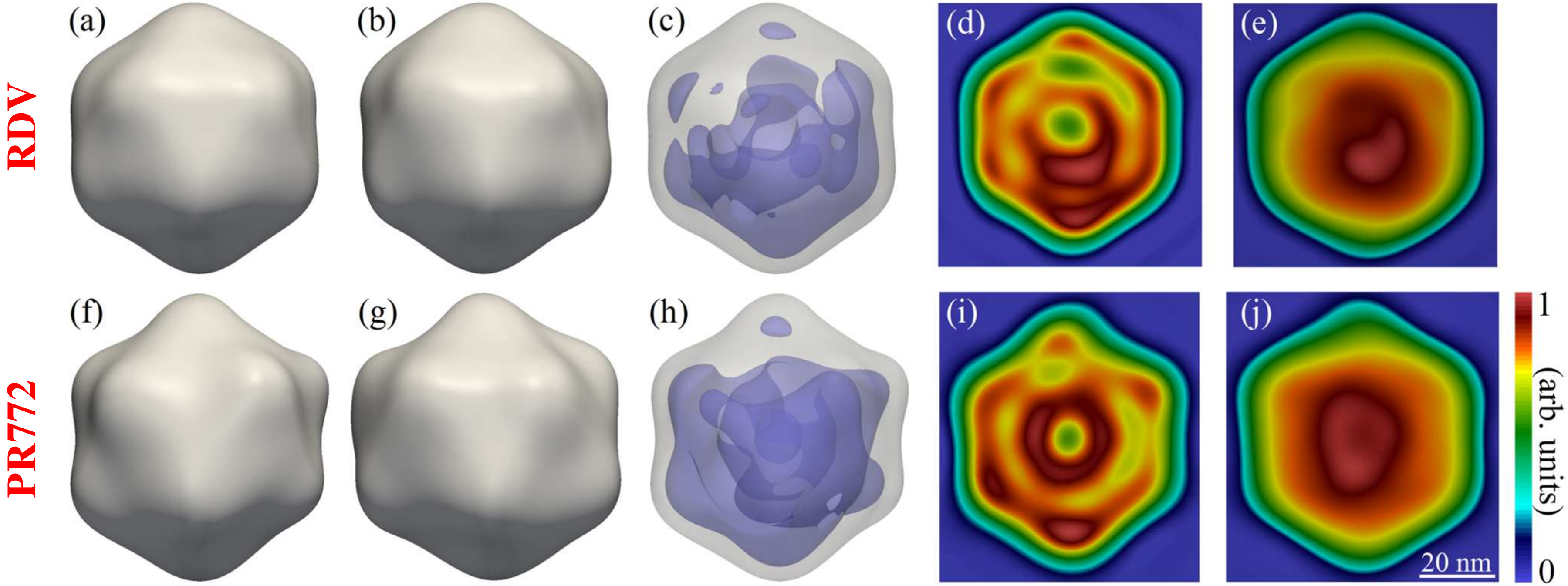


R.P. Kurta *et al.*, PRL 119, 158102 (2017)

Model-based structure analysis



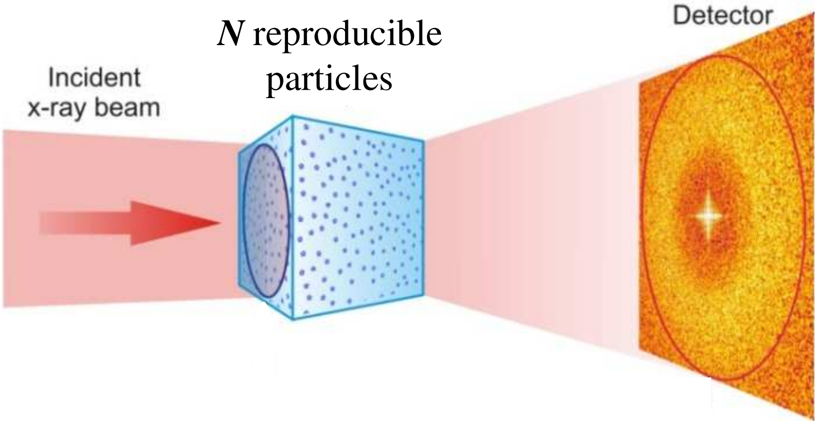
Multitiered Iterative Phasing (MTIP) reconstructions from single-particle scattering



Reconstructed images of RDV and PR772.
Two different views (corresponding to a 72 degree rotation about the top axis) of the reconstructed RDV (a,b) and PR772 (f,g) particles, as well as density plots showing nonuniformities in the internal distribution of material inside RDV (c) and PR772 (h), 2D slices through the center of the reconstructed densities for RDV (d) and PR772 (i), and 2D projections of the reconstructed densities for RDV (e) and PR772 (j).

J. J. Donatelli, P. H. Zwart, J. A. Sethian, PNAS 112 (33), 10286 (2015)
R.P. Kurta *et al.*, PRL 119, 158102 (2017)

Fluctuation X-Ray Scattering (FXS)



Z. Kam, Macromolecules 10, 927 (1977)

- The aim is to increase information content of x-ray scattering measurements from bioparticles in solution to facilitate structure recovery.
- Employs angular cross-correlation functions to extract structural information from the measured intensity fluctuations.
- Can be applied to a finite number (N≥1) of reproducible particles with a uniform distribution of orientations.
- Requires short x-ray exposures to bypass orientational averaging of fluctuations due to rotational diffusion of particles.

  - “old” approach: freeze solution of particles to slow down their diffusion
  - “modern” approach: illuminate solution with ultrashort intense XFEL pulses.

Similar approaches with different names

Fluctuation X-Ray Scattering (FXS)

Z. Kam, M.H. J. Koch and J, Bordas, PNAS 78, 3559 (1981)
E. Malmerberg et al, IUCrJ 2, 309 (2015)
A.V. Martin, IUCrJ 4, 24 (2017)
J. J. Donatelli, P. H. Zwart, J. A. Sethian, PNAS 112 (33), 10286 (2015)

$$C(q,\Delta) = \langle I(q,\varphi) I(q,\varphi+\Delta)\rangle_{\varphi} \quad (1)$$

$$C(q,\Delta) = \langle \delta I(q,\varphi) \delta I(q,\varphi+\Delta)\rangle_{\varphi} \quad (2)$$

$$C(q,\Delta) = \langle I(q,\varphi) I(q,\varphi+\Delta)\rangle_{\varphi} - \langle I(q,\varphi)\rangle_{\varphi}^{2} \quad (3)$$

$$C(q,\Delta) = \frac{\langle I(q,\varphi) I(q,\varphi+\Delta)\rangle_{\varphi} - \langle I(q,\varphi)\rangle_{\varphi}^{2}}{\langle I(q,\varphi)\rangle_{\varphi}^{2}} \quad (4)$$

Correlated X-Ray Scattering (CXS)

R.A. Kirian, J. Phys. B: At. Mol. Opt. Phys. 45, 223001 (2012)
D. Mendez et al., IUCrJ 3, 420 (2016)
A. Niozu et al., IUCrJ 7, 276 (2020)

Cross-Correlation Intensity Fluctuation Spectroscopy (CCIFS)

N.A. Clark, B.J. Ackerson, A.J. Hurd, Phys. Rev. Lett. 50, 1459 (1983)
B.J. Ackerson, N.A. Clark, Faraday Discuss. Chem. Soc. 76, 219 (1983)

Apertured Cross-Correlation Function (ACCF)

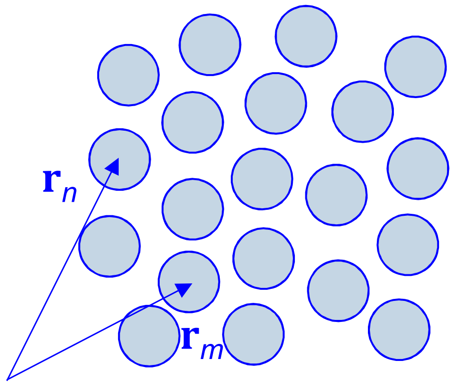
B.J. Ackerson et al., Phys. Rev. A 31, 3183 (1985)
A. S.-Y. Sheu and S. A. Rice, J. Chem. Phys. 129, 124511 (2008)
Z. Krebs et al., J. Chem. Phys. 149, 034503 (2018)

$$I(\mathbf{q}) = \sum_n |F_n(\mathbf{q})|^2 + \sum_n \sum_{m, m\neq n} F_n(\mathbf{q}) F_m^*(\mathbf{q}) e^{i\mathbf{q}(\mathbf{r}_n - \mathbf{r}_m)}$$

X-Ray Cross-Correlation Analysis (XCCA)

P. Wochner et al., PNAS, 106, 11511 (2009)
R.P. Kurta, M. Altarelli, I.A. Vartanyants, Adv. Chem. Phys. 161, Ch.1 (2016)
F. Lehmkühler et al., IUCrJ 5, 354 (2018)
I. A. Zaluzhnyy et al., Materials 12, 3464 (2019)

X-ray scattering from a monoatomic amorphous material

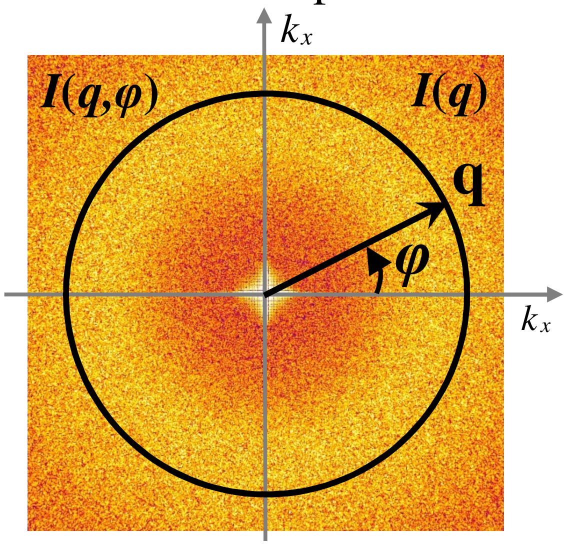


Debye scattering equation: $I(q) = f(q)^2 \sum_n \sum_m \frac{\sin(qr_{nm})}{qr_{nm}}$

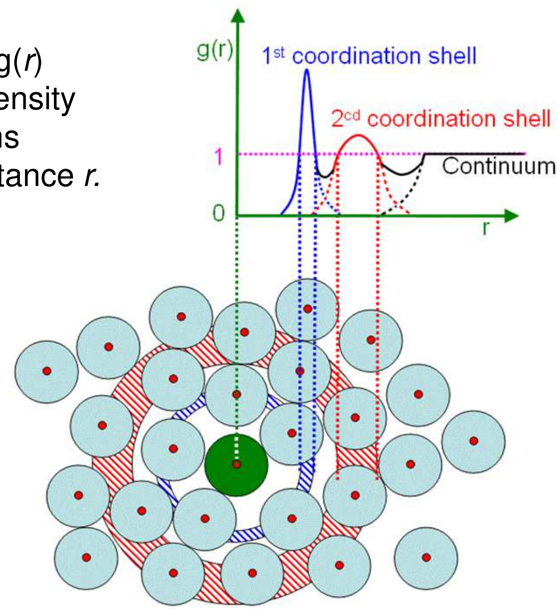
Structure factor: $S(q) = I(q)/Nf(q)^2$

Radial distribution function: $g(r) = 1 + \frac{1}{\langle\rho\rangle 2\pi^2 r}\int_0^\infty q[S(q)-1]\sin(qr)dq$

Diffraction pattern



The RDF function g(*r*) is the probability density of finding two atoms separated by a distance *r*.



J. Als-Nielsen, Elements of Modern X-ray Physics, 2nd Ed. (2011)

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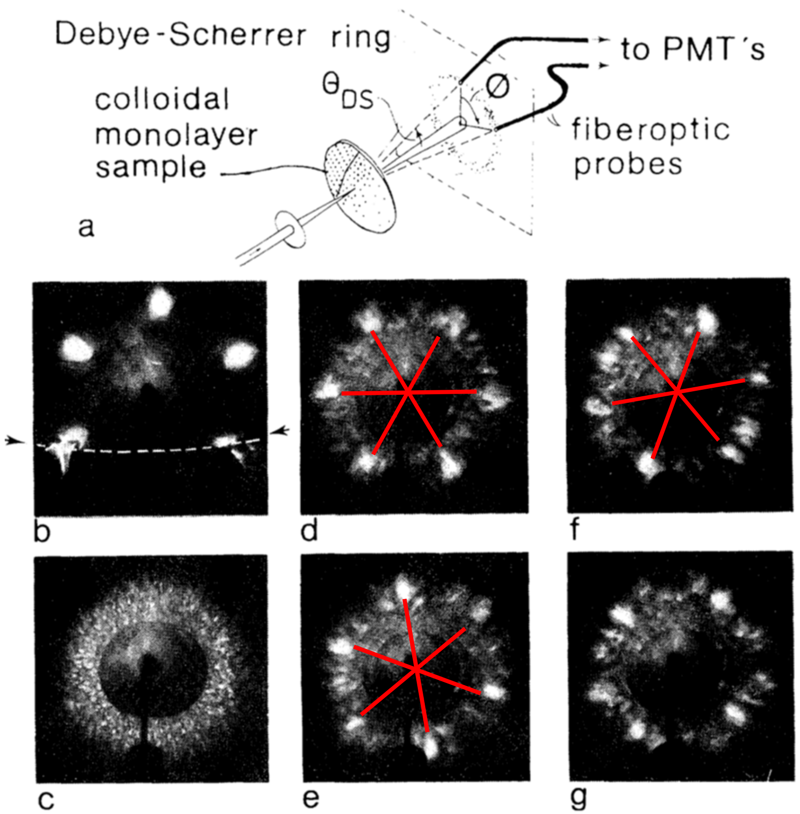
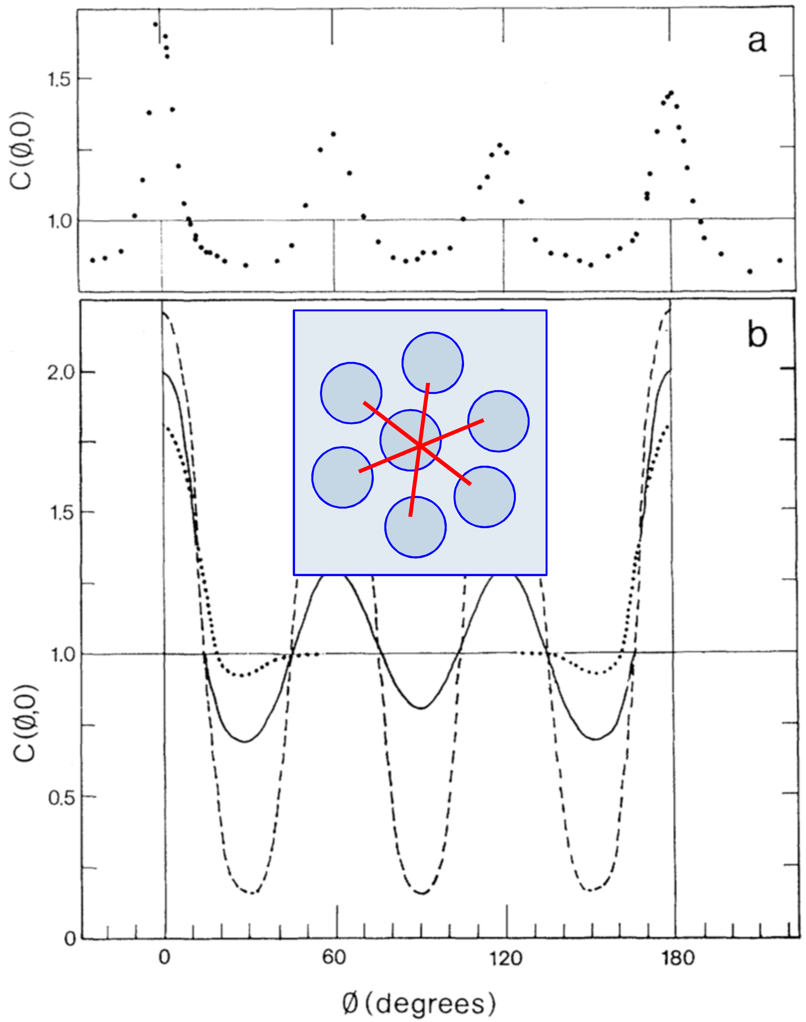
Light scattering from colloidal suspensions shows strong angular correlations (CCIFS)

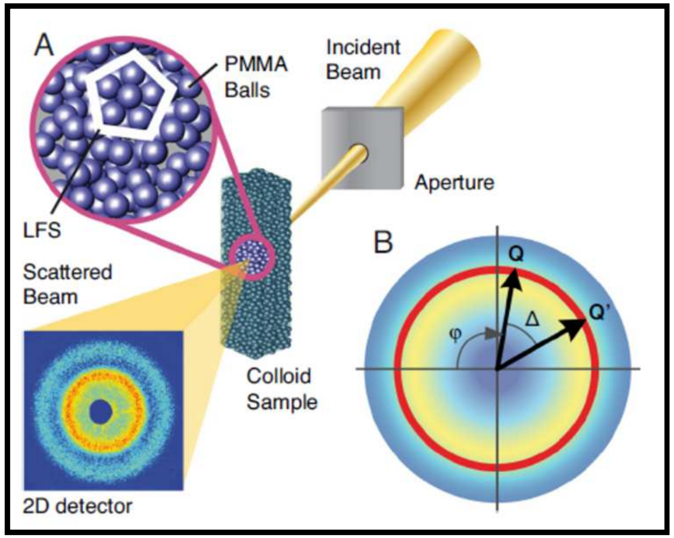


CCF



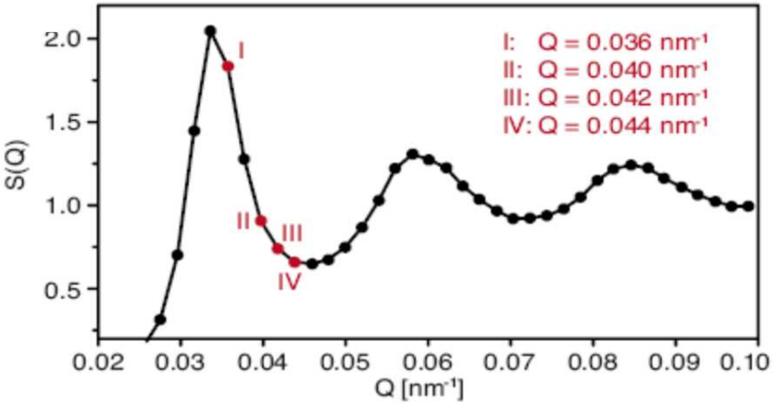
N. A. Clark, B. J. Ackerson, A. J. Hurd, PRL 50, 1459 (1983)

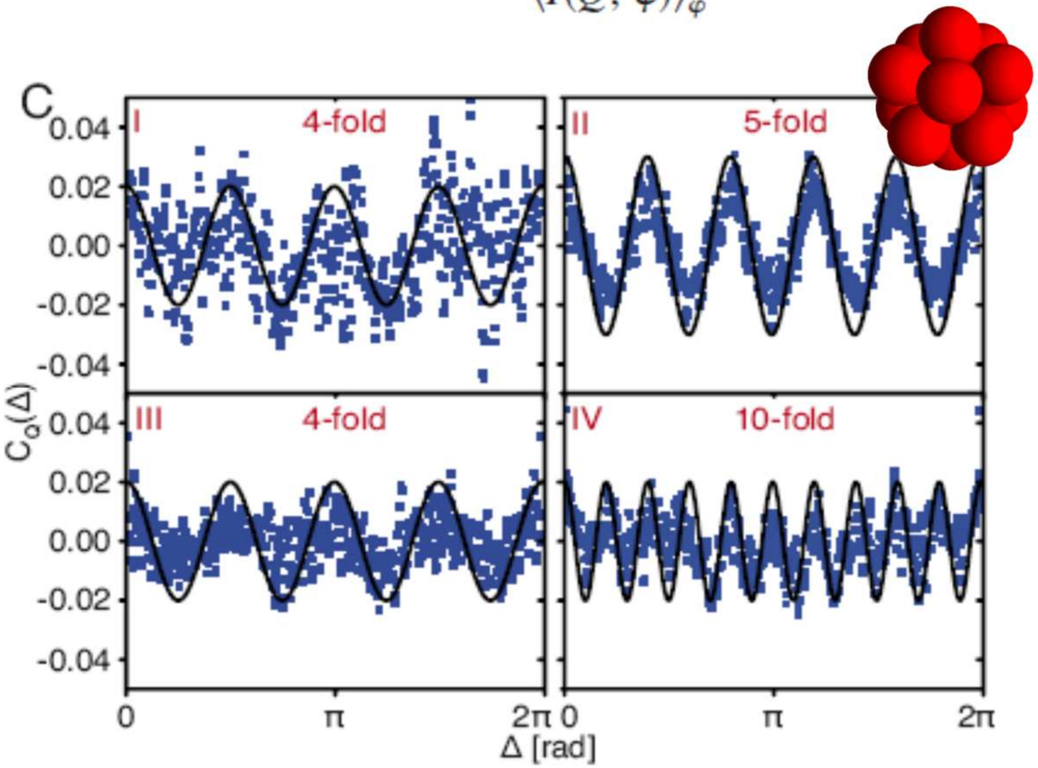
Local structures in a colloidal glass studied by angular x-ray cross-correlation analysis (XCCA)



Normalized CCF:

$$C_Q(\Delta) = \frac{\langle I(Q,\varphi) I(Q,\varphi+\Delta)\rangle_\varphi - \langle I(Q,\varphi)\rangle_\varphi^2}{\langle I(Q,\varphi)\rangle_\varphi^2}$$





Wochner P. *et al*, PNAS, 106, 11511 (2009)

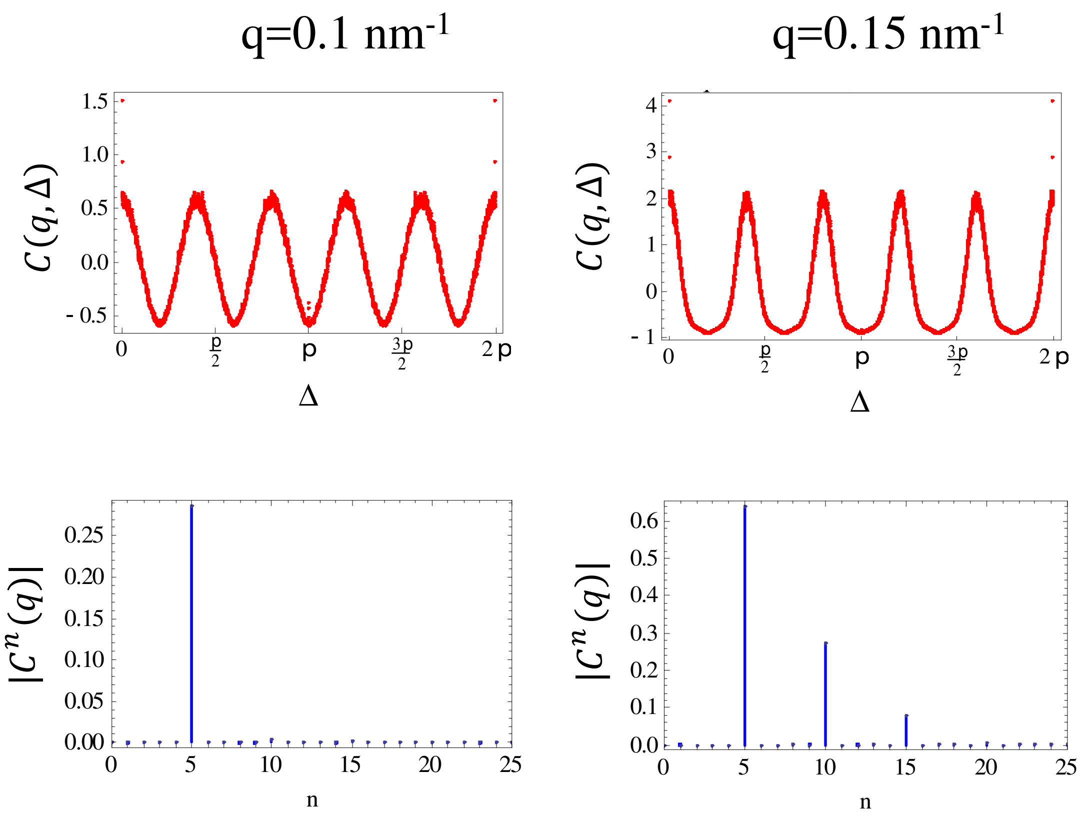
Fourier series expansion of the CCF

Angular cross-correlation function (CCF):

$$C(q,\Delta)$$

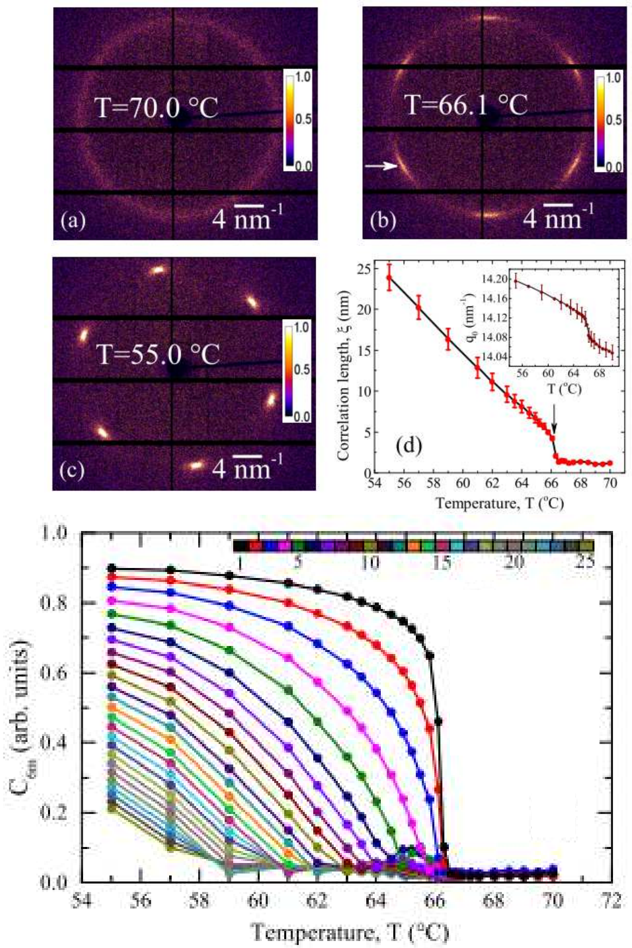
Fourier coefficients of the CCF:

$$C^n(q) = \frac{1}{2\pi}\int_0^{2\pi} C(q,\Delta)\exp(-in\Delta)\,d\Delta$$

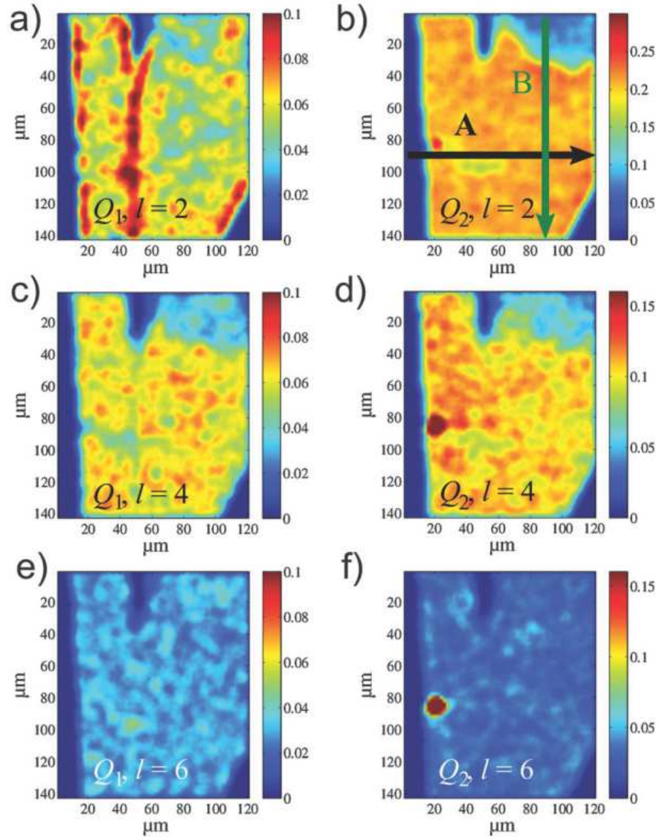


M. Altarelli, R.P. Kurta, I.A. Vartaniants, Phys. Rev. B 82, 104207 (2010)

Analysis of the Fourier coefficients of the CCF



R.P. Kurta, *et al*, Phys. Rev. E 88, 044501 (2013)
I.A. Zaluzhnyy, *et al.*, Phys. Rev. E 91, 042506 (2015)



M. A. Schroer et al., Soft Matter 11, 5465 (2015)

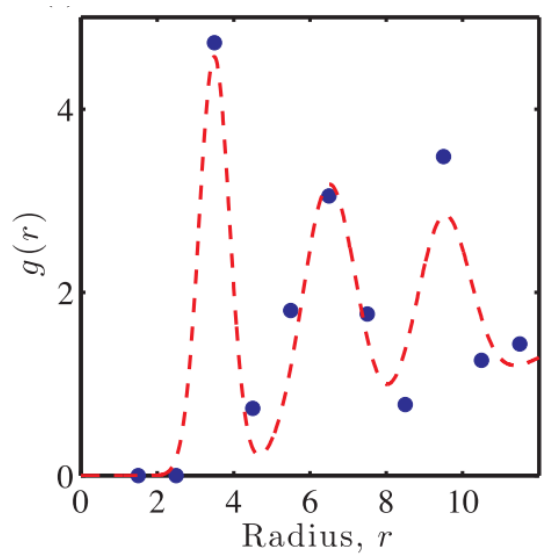
Pair Angle Distribution Function (PADF)

- **Scattered intensity**

↓

- **Pair Distribution Function (PDF)**

$$g(r)$$



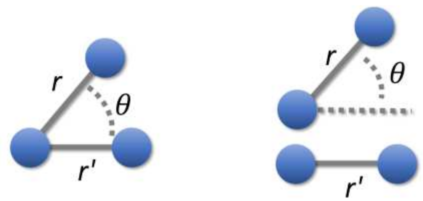
J. Als-Nielsen, Elements of Modern X-ray Physics, 2nd Ed. (2011)

- **Angular Cross-Correlation Function**

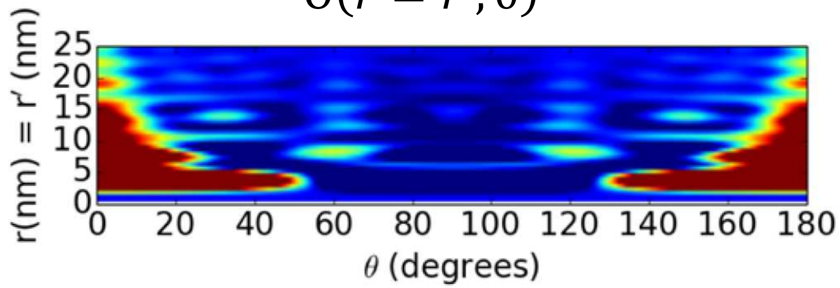
↓

- **Pair Angle Distribution Function (PADF)**

$$\Theta(r, r', \theta)$$



$$\Theta(r = r', \theta)$$



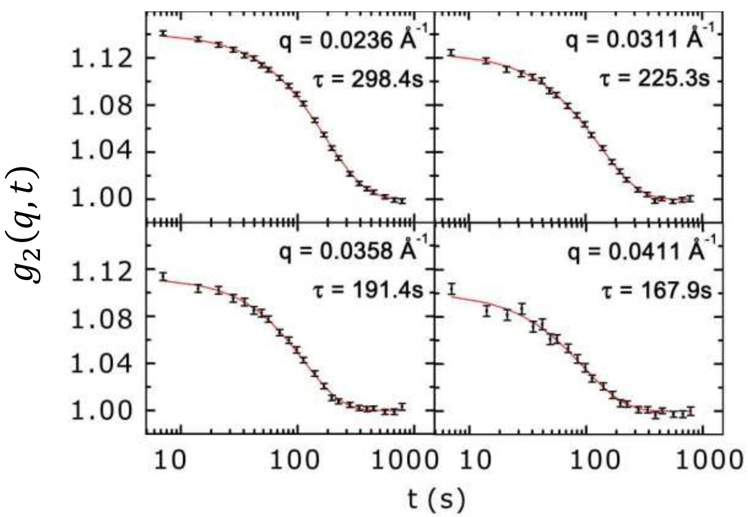
A.V. Martin, IUCrJ 4, 24 (2017)
A.V. Martin, Communication Materials 1, 40 (2020)

XPCS vs XCCA

- **X-Ray Photon Correlation Spectroscopy (XPCS)**
- Time correlation function

$$g_2(q,t) = \frac{\langle I(q,t')I(q,t'+t)\rangle_{t'}}{\langle I(q,t')\rangle_{t'}^2}$$

- Access to dynamics of materials @ different lengthscales



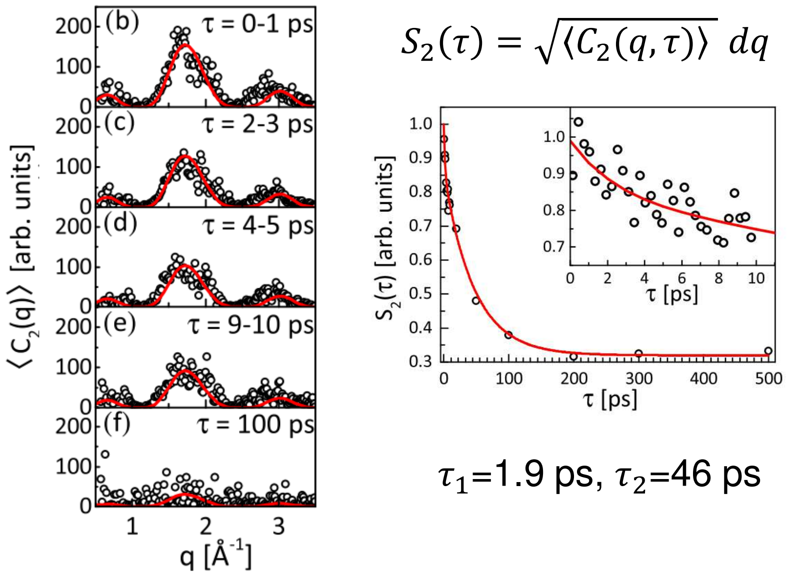
A. Madsen *et al.*, New J. Phys. 12, 055001 (2010)
J. Carnis, et al., Sci. Rep. 4, 6017 (2014)

- **X-Ray Cross-Correlation Analysis (XCCA)**
- Angular (spatial) correlation function:

$$C(q,\Delta) = \frac{\langle \delta I(q,\varphi)\delta I(q,\varphi+\Delta)\rangle_{\varphi}}{\langle I(q,\varphi)\rangle_{\varphi}^2}$$

- Access to material structure & dynamics

$$S_2(\tau) = \sqrt{\langle C_2(q,\tau)\rangle}\ dq$$



τ_1=1.9 ps, τ_2=46 ps

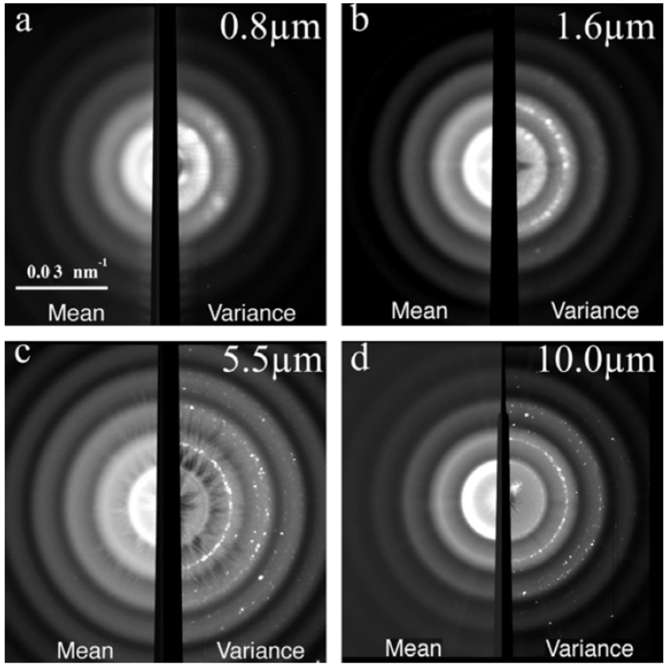
P. Vester et al., Struct. Dyn. 6, 024301 (2019)

Fluctuation microscopy

Normalized intensity variance:

$$V(\boldsymbol{q},R) = \frac{\langle I^2(r_p,\boldsymbol{q},R)\rangle_{r_p}}{\langle I(r_p,\boldsymbol{q},R)\rangle_{r_p}^2} - 1$$

$I(r_p,\boldsymbol{q},R)$ is the scattered intensity,
q is the scattering vector,
r_p is the location of the probe of width R.



Mean intensity and variance for different probe sizes R=0.8÷10.0 µm (STXM experiment)

M. M. J. Treacy et al., Rep. Prog. Phys. 68, 2899 (2005)
M. M. J. Treacy, K. B. Borisenko, Science 385, 950 (2012)
S. Im, et al., Ultramicroscopy 195, 189 (2018)

Summary

- Fluctuation scattering approaches (FXS, CXS, XCCA, etc) rely on the ability to measure intensity fluctuations about the average scattered intensity.

- Structural information is accessed via the spatial (angular) cross-correlation functions, and their decomposition in terms of suitable basis functions.

- Applicable to x-ray, neutron, electron or light scattering.

- Fluctuation x-ray scattering is especially suitable for XFELs, which produce ultrashort and ultrabright x-ray pulses.