Peccei - Quinn quality problem - Workshop seminar -

Hot topics on axions - DESY (2021) - Pablo Quilez

Main references

Kamionkowski et al, hep-th/9202003

Landscape of QCD axion models, 2003.01100 (Secs 2.11, 7.6)

Kallosh et al, hep-th/9502069

Alonso + Urbano, 1706.07415

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1) Strong CP problem and axion solution in a nutshell

→ Why is CP such a good symmetry of the strong interactions?

The so-called θ-term would contribute to CP violating observables such as the Electric Dipole Moment (nEDM) of the neutron but it is experimentally bounded:

$$\mathcal{L} \supset \bar{\theta} \frac{\alpha_s}{8\pi} G^a_{\mu\nu} \tilde{G}^{a\,\mu\nu} \quad ; \quad \bar{\theta} = \theta + \underbrace{\mathrm{Arg}(\det M)}_{\text{quark mass Matrix}}$$

nEDM experiments ⇒ $\boxed{\bar{\theta} \lesssim 10^{-10}}$

↳ Why so small?

→ Peccei-Quinn solution: They introduced a new global symmetry: $U(1)_{PQ}$ that renders the θ-parameter unphysical.

$U(1)_{PQ}$ properties

- Exact at the classical level: $\partial_\mu j^\mu_{PQ}\big|_{class} = 0$
- Anomalous under QCD ≡ explicitely broken by QCD instantons

$$\partial_\mu j^\mu_{PQ} = - \frac{\alpha_s}{8\pi} G\tilde{G} = \delta\mathcal{L} \qquad [1]$$

- Spontaneously broken ⇒ pseudo-Goldstone boson ≡ AXION

* In order to solve the strong CP, only the QCD anomaly breaks the PQ symmetry.

For concreteness, $\Phi = \frac{1}{\sqrt{2}}(\rho + f_a)\, e^{i a/f_a}$

$\Phi \xrightarrow{U(1)_{PQ}} e^{i\beta}\, \Phi$

Φ complex scalar field "PQ field"
- a → axion, real pseudo-scalar
- ρ → radial mode
- f_a → axion decay constant

From the low energy EFT of the axion

$$\mathcal{L}_a = \frac{1}{2}(\partial_\mu a)^2 + \mathcal{L}(\partial_\mu a, \psi) + \frac{\alpha}{8\pi}\frac{a}{f_a} G\tilde{G} + \frac{\alpha}{8\pi}\bar{\theta} G\tilde{G}$$

where again the only term breaking the PQ symmetry $\equiv$
$\equiv$ shift symmetry of the axion $a \xrightarrow{PQ} a + \beta f_a$ is

$$\delta\mathcal{L} = \beta \frac{\alpha}{8\pi} G\tilde{G}$$

and therefore is responsible for the axion mass/potential.

Explicit breaking $\Longleftrightarrow$ mass for the pGB (axion)

The Vafa-Witten theorem:

"In vector-like $\underbrace{\text{theories}}_{\text{CP conserving}}$, CP cannot be spontaneously broken"

implies that the axion vev ($\equiv$ minimum of the potential)
corresponds to the CP conserving point $\langle \frac{a}{f_a} + \bar{\theta} \rangle = \bar{\theta}_{eff} = 0$

(From now on we shift $\frac{a'}{f_a} \equiv \frac{a}{f_a} + \theta$ and drop the prime)

2) PQ quality problem

" Strong sensitivity of the minimum of the axion potential to any extra breaking of the PQ symmetry (apart from the QCD anomaly) "

2.1) PQ quality naively: $\cancel{PQ}$ EFT approach

Let us assume that there is some UV physics at scale $\Lambda \gg f_a$ that breaks the PQ symmetry. Let us also assume that below Λ this breaking can be parametrized with high dimensional operators suppressed by powers of the UV scale Λ (which is taken to be the largest $\Lambda = M_{Pl}$). The lowest dimensional non-renormalizable operator that breaks PQ is

$$\mathcal{L}_{\cancel{PQ}} \supset g_{d=5,n=1} \frac{|\Phi|^4 \phi}{M_{Pl}} + h.c. \simeq |g_{5,1}| \frac{f_a^5}{2\sqrt{2} M_{Pl}} \cos\left(\frac{a}{f_a} + \delta\right)$$

where $\delta = \mathrm{Arg}(g_{5,1})$. Now there are two compiting terms in the axion potential

$$V_a(a) = -m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4 m_u m_d}{(m_u + m_d)^2} \sin^2\left(\frac{a}{2 f_a}\right)} - \overbrace{|g_{5,1}| \frac{f_a^5}{2\sqrt{2} M_{Pl}}}^{\Lambda_5^4} \cos\left(\frac{a}{f_a} + \delta\right)$$

Assuming that the QCD potential dominates one can compute the displacement of the axion vev from the CP conserving point, i.e. the effective theta-parameter $\bar{\theta}_{eff} = \langle \frac{a}{f_a} \rangle$

$$\left.\frac{dU}{da}\right|_{a=\langle a\rangle} = 0 \Rightarrow |\bar{\theta}_{eff}| = \left|\langle \frac{a}{f_a} \rangle\right| = |\sin\delta| \frac{\Lambda_5^4}{m_a^2 f_a^2 + \Lambda_5^4 \cos\delta}$$

$$\left[\Lambda_5^2 \ll m_a f_a\right] \simeq |\sin\delta| \frac{\Lambda_5^4}{m_a^2 f_a^2}$$

Finally

$$|\bar{\theta}_{eff}| = \left|\langle \frac{a}{f_a} \rangle\right| = |g_{5,1} \sin\delta| \frac{f_a^3}{2\sqrt{2}\, m_a^2 m_{Pl}} = \frac{|g_{5,1} \sin\delta|}{2\sqrt{2}} \frac{f_a^5}{m_\pi^2 f_\pi^2 m_{Pl}} \frac{(m_u+m_d)^2}{m_u m_d}$$

Imposing the nEDM bound and for $f_a \sim 10^{12}$ GeV

$$|\bar{\theta}_{eff}| \lesssim 10^{-10} \Rightarrow |g_{5,1} \sin\delta| \lesssim 10^{-55} \,!!$$

In order to explain the smallness of a parameter at the order of $\theta \lesssim 10^{-10}$ we need another parameter to be $\lesssim 10^{-55}$: "This does not look like a fair trade"

Kallosh et al.

However, the situation is more complicated and this argument is full of caveats and possible solutions as we will see next.

Before that let us write a general d-dimensional operator breaking the PQ symmetry by n-units

$$\mathcal{L}_{\cancel{PQ}} = g_{d,n} \frac{|\phi|^{d-n} \phi^n}{m_{Pl}^{d-4}} + h.c.$$

which translates in

$$|\bar{\theta}_{eff}| \simeq \frac{|g_{nd} \sin\delta|}{2n} \frac{f_a^2}{m_a^2} \left(\frac{f_a}{\sqrt{2} M_{Pl}}\right)^{d-4} = \frac{|g_{dn} \sin\delta|}{2^{1+\frac{(d-4)}{2}} n} \frac{f_a^4}{m_\pi^2 f_\pi^2} \left(\frac{f_a}{M_{Pl}}\right)^{d-4} \times \frac{(m_u+m_d)^2}{m_u m_d}$$

so parametrically

$$|\bar{\theta}_{eff}| \simeq g_{dn} \frac{f_a^4}{m_\pi^2 f_\pi^2} \left(\frac{f_a}{M_{Pl}}\right)^{d-4}$$

2.1) PQ motivation

→ Any New Physics could violate PQ as long as it is not forbidden by gauge symmetries and Lorentz inv.

→ "Quantum gravity breaks all global symmetries"
General argument based on black hole physics (semi-classical)
If particles are thrown into a BH (with some amount of global charge) and then the BH evaporates emitting Hawking radiation then the global symmetry is violated. * No global sym. in String Theory nor in AdS/CFT

How to quantify the breaking?
+ Connections with the Weak gravity conjecture
+ Wormhole physics (Euclidean)

Caveat of naive EFT: BH are non-perturbative and therefore one should expect $|\bar{\theta}_{eff}| \propto e^{-S} \sim e^{-M_{Pl}/f_a}$

2.3) PQ quality from gravity quatitatively: Wormhole solution

Wormhole: "Euclidean solution of the classical field equations for some field theory containing gravity, with geometry consisting of two asymptotically flat regions connected by a tube or throat."

Coleman-Lee

Sometimes they are called gravitational instantons but $\neq$ Eguchi-Hanson (self-dual)

Euclidean wormholes $\neq$ $\begin{cases} \text{Einstein-Rosen bridge} \\ \text{Traversable wormholes} \\ \text{Schwarzchild wormholes} \end{cases}$

Euclidean wormholes describe non-perturbative gravitational processes that violate global symmetries, in our case by swallowing axionic charge.

But... why Euclidean? We live in Minkowski signature

why classical? We want to describe non-perturbat. quantum processes.

One of the few (and best) methods that we have in QFT in order to study non-perturbative quantum processes is the Eclidean path integral, which, if we apply the steepest-descent method, will be dominated by the Euclidean classical solutions.

The Euclidean path integral method is also used to compute barrier penetration processes (quantum tunneling), instantons for the θ-vacuum etc.

Let's refresh path integral formalism: a quantum amplitude can be expressed as the sum over all possible paths from $|i\rangle$ to $|f\rangle$:

$$\langle i | \underbrace{e^{-iHt}}_{\text{Operator of time evolution.}} | f \rangle = N \int [d\phi] \, e^{iS/\hbar} \rightarrow \text{steepest descent method SDM}$$

Let's now consider a completely different matrix element (not time evolution)

$$Z_{fi}(\tau) = \langle i | \underbrace{e^{-H\tau/\hbar}}_{\underline{\text{no}} \text{ factor of } i, \ \tau \text{ is } \underline{\text{NOT}} \text{ usual time}} | f \rangle$$

$$Z_{fi}(\tau) = \sum_n \langle i | n \rangle \langle n | f \rangle \, e^{-E_n \tau/\hbar} \xrightarrow{\tau \to \infty} \langle i | \Omega \rangle \langle \Omega | f \rangle \, e^{-iE_0/\hbar}$$

The matrix element for $\tau \to \infty$ is dominated by the lowest energy state $\equiv$ the vacuum or ground state

But how do we compute $Z_{fi}(\tau)$? If we look carefully it is nothing but the Euclidean path integral of a theory with no time an 4 spatial dimensions

$$\lim_{\tau \to \infty} Z_{fi}(\tau) = \langle i | \Omega \rangle \langle \Omega | f \rangle \, e^{-E_0 \tau/\hbar} = N \int [d\phi] \, e^{-S_E/\hbar}$$

Again one can compute the Euc. path. Int. using the SDM and therefore it will be dominated by the finite ~~ener~~ action solutions to the classical theory $\equiv$ stationary points//saddle point.

This is in complete analogy with quantum tunneling and instantons

Minkowski	Euclidean
Quantum tunneling with WKB $\Gamma \propto e^{-\int dq\sqrt{2m(V(q)-E)}}$	Euclidean path integral $\Gamma \propto e^{-S_E}\Big\vert_{\text{class. sol}} \propto e^{-\int dq\sqrt{2m(V-E)}}$
Quantum tunneling among classical vacua with different winding numb. θ-vacuum.	Instantons: classical sol to the Euclidean field eq. with non-trivial topology
Axion potential/mass	** Instanton sum DIGA (only qualitatively) $e^{-S_E} = e^{-\frac{2\pi}{\alpha_S}}$
Axion potential from gravity $\cancel{P}Q$ operators	Euclidean wormholes. $\propto e^{-S_E}$

A) Giddings - Strominger - Lee Wormhole ≡ Axionic wormhole

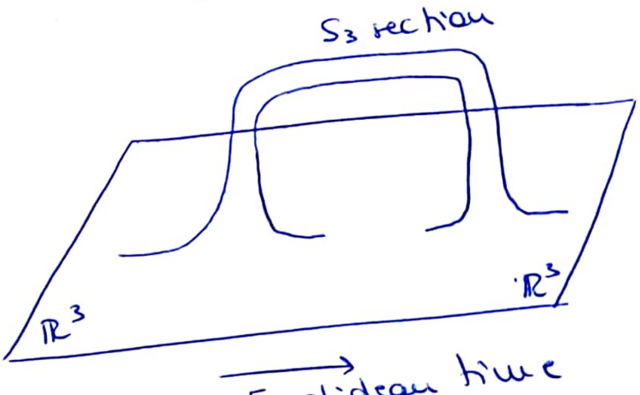
→ Real (pseudo)-scalar field minimally coupled to Einstein gravity. $\theta_a = \frac{a}{f_a}$

$$S_E = \int d^4x \sqrt{g} \left[- \frac{M_{Pl}^2}{16\pi} R + \frac{f_a^2}{2} (\partial_\rho \theta_a)(\partial^\rho \theta_a) \right]$$

→ The solution for the wormhole metric reads

$$ds^2 = \left(\frac{1}{1 - L^4/r^4} \right) dr^2 + r^2 d\Omega_{3,k=1}$$

with $L = \left(\frac{n^2}{3\pi^2 M_{Pl}^2 f_a^2} \right)^{1/4}$ for any integer n



Slicing the solution at constant Euclidean time we find that the wormhole changes the topology

$$\mathbb{R}^3 \longrightarrow \underbrace{\mathbb{R}^3 \oplus S_3}_{\text{Baby Universe}} \longrightarrow \mathbb{R}^3$$

The wormhole interpolates between two $\mathbb{R}^3$ geometries (asymptotically flat).

It can be shown that n corresponds to the number of units of global charge flowing through the throat of the wormhole

$$Q = \int_{S^3} f_a^2 \partial^r \theta \, r^3 \, d\Omega_{3,r=1} = \dots = n$$

and it can also be shown that n is quantized.

And the final result for the Euclidean action of the wormhole solution is

$$\boxed{S_E = \frac{\sqrt{3\pi}}{8}\left(1 - \frac{2}{\pi}\right) \frac{M_{Pl}}{f_a} n}$$

After summing over all possible chains of instantons/antinst. we can obtain

$$\mathcal{L}_{\cancel{PQ}, \text{wormholes}} \sim \frac{1}{L_n^4} e^{-S_{E,n}} \cos\left(n \frac{a}{f_a} + \delta\right)$$

$$\sim M_{Pl}^2 f_a^2 \, e^{-\frac{M_{Pl} n}{f_a}} \cos\left(n \frac{a}{f_a} + \delta\right)$$

For most relevant values of $f_a \lesssim 10^{16}$ GeV, there is no PQ-quality problem!

The characteristic non-perturbative factor e^{-S_E} makes the contribution from the GSL wormholes negligibly small.

> "A wormhole is not a naturally occurring phenomenon"
>
> Christopher Nolan
>
> via [Alonso+Urbano 17]

B) Wormholes with dynamical radial component $\left(\equiv \begin{matrix}\text{unfrozen}\\ \text{wormholes}\end{matrix}\right)$

→ The axion EFT is only valid bellow f_a

→ The characteristic energy scale of the wormhole is the throat size $\frac{1}{L} \sim \sqrt{f_a M_{Pl}} > f_a$. Therefore the radial component of the PQ field needs to be taken into account.

[Abbot + Wise, 89] [Kallosh et al, 95]

→ The Euclidean action in this case reads

$$S_E \simeq n \cdot \log(M_{Pl}/f_a)$$

$$\Rightarrow e^{-S_E} \sim \left(\frac{f_a}{M_{Pl}}\right)^n \Rightarrow$$ We go back to power-like suppression and the PQ quality problem arises again.

> "An unfrozen wormhole may be a naturally ocurring phenomenon"
>
> C. Nolan / P. Quilez

Strengths:

→ Stable under modification of the radial mode potential

Caveats:

→ Wormhole solutions may disappear when introducing R^2 corrections (quadratic gravity)

→ Topological suppression: One can introduce two different topological terms

$$S = - \int d^4x \sqrt{g} \left(\frac{M_p^2}{16\pi} R + \frac{\theta_g}{32\pi^2} R_{\mu\nu\lambda\delta} {}^*R^{\mu\nu\lambda\delta} + \frac{\gamma}{32\pi^2} {}^*R_{\mu\nu\lambda\delta} {}^*R^{\mu\nu\lambda\delta} \right)$$

with ${}^*R_{\mu\nu\lambda\delta} = \frac{1}{2} \epsilon_{\mu\nu\mu'\nu'} R^{\mu'\nu'}{}_{\lambda\delta}$

Gravitational θ-~~term~~. Gauss-Bonet term.

They do not contribute to the EOM (they are total deriv.) but may contribute to the action.

- Grav. θ-term: Vanishes for Wormhole. It is only relevant for Eguchi-Hanson instantons.
- Gauss-Bonet:

$$S_{E,GB} \simeq \gamma$$

Therefore the operators are further suppressed by

$e^{-\gamma}$; γ is a priori unknown.

→ String theoretical set ups may inherit additional suppression

$$e^{-\frac{8\pi^2}{g_{str}^2}}.$$

→ Euclidean methods may be unvalid to estimate these effects since the euclidean action is unbounded from bellow.

3) Solutions to the PQ quality problem.

Let us remind the reader that the effective θ-param. generated by higher dimensional operators scales as

$$|\bar{\theta}_{eff}| \simeq g_{d,n} \frac{f_a^4}{m_\pi^2 f_\pi^2} \left(\frac{f_a}{M_{Pl}}\right)^{d-4}$$

3.1) Arguably small $g_{d,n} \propto e^{-\frac{M_{Pl}}{f_a}}$

{ WGC
Wormhole arguments

3.2) Large dimension d as the first NR operator

$\equiv$ PQ as an accidental symmetry

→ Connection with PQ origin.

→ Analogous to Baryon number in SM

→ Tons of model building:

• Discrete Z_N symmetry (gauged) one typically needs $N \gtrsim 8, 10, 21$ for $f_a \sim 10^8, 10^{10}, 10^{15}$ GeV

• Extra gauge symmetries rendering PQ accidental. Particularly interesting are composite axion models.

• Extra dimensions + gauge symm.

3.3) Low decay constant in heavy axion models.

For the canonical QCD axion astrophysical bounds imply $f_a \gtrsim 10^8$ GeV. However if the "heavy axion" is heavy enough not to be kinematically accessible in the stellar medium, the bound on f_a disappears and one can avoid the PQ quality problem with $f_a \sim 10$ TeV.