Introduction to the cosmological relaxation.

This note will be about so-called relaxion. It is one of proposed solution to the EW hierarchy problem in the SM. It is a relatively new idea with a totally different approach, compared to a traditional approach to hierarchy problems. To appreciate how it works and how it is different from others, let us start with a few words about hierarchy problem in SM.

Let us consider a few terms in the Standard model. We consider two terms.

$$\mathcal{L} \supset \Lambda_{cc} + m_H^2 |H|^2$$

What are they? They are cosmological constant and the Higgs doublet mass. They are the only relevant operators in SM. They are the only coefficients with positive mass dimension in SM.

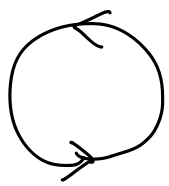
The observed values for these two parameters are theoretically intriguing as they are UV-sensitive in QFT.

The measured values for these parameters are

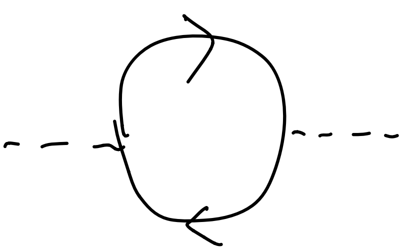
$$\Lambda_{c.c} \sim (10^{-3}\ eV)^4$$

$$m_H^2 \sim (10^2\ GeV)^2$$

On the other hand, any state in (B)SM contributes



$$\delta\Lambda_{cc} \sim \frac{M^4}{16\pi^2} + \cdots \quad (M: \text{mass of state})$$



$$\delta m_H^2 \sim g\,\frac{M^2}{16\pi^2} + \cdots$$

Therefore, a naive comparison between observed values and renormalized values in QFT yields

$$\begin{cases} \dfrac{\delta\Lambda_{cc}}{\Lambda_{cc.obs}} \sim \dfrac{M_{GUT}^4}{(10^{-3} eV)^4} \sim \dfrac{(10^{16} GeV)^4}{(10^{-3} eV)} \sim O(10^{100}) \\[2ex] \dfrac{\delta m_H^2}{m_{H.obs}^2} \sim \dfrac{M_{GUT}^2}{(10^2 GeV)^2} \sim 10^{28} \end{cases}$$

This seems odd and "unnatural".
In QFT framework, a fine-tuning at UV scale is required to explain what we see for (Λ_{cc}, m_H^2).
A lot of theoretical efforts have been given to explain these small numbers and to make them look more natural. This 'naturalness criteria' has been guiding the community for a few decades.

[See Giudice 0801.2562, 1307.7879 for an overview.]

Before we talk about relaxation mechanism, let me summarize (roughly) proposed solutions to hierarchy problem. This will give a context to the relaxion. There are roughly speaking three classes of solutions, each of them based on

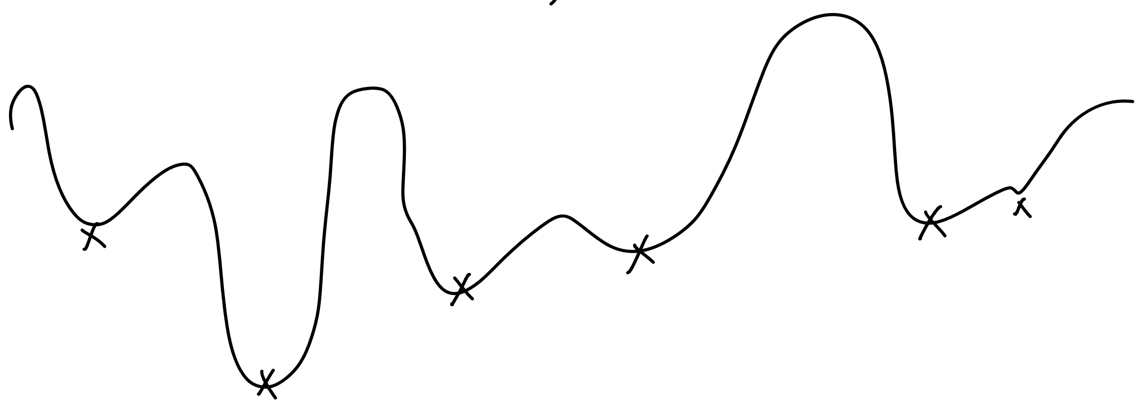
(i) Symmetry, e.g. SUSY, technicolor, ...
$\Rightarrow$ there exists a symmetry, softening UV-sensitivity of these parameters

(ii) Anthropics (multiverse)

$\Rightarrow$ Fundamental constants are actually a function of dynamical fields,

$$\Lambda_{cc} = \Lambda_{cc}(\{\phi_i\}),$$

and there are a number of stable vacua for $\{\phi_i\}$,



each represents a universe with different fundamental constant. The reason why we see what we see is that it's more likely for an observer to observe those fundamental constant due to anthropic reasons.

{ Galactic principle for C.C, Weinberg (87)

{ Atomic principle for EW, Donoghue et al. (hep-ph/9707380)

(iii) Dynamics

Similar to anthropic solution, it begins with recognizing that fundamental constant might be a function of fields.

$$\Lambda_{cc} = \Lambda_{cc}(\{\phi_i\}),$$

but the potential for the field is "engineered" such that a parametrically smaller valued is much more preferred.

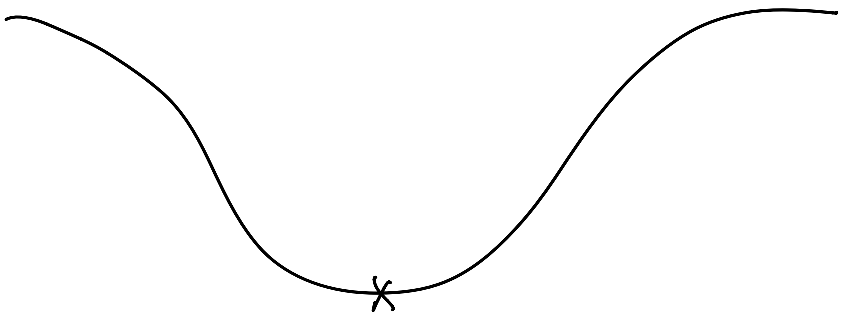
- QCD axion solution to strong CP
- Abbott (85) for C.C.
- Relaxion, Graham, Rajendran, Kaplan (15)

This is only rough classification. There are recent activities, that may be understood as combination of more than two above principles.

How does relaxation works?

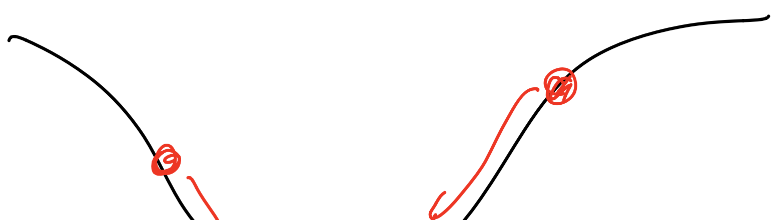
Consider first an example of QCD axion.

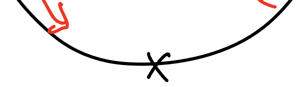
$$e^{-\int dx\, V(\theta)} = \int \mathcal{D}A\,\mathcal{D}\psi\,\mathcal{D}\bar{\psi}\, e^{-S_{QCD}(\theta)}$$



If θ is just a constant, then different value of θ is just different theory with different vacuum energy. But if we promote $\theta \to (a/f)$ dynamical field by introducing PQ, then

$$\int \mathcal{D}a\, e^{-\int dx\,(T+V)} = \int \mathcal{D}a\,\mathcal{D}A\,\mathcal{D}\psi\,\mathcal{D}\bar{\psi}\, e^{-S_{QCD}(\theta)}$$





$\theta = 0$

So, QCD axion relaxes strong CP angle.

Now we know what to do for the Higgs mass. The first thing to do is to promote the Higgs mass as a dynamical field, and we do that by introducing "relaxion",

$$V(\phi, H) = \boxed{(M^2 - gM\phi)}\,|H|^2 + |H|^4 + V(\phi)$$

Higgs mass

UV cutoff for the Higgs mass.

The relaxion potential.

What we require

- ϕ relax the Higgs mass from $M^2 \to m_H^2$.

An immediate difficulty is that there is no apriori reason for $V(\phi)$ to have $\langle \phi \rangle$ such that

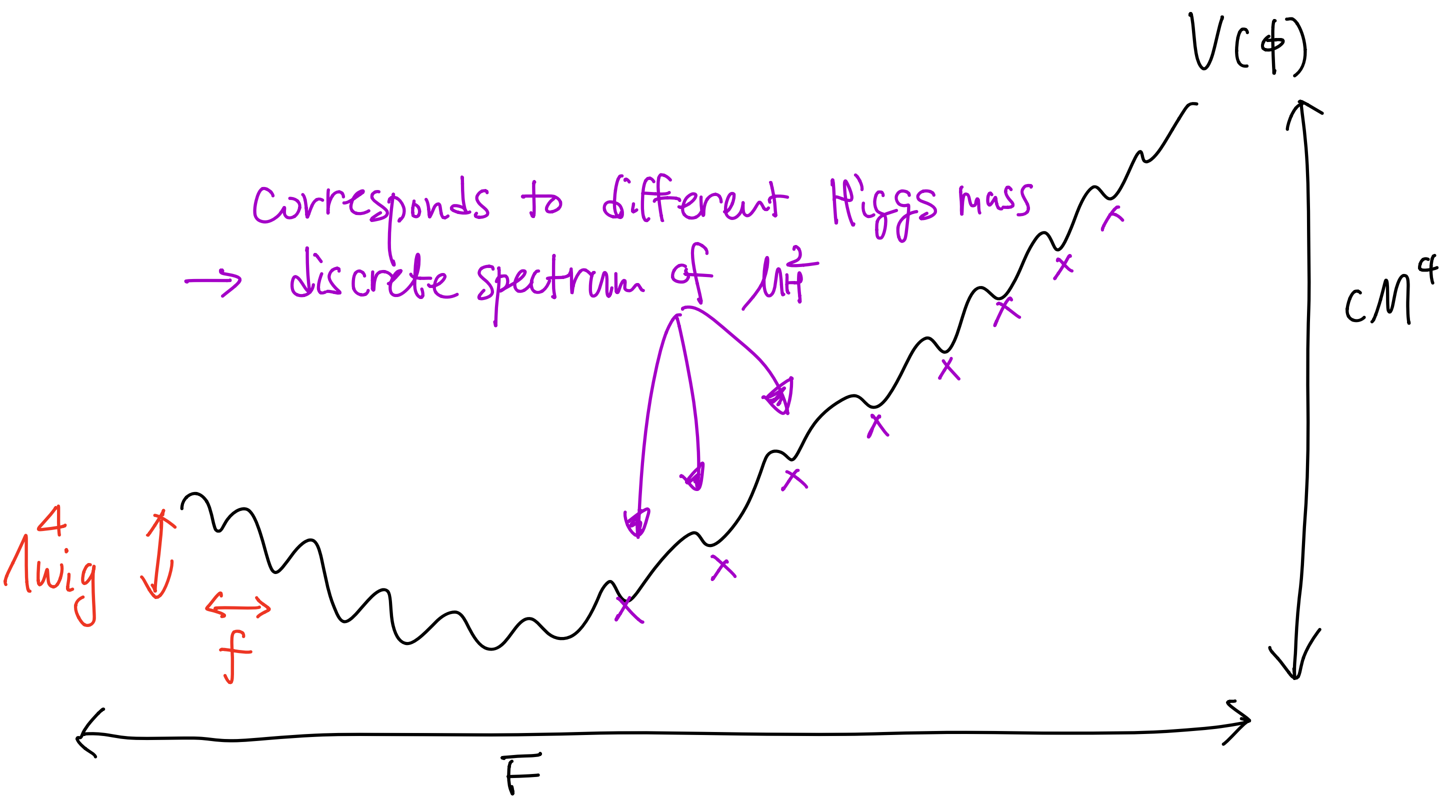
$$\mu_H^2 \equiv M^2 - gM\langle \phi \rangle \ll M^2.$$

This is again the same kind of fine-tuning problem. There's nothing special about $\mu_H^2 = 0$.

Since there's no symmetry associated with $\mu_H^2(\phi)$, what we can do is to give a landscape like structure to the potential, so that there's discrete spectrum of $\mu_H^2(\langle\phi\rangle)$. For that, we introduce an axion like particle,

$$V(\phi, H) = (M^2 - cM^2 \cos(\tfrac{\phi}{F}))\,|H|^2 + |H|^4 + V(\phi)$$

$$V(\phi) = -c\,M^4 \cos(\tfrac{\phi}{F}) + \Lambda_{wig}^4 \cos(\tfrac{\phi}{f})$$



For each steps $\Delta\phi \sim f$, the change of Higgs mass is

$$\Delta m_h^2 \sim c M^4 \frac{f}{F}$$

so, as long as,

$$f/F \sim \frac{m_{EW}^4}{M^4} \ll 1$$

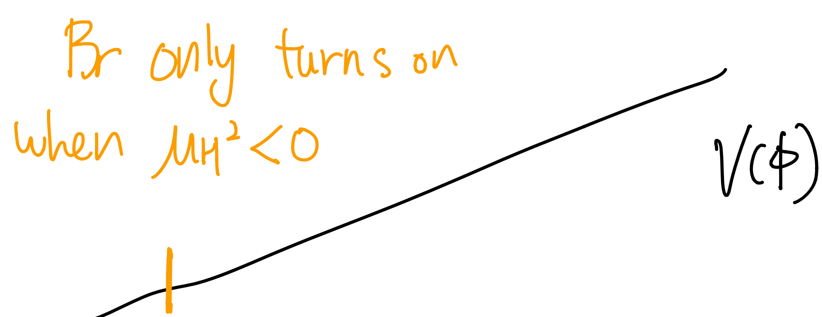
(Choi Kim Yun 14
Choi Im 15
Kaplan Rattazzi 15)

this landscape has enough resolution to identify the EW Higgs mass.

But this potential structure is more like that for anthropic solution. Small Higgs mass is not particularly preferred over the large Higgs mass, and therefore, it doesn't seem like different from anthropic solution.

Additional ingredient is required. What we are missing is a proper feedback mechanism, that enforces ϕ to stop only when m_H^2 is small.

For instance,



The potential that does this might be

$$V(\phi) = -c\,M^4 \cos\left(\frac{\phi}{F}\right) + \underline{\underline{\Lambda_{br}^4(h) \cos\left(\frac{\phi}{f}\right)}}$$

An introduction of "back reaction" potential, that's proportional to Higgs vev.

And this backreaction potential is chosen such that

(i) $\Lambda_{br}^4 \propto h^n \quad (n > 0)$

(ii) $\dfrac{M^4}{F} \sim \dfrac{\Lambda_{br}^4(\langle h_{ew} \rangle)}{f}$

The first condition is clear, and the second term is to make it stop only when Higgs mass is EW scale.

We are almost there, but there are still two questions to answer.

(1) Where does V_{br} come from?

(2) When does this Happen?

For the first question, there are multiple answers

(a) QCD

$$\frac{\phi}{f} G\tilde{G} + (m_q q q^c + h.c)$$

$$\to m_q \Lambda_{QCD}^3 \cos\frac{\phi}{f} = y_q h \Lambda_{QCD}^3 \cos\frac{\phi}{f}$$

(b) From new strong sector

$$\frac{\phi}{f} H\tilde{H} + m_N N N^c + m_L L L^c + y h L N^c + \tilde{y} h^+ L^c N$$

$$\to \frac{a}{f} H\tilde{H} + (m_N + \frac{y\tilde{y}}{m_L} |h|^2) N N^c$$

$$\to \frac{y\tilde{y}}{m_L} |h|^2 \Lambda^3 \cos\frac{\phi}{f}$$

So, for any models, we may parametrize the backreaction potential as

$$V_{br} = \Lambda_{br}^4 \left(\frac{h}{v}\right)^n \cos\frac{\phi}{f}$$

(for details see Graham Kaplan Rajendran (GKR) and also Espinosa, Grojean, Panico, Pomarol, Servant, Pujolas) ('15)

For the second question, the original GKR scenario use Hubble friction during inflation. So, the relaxion can adiabatically settle down at the minimum.

Now let's summarize.

$$V(\phi, H) = (M^2 - cM^2 \cos\frac{\phi}{F}) |H|^2 + \lambda |H|^4 + V(\phi)$$

$$V(\phi) = -cM^4 \cos\left(\frac{\phi}{F}\right) + \Lambda_{br}^4 \left(\frac{h}{v}\right)^n \cos\frac{\phi}{f}$$

(1) $\dfrac{M^4}{F} \sim \dfrac{\Lambda_{br}^4}{f}$ Stable point at $h = v$

(2) $N_{efold} \sim H \dfrac{\Delta\phi}{\dot{\phi}} \sim \dfrac{H^2 F^2}{M^4} \sim \dfrac{H^2 f^2}{\Lambda_{br}^4} \left(\dfrac{M^4}{\Lambda_{br}^4}\right)$

Required efolding # to scan Higgs mass

(3) $H_I > \frac{M^2}{M_{pl}}$ inflaton sector dominate total energy den.

(4) $H_I < \Lambda_{QCD}$ or other strong Dynamics

(5) $\Delta\phi_{classical} > \Delta\phi_{quantum}$

$$\Rightarrow \frac{\dot{\phi}}{H} = \frac{V'}{H^2} > H$$

$$\Rightarrow H < \left(\frac{M^4}{F}\right)^{1/3} = \left(\frac{\Lambda_{br}^4}{f}\right)^{1/3}$$

Combining (3) and (5), and taking smallest possible $f \simeq M$,

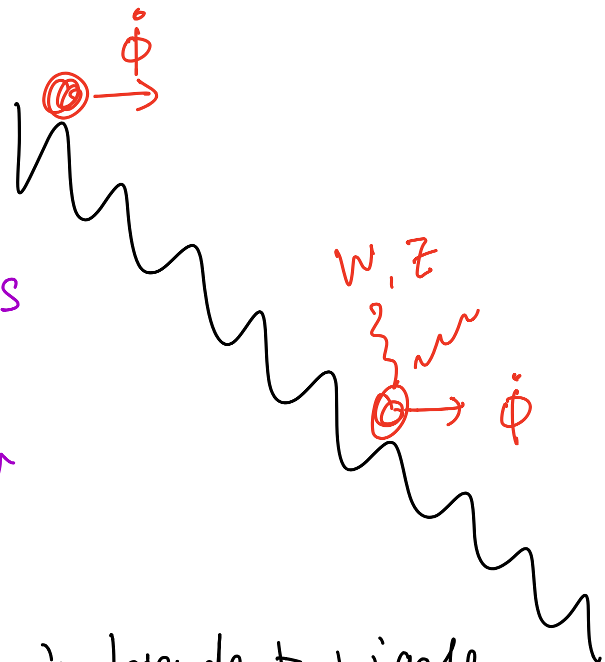
$$M < \left(M_{pl}^3 \Lambda_{br}^4 / f\right)^{1/6}$$

$$\Rightarrow M \lesssim \left(M_{pl}^3 \Lambda_{br}^4\right)^{1/7} \simeq 10^9\,\mathrm{GeV}\left[\frac{\Lambda_{br}}{10^2\,\mathrm{GeV}}\right]^{4/7}$$

So, relaxion can relax the Higgs mass at most from 10^9 GeV.

Remarks

1. Alternative friction: particle production



$\phi F \tilde{F}$ (W or Z)

(Hook, Tavares 16)

Large Higgs
$\begin{cases} m_H^2 < 0 \\ |m_H^2| \gg v \end{cases}$

Higgs-independent wiggle

The particle production only happens when the gauge bosons are light enough (at EW scale)

$\Rightarrow$ Excitations of $\delta\phi(\vec{x},t)$ from self-interaction dramatically changes the picture.

(Fonseca, Morgante, Sato, Servant 19)

2. Dark matter

- keV-scale particle dark matter

(Fonseca & Morgante 18)

(thermal production after reheating)

- Sub-eV scale axionlike dark matter

  (Banerjee, Kim, Perez 18)

  (misalignment mechanism)

3. Phenomenology

Unlike the other traditional solutions to hierarchy problem, GKR relaxion predicts a light new state, lighter than EW scale.

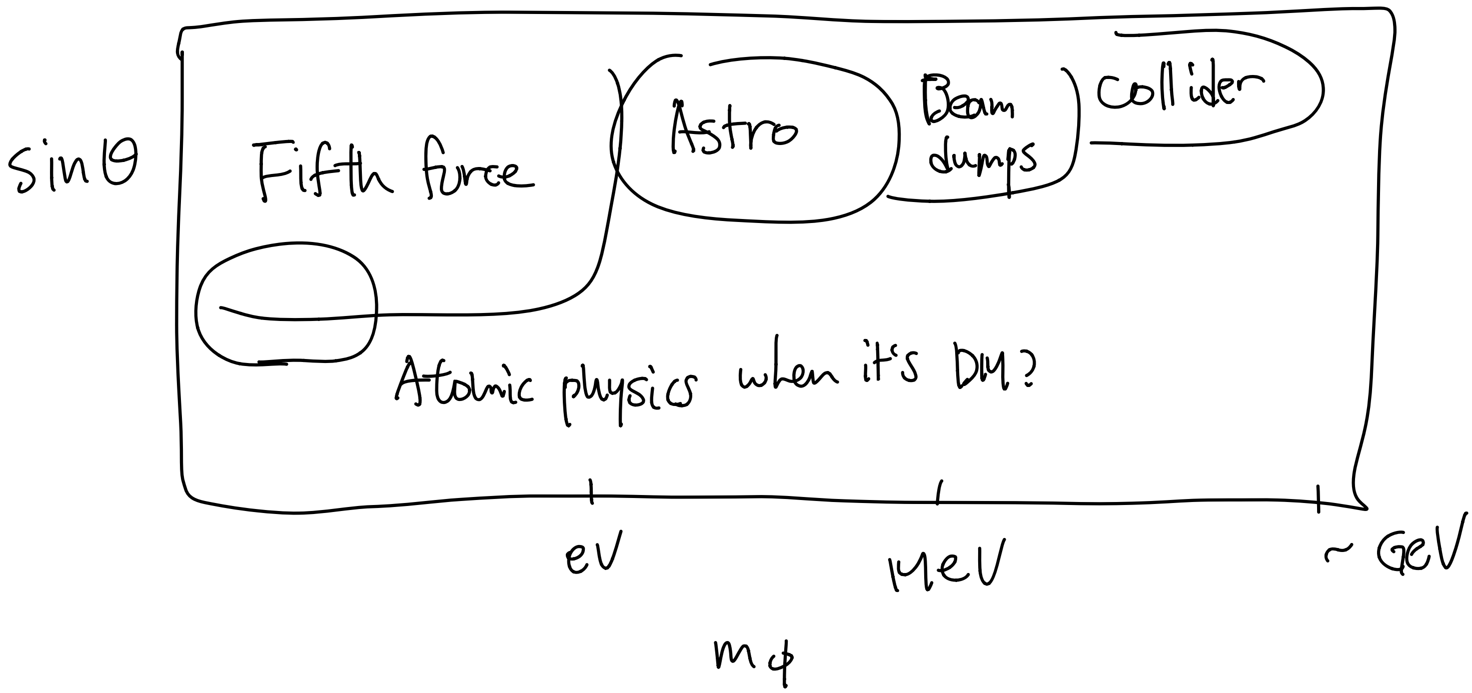
From its backreaction potential,

$$V_{br} = \Lambda_{br}^4 \left(\frac{h}{v}\right)^n \cos(\phi/f)$$

it mixes with higgs, and low energy pheno is pretty similar to Higgs portal models although ϕ is axionlike particle

(Choi & Im 16

Flacke, Frugiuele, Fuchs, Gupta, Perez 16, 18)



Another interesting point is that the relaxion mass is also generically relaxed in a similar way. Naively speaking

$$V_{br} \sim \Lambda_{br}^4 \cos\frac{\phi}{f}$$

$$m_\phi^2 \sim \frac{\Lambda_{br}^4}{f^2}$$

But due to fine-scanning of higgs mass

Resolution of Higgs mass scanner.

$$\delta^2 = \frac{\delta m_h^2}{m_h^2} \sim \frac{M^2}{m_h^2}\frac{f}{F} \ll 1$$

The first stopping point is not just

$$\phi/f \sim \mathcal{O}(1)$$

but $(\phi/f) \sim \pi/2$ where the second derivative is suppressed. So actually

$$m_\phi^2 \sim \delta \, (\Lambda_{br}^4 / f^2)$$

(Banerjee, Kim, Matsedonskyi, Perez, Safronova 20)

while mixing angle is just the same. Because of this relaxation of relaxion mass, low energy observer (EFT theorists) may find relaxion parameter unnatural although everything is constructed in a technically natural way.