Axion Miniclusters and Structure Formation

Henrique Rubira

henrique.rubira@desy.de

Preamble

Let's talk about three observables

The matter power spectrum

The halo mass function

The halo profile

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Let's talk about three observables

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~~Let's not talk about problems to connect those observables to galaxy surveys~~
e.g., galaxy bias, completeness, purity, baryonic feedback

Structure formation (LCDM)

(review 0112551)

Structure formation (LCDM)

Continuity

$$\boxed{\frac{\partial \delta(\mathbf{x},\tau)}{\partial \tau} + \nabla \cdot \{[1+\delta(\mathbf{x},\tau)]\,\mathbf{u}(\mathbf{x},\tau)\} = 0,}$$

Euler

$$\boxed{\begin{aligned}\frac{\partial \mathbf{u}(\mathbf{x},\tau)}{\partial \tau} + \mathcal{H}(\tau)\ \mathbf{u}(\mathbf{x},\tau) + \mathbf{u}(\mathbf{x},\tau)\cdot\nabla\mathbf{u}(\mathbf{x},\tau) = \\ -\nabla\Phi(\mathbf{x},\tau) - \frac{1}{\rho}\nabla_j\left(\rho\,\sigma_{ij}\right),\end{aligned}}$$

Poisson

$$\boxed{\nabla^2\Phi(\mathbf{x},\tau) = \frac{3}{2}\Omega_m(\tau)\ \mathcal{H}^2(\tau)\ \delta(\mathbf{x},\tau).}$$

*** During radiation domination we need to include photons

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$$\delta = \frac{2}{5} \frac{k^2}{H_0^2 \Omega_m} T(k) D(z) \mathcal{R}(k)$$

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Initial conditions
(inflation + adiabatic)

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$$T(k)$$

Transfer function: Behaviour of the mode by entering the horizon

(review 0112551)

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$D(z)$ Growth rate: late time linear growth

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Please, remind those guys

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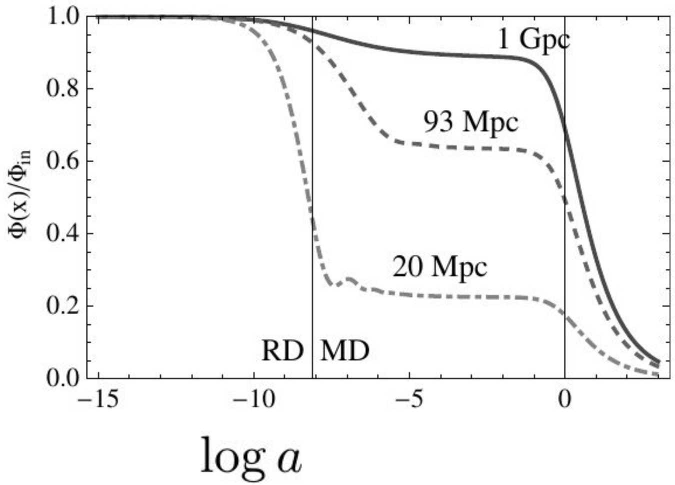
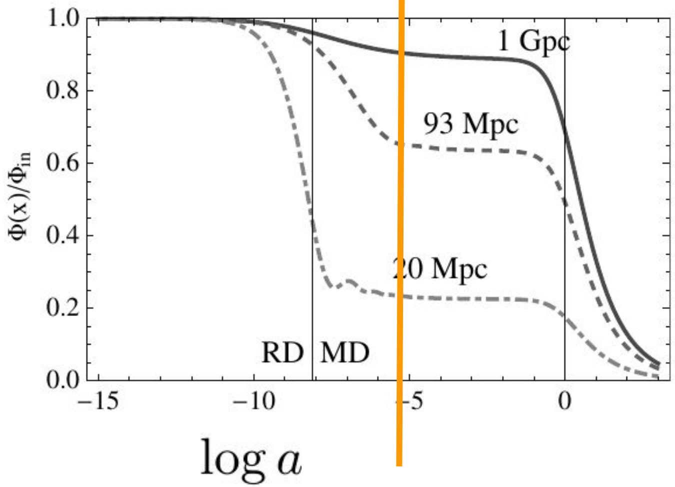


(Maggiore 2)

Structure formation (LCDM)

$\boxed{T(k)}$



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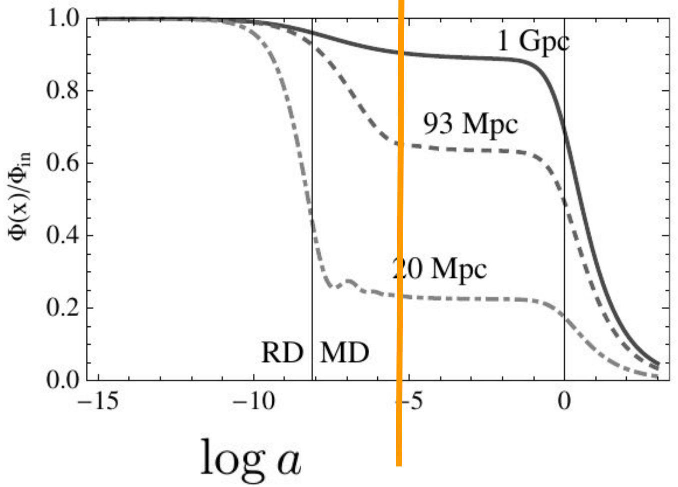

You get the transfer function from Boltzmann solvers (CAMB/CLASS)

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$$D(a) = \frac{5\Omega_M}{2} h(a) \int_0^a \frac{da'}{[a'h(a')]^3}.$$

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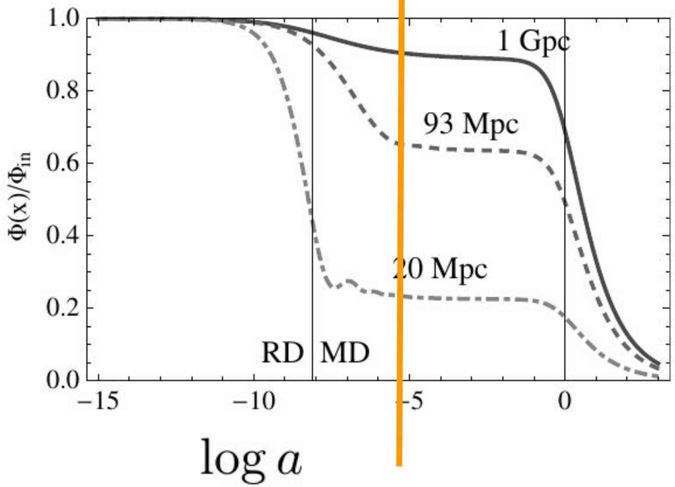

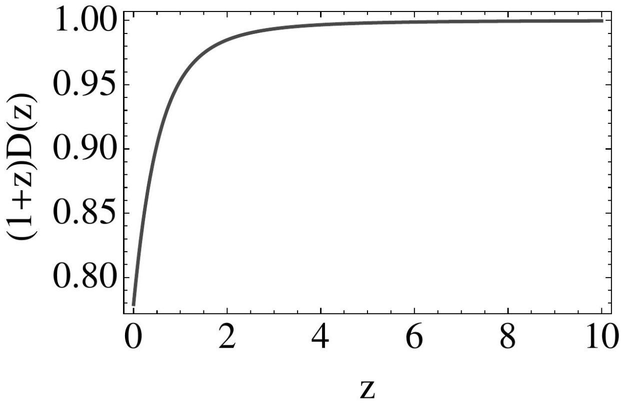

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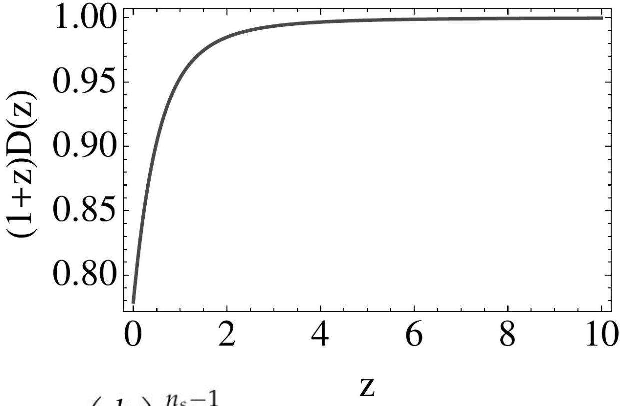

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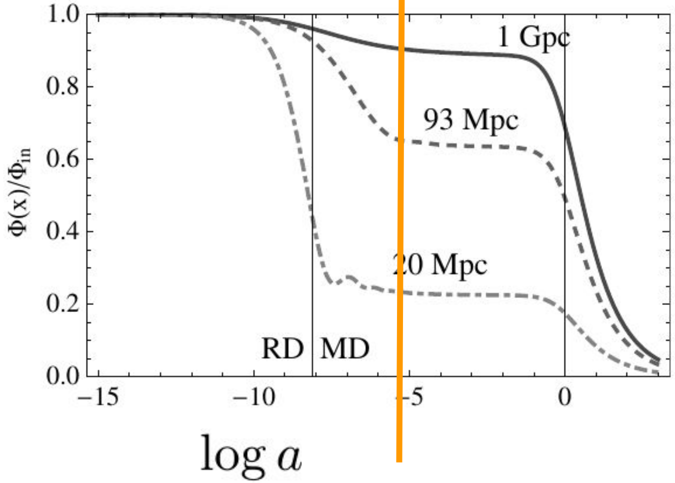
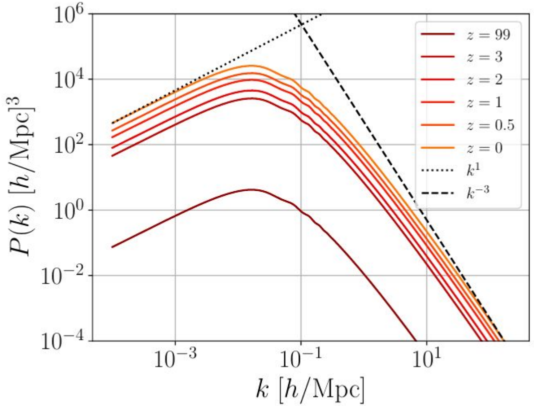


$$P_L(z,k) = \frac{8\pi^2}{25} A_s \left(\frac{D(z)}{H_0^2\Omega_m}\right)^2 k T^2(k) \left(\frac{k}{k_\star}\right)^{n_s-1}$$



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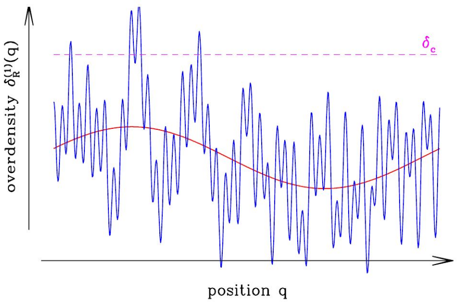
The Halo Mass Function

(1611.09787, Dodelson and Schmidt,
0611454, 2003.06411)

The Halo Mass Function

The Press-Schechter formalism



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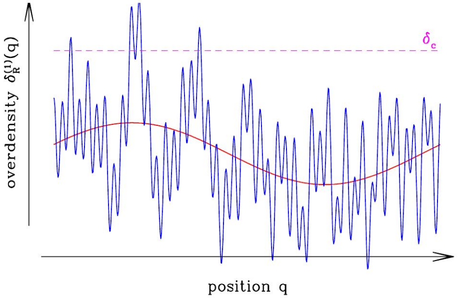

Fraction of collapsed objects

$$F_{\mathrm{coll,PS}}(M,z) = 2 \times \frac{1}{\sqrt{2\pi}\sigma(R_L[M],z)} \int_{\delta_{\mathrm{cr}}}^{\infty} d\delta e^{-\delta^2/2\sigma^2(R_L[M],z)}$$

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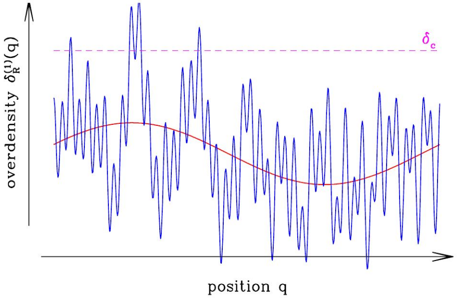

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Delta is a Gaussian variable with variance sigma(R)

$$\sigma^2(R,z) = \langle \delta_R^2(\mathbf{x}) \rangle = \int_{\mathbf{k}} P_L(z,k)|W_R(k)|^2$$

Filter

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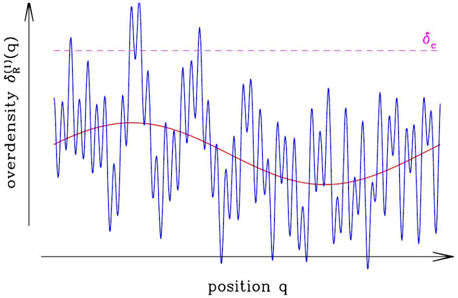

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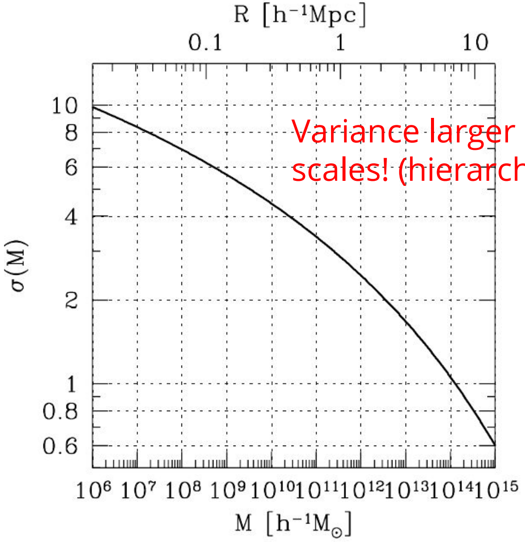

Variance larger on small scales! (hierarchical)

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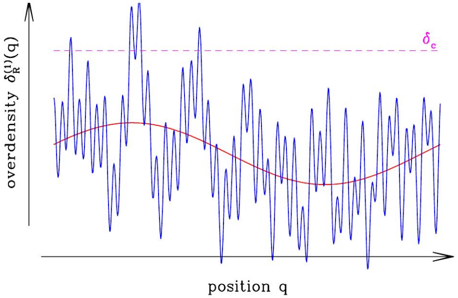

The Press-Schechter formalism

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$$\frac{dn(M,z)}{d\ln M} = \frac{\rho_m(t_0)}{M}\left|\frac{dF_{\rm coll,PS}}{d\ln M}\right|$$

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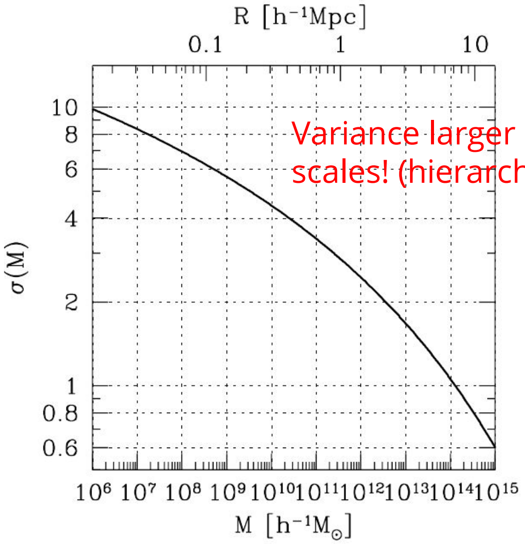

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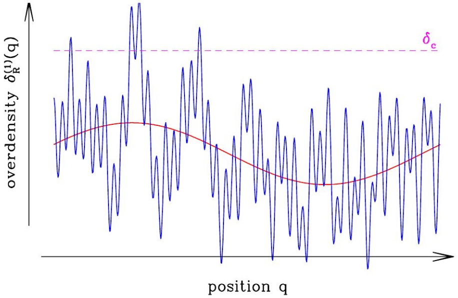

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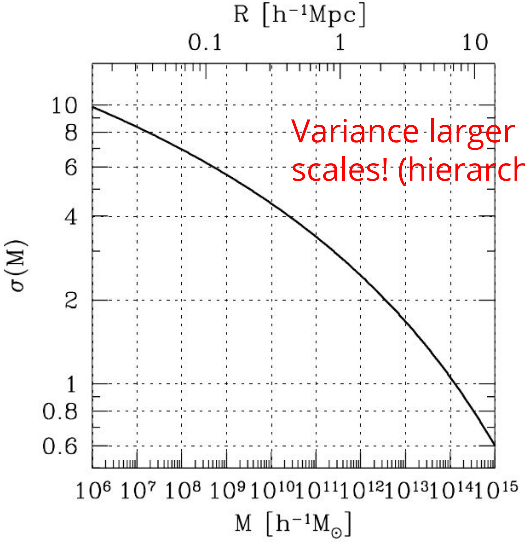

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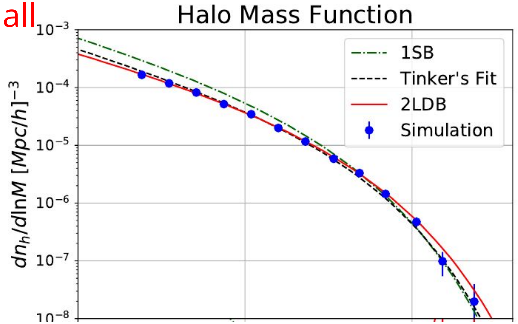

(1611.09787, Dodelson and Schmidt, 0611454, 2003.06411)

The Halo profile

$$\boxed{\frac{\rho_{\rm NFW}(r)}{\rho_{\rm crit.}} = \frac{\delta_{\rm NFW}}{r/r_s(1+r/r_s)^2}\,,}$$

(from v. d. Bosch
slides, 2003.06411)

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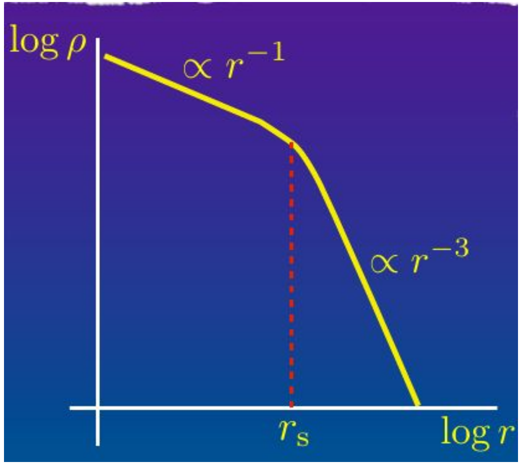

r_s is defined with respect to concentration and some specific mass scale

(from v. d. Bosch slides, 2003.06411)

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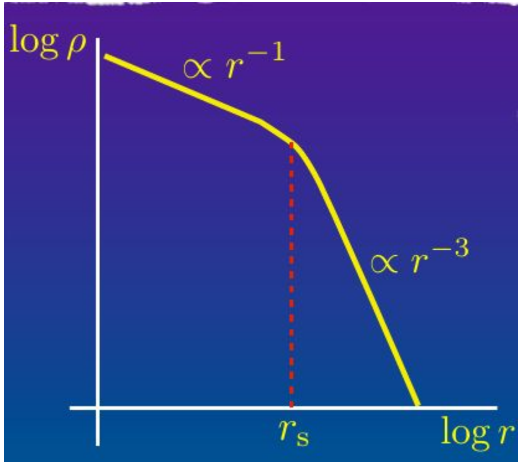

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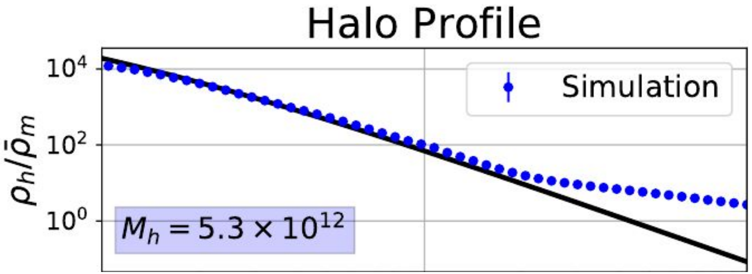

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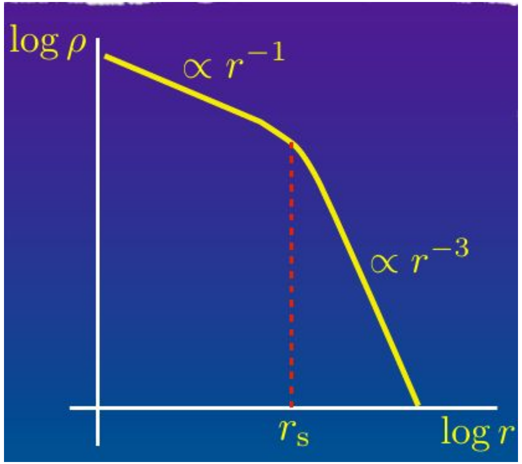

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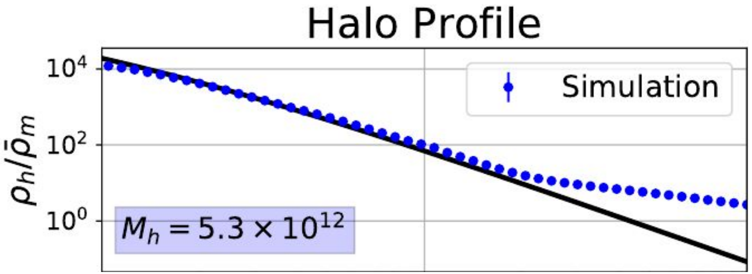

Problem: observations favor core over cusp profiles (0910.3538 for a review). More later

(from v. d. Bosch slides, 2003.06411)

I will address in this talk

How those observables change for ULA ...

... within the misalignment mechanism?

+ Mini-halos

... within the LARGE misalignment mechanism?

... within monodromies?

Misalignment

(1510.07633)

$$\ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0,$$

Misalignment

Initial condition for ϕ close to the minimum of its potential

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Misalignment

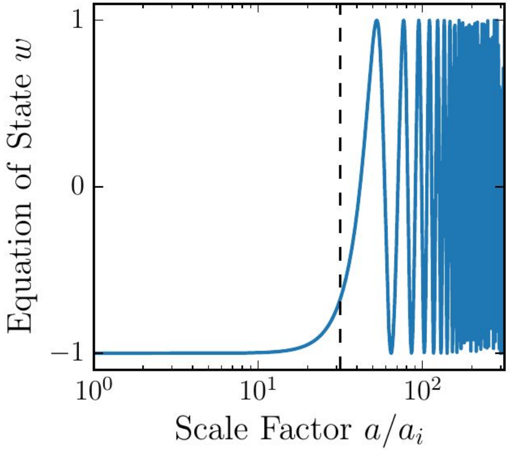
Initial condition for ϕ close to the minimum of its potential

Two regimes:

1) Overdamped $H > m_a$

2) Underdamped $H < m_a$

~ DE ~ DM



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Misalignment

Easy to calculate ULA abundance

$$\rho_a(a) \approx \rho_a(a_{\rm osc})(a_{\rm osc}/a)^3$$

Initial condition for ϕ close to the minimum of its potential

$$\Omega_a \approx \begin{cases} \frac{1}{6}(9\Omega_r)^{3/4}\left(\frac{m_a}{H_0}\right)^{1/2}\left\langle\left(\frac{\phi_i}{M_{pl}}\right)^2\right\rangle \text{if } a_{\rm osc} < a_{\rm eq}\,, \\ \frac{9}{6}\Omega_m\left\langle\left(\frac{\phi_i}{M_{pl}}\right)^2\right\rangle \text{if } a_{\rm eq} < a_{\rm osc} \lesssim 1\,, \\ \frac{1}{6}\left(\frac{m_a}{H_0}\right)^2\left\langle\left(\frac{\phi_i}{M_{pl}}\right)^2\right\rangle \text{if } a_{\rm osc} \gtrsim 1\,, \end{cases}$$

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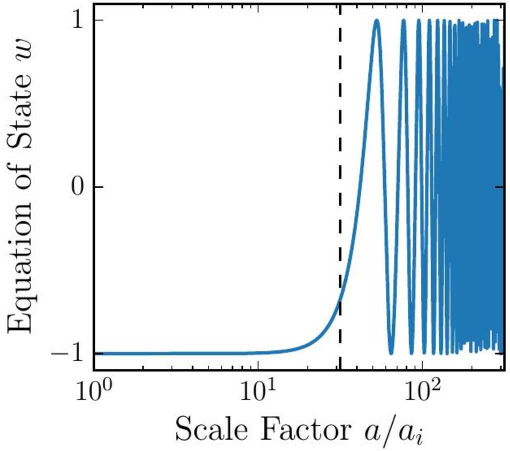

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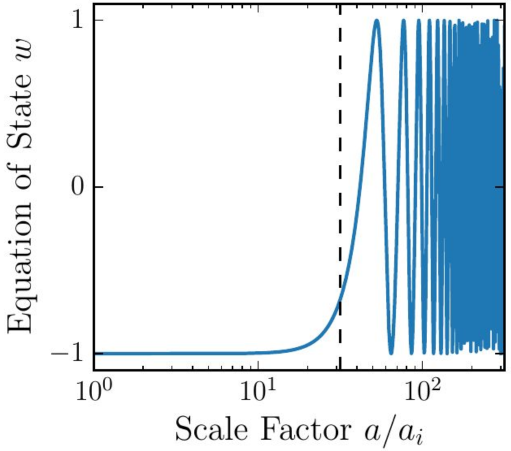

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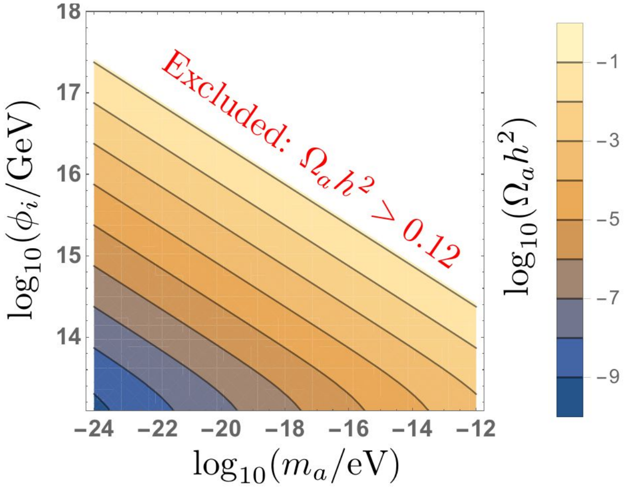

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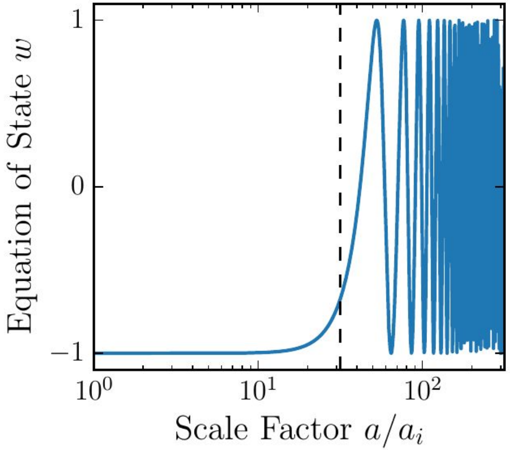
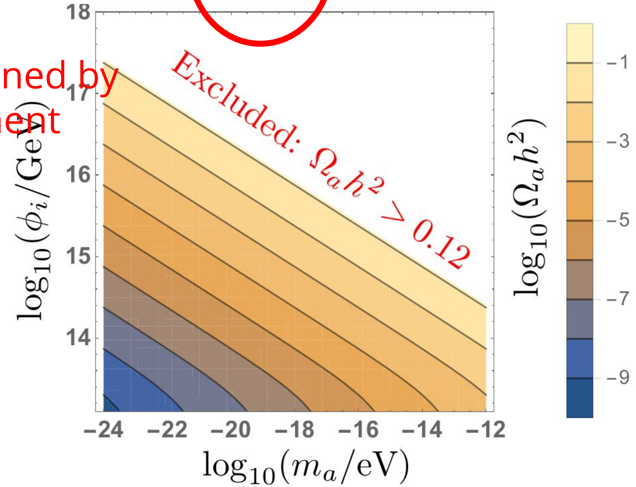


Abundance determined by the initial displacement



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Misalignment

Now we are done with bkg,
let's analyse fluctuations

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Now we are done with bkg,
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I hope you remind those guys
from 15 min ago

$\mathcal{R}(k)$ Initial conditions (inflation)

$T(k)$ Transfer function: Behaviour of the mode by entering the horizon

$D(z)$ Growth rate: late time linear growth

Misalignment - Initial conditions

(1510.07633)

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$$S_{ij} = 3(\zeta_i - \zeta_j)$$

$$\zeta_i = -\Psi - H\frac{\delta\rho_i}{\dot{\rho}_i} \, .$$

** They will be relevant for miniclusters, as we will see

(1510.07633)

Misalignment - Transfer function

(Maggiore 2, 1510.07633,
1410.2896)

Misalignment - Transfer function

CDM

$$\delta_{\rm c}{}' + iku_{\rm c} = -3\Phi',$$

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$\longrightarrow$

ULA

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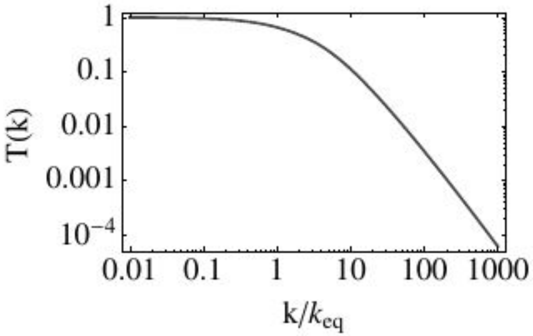

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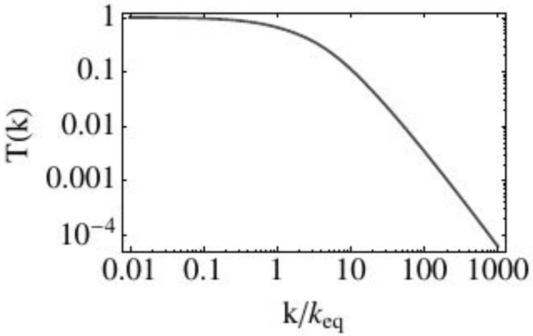

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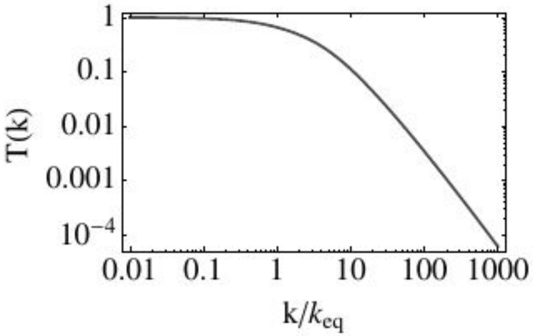

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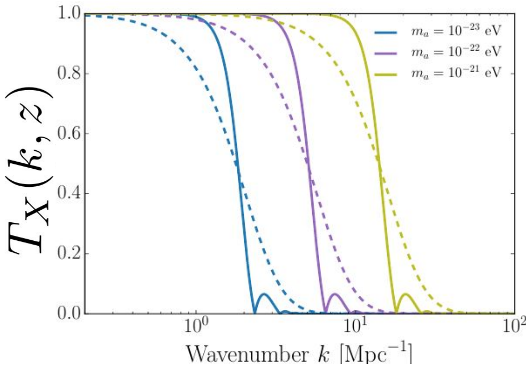

ULA (solid) and WDM (dashed)

(Maggiore 2, 1510.07633, 1410.2896)

Misalignment - Growth function

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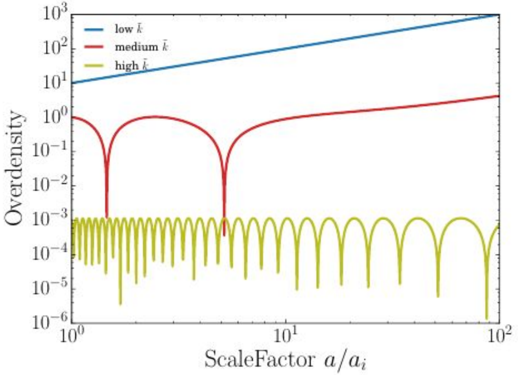
Scale dependent growth



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Misalignment - LSS + CMB Constraints

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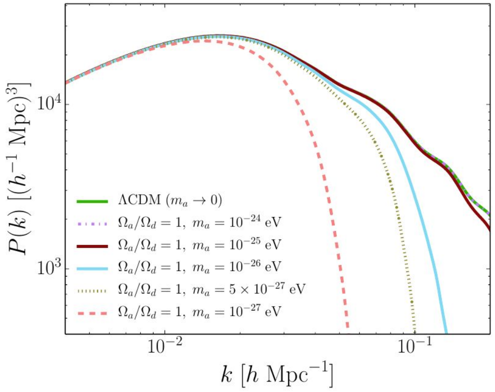

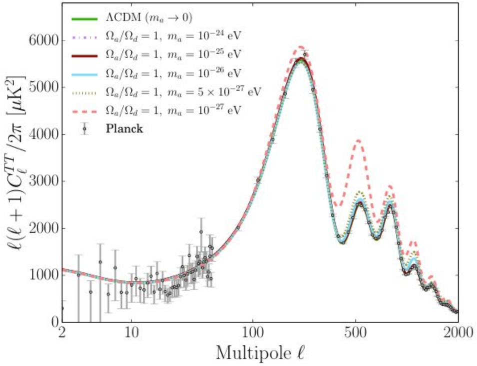

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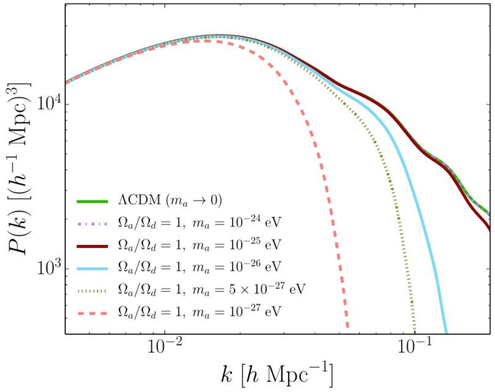

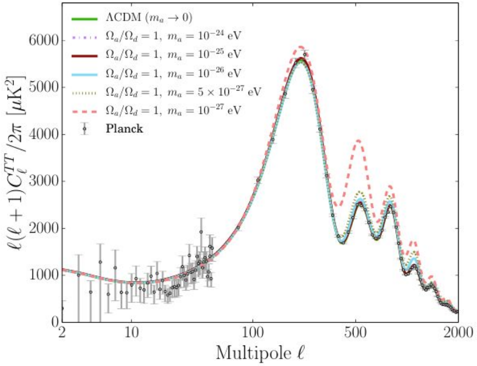

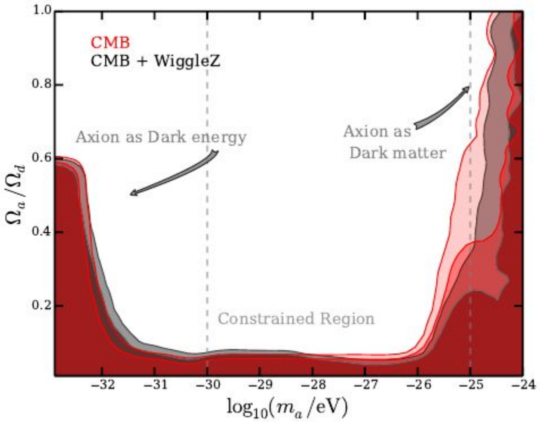

(1410.2896)

Non-linear evolution

(2101.11735, 1911.04505)

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Factor out time part that oscillates fast (WKB)

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(2101.11735, 1911.04505)

Misalignment - Halo Profile

(1407.7762)

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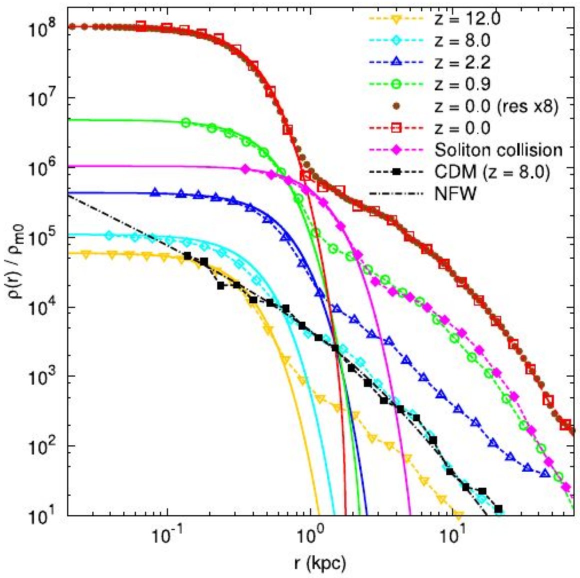
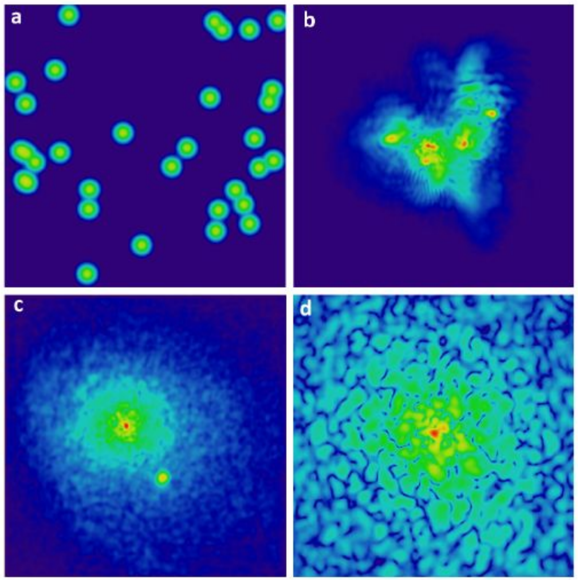
From numerical simulations





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Misalignment - Halo Mass function

(1409.3544)

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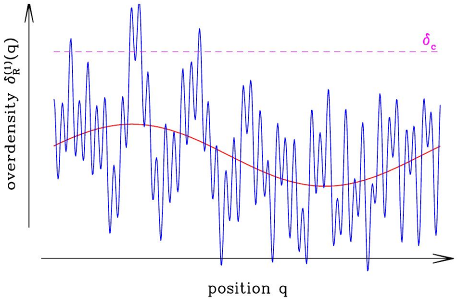

Jeans scale introduces threshold dependence

(1409.3544)

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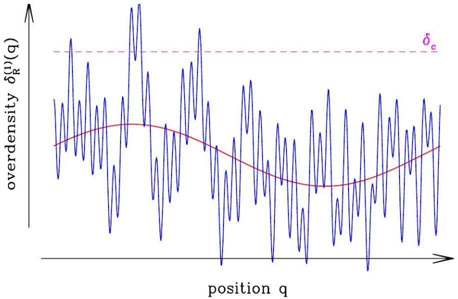

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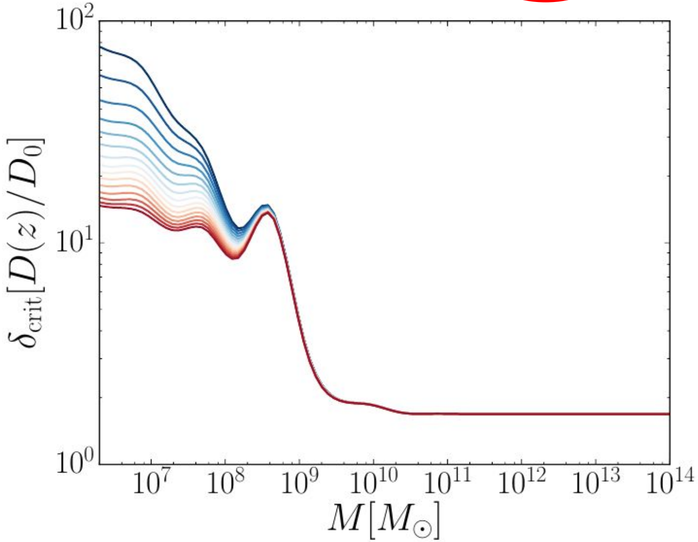

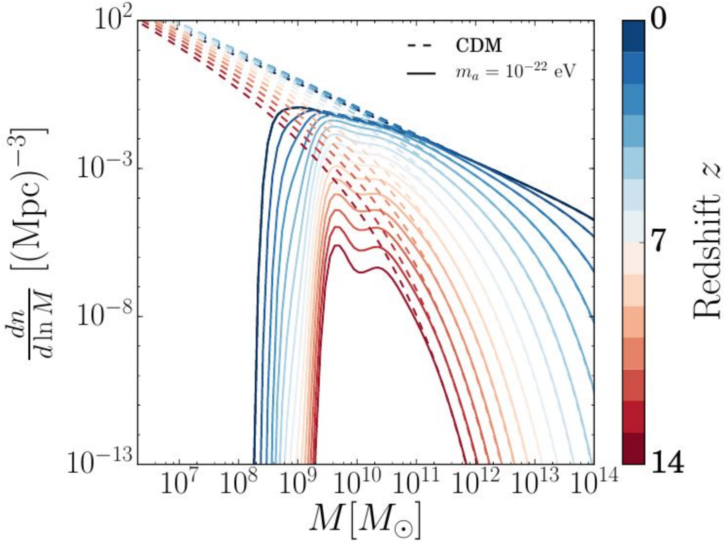

(1409.3544)

Misalignment: Mini-Halos

(1707.03310)

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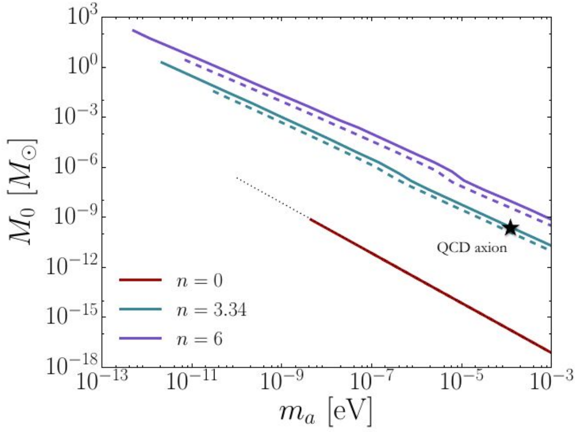
$$M_0(m_a, n=0) \approx 2.3 \times 10^{-7} M_\odot \left(\frac{m_a}{10^{-10}\ \mathrm{eV}}\right)^{-3/2} \left(\frac{\Omega_c h^2}{0.12}\right)\left(\frac{\Omega_m}{0.32}\right)^{-3/4}\left(\frac{1+z_{\rm eq}}{3403}\right)^{3/4}$$



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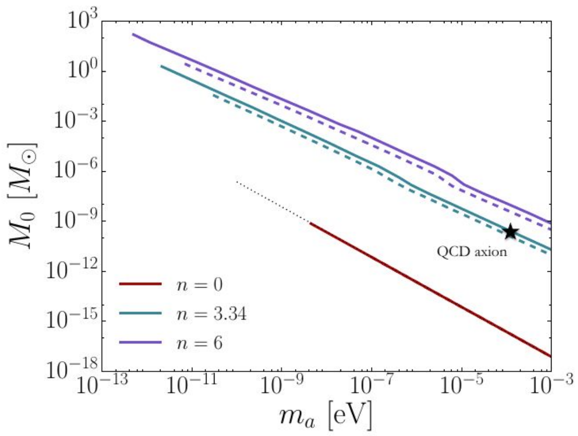

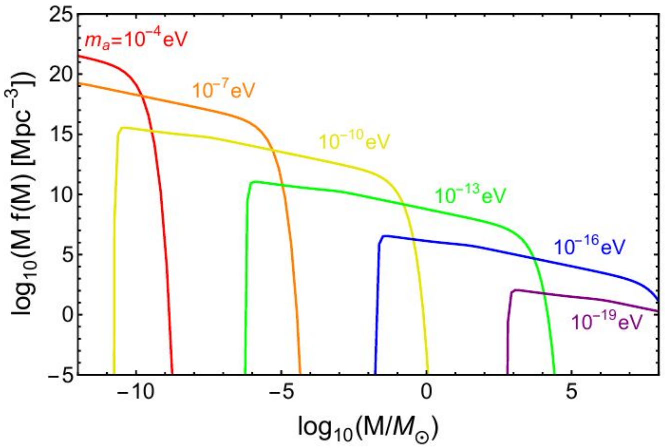

(1707.03310)

(1708.04466)

Misalignment: MiniHalos, Miniclusters for QCD axion

$M^2/\overline{\rho}(dn/dM)$

(1708.04466)

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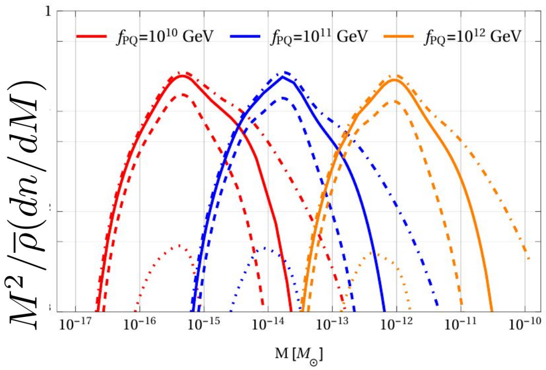

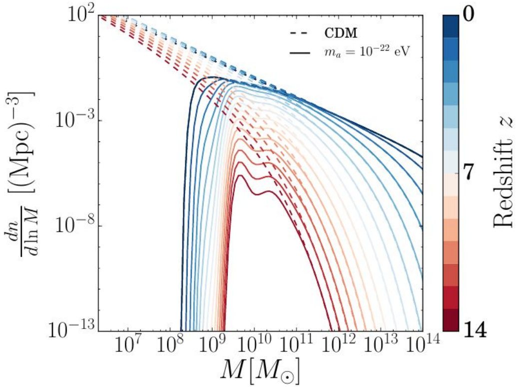

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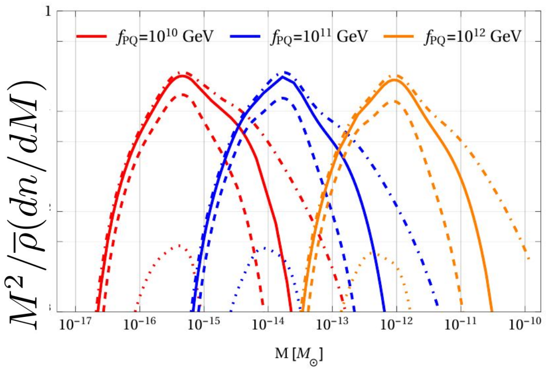

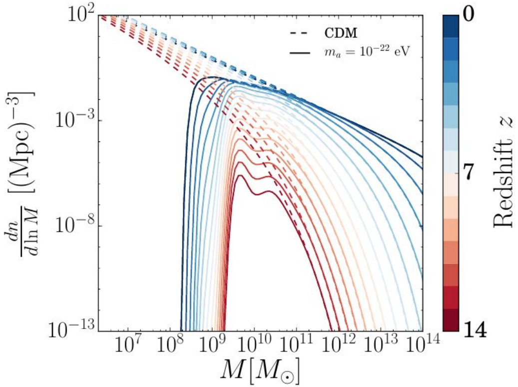

PQ broken after inflation (f < H_I):

Different Hubble patches with different θ_{PQ}

Topological defects that decay generating axions (complicated stuff)

(1708.04466)

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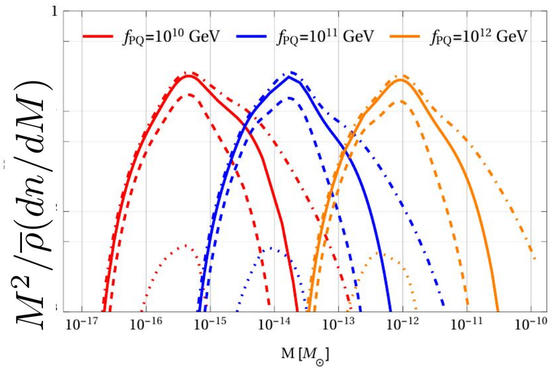

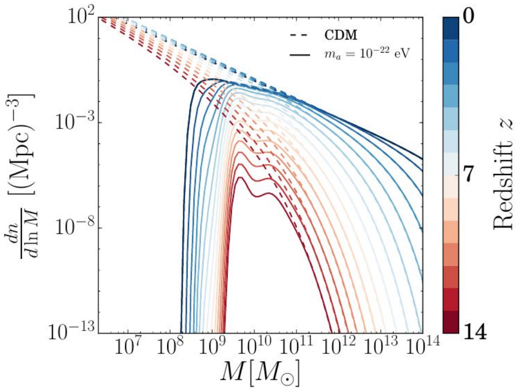

PQ broken after inflation (f < H_I):

Different Hubble patches with different θ_{PQ}

Topological defects that decay generating axions (complicated stuff)

$$P_\theta^{\mathrm{TH}}(k) = \frac{2\pi^4}{K^3}\Theta(K - k)\,.$$

K the typical comoving Hubble at ~ $1\ \mathrm{GeV}$

(1708.04466)

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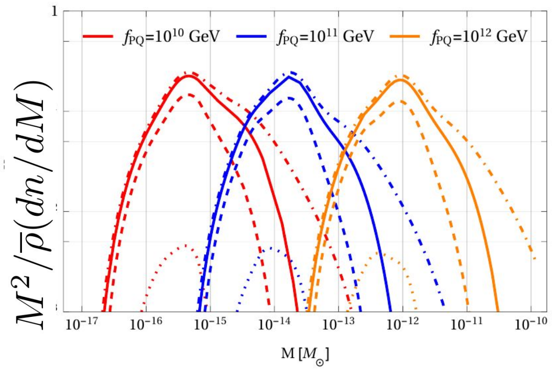

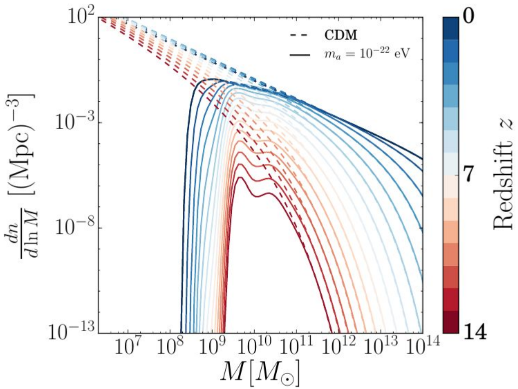

We can write down the energy density as

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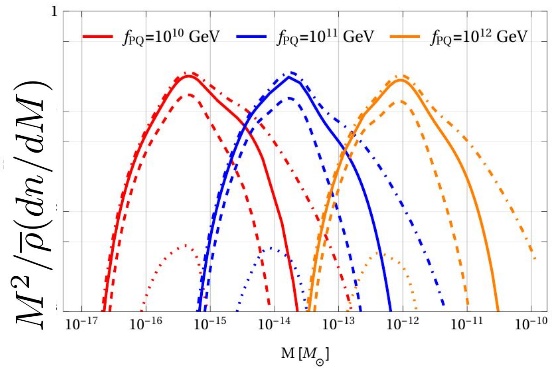

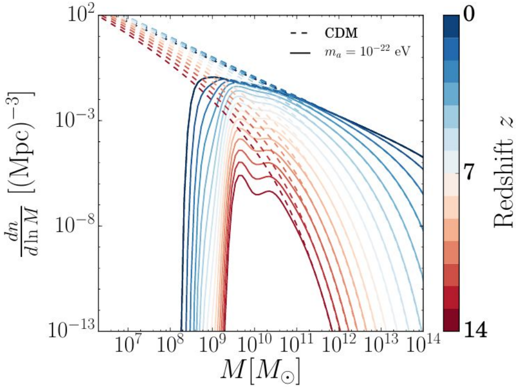

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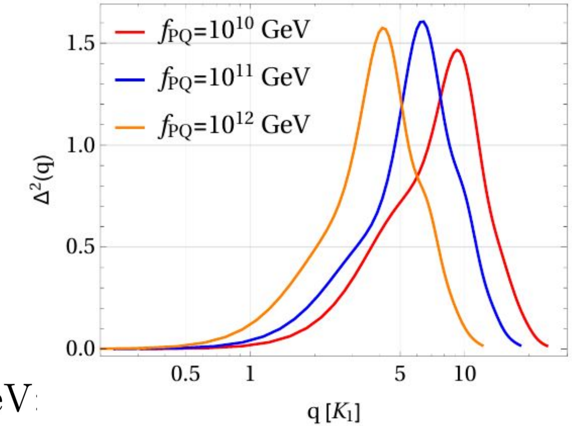
After some developments (Wick's theo, etc.)



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Large misalignment

(mostly from 1909.11665)

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self-interaction

$$V = m^2 f^2 \left[1 - \cos\left(\frac{\phi}{f}\right)\right] \qquad \ddot{\phi} + 3H\dot{\phi} + m^2\phi - \lambda\phi^3 + \cdots = 0$$

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Lets analyse the fluctuations

(mostly from 1909.11665)

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$$\theta''_{\mathbf{k}} + \frac{3}{2t_m}\theta'_{\mathbf{k}} + \left[\cos(\Theta) + \frac{\tilde{k}^2}{t_m}\right]\theta_{\mathbf{k}} = S\left(\tilde{k}, t_m\right)$$

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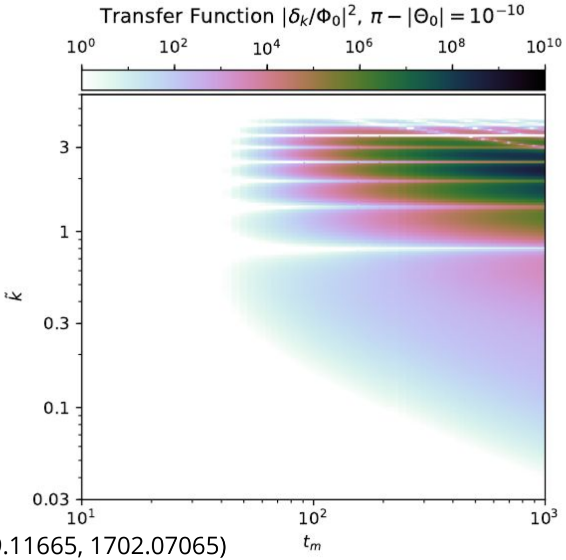

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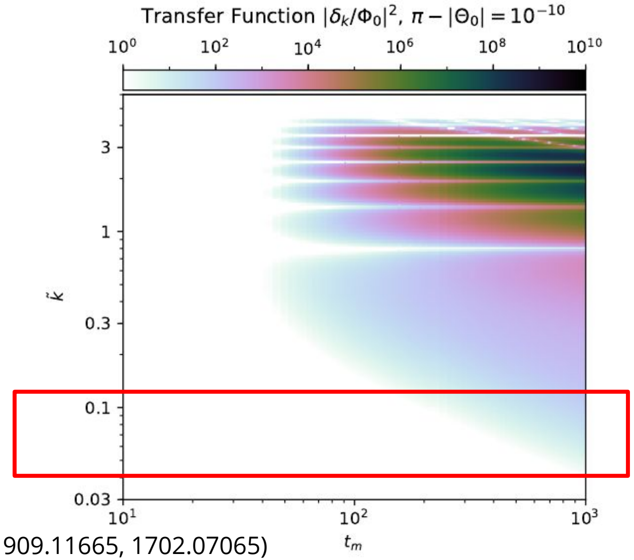

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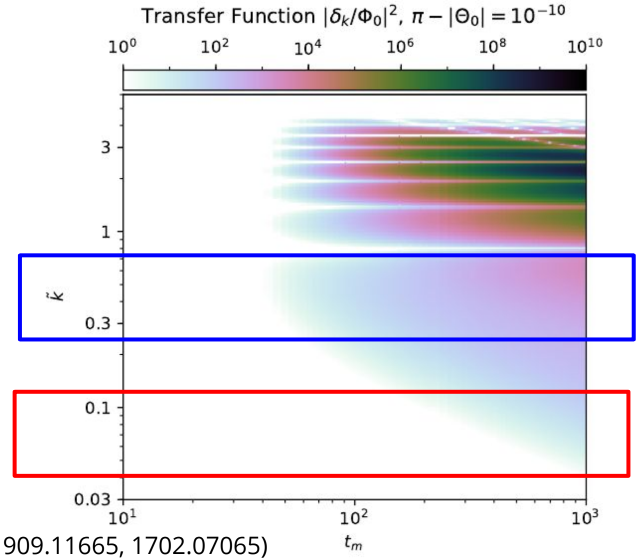

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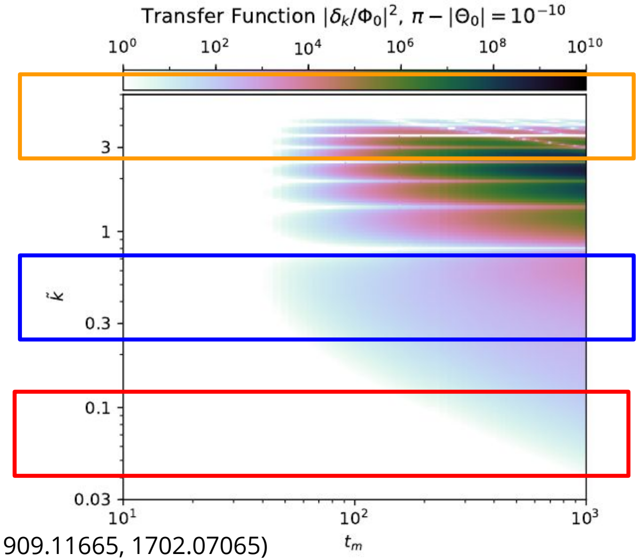

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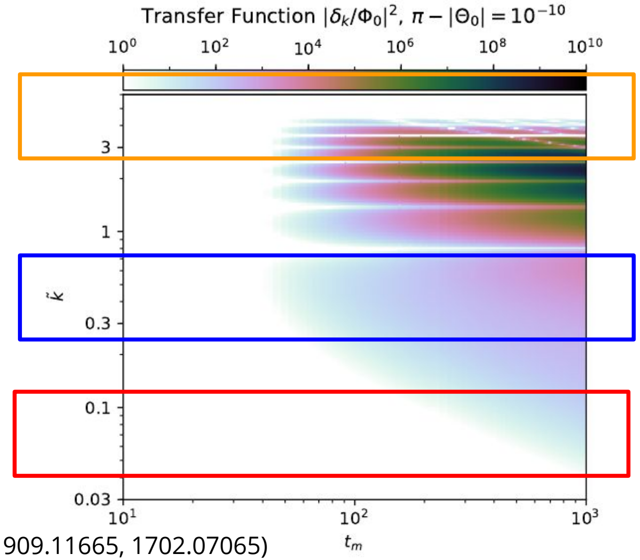

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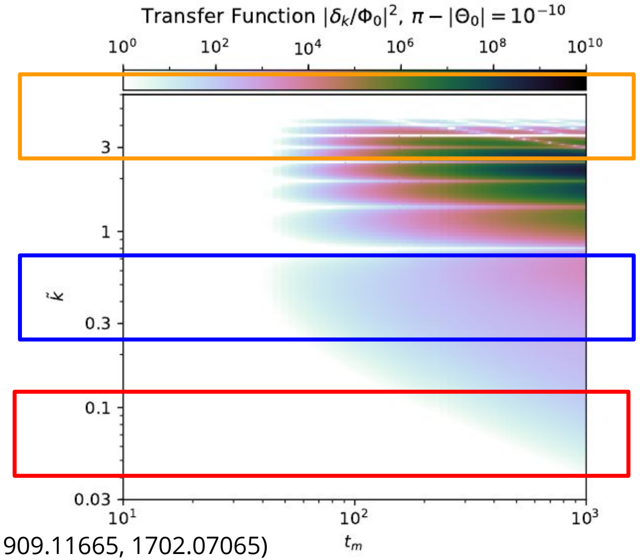

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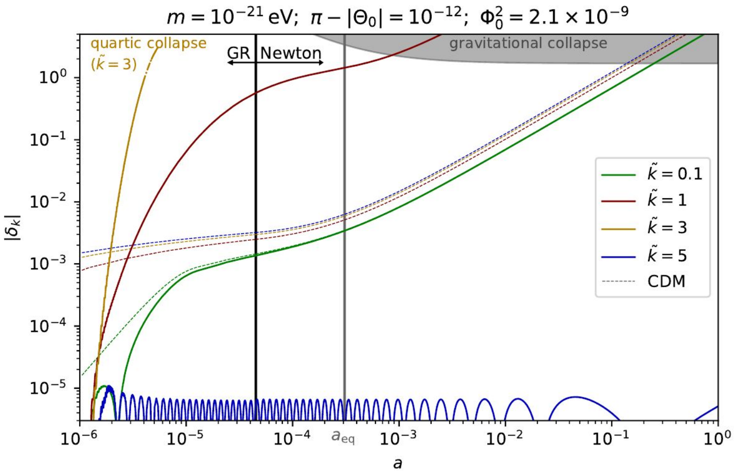

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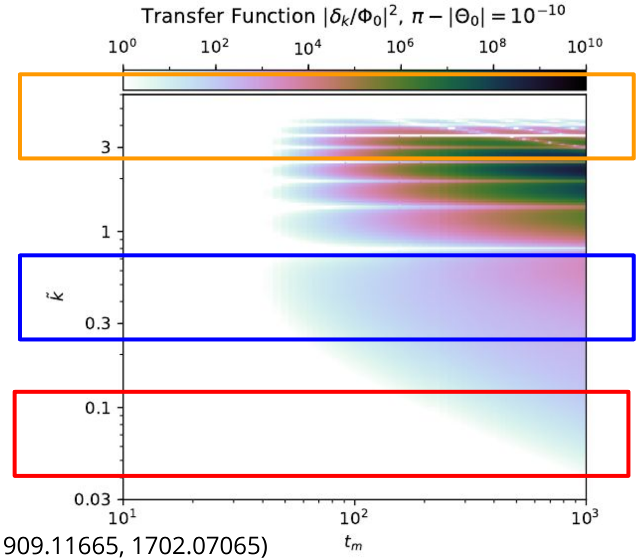

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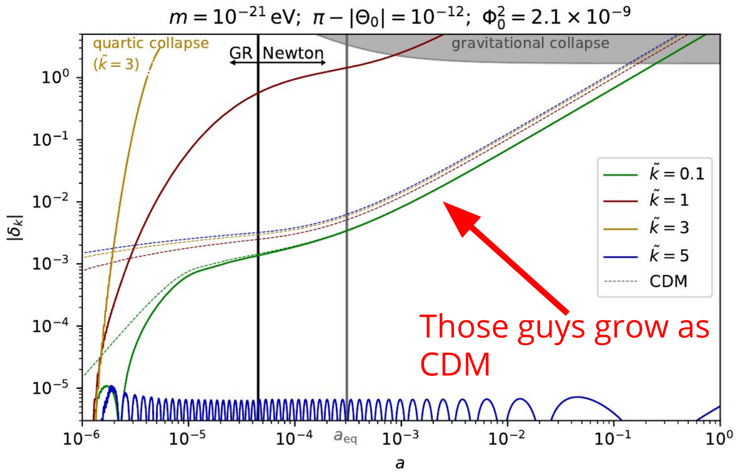

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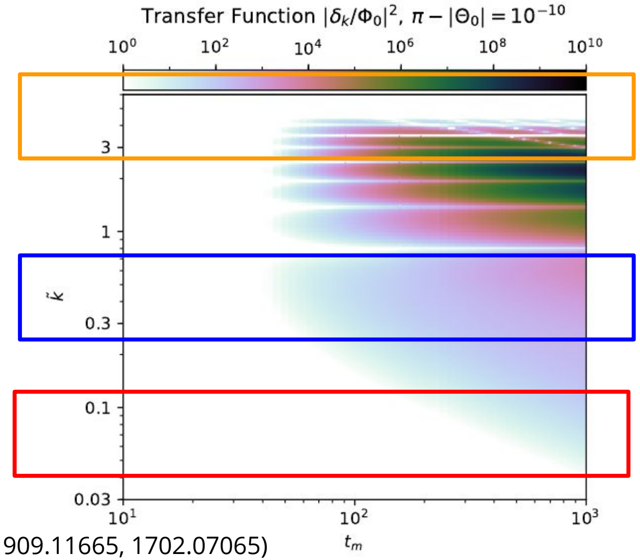

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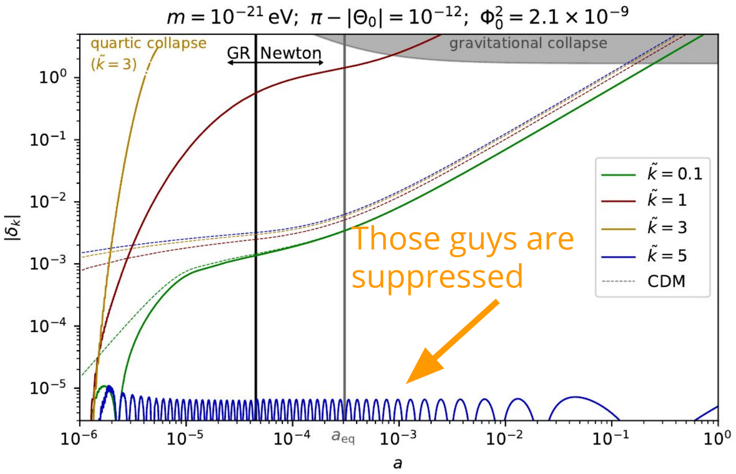

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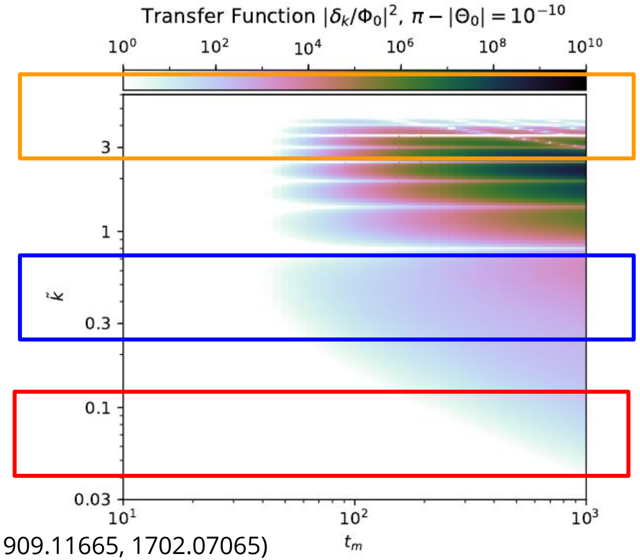

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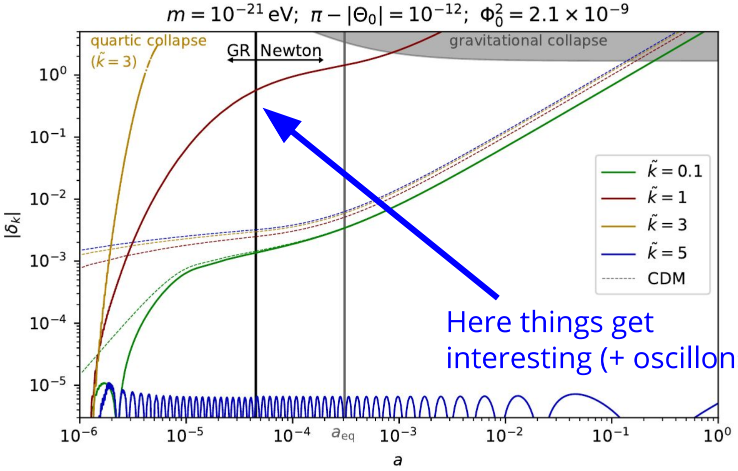

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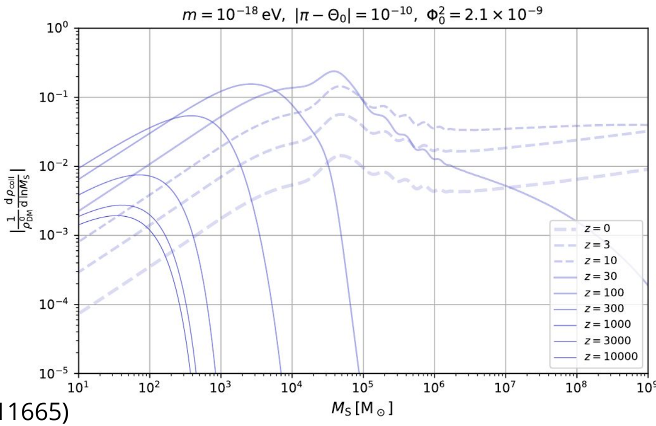

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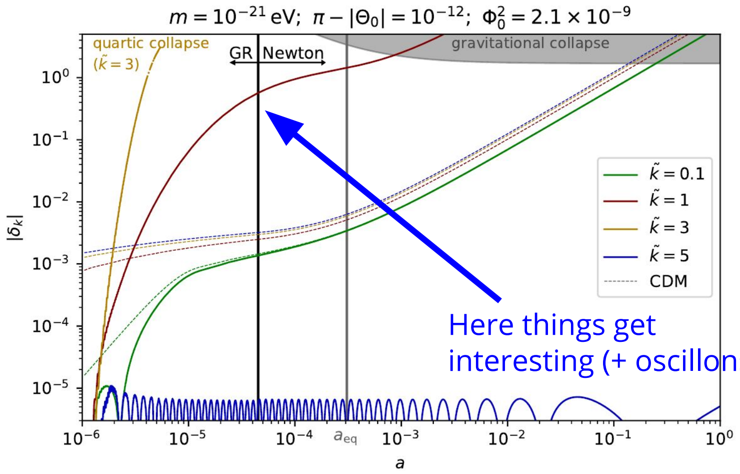

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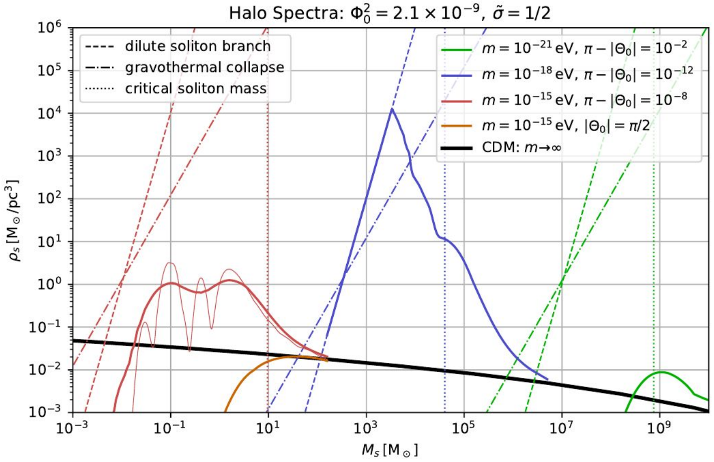

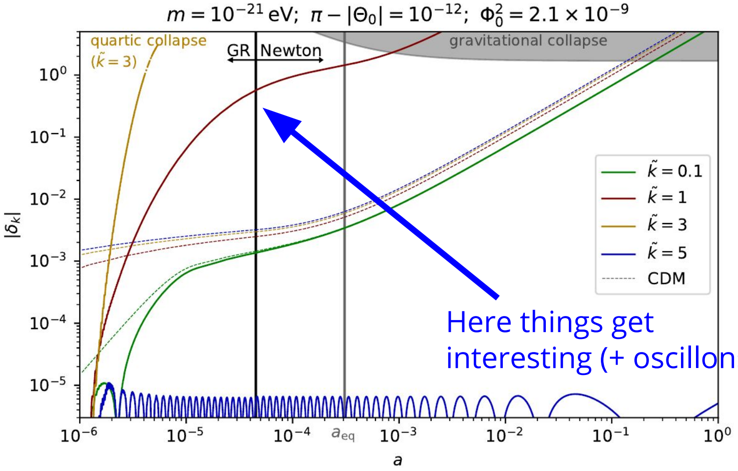

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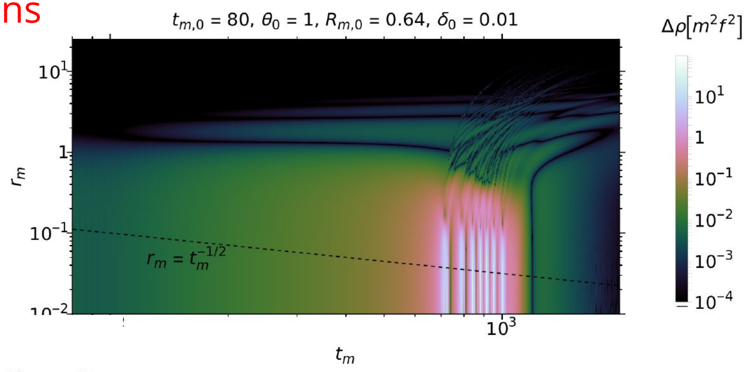

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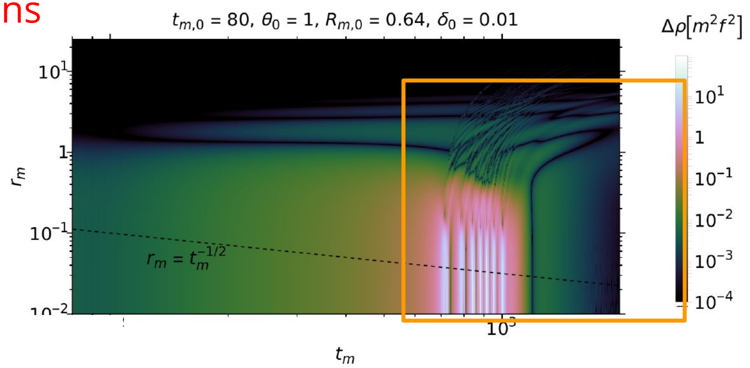

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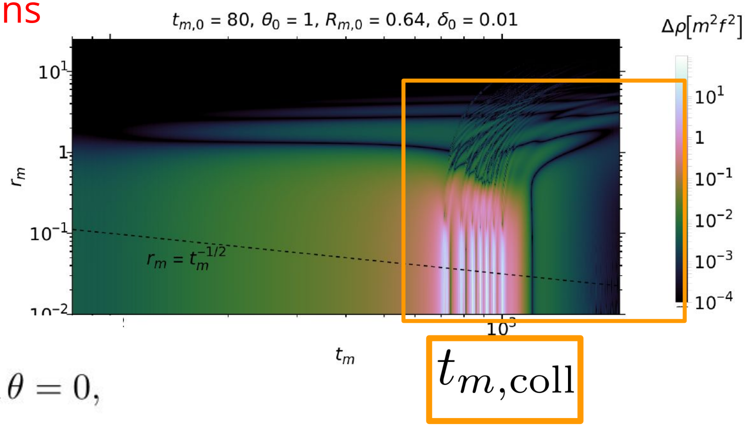

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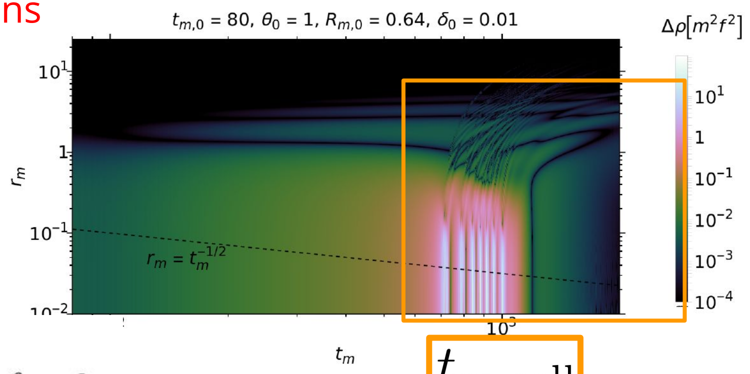

$t_{m,\mathrm{coll}}$

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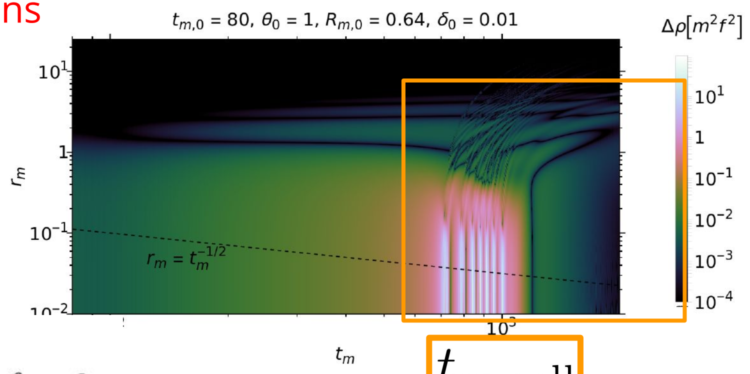
(1909.11665)

Oscillons

Metastable and compact, bound by self-interaction (different than solitons that are bound by gravity)

Decay: emission of scalar + tensor waves



$t_{m,\mathrm{coll}}$

Numerical simulations

Initial condition (gaussian)

$$\theta(t_{m,0}, \mathbf{x}_m) = \theta_0 \left[1 + \frac{1}{2}\delta_0 \exp\left(-\frac{r_m^2}{2R_{m,0}^2}\right)\right],$$

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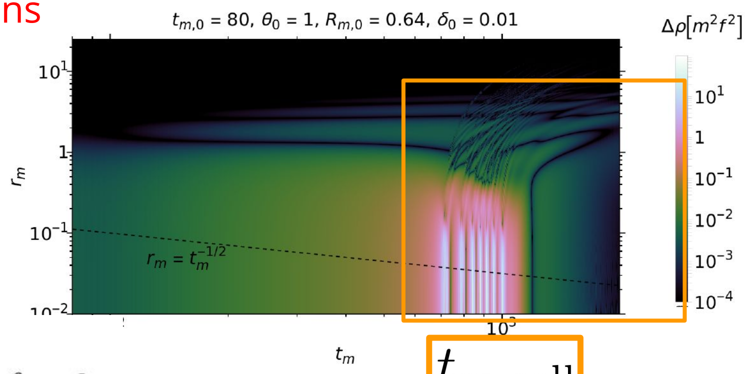

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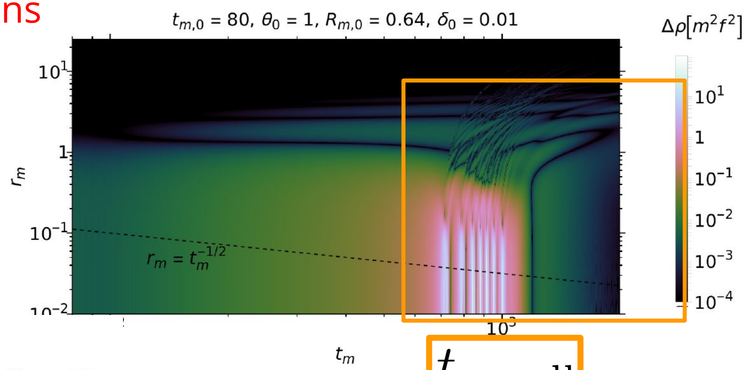

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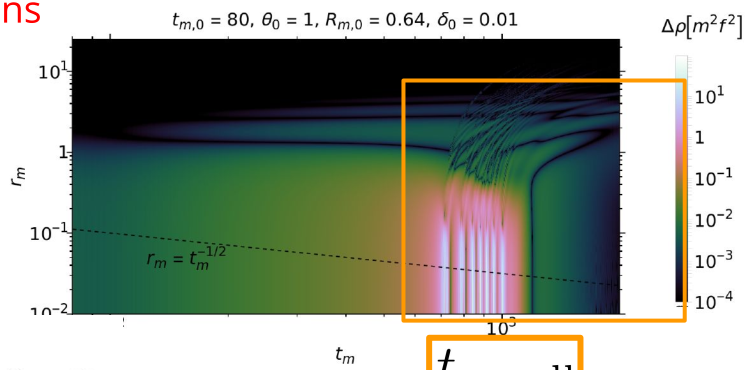

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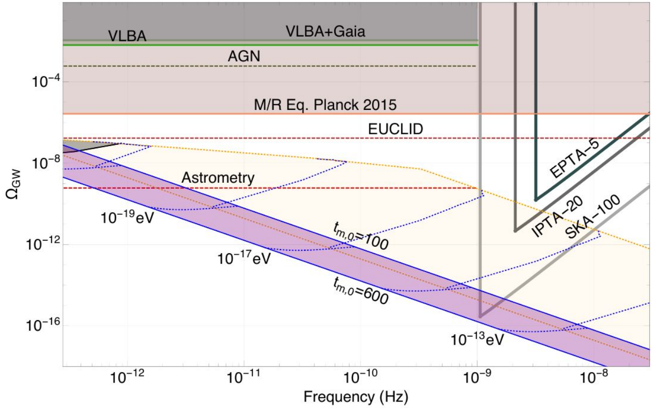

(1909.11665)

QCD axion

(1909.11665)

Mass and f constrained

QCD axion

$$m_a(T=0) = 5.70\,\mu\mathrm{eV}\left(\frac{10^{12}\,\mathrm{GeV}}{f_a}\right)$$

Temperature corrections from light quarks

$$V(\phi) = -m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4m_u m_d}{(m_u+m_d)^2}\sin^2\left(\frac{\phi}{2}\right)}$$

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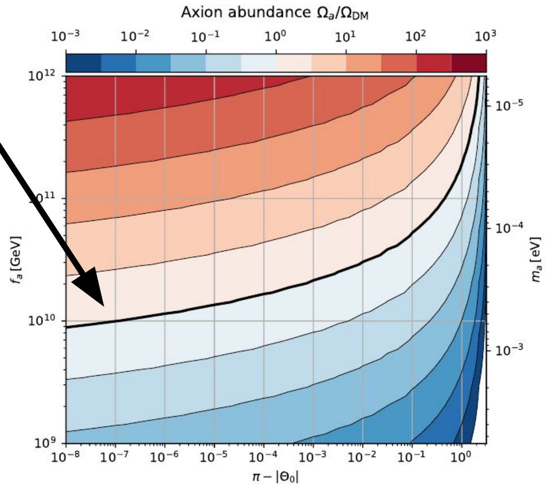

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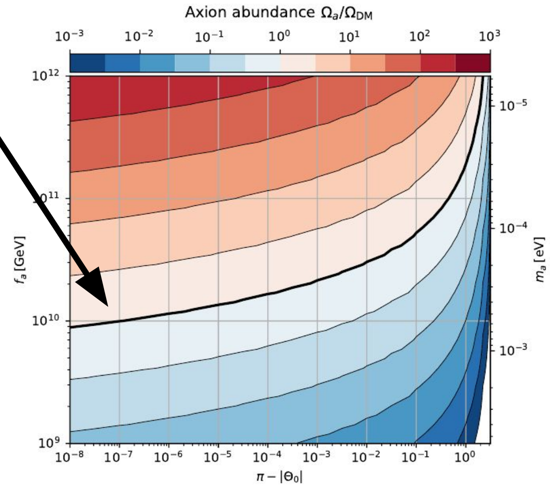

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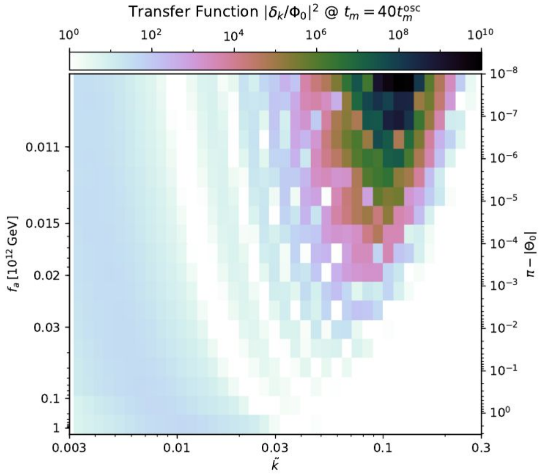

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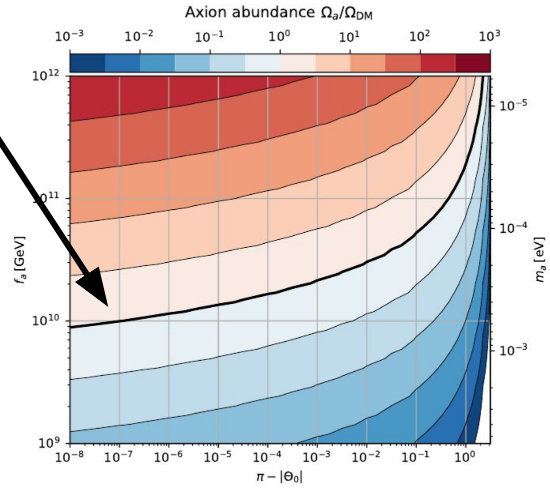

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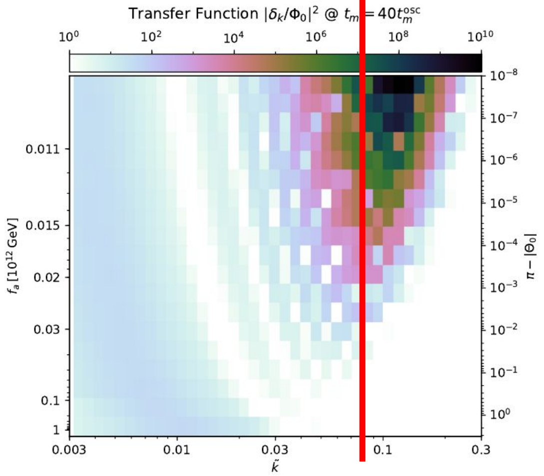

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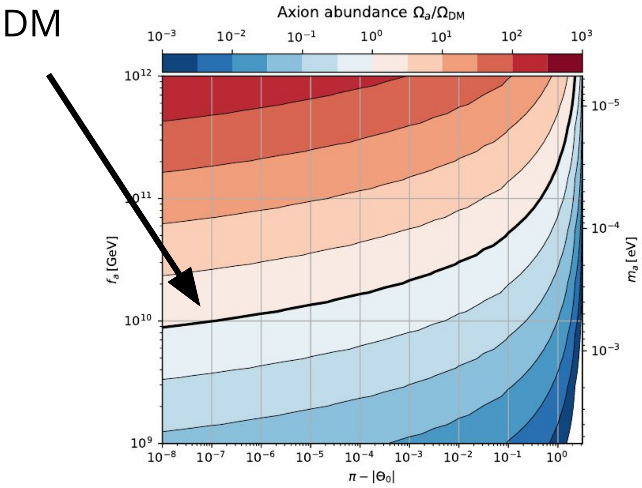

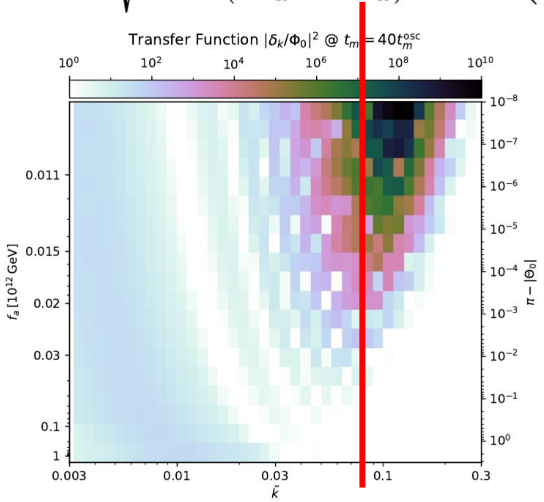

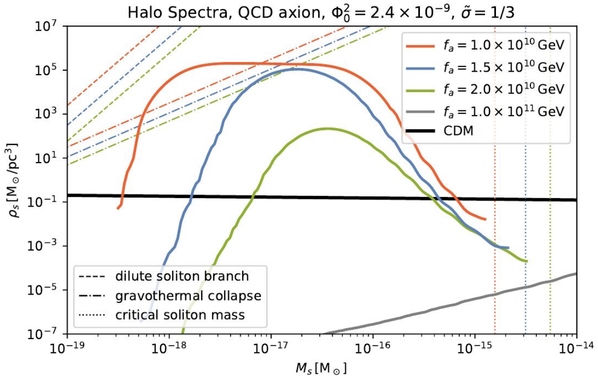

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IC fine tuning

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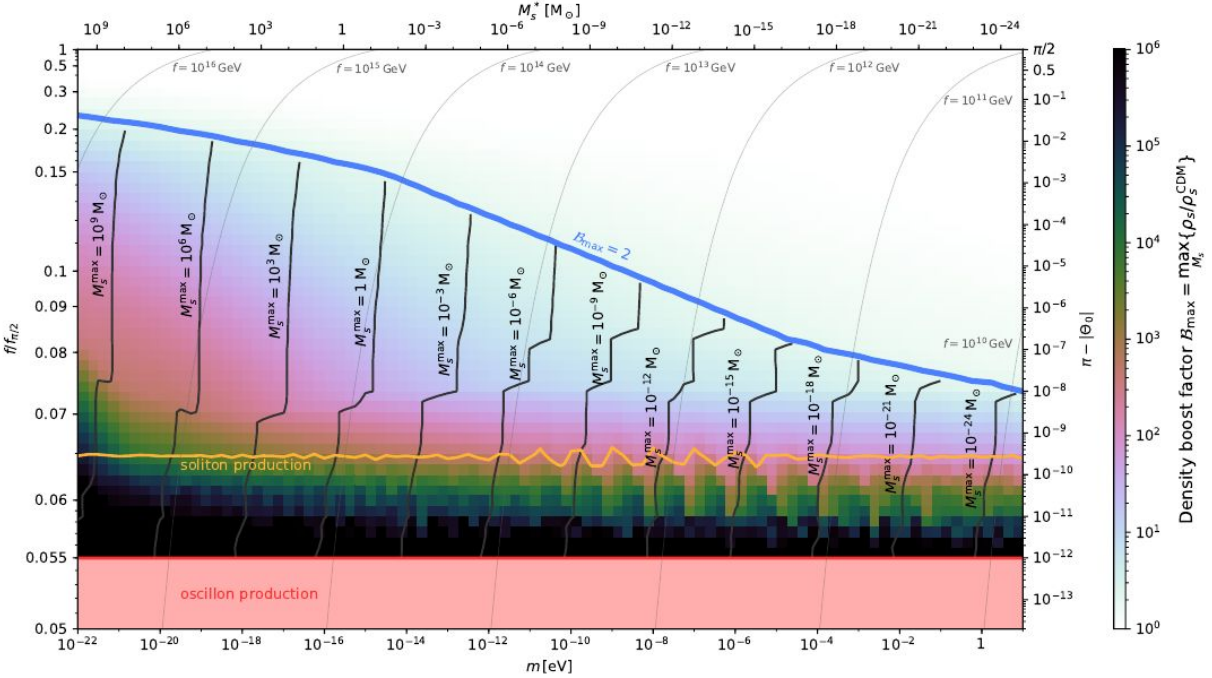

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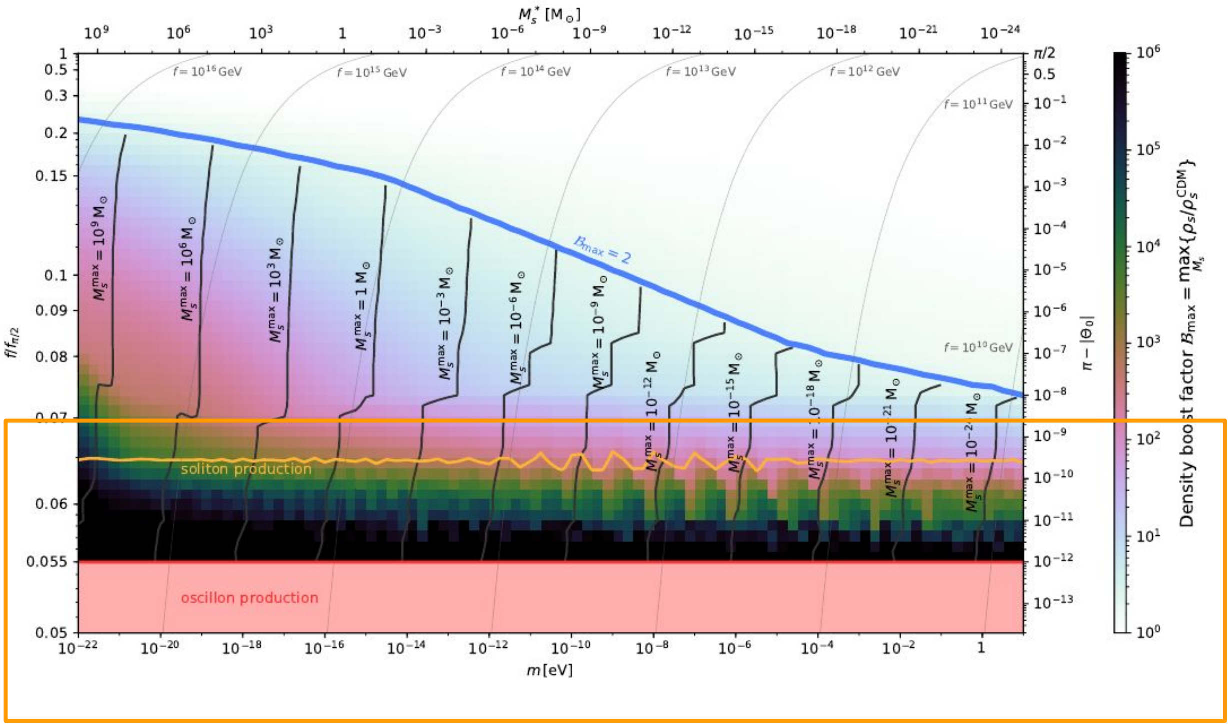
IC fine tuning

To have a large effect we need theta~pi



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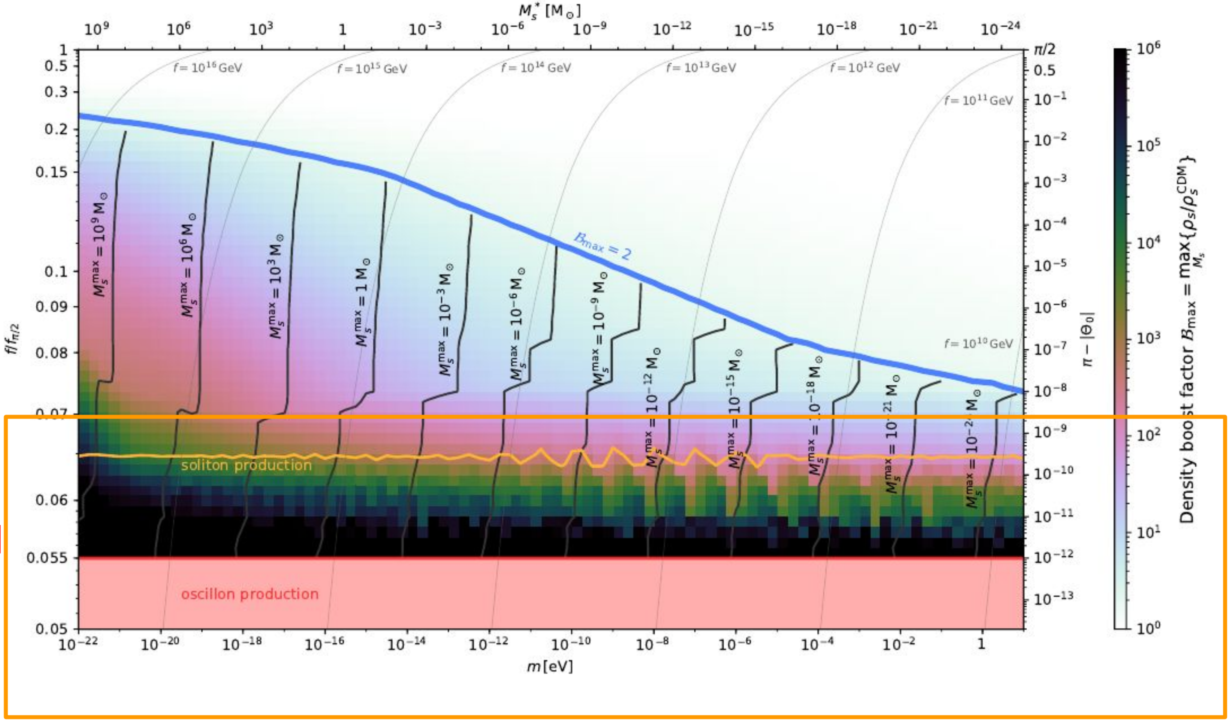
People have cook up inflation-based/anthropic-based mechanisms to get theta~pi (1812.11192, 0810.0703)



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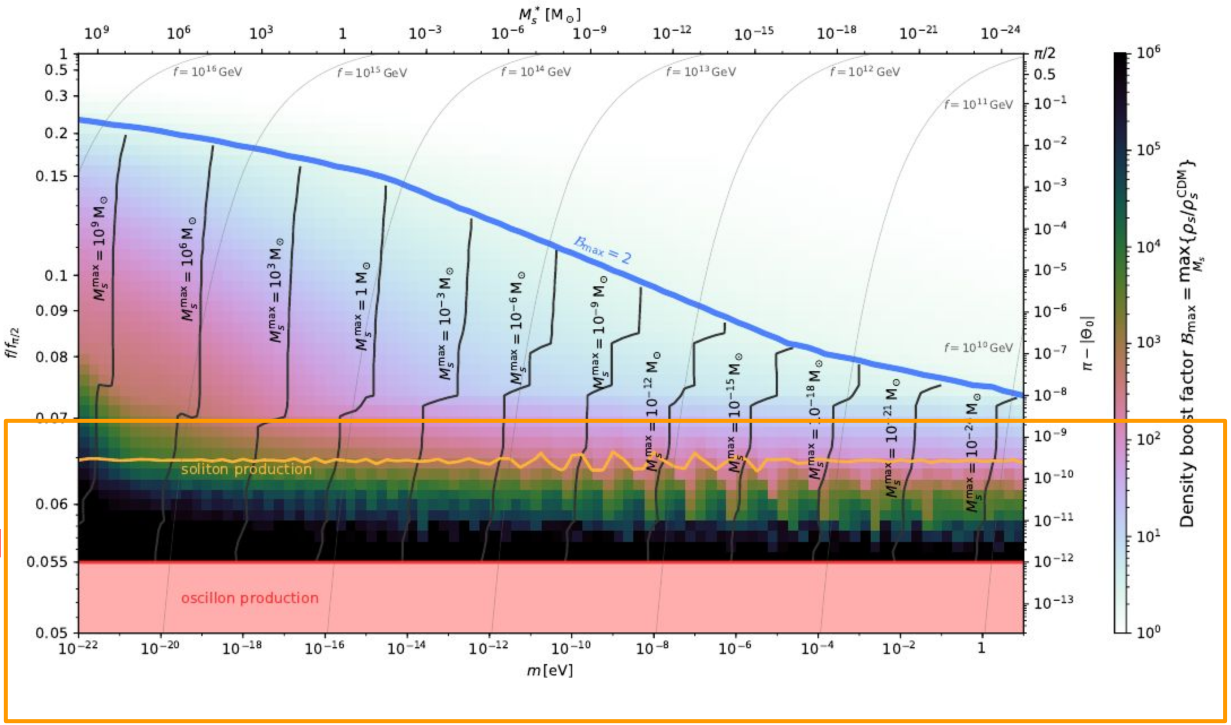

*** For more about astrophysical effects (star formation, lensing), see 1909.11665

(1909.11665)

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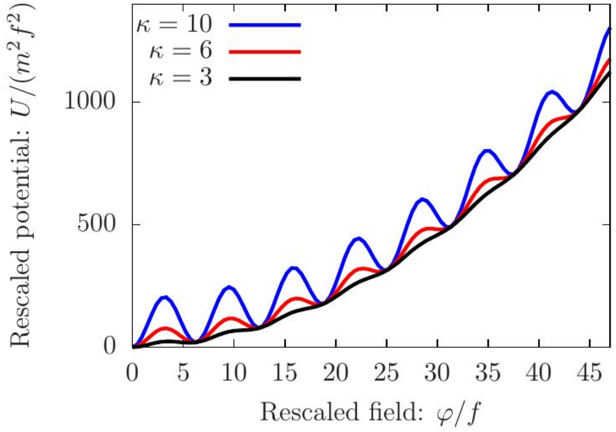

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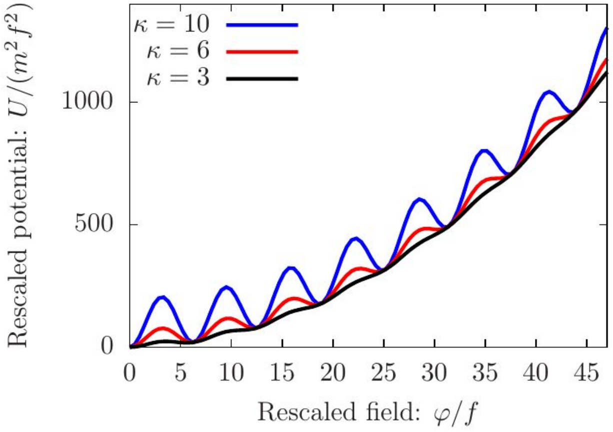

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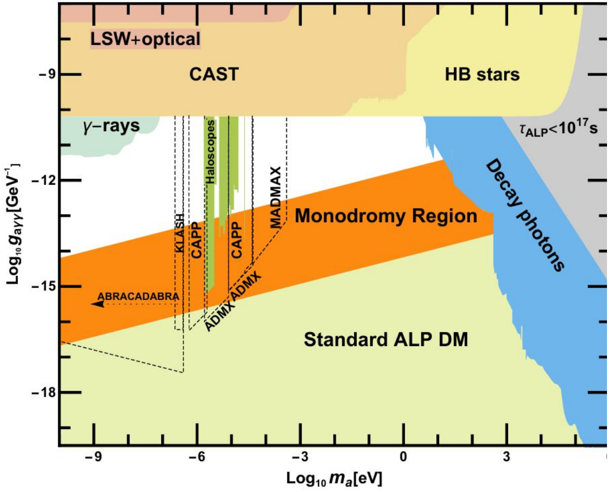

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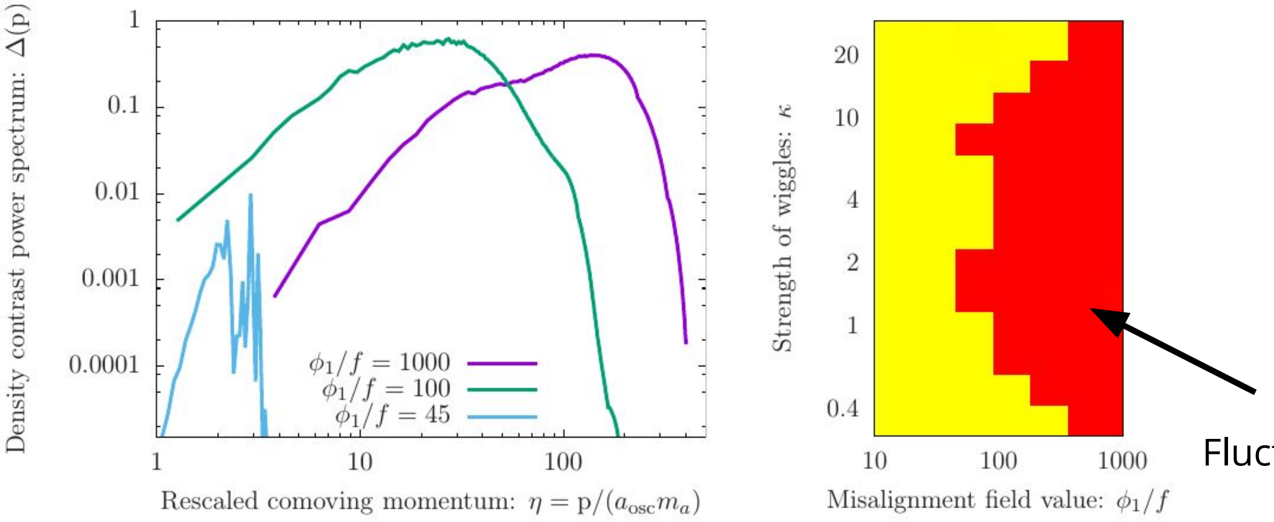

Fluctuations ~1

Monodromies

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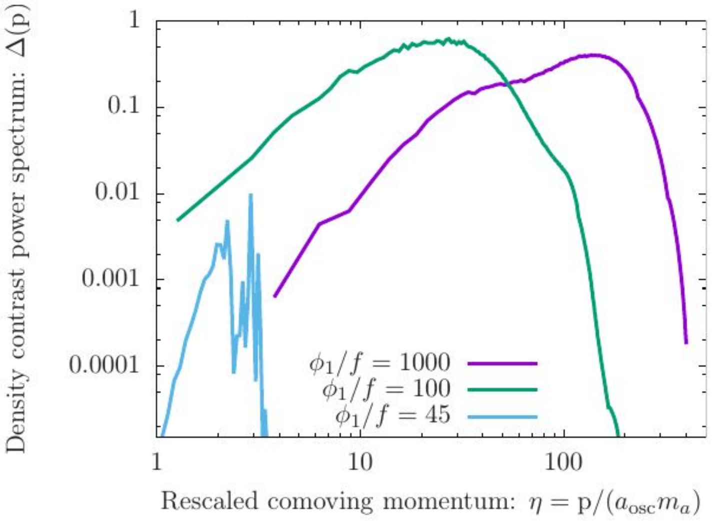

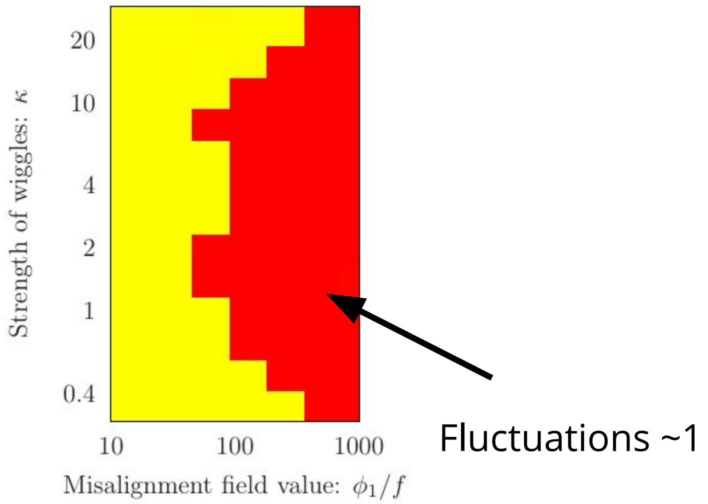

Using Press-Schechter formalism (linear evolution in time), they show it didn't form miniclusters yet

Too small, pressure is too high

Final remarks

Depending on axion potential and initial conditions, we have a whole zoo of structures that can form in a broad range of masses

Rich pheno and observational prospects (GW, lensing, star formation, tidal disruption, ...)

Non-linear evolution? SP solver

How baryons affect this picture?

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Such a broad area with many directions to follow.
I would be glad to chat more ;)