

GRAVITATIONAL WAVES AT PARTICLE STORAGE RINGS



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ω_g



10^{-10} nHz

10kHz

...

Size of the Universe

Largest from Astrophysics

ω_g

10 Hz

kHz

LIGO-VIRGO

ω_g

nHz

μ Hz

10 Hz

kHz

PULSAR TIMING

LIGO-VIRGO



Any new idea for detection is interesting, even the most “ambitious” ones

ARIES WP6 Workshop: Storage Rings and Gravitational Waves – SRGW2021



<https://indico.cern.ch/event/982987/>

<https://arxiv.org/pdf/2105.00992.pdf>

Sensitivity to Deformations

LIGO

10^{-15} mm

LHC

mm



Sensitivity to Deformations

Effective Size

LIGO

$$10^{-15} \text{ mm}$$

$$10^3 \text{ km}$$

$$(4 \text{ km}) \times 300$$

LHC

$$\text{mm}$$

$$10^5 \text{ km} \left(\frac{\tau}{\text{s}} \right)$$



Everything interacts with the gravitational wave: magnets, tunnel, protons, EM fields, us observers, ...





CAVEAT: large gauge symmetry of GR (i.e. rigid objects in flat space can move a lot in certain coordinate systems)

$$x^\mu \rightarrow x^{\mu'}(x^\mu)$$

Any change of coordinates still describes the same physics






We can use this symmetry to our advantage: choose a frame where our Newtonian intuition applies (**proper detector frame**) at least for small distances compared to the wavelength

$$\frac{L}{\lambda_g} \ll 1$$

This is the frame of an observer standing still next to the LHC and watching the protons fly by, i.e. exactly what we want



Everything interacts with the gravitational wave: magnets, tunnel, protons, EM fields, us observers, ...



However if we focus on a **small frequency range**, the **majority of potential detectors are rigid**



Any system in equilibrium if displaced by a small amount δ
responds as a harmonic oscillator

$$\ddot{\delta}(t) + \frac{\dot{\delta}(t)}{\tau_s} + \omega_s^2 \delta(t)$$





Everything interacts with the gravitational wave: magnets, tunnel, protons, EM fields, us observers, ...

$$\ddot{\delta}(t) + \frac{\dot{\delta}(t)}{\tau_s} + \omega_s^2 \delta(t) = \omega_g^2 f(\omega_g, t)$$



An aerial photograph of the ATLAS particle detector at CERN, showing the circular tunnel and various detector components. The word "ATLAS" is visible in the top right and bottom left corners.

Everything interacts with the gravitational wave: magnets, tunnel, protons, EM fields, us observers, ...

$$\tilde{\delta}(\omega) = \tilde{f}(\omega_g, \omega) \frac{\omega_g^2}{(\omega_s^2 - \omega^2) + i\tau_s \omega}$$

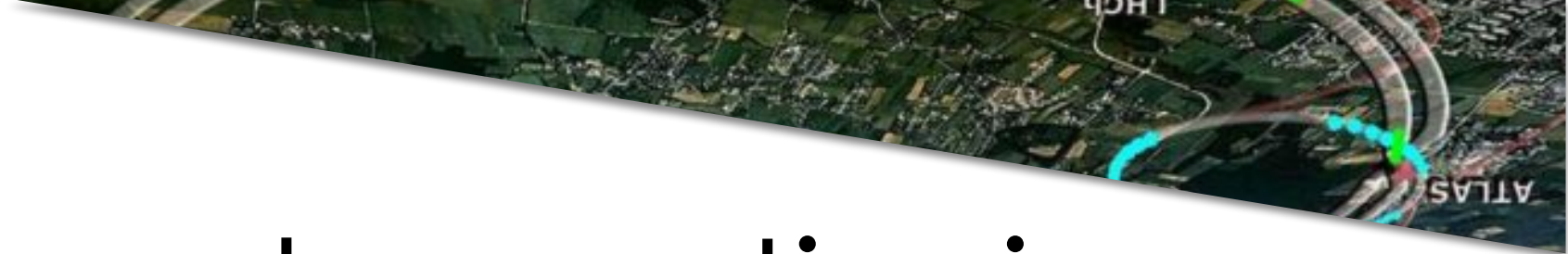
An aerial photograph of the ATLAS detector at CERN, showing the circular structure of the LHC tunnel and the ATLAS detector itself. The word "ATLAS" is visible in the top right corner.

Everything with $\omega_s \gg \omega_g$

Has a very suppressed response

$$\tilde{\delta}(\omega_g) \simeq \frac{\omega_g^2 \tilde{f}(\omega_g, \omega_g)}{\omega_s^2} \ll \tilde{f}(\omega_g, \omega_g)$$





This is useful also if the wave is not at all monochromatic, i.e. we can always ignore objects with characteristic frequencies much larger than our readout frequency

$$\omega_s \gg \omega_r$$

$$\tilde{\delta}(\omega_r) \simeq \frac{\omega_r^2 \tilde{f}(\omega_g, \omega_r)}{\omega_s^2}$$

CAVEAT: large gauge symmetry of GR (i.e. rigid objects in flat space can move a lot in certain coordinate systems)



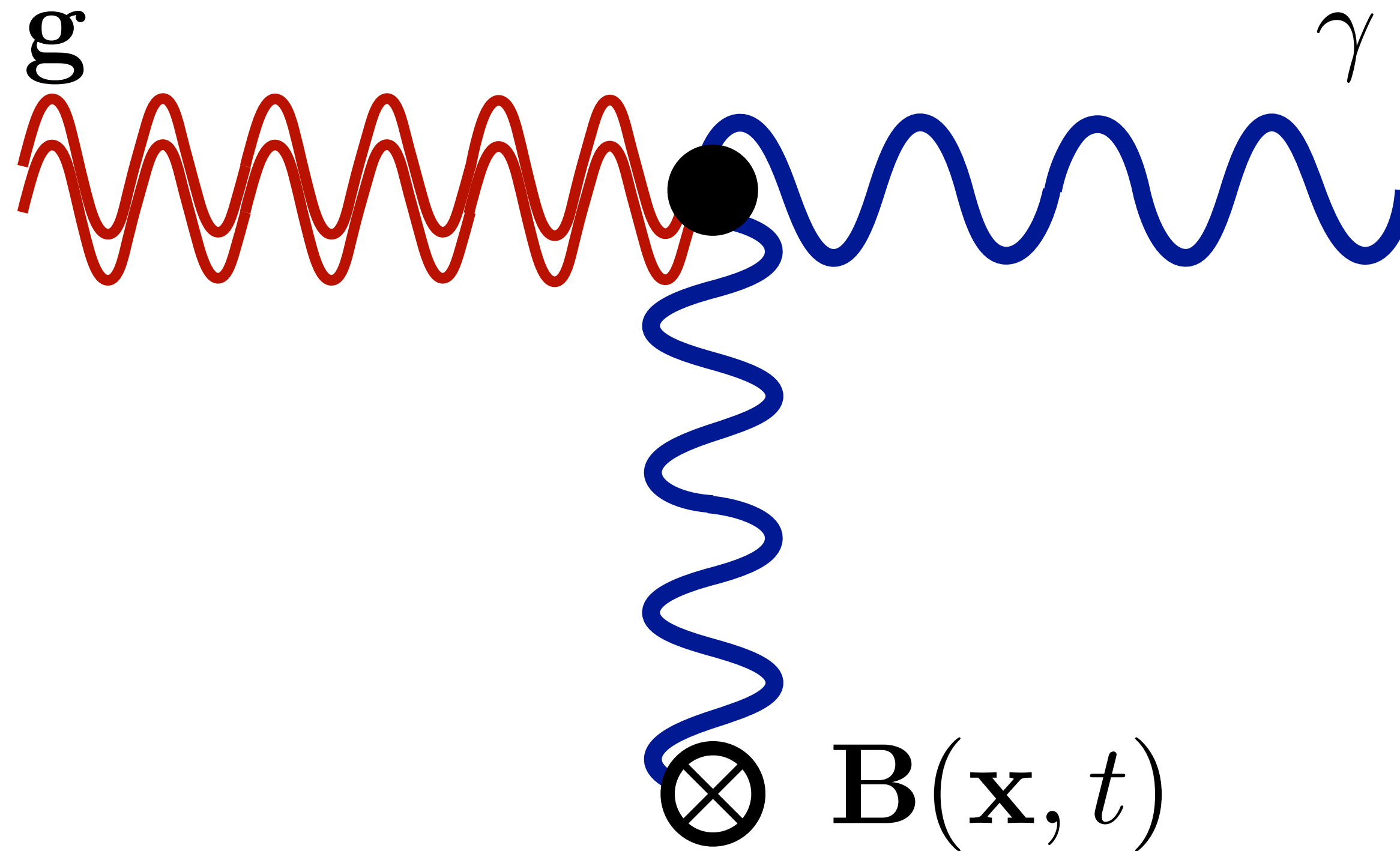


In the **proper detector frame** rigid objects don't move (magnets, tunnel, measuring devices, ...) so focus on **protons** and **EM fields**

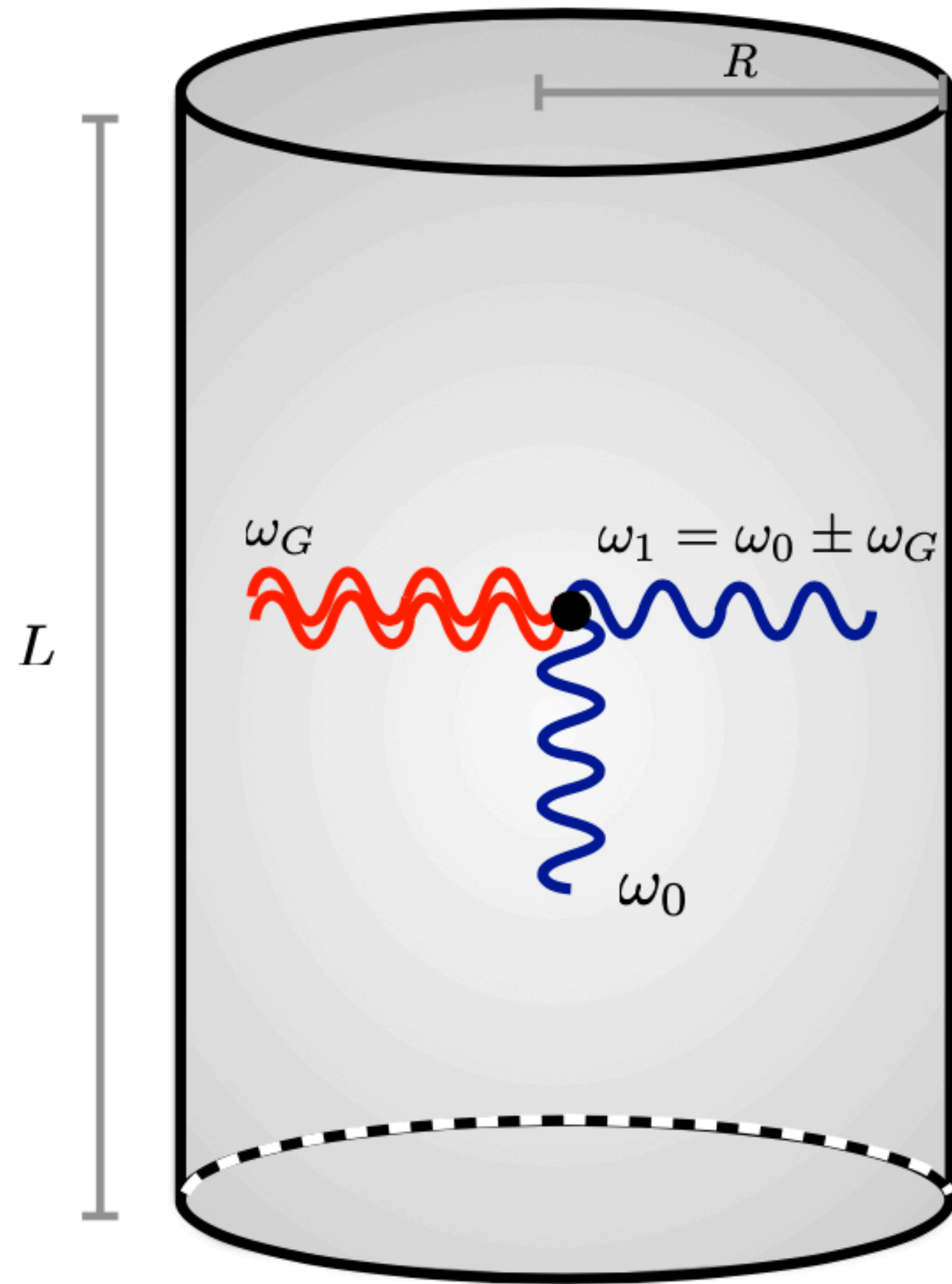
- **EM fields**
- **Radial**
- **Vertical**
- **Longitudinal**



EM fields



Braginskii & Menskii, 1971



Superconducting RF Cavity

~ Proton Acceleration

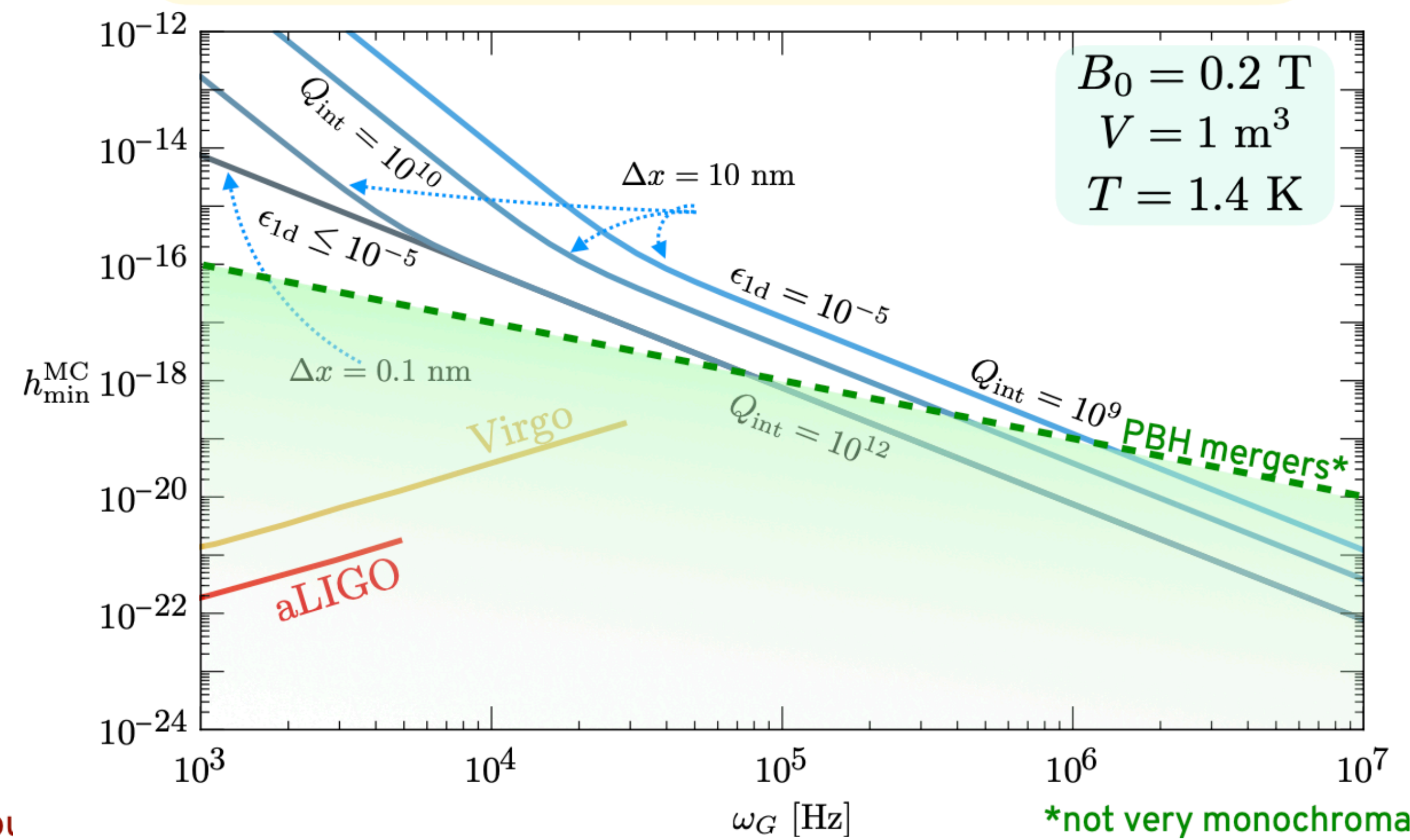
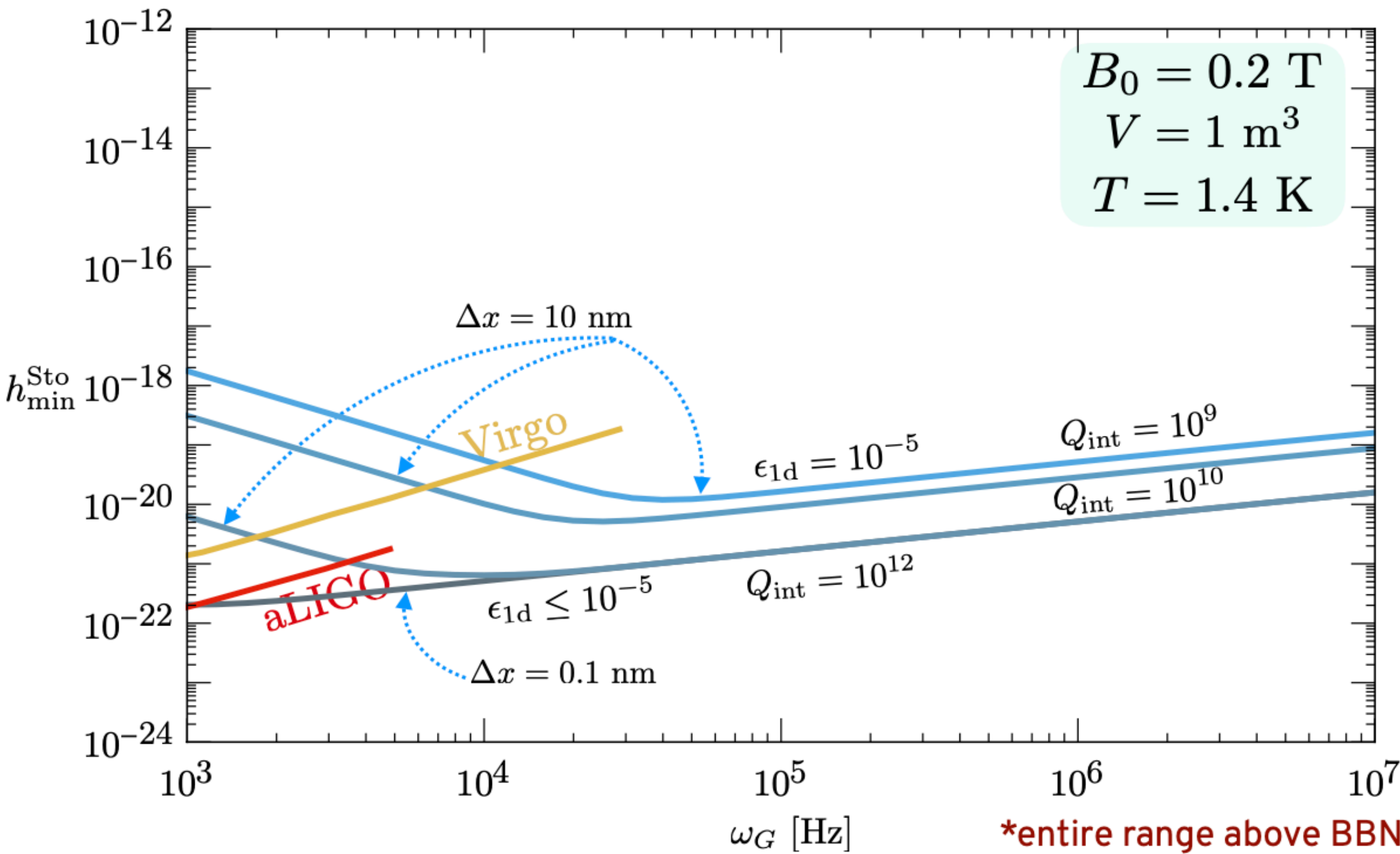
$$\omega_0 \simeq \text{GHz}$$

$$\omega_G \simeq \text{kHz} \div \text{GHz}$$

Pegoraro, Picasso & Radicati, 1978

Pegoraro, Radicati, Bernard & Picasso, 1978 and MAGO collaboration

Ultra-preliminary



Many other ideas in this range: See F. Muia's talk
and 2011.12414

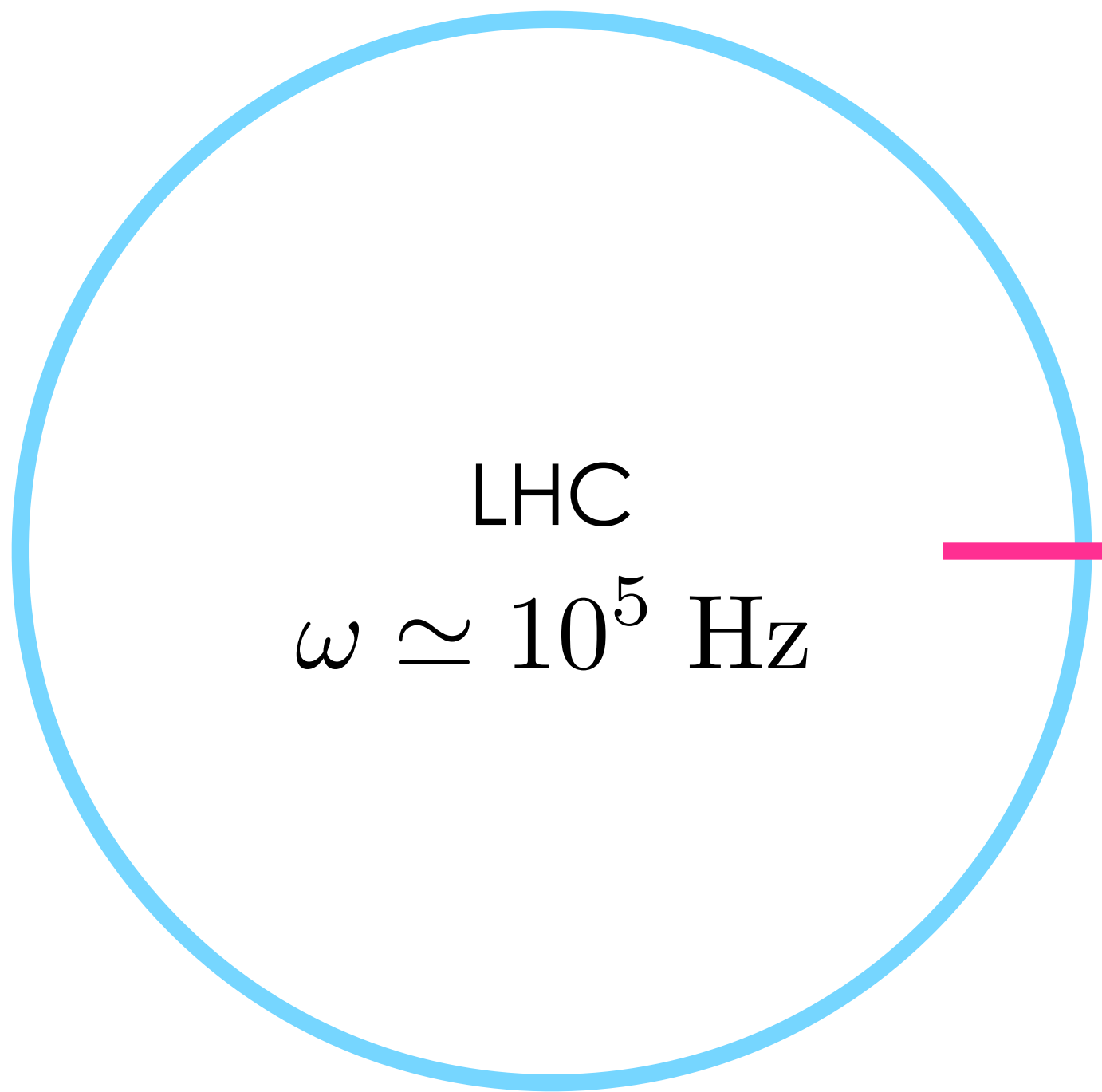


In the **proper detector frame** rigid objects don't move (magnets, tunnel, measuring devices, ...) so focus on **protons** and **EM fields**

- **EM fields** ✓
- **Radial**

First record of the idea that I could find: D. Zer Zion (n.b. very optimistic damping time)


<https://inspirehep.net/literature/533999>
<https://inspirehep.net/literature/529085>
<https://inspirehep.net/literature/470998>



See talk by K.Oide,

<https://indico.cern.ch/event/982987/>





In the **proper detector frame** rigid objects don't move (magnets, tunnel, measuring devices, ...) so focus on **protons** and **EM fields**

- **EM fields** ✓
- **Radial** ✓
- **Vertical**

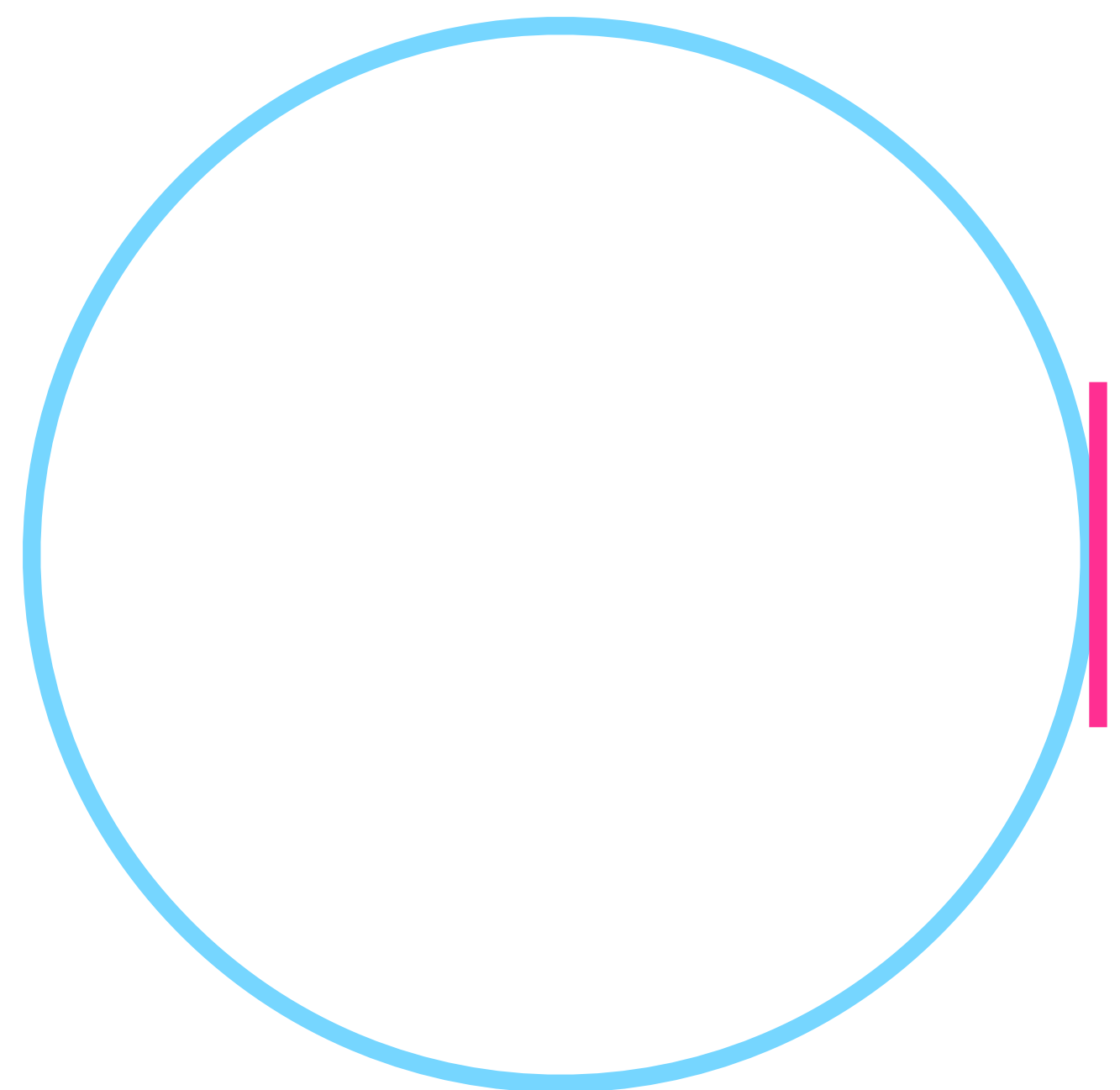
$$\Delta z \simeq h z \simeq h \times \text{cm} \ll h \times \text{km}$$

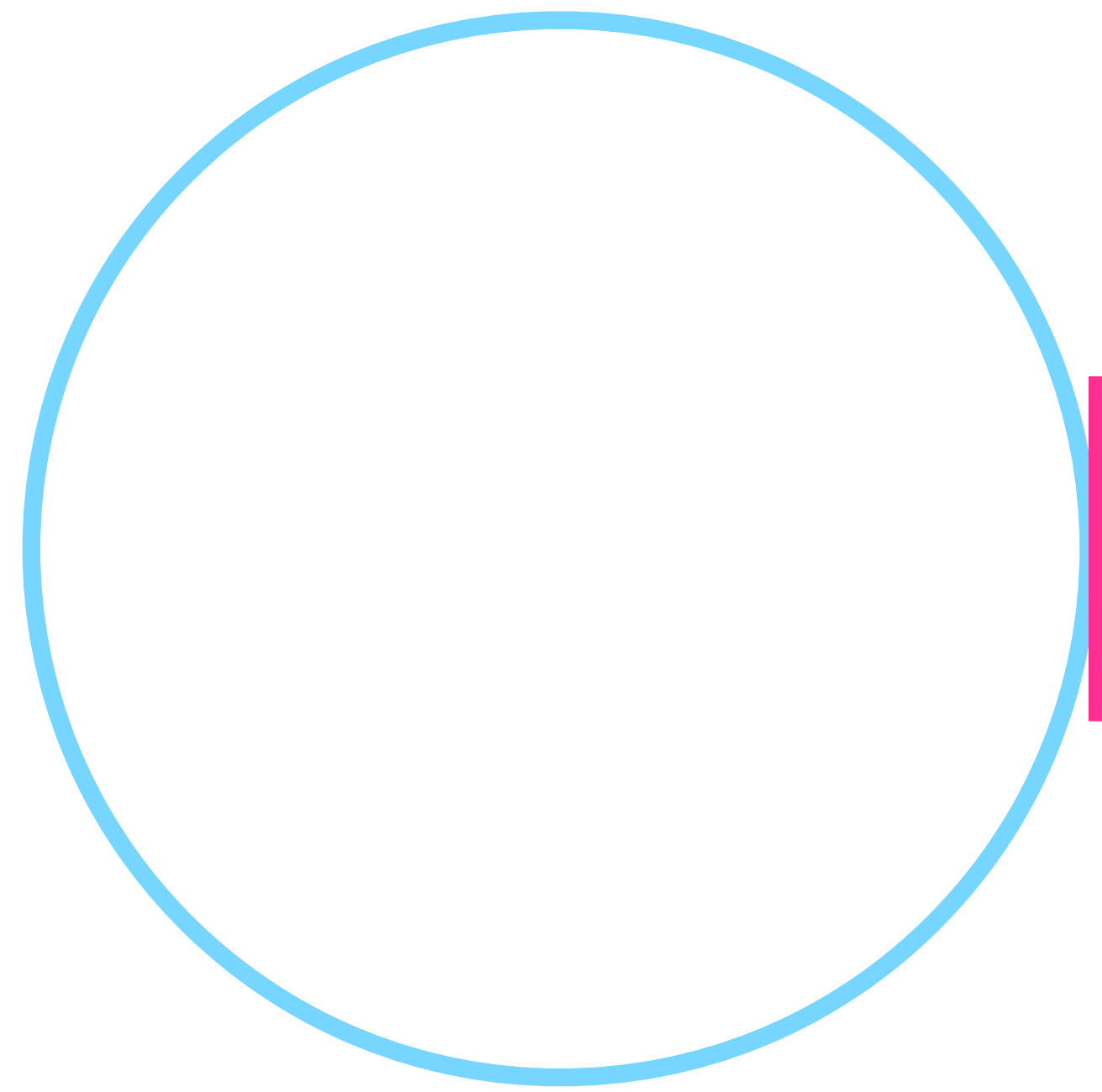




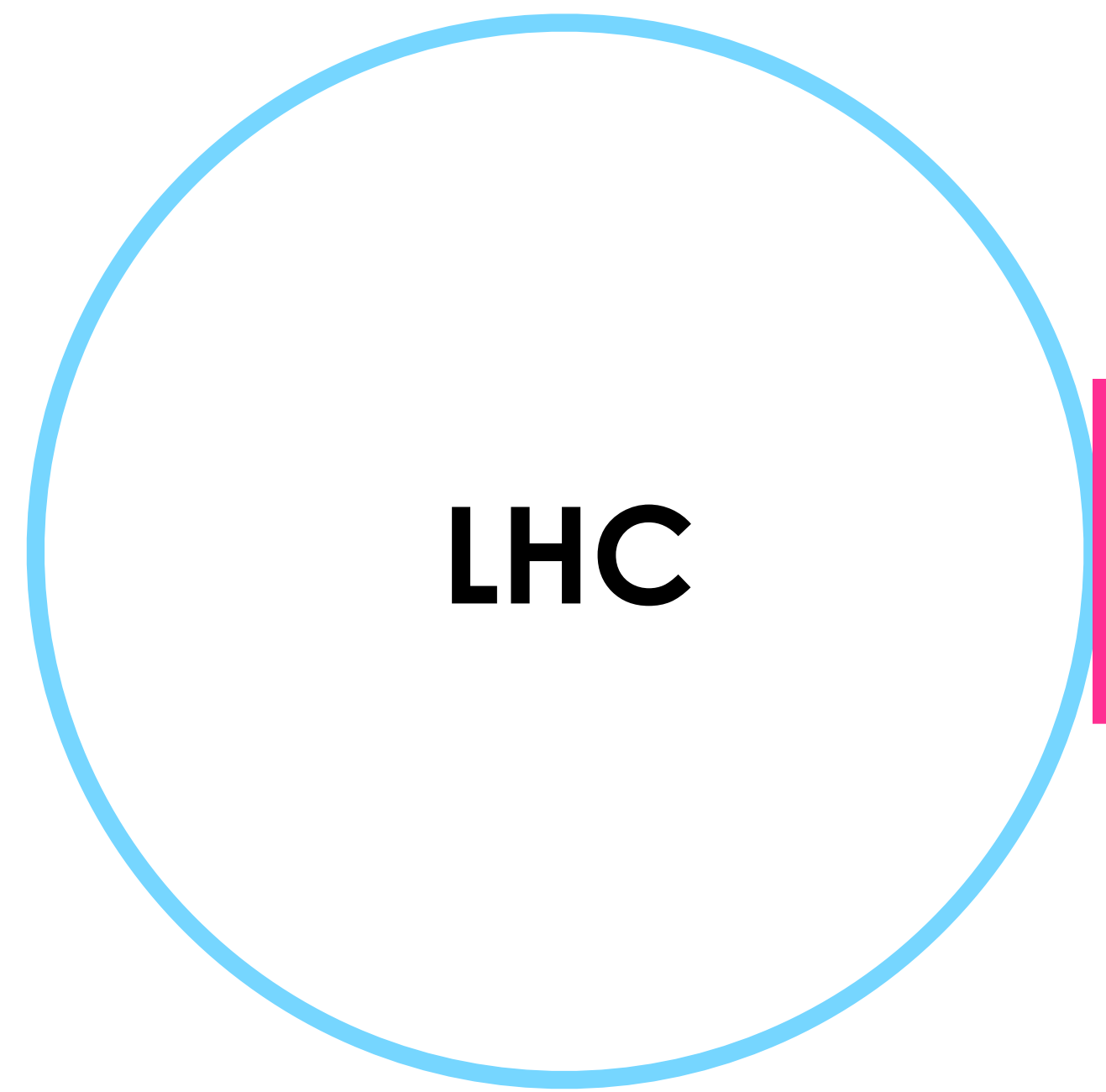
In the **proper detector frame** rigid objects don't move (magnets, tunnel, measuring devices, ...) so focus on **protons** and **EM fields**

- **EM fields** ✓
- **Radial** ✓
- **Vertical** ✓
- **Longitudinal**



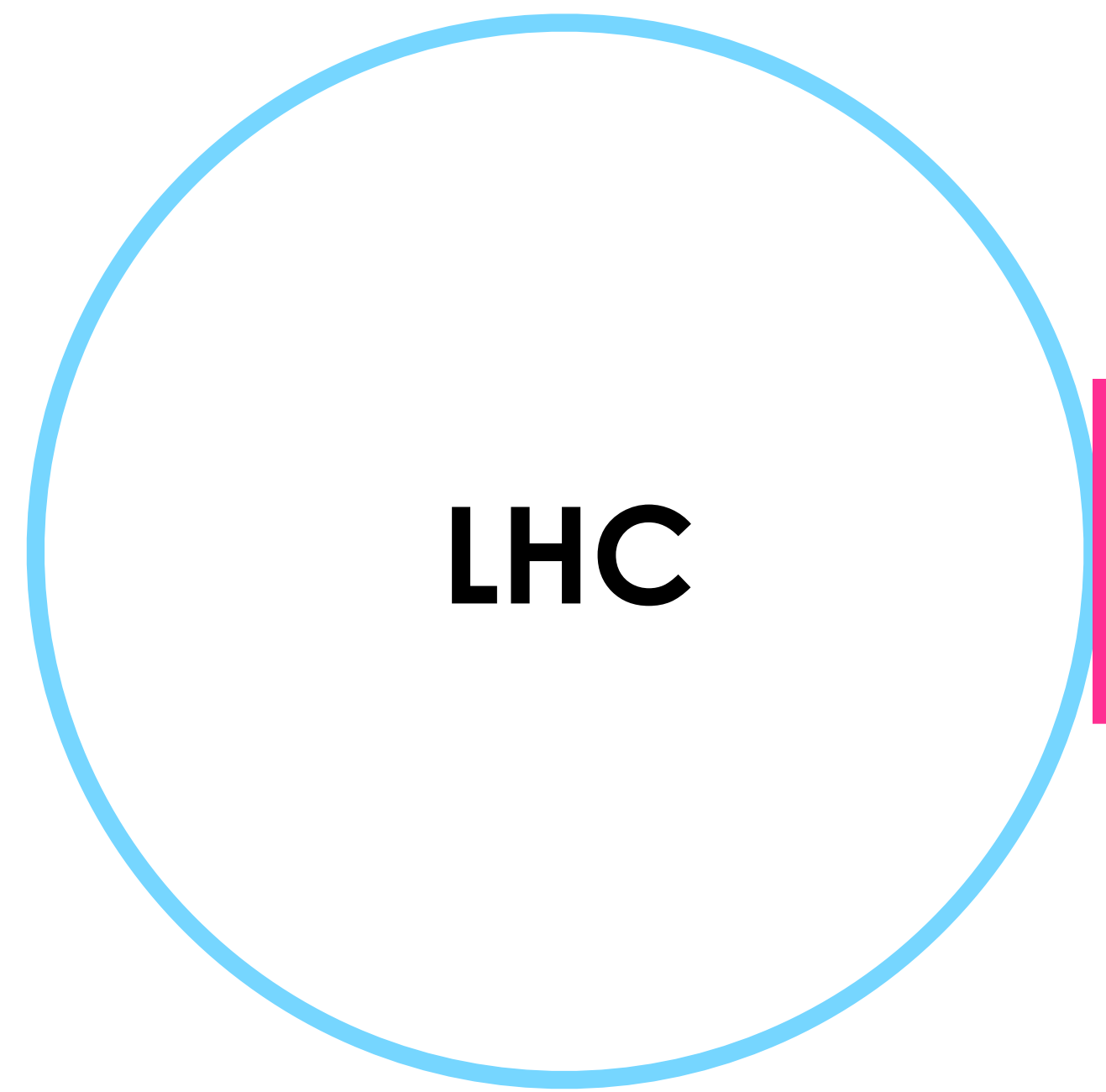


$$\ddot{\delta}_l + \frac{\dot{\delta}_l}{\tau_l} + \omega_l^2 \delta_l = \omega_g^2 f(\omega_g, t)$$



$$\ddot{\delta}_l + \frac{\dot{\delta}_l}{\tau_l} + \omega_l^2 \delta_l = \omega_g^2 f(\omega_g, t)$$

$$\omega_l \approx \omega_0 \sqrt{\frac{\tilde{h} \alpha_C q V_{\text{RF}}}{R}} \approx 2\pi \times 10 \text{ Hz}$$



$$\ddot{\delta}_l + \frac{\dot{\delta}_l}{\tau_l} + \omega_l^2 \delta_l = \underline{\omega_g^2 f(\omega_g, t)}$$

Compute

$$\ddot{\delta}_l + \frac{\dot{\delta}_l}{\tau_l} + \omega_l^2 \delta_l = \omega_g^2 f(\omega_g, t)$$

DIMENSIONAL ANALYSIS

$$f(\omega_g, t) \simeq h \times L \times \cos(\omega_g t + \phi)$$

$$\ddot{\delta}_l + \frac{\dot{\delta}_l}{\tau_l} + \omega_l^2 \delta_l = \omega_g^2 f(\omega_g, t)$$

DIMENSIONAL ANALYSIS

$$f(\omega_g, t) \simeq \underline{h} \times L \times \cos(\omega_g t + \phi)$$

Dimensionless strength of the wave

$$\ddot{\delta}_l + \frac{\dot{\delta}_l}{\tau_l} + \omega_l^2 \delta_l = \omega_g^2 f(\omega_g, t)$$

DIMENSIONAL ANALYSIS

$$f(\omega_g, t) \simeq h \times \underline{L} \times \cos(\omega_g t + \phi)$$

Length of the ring

$$\ddot{\delta}_l + \frac{\dot{\delta}_l}{\tau_l} + \omega_l^2 \delta_l = \omega_g^2 f(\omega_g, t)$$

DIMENSIONAL ANALYSIS

$$f(\omega_g, t) \simeq h \times L \times \underline{\cos(\omega_g t + \phi)}$$

For simplicity: perfectly monochromatic

$$\ddot{\delta}_l + \frac{\dot{\delta}_l}{\tau_l} + \omega_l^2 \delta_l = \omega_g^2 f(\omega_g, t)$$

ON RESONANCE

$$f(\omega_g, t) \simeq h \times L \times \cos(\omega_g t + \phi)$$

$$\delta_t = \frac{\delta_l}{c} \simeq (hT)(\omega_l \tau_l)$$

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$$\left(\frac{\Delta T}{T} \right)_{\text{exp}} \simeq 10^{-7}$$

PHASE MEASUREMENT IN THE RF CAVITY

$$\delta_t = \frac{\delta_l}{c} \simeq (hT)(\omega_l \tau_l)$$

$$\left(\frac{\Delta T}{T}\right)_{\text{exp}} \simeq 10^{-7}$$

$$\omega_l \simeq 2\pi \times 10 \text{ Hz}$$

$$\delta_t = \frac{\delta_l}{c} \simeq (hT)(\omega_l \tau_l)$$

$$\left(\frac{\Delta T}{T}\right)_{\text{exp}} \simeq 10^{-7}$$

$$\omega_l \simeq 2\pi \times 10 \text{ Hz}$$

$$\tau_l \simeq 10 \text{ hours}$$

$$\delta_t = \frac{\delta_l}{c} \simeq (hT)(\omega_l \tau_l)$$

$$h \gtrsim 10^{-13}$$

$$\left(\frac{\Delta T}{T}\right)_{\text{exp}} \simeq 10^{-7}$$

0.2 deg @ 400 MHz

$$h \gtrsim 10^{-13}$$

$$\delta_t = \frac{\delta_l}{c} \simeq (hT)(\omega_l \tau_l)$$

- AT LEAST 7 ORDERS OF MAGNITUDE ABOVE KNOWN SOURCES
- BUT THERE IS A WAY FORWARD

$$h \gtrsim 10^{-13}$$

$$\delta_t = \frac{\delta_l}{c} \simeq (hT)(\omega_l \tau_l)$$

- WAY FORWARD

- (MUCH) SLOWER PROTONS. Saturation:

$$T \simeq \frac{1}{\omega_g} = \frac{1}{\omega_l}$$

$$h \gtrsim 10^{-13}$$

$$\delta_t = \frac{\delta_l}{c} \simeq (hT)(\omega_l \tau_l)$$

- WAY FORWARD
 - (MUCH) SLOWER PROTONS [**3 orders of magnitude better ?**]
 - IMPROVE TIME RESOLUTION (Electro-Optical Sampling),

$$\left(\frac{\Delta T}{T}\right)_{\text{exp}} \simeq 10^{-8}$$

$$h \gtrsim 10^{-13}$$

$$\delta_t = \frac{\delta_l}{c} \simeq (hT)(\omega_l \tau_l)$$

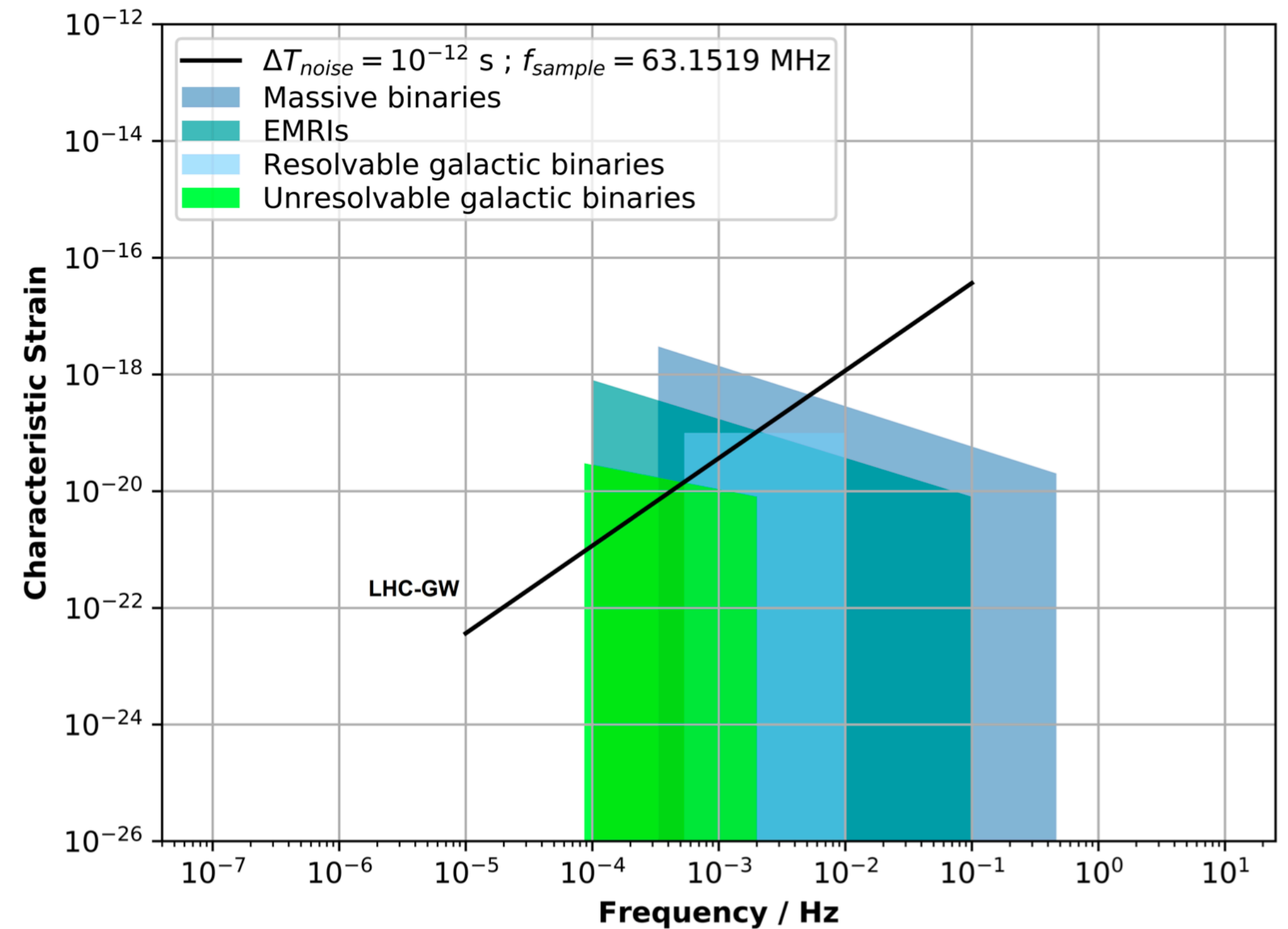
- WAY FORWARD
 - (MUCH) SLOWER PROTONS **[3 OM]**
 - IMPROVE TIME RESOLUTION **[1 OM]**
 - IMPROVE BEAM STABILITY
 - THE BEAM INTENSITY CAN BE CONSIDERABLY REDUCED
 - THE LOWER THE ENERGY THE BETTER
 - A SMALLER RING WITH SLOWER PROTONS CAN HAVE THE SAME SENSITIVITY

- MORE RADICAL WAY FORWARD:
TURN OFF THE RF (coasting beam)

$$\omega_l \rightarrow 0$$

Rao, Bruggen, Liske

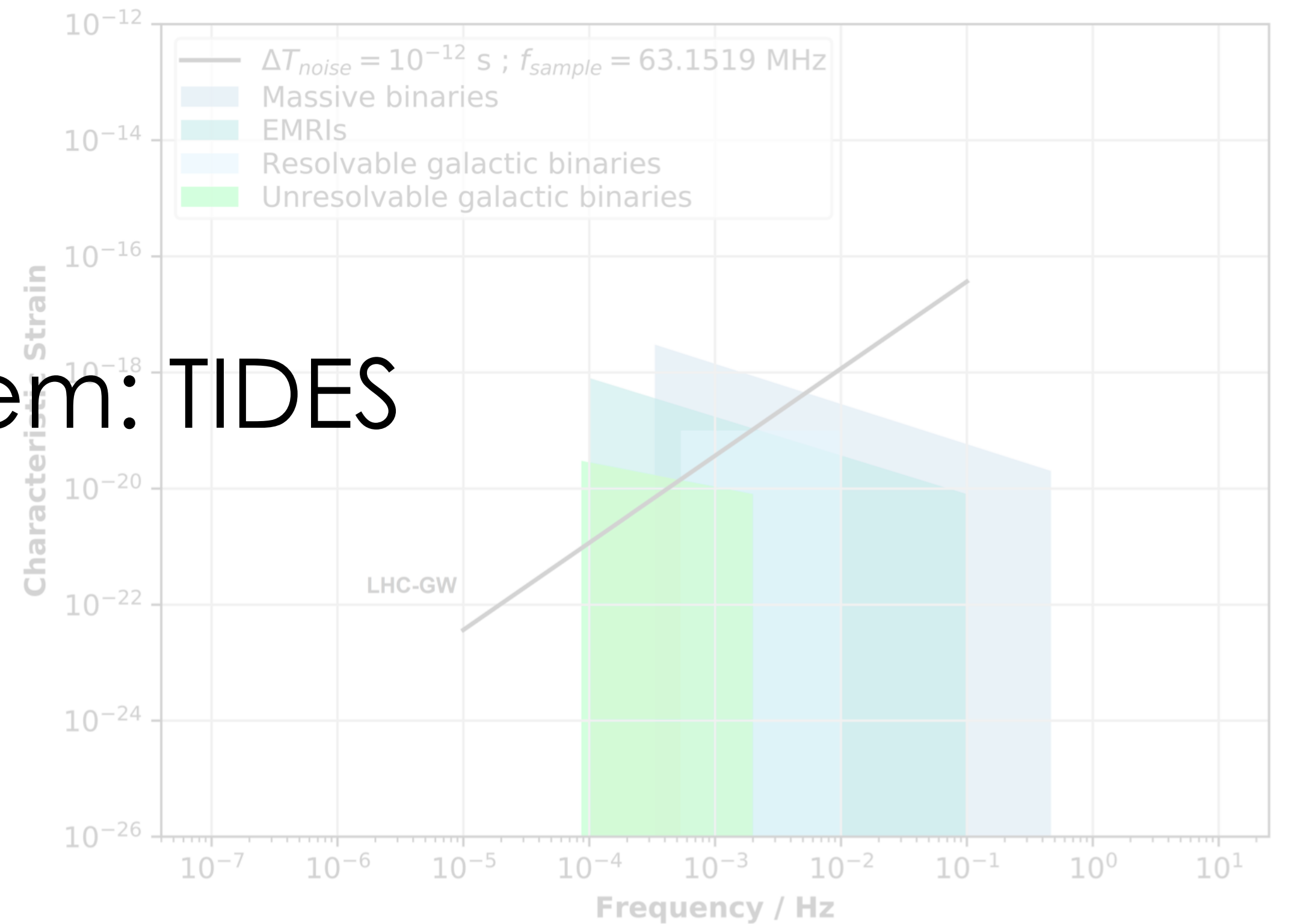
Phys.Rev.D 102 (2020) 12, 122006 [2012.00529](https://arxiv.org/abs/2012.00529)

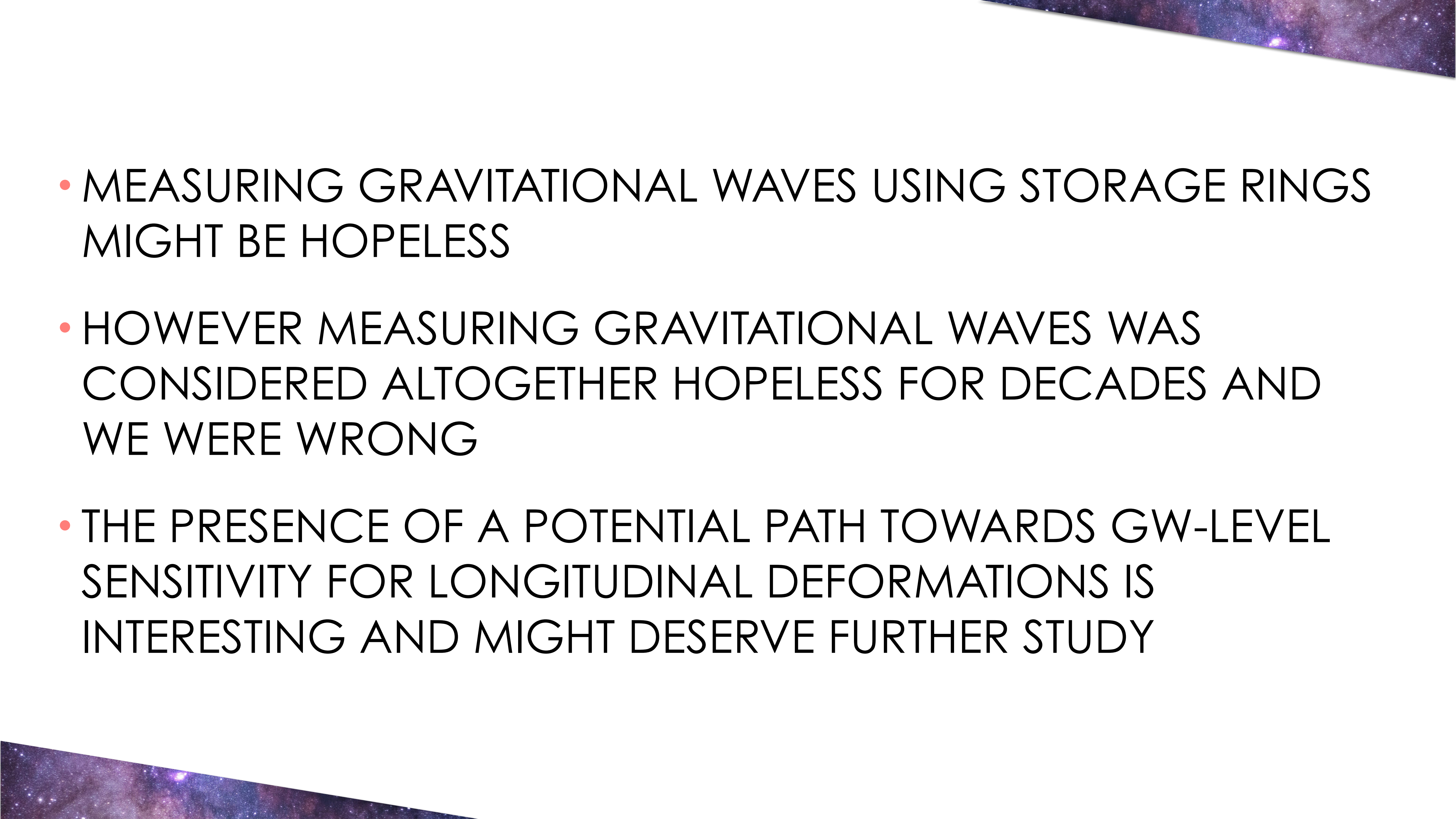


- MORE RADICAL WAY FORWARD:
TURN OFF THE RF (coasting beam)

$\omega_l \rightarrow 0$ Potential Problem: TIDES

Rao, Bruggen, Liske
Phys.Rev.D 102 (2020) 12, 122006 [2012.00529](#)



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- A decorative background featuring a vibrant, multi-colored galaxy with purple, blue, and pink hues, visible in the top-right and bottom-left corners of the slide.
- MEASURING GRAVITATIONAL WAVES USING STORAGE RINGS MIGHT BE HOPELESS
 - HOWEVER MEASURING GRAVITATIONAL WAVES WAS CONSIDERED ALTOGETHER HOPELESS FOR DECADES AND WE WERE WRONG
 - THE PRESENCE OF A POTENTIAL PATH TOWARDS GW-LEVEL SENSITIVITY FOR LONGITUDINAL DEFORMATIONS IS INTERESTING AND MIGHT DESERVE FURTHER STUDY