Four-dimensional conformal supergravity from $N=4$ gauged supergravity on asymptotically AdS_5

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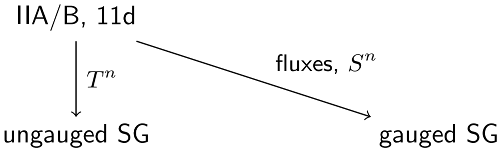
$N{=}4$ SU(2)$\otimes$U(1) gauged supergravity


IIA/B, 11d
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fluxes, S^n
ungauged SG
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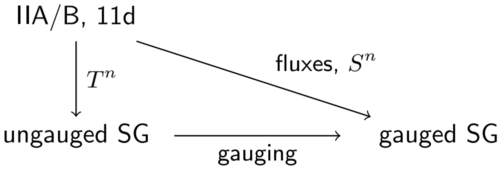
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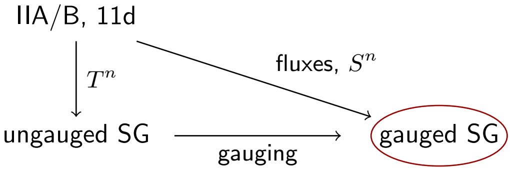

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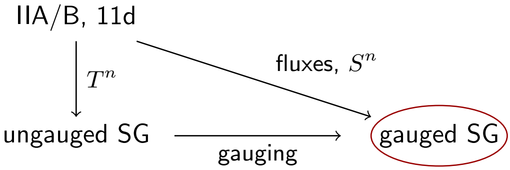

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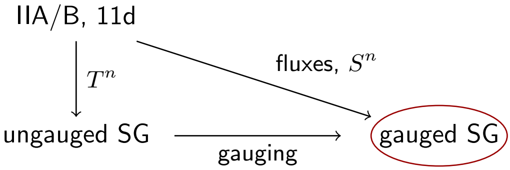

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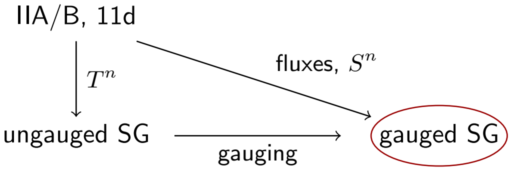
AdS$_5$ vacuum



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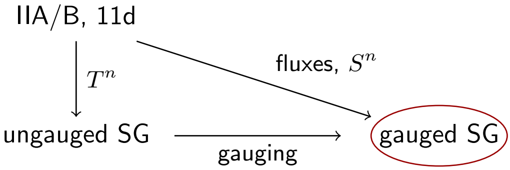

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e.g. AdS$_5$ vacuum $\to$ near-horizon limit M5-M5' in 11d
dual to $N{=}2$ SCFT on intersection

N=4 SU(2)⊗U(1) GSG Lagrangian: [Romans '86]

$$\begin{aligned}\mathcal{L} = &-\frac{1}{4}e\mathcal{R}(\omega) - \frac{1}{2}ie\bar{\psi}^i_\mu\gamma^{\mu\nu\rho}D_\nu\psi_{\rho i} + \frac{3i}{2}eT_{ij}\bar{\psi}^i_\mu\gamma^{\mu\nu}\psi^j_\nu - ieA_{ij}\bar{\psi}^i_\mu\gamma^\mu\chi^j + \frac{1}{2}ie\bar{\chi}^i\gamma^\mu D_\mu\chi_i \\ &+ ie\left(\frac{1}{2}T_{ij} - \frac{1}{\sqrt{3}}A_{ij}\right)\bar{\chi}^i\chi^j + \frac{1}{2}eD^\mu\phi D_\mu\phi + eP(\phi) - \frac{1}{4}e\,\xi^2 B^{\mu\nu\alpha}B_{\mu\nu}^{\;\alpha} \\ &+ \frac{1}{4g_1}\epsilon^{\mu\nu\rho\sigma\tau}\epsilon_{\alpha\beta}B^{\;\alpha}_{\mu\nu}D_\rho B^{\;\beta}_{\sigma\tau} - \frac{1}{4}e\,\xi^{-4}f^{\mu\nu}f_{\mu\nu} - \frac{1}{4}e\,\xi^2F^{\mu\nu I}F^{\;I}_{\mu\nu} - \frac{1}{4}\epsilon^{\mu\nu\rho\sigma\tau}F^{\;I}_{\mu\nu}F^{\;I}_{\rho\sigma}a_\tau \\ &+ \frac{1}{4\sqrt{2}}ie\Big(H^{ij}_{\mu\nu} + \frac{1}{\sqrt{2}}h^{ij}_{\mu\nu}\Big)\bar{\psi}^\rho_i\gamma_{[\rho}\gamma^{\mu\nu}\gamma_{\sigma]}\psi^\sigma_j + \frac{1}{2\sqrt{6}}ie\Big(H^{ij}_{\mu\nu} - \sqrt{2}h^{ij}_{\mu\nu}\Big)\bar{\psi}^\rho_i\gamma^{\mu\nu}\gamma_\rho\chi_j \\ &- \frac{1}{12\sqrt{2}}ie\Big(H^{ij}_{\mu\nu} - \frac{5}{\sqrt{2}}h^{ij}_{\mu\nu}\Big)\bar{\chi}_i\gamma^{\mu\nu}\chi_j + \frac{1}{\sqrt{2}}ie\,(\partial_\nu\phi)\,\bar{\psi}^i_\mu\gamma^\nu\gamma^\mu\chi_i\end{aligned}$$

Susy transformations:

$$\delta_\epsilon e^a_\mu = i\bar{\psi}^i_\mu\gamma^a\varepsilon_i\,, \qquad \delta_\epsilon A^I_\mu = \theta^{ij}_\mu\left(\Gamma^I\right)_{ij}\,, \qquad \delta_\epsilon a_\mu = \frac{1}{2}i\xi^2\left(\bar{\psi}^i_\mu\varepsilon_i + \frac{2}{\sqrt{3}}\bar{\chi}^i\gamma_\mu\varepsilon_i\right)$$

$$\delta_\epsilon\psi_{\mu i} = D_\mu\varepsilon_i + \gamma_\mu T_{ij}\varepsilon^j - \frac{1}{6\sqrt{2}}\left(\gamma_\mu^{\;\nu\rho} - 4\delta_\mu^{\;\nu}\gamma^\rho\right)\left(H_{\nu\rho ij} + \frac{1}{\sqrt{2}}h_{\nu\rho ij}\right)\varepsilon^j$$

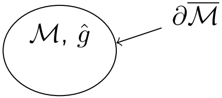
$$\delta_\epsilon\chi_i = \frac{1}{\sqrt{2}}\gamma^\mu\left(\partial_\mu\phi\right)\varepsilon_i + A_{ij}\varepsilon^j - \frac{1}{2\sqrt{6}}\gamma^{\mu\nu}\Big(H_{\mu\nu ij} - \sqrt{2}h_{\mu\nu ij}\Big)\varepsilon^j\,, \qquad \delta_\epsilon\phi = \frac{1}{\sqrt{2}}i\bar{\chi}^i\varepsilon_i$$

$$\delta_\epsilon B^\alpha_{\mu\nu} = 2D_{[\mu}\Theta^{ij}_{\nu]}\left(\Gamma^\alpha\right)_{ij} - \frac{ig_1}{\sqrt{2}}\epsilon^{\alpha\beta}\left(\Gamma_\beta\right)_{ij}\xi\left(\bar{\psi}^i_{[\mu}\gamma_{\nu]}\epsilon^j + \frac{1}{2\sqrt{3}}\bar{\chi}^i\gamma_{\mu\nu}\epsilon^j\right)$$

asymptotically AdS_5
partial gauge fixing

Asymptotically AdS:

- $(\mathcal{M}, \hat{g})$ conformally compact:



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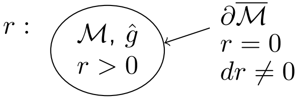

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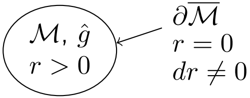

$$\mathcal{R}_{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}} = |dr|_g^2 \big(\hat{g}_{\hat{\mu}\hat{\rho}}\hat{g}_{\hat{\sigma}\hat{\nu}} - \hat{g}_{\hat{\mu}\hat{\sigma}}\hat{g}_{\hat{\rho}\hat{\nu}}\big) + \mathcal{O}(r^{-3})$$

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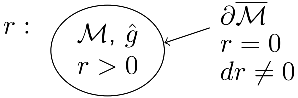

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$$\hat{g} = r^{-2}\left(g_{\mu\nu}(x,r)dx^\mu dx^\nu - dr^2\right)$$

[Fefferman Graham]

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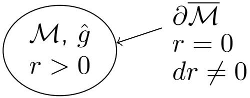

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Partial gauge fixing:

▷ local Lorentz: $\hat{e}_r^a = \hat{e}_\mu^r = 0 \quad \longrightarrow \quad \hat{e}^{\hat{a}} = r^{-1}\big(e_\mu^a dx^\mu,\, dr\big)$

▷ susy + YM: $\hat{\psi}_r^i = \hat{A}_r^I = \hat{a}_r = 0$

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Recipe: for $\hat{\psi}$ find f s.t. $\lim_{r\to 0} f(r)^{-1}\hat{\psi} =: \psi(x)$ finite boundary field

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▷ gravitinos $\hat{\psi}_\mu^i$: 4 Dirac / symplectic Majorana = 2 Dirac
$\to$ 2 chiral boundary gravitinos $\propto r^{-1/2}$

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N=2 Weyl multiplet

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Scaling from linearized asymptotic eom consistent in nonlinear theory?

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⇒ obtained scalings consistent in nonlinear theory ✓

boundary symmetries

Residual bulk symmetries:

- solutions to

$$\Big(\hat{\delta}_{\text{diffeo}} + \hat{\delta}_{\text{Lorentz}} + \hat{\delta}_{\text{susy}} + \hat{\delta}_{\text{YM}}\Big)\big\{\hat{e}_r^{\,r}, \hat{e}_r^{\,a}, \hat{e}_\mu^{\,r}, \hat{a}_r, \hat{A}_r^I, \hat{\psi}_r^i\big\} = 0$$

  coupled diff. eq.(r), solved by combinations $\hat{\delta}_{\text{diffeo}} + \hat{\delta}_{\text{Lorentz}} \dots$

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	e^a_μ	$\psi^L_{\mu i_+}$	a_μ, A^I_μ	$\chi^L_{i_+}$	$C^-_{\mu\nu}$	ϕ
w	-1	$-\frac{1}{2}$	0	$\frac{3}{2}$	-1	2
U(2)	-	$\mathbf{2}_{\frac{1}{2}}$	$\mathbf{1}_0$, $\mathbf{3}_0$	$\mathbf{2}_{\frac{1}{2}}$	$\mathbf{1}_1$	-

$\to$ $N=2$ Weyl multiplet, bosonic part of local superconformal symmetry

Boundary symmetries (fermionic):

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▷ N=2 super-Weyl δ_η: $\delta_\eta e_\mu^a = 0$, $\delta_\eta\psi_{\mu i_+} = -i\gamma_\mu\eta_{i_+}$ etc.

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▷ $N{=}2$ susy δ_ζ:

$$\delta_\zeta e_\mu^a = i\bar{\psi}_\mu^{i_+}\gamma^a\zeta_{i_+} + \text{c.c.}$$

$$\delta_\zeta \psi_{\mu i_+} = D_\mu \zeta_{i_+} - \tfrac{1}{4}\gamma\cdot C^-_{i_+j_+}\gamma_\mu\zeta^{j_+} \qquad \text{etc.}$$

▷ $N{=}2$ super-Weyl δ_η: $\delta_\eta e_\mu^a = 0$, $\delta_\eta \psi_{\mu i_+} = -i\gamma_\mu \eta_{i_+}$ etc.

Symmetry algebra: $[\delta_\zeta, \delta_{\zeta'}] = \delta_{\text{diffeo}} + \delta_{\text{Lorentz}} + \delta_{\text{U(2)}}$, $[\delta_\eta, \delta_{\eta'}] = 0$

$$[\delta_\eta, \delta_\zeta] = \delta_{\text{Weyl}} + \delta_{\text{Lorentz}} + \delta_{\text{U(2)}}$$

Conclusion

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- ▷ boundary fields from linearized theory, consistent w/ interactions
- ▷ residual bulk symmetries on boundary:
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AdS/CFT: $S_{\text{sugra}}^{\text{on-shell}}\big[\hat{\varphi}[\varphi]\big] - S_{\text{ct}}[\varphi] = \ln \mathcal{Z}_{\text{CFT}}[\varphi]$

$\longrightarrow$ $N{=}4$ invariant S_{ct} from pure-gravity $S_{\text{ct}}[g]$

$\longrightarrow$ conformal anomaly of dual $N{=}2$ SCFT from pure-metric part